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Pion-nucleus optical potential in the isobar-doorway model

A. N. Saharia* and R. M. Woloshyn
TRIUMF, Vancouver, British Columbia, Canada, V6T 2A3

L. S. Kisslinger
Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
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The isobar-doorway model is used to parametrize the pion-nucleus optical potential so as to take into account the interaction of the Δ with other nucleons as well as coupling to inelastic and reaction channels. The parameters of the model are determined by fitting elastic scattering angular distributions. It is found that the nonlocality associated with the isobar propagation is important in the resonance energy region. This may indicate that the use of impulse approximation in obtaining the pion optical potential may be doubtful even if we introduce a complex shift in the energy at which the pion-nucleon scattering amplitude is evaluated to get a first-order optical potential.

[NUCLEAR REACTIONS Pion-nucleus optical potential, parametrization of many-body modifications to the isobar propagator, $\sigma(\theta)$ and σ for ^{16}O, ^{12}C, and ^{4}He in the energy range 110–250 MeV.]

I. INTRODUCTION

In recent years there have been various studies of pion-nucleus interaction. (For a recent review of pion-nucleus scattering see Ref. 1 and references cited therein.) In calculations based on multiple scattering theories,[2,3] the starting point is the truncation of the series for the pion-nucleus optical potential at the first term, which is then evaluated using various approximations.[4] Even though such a first-order optical potential gives a reasonable description of pion elastic scattering data in the intermediate-energy region, its success is rather surprising. Multiple scattering theories are based on the assumption that projectiles interact with nucleons in the target through a potential, and hence do not take into account the fact that pions interact with nucleons through absorption and emission. Furthermore, one has to use closure over intermediate nuclear states to identify the Watson operator with a free pion-nucleon scattering amplitude—an approximation whose validity may be doubtful[5,6] due to strong energy dependence of the pion-nucleon interaction in the intermediate energy region. To compensate for these inadequacies, one then adds terms proportional to ρ^2, where ρ is the nuclear single particle density distribution, which are presumed to take into account the effects of pion true absorption, inelastic scattering, etc. Potentials of this type (with phenomenological ρ^2 terms) are currently being used to fit the accurate experimental data now available.[7-10]

The isobar-doorway theory has been proposed as an alternative approach for describing the meson-nucleus interactions.[11] In this approach, one introduces nuclear states of the A-baryon system with $(A-1)$ nucleons and an isobar [$\Delta(1232)$ for pion-nucleus system] called the doorway states. All pionic scattering and reactions mediated by Δ formation must proceed through these doorway states. The many-body effects are then taken into account by allowing the doorway states to couple to the states in the inelastic and reaction channels. Recently a number of microscopic calculations[12-15] have used a T-matrix formulation of the isobar-doorway model to calculate pion-nucleus scattering and reactions. In such calculations 1Δ-1h states are used as the basis set and the modification to the isobar propagator due to the one-pion exchange potential are calculated—thereby including elastic scattering, inelastic scattering, Pauli blocking, etc. The effect of coupling to other reaction channels is included phenomenologically. For example, this is done by introducing a complex "spreading potential" for the Δ-$(A-1)$ system in Refs. 12 and 15 and by introducing a damping width in Ref. 13. Within this framework, there also have been purely phenomenological calculations[16,17] in which a resonance form for the pion-nucleus T matrix is introduced. The parameters are determined by fitting elastic scattering data. These are then used to calculate cross sections for pionic reactions to isobaric analog states, where we expect the isobar-doorway states to be the same as for elastic scattering.

A microscopic calculation of the optical potential can be carried out in a similar way by introducing a basis set of 1Δ-1h states and calculating the energy shifts and widths due to coupling to the inelastic scattering and reaction channels.[11] To the extent that one or a few doorway states dominate, one obtains a closed form of the potential corresponding to the pion-nucleon 3-3 channel which not only represents the entire multiple scattering series but also takes into account the various many-body effects such as true absorption, ρ-meson exchange, etc., which have to be set by hand if we use a multiple scattering theory. Such an optical potential would provide not only a better description of pion scattering data but also important insight into the interaction and propagation of the isobar in nuclear matter.

In this work we treat the optical potential given by the isobar-doorway theory as a phenomenological quantity. (A preliminary version is given in Ref. 18.) Our objective here is to use the form provided by theory, with certain quantities defined by theory taken as parameters. We should emphasize that approximate fits to elastic scattering data can be obtained with widely different potentials.[19] It is essential to have a theoretical basis for the form of the optical potential if the parameters determined by fits to the data are to be meaningful.

In Sec. II we give a brief derivation of the pion optical potential in the isobar-doorway model (see Refs. 11 and 20 for details). Section III describes our parametrization. Details of the calculation together with our fits to elastic scattering data are given in Sec. IV. The summary of our results and conclusions is given in Sec. V.

II. DERIVATION

The starting point of the isobar-doorway model is the introduction of states which explicitly contain isobars. Let us use the standard notation[11,21] of the doorway picture with the subspaces: P containing states of a pion and nucleus in its ground state, D containing states with one Δ and $(A$-1) nucleons, and Q containing all other states. The model makes implicit use of the interaction $V_{\pi N\Delta}$, which couples the Δ to πN scattering states in the 3-3 channel:

$$V_{\pi N\Delta}(\vec{\kappa})=\frac{f_{\pi N\Delta}}{m_\pi}\vec{S}\cdot\vec{\kappa}T_\alpha g(\kappa)\,, \tag{1}$$

where $\vec{S}(\vec{T})$ is the transition operator between spin (isospin) $\frac{1}{2}$ and spin (isospin) $\frac{3}{2}$ objects, α the pion isospin index, $g(\kappa)$ the $\pi N\Delta$ vertex function $[g(0)=1]$, and $\vec{\kappa}$ the pion-nucleon relative momentum. The matrix elements of $V_{\pi N\Delta}$ are taken as basic input to the theory, much as two body matrix elements are used as the basic input for nuclear many-body calculations.

Using the standard projection operator algebra and making the doorway assumption that the states in Q space do not couple to the states in P space except through the states in D space, the effective Hamiltonian for the scattering states is given by

$$\mathcal{H}=H_{PP}+H_{PD}\frac{1}{E^+-H_{DD}-H_{DQ}[1/E^+-H_{QQ})]H_{QD}}H_{DP}\,, \tag{2}$$

with

$$H_{PD}=H_{DP}=V_{\pi N\Delta}+\text{H.c.} \tag{3}$$

The pion-nucleus optical potential is then given by

$$\langle\vec{k}'|V|\vec{k}\rangle=\langle\vec{k}'|V^{\rm NR}|\vec{k}\rangle+\langle\vec{k}'|V^{\rm R}|\vec{k}\rangle\,, \tag{4}$$

where $V^{\rm NR}=H_{PP}-H_0$, H_0 being the free pion-nucleus Hamiltonian, is the nonresonant optical potential generated by the interactions in P space. Since the nonresonance interactions vary slowly with energy, $V^{\rm NR}$ is taken to be the first order optical potential with the contribution of the pion-nucleon 3-3 channel excluded. For the resonant part we introduce in Eq. (2) a complete set of states $|D_i\rangle$ which diagonalize the energy denominator so that

$$\langle\vec{k}'|V^{\rm R}|\vec{k}\rangle=\sum_i\frac{\langle 0,\vec{k}'|V^\dagger_{\pi N\Delta}|D_i\rangle\langle D_i|V_{\pi N\Delta}|0,\vec{k}\rangle}{E-E_i-\epsilon_i^{\rm in}+i\Gamma_i^{\rm in}/2}\,, \tag{5}$$

where $|0\rangle$ is the nuclear ground state and

$$E_i=\langle D_i|H_{DD}|D_i\rangle \tag{6}$$

and

$$\epsilon_i^{\rm in}-i\Gamma_i^{\rm in}/2=\left\langle D_i\left|H_{DQ}\frac{1}{E^+-H_{QQ}}H_{DQ}\right|D_i\right\rangle. \tag{7}$$

In general the doorway states $|D_i\rangle$ are complicated A-baryon states. However, due to the single particle coupling interaction $V_{\pi N\Delta}$, only the 1Δ-1h components of these states contribute to the matrix elements in the numerator in Eq. (5). We now make the assumption that the spin-spin and spin-orbit interactions for the Δ-(A-1) system can be neglected. [Although never specified explicitly in Ref. 11, this turns out to be a necessary condition for using the Kisslinger-Wang model defined by Eq. (9).] The states $|D_i\rangle=|D_{i,\alpha}\rangle$ with the same Δ-orbital angular momentum but with different total angular momentum corresponding to different α, the z component of Δ spin, will be degenerate. We group these terms for a given orbital angular momentum together and sum over the spin quantum number α. We can then use the definition of $V_{\pi N\Delta}$ to write

$$\sum_{\alpha=-3/2}^{3/2} V^{\dagger}_{\pi N\Delta}(\vec{\kappa}')|\chi_\alpha\rangle\langle\chi_\alpha|V_{\pi N\Delta}(\vec{\kappa})$$
$$= V^{\dagger}_{\pi N\Delta}(\vec{\kappa}')V_{\pi N\Delta}(\vec{\kappa})$$
$$= (E - M_\Delta + i\Gamma_\Delta/2)\langle\vec{\kappa}'|t^R_{\pi\pi}(E)|\vec{\kappa}\rangle, \qquad (8)$$

where $|\chi_\alpha\rangle$ are the spin $\frac{3}{2}$ eigenvectors, M_Δ is the mass of Δ, Γ_Δ its width and $\langle\vec{\kappa}'|t^R_{\pi\pi}(E)|\vec{\kappa}\rangle$ is the (off-shell) pion-nucleon scattering matrix in the 3-3 channel. The numerator in Eq. (5), which is the sum over all such degenerate states, then reduces to

$$N_i = \sum_\alpha \langle 0,\vec{k}'|V^{\dagger}_{\pi N\Delta}|D_{i,\alpha}\rangle\langle D_{i,\alpha}|V_{\pi N\Delta}|0,\vec{k}\rangle \qquad (9a)$$

$$= \sum_N \int \phi^*_N(\vec{p})V^{\dagger}_{\pi N\Delta}(\vec{\kappa}')\phi^i_\Delta(\vec{p}' + \vec{k}') \times \phi^{i*}_\Delta(\vec{p} + \vec{k})V_{\pi N\Delta}(\vec{\kappa})\phi_N(\vec{p})d^3p\, d^3p' \qquad (9b)$$

$$= \sum_N \int (E - M_\Delta + i\Gamma_\Delta/2)\phi^*_N(\vec{p}')\langle\vec{\kappa}'|t^R_{\pi\pi}(E)|\vec{\kappa}\rangle \times \phi_N(\vec{p})\phi^i_\Delta(\vec{p}' + \vec{k}')\phi^{i*}_\Delta(\vec{p} + \vec{k})d^3p'd^3p\,, \qquad (9c)$$

where ϕ_N is the nuclear single particle wave function and ϕ^i_Δ is the radial part of the Δ wave function in the doorway state $|D_{i,\alpha}\rangle$. $t^R_{\pi\pi}$ in the above equation is an operator in the spin space of the nucleon. However, for a closed j-shell nucleus, the spin-flip term will not contribute and $t^R_{\pi\pi}$ can be treated as a c number depending on momenta only. The relative momenta in the pion-nucleon center of mass system are defined by

$$\vec{\kappa} = \frac{M_N}{(S_{\pi N})^{1/2}}\left\{\vec{k} - \frac{\vec{p}}{M_N}\left[E_\pi(k) - \frac{\vec{p}\cdot\vec{k}}{E_N(p) + M_N}\right]\right\}$$
$$= \frac{M_N}{(S_{\pi N})^{1/2}}(\vec{k} - a\vec{p}) \qquad (10a)$$

and

$$\vec{\kappa}' = \frac{M_N}{(S_{\pi N})^{1/2}}\left\{\vec{k}' - \frac{\vec{p}'}{M_N}\left[E_\pi(k') - \frac{\vec{p}'\cdot\vec{k}'}{E_N(p') + M_N}\right]\right\}$$
$$= \frac{M_N}{(S_{\pi N})^{1/2}}(\vec{k}' - b\vec{p}')\,. \qquad (10b)$$

We now make the approximation that $V^{\dagger}_{\pi N\Delta}(\vec{\kappa}')$ $V_{\pi N\Delta}(\vec{\kappa})$ can be factored out of the integration on the right hand side of Eq. (8). (We give a justification for this approximation in the next section.) The resonant part of the optical potential is then given by

$$\langle\vec{k}'|V^R|\vec{k}\rangle = \sum_i \frac{(E - M_\Delta + i\Gamma_\Delta/2)}{E - E_i - \epsilon_i + i\Gamma^{\rm in}_i/2}\langle\vec{k}'|\bar{t}^R_{\pi\pi}|\vec{k}\rangle F_i(\vec{k}',\vec{k})\,, \qquad (11)$$

where $\langle\vec{k}'|\bar{t}^R_{\pi\pi}|\vec{k}\rangle$ is the pion-nucleon scattering amplitude in the 3-3 channel in the pion-nucleus center of mass frame, and

$$F_i(\vec{k}',\vec{k}) = \sum_N \int \phi^*_N(\vec{p})\phi^i_\Delta(\vec{p}' + \vec{k}')\phi^{i*}_\Delta(\vec{p} + \vec{k})\phi_N(\vec{p})d^3p\, d^3p' \qquad (12)$$

is a modified nuclear form factor which takes into account nonlocality associated with Δ propagation in the doorway state $|D_i\rangle$. The above equation is the defining expression for the optical potential in the isobar-doorway model. As mentioned earlier, a theoretical determination of the optical potential requires evaluating the quantities F_i, E_i, $\epsilon^{\rm in}_i$, and $\Gamma^{\rm in}_i$ for each doorway state. This requires not only a complete understanding of the interactions in D space, but also a coupling of $|D_i\rangle$ to states in Q space. As these features of the isobar dynamics are still not well understood,[22,23] we look for a convenient parametrization of the optical potential. Before discussing our phenomenological model we point out that the optical potential given by Eq. (11) not only includes higher order scattering terms but also the many-body effects such as ρ-meson exchange, pion true absorption, Pauli blocking, etc., in the pion-nucleon 3-3 channel. This is shown schematically in Fig. 1. Of course we still have to add by hand corresponding correction terms for the nonresonant part. In the language of the isobar-doorway model, these correspond to lifting the doorway assumption $H_{PQ} = 0$. The most important in this category is probably the "S-wave absorption" term where the scattering states couple to A-baryon states with no pion in the asymptotic region. Although there have been many

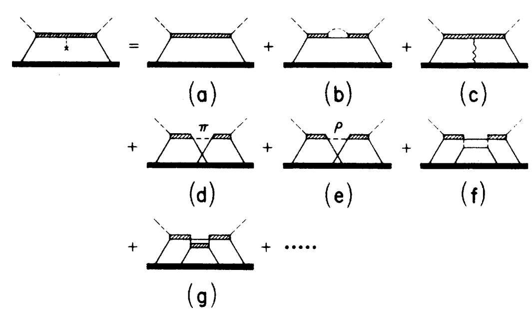

FIG. 1. Left hand side: pion optical potential in the isobar-doorway model. ---× represents all the many-body modifications to the isobar propagator except those due to coupling to P space. Right hand side: some of the lowest order terms contributing to the optical potential: (a) single scattering term, (b) quasielastic scattering term, (c) Δ- $(A-1)$ binding correction term, (d) higher order scattering term, (e) ρ exchange term, (f) two nucleon absorption term, and (g) local field correction term.

estimates of this correction term,[24,25] we feel that there are some unresolved questions which need a more careful treatment. For example, the transition operator in the two nucleon absorption model is taken to be scattering from the first nucleon followed by absorption on the second so as to provide energy-momentum sharing. As this correction is to be added to an (uncorrected) optical potential which generates scattering from individual nucleons, the rescattering may be counted twice—once in the correction term and again in the optical potential. Also, in some cases a closure over the intermediate 2-nucleon 2-hole states is used without proper orthogonalization with the nuclear ground state, which may lead to further double counting.[26] Moreover, the estimates of the effect of ρ-meson exchange for absorption are in considerable doubt.[27] Because of these and other problems associated with estimating the nonresonance absorption terms, we have confined our analysis to the resonance energy region, i.e., for pion kinetic energies lying between 100 and 250 MeV. In this energy region the contribution of the S-wave absorption terms to elastic scattering is less important and does not affect the conclusions reached here.

III. PARAMETRIZATION

In our attempt to parametrize the optical potential given by Eq. (11), we notice that the important differences between this expression and the corresponding one for the first-order optical potential are (a) inclusion of Δ-(A-1) binding through $E_i=\langle D_i|H_{DD}|D_i\rangle$, (b) inclusion of coupling to reaction channels (except coupling to the quasielastic scattering channel which is included in both) through

$$\epsilon_i - i\Gamma_i^{\text{in}}/2=\left\langle D_i\left|H_{DQ}\frac{1}{E-H_{QQ}}H_{DQ}\right|D_i\right\rangle,$$

and (c) inclusion of nonlocality due to Δ propagation through $F_i(\vec{k}',\vec{k})$. The nonlocality contained in F_i is in addition to that given by the finite range off-shell extrapolation of the pion-nucleon scattering amplitude which has been included in most of the momentum space calculations of pion-nucleus scattering.[9,10,28] Effects (a) and (b) are sometimes included in conventional calculation phenomenologically, by introducing a shift in the energy parameter at which the pion-nucleon T matrix is evaluated, by smearing the resonance (collision broadening due to Fermi motion), and by adding "ρ^2 terms" (presumably due to true absorption, etc.). However, due to the static limit used in deriving the optical potential (i.e., using closure over the nuclear Hamiltonian in identifying Watson's operator with a free pion-nucleon scattering amplitude), the nonlocality associated with Δ propagation is missed completely. It is our aim here to include all these dynamical effects through a small number of parameters. To this effect we note that even in the absence of coupling to the reaction channels, we expect the inelastic width of the doorway states Γ_i^{in} to be very large due to coupling to the quasielastic channels. Thus to the extent that the average width of the doorway states is larger than the average separation energy $\overline{|E_i+\epsilon_i-E_{i\pm1}-\epsilon_{i\pm1}|}$, we can replace the energy denominator by an appropriate average. We therefore write Eq. (11) as

$$\langle\vec{k}'|V^R|\vec{k}\rangle=\frac{E-M_\Delta+i\Gamma_\Delta/2}{E-M_\Delta-\Delta E+i\beta\Gamma_\Delta/2}\langle\vec{k}'|\bar{t}^R_{\pi\pi}|\vec{k}\rangle F(\lambda;\vec{k}',\vec{k}), \tag{13}$$

where

$$F(\lambda;\vec{k}'\vec{k})=\sum\int\phi_N^*(\vec{p}')\rho_\Delta(\lambda;\vec{p}'+\vec{k}',\vec{p}+\vec{k})\phi_N(\vec{p})d^3p\,d^3p', \tag{14}$$

with

$$\rho_\Delta(\lambda;\vec{p}'+\vec{k}',\vec{p}+\vec{k})=\sum_i\phi_\Delta^i(\vec{p}'+\vec{k}')\phi_\Delta^{i*}(\vec{p}+\vec{k}), \tag{15}$$

where the sum extends over the states which contribute at a given energy. In Eq. (13) we have introduced the following parameters: ΔE—the average energy shift, β—the average ratio of the width of Δ in the nuclear medium to the free width, and λ—a nonlocality parameter. The modified form factor F for light nuclei ($A\leqslant 16$) is chosen to be

$$F(\lambda;\vec{k}',\vec{k})=2\left\{1+\alpha\left[\left(\frac{3}{2}-\frac{Q^2c^2}{4}\right)-\frac{\beta_2^2}{4c^2}\left(\frac{3}{2}-\frac{K^2\beta_2^2}{4}\right)\right]\right\}\times e^{-Q^2\beta_1^2/4}e^{-K^2\beta_2^2/4}F_{\text{c.m.}}(Q), \tag{16}$$

with

$$\beta_1^2=\frac{c^2\lambda_1^2}{c^2+\lambda_1^2}\text{ and }\beta_2^2=\frac{c^2\lambda_2^2}{c^2+\lambda_2^2/4}, \tag{17}$$

c being the oscillator parameter, $\vec{Q}=\vec{k}'-\vec{k}$, $\vec{K}=(\vec{k}'+\vec{k})/2$, and $\alpha=(N-2)/3$ $[(Z-2)/3]$ for neutrons (protons). $F_{\text{c.m.}}(Q)$ is the correction due to the center of mass motion of the target, which we take from the static limit[29] to be $F_{\text{c.m.}}(Q)=\exp(Q^2c^2/4A)$. λ_1 and λ_2 are given in terms of the nonlocality parameter λ by

$$\lambda_1=\frac{1}{k_0\lambda}\text{ and }\lambda_2=\frac{\lambda}{k_0}, \tag{18}$$

k_0 being the on-shell pion-nucleus relative momentum. The form factor given by Eq. (16) is a generalization of the one used in earlier works.[18,30]

This corresponds to the Δ-density matrix being

$$\rho_\Delta(\vec{p},\vec{p}\,')=e^{-p^2\lambda_2^2/4}\delta^3(\vec{p}-\vec{p}\,')\,. \tag{19}$$

By introducing the parameter λ_1 we allow for a change in the three-momentum of the Δ due to interactions with the nuclear medium.

The parametrization of the optical potential as given by Eq. (13) is not unique (or necessarily optimal). One can conceivably keep the averaging interval small enough so that an intermediate structure of the optical potential shows up in terms of there being more than one term on the right hand side of Eq. (13), each being significant at a different energy. Or one can possibly construct a better parametrization of the form factor to describe the propagation of the Δ in the nuclear medium. One such improvement which we did investigate is the channel dependence of the parameters. In general, we expect the parameters of the isobar-doorway model to be dependent on the pion-nucleus channel quantum numbers such as l, J, T, etc. For spin-zero–isospin-zero targets such as ^{12}C, ^{16}O, etc., this channel dependence can be included by allowing ΔE_l and β_l to go to the asymptotic values 0 and 1, respectively, through a Fermi cutoff:

$$\Delta E_l=\frac{\Delta E}{1+\exp[(l-l_0)/\delta l_0]}\,, \tag{20a}$$

$$\beta_l=1+\frac{(\beta-1)}{1+\exp[(l-l_0)/\delta l_0]}\,, \tag{20b}$$

where $l_0=k_0R$ and $\delta l_0=k_0t$, R and t being nuclear rms radius and surface thickness, respectively. In the next section we compare cross sections for the channel independent and channel dependent parametrizations. In view of the similar quality of fit to experimental data obtained for the two cases, and also the fact that the inelastic width $\beta_l\Gamma_\Delta$ is much larger compared to the variations in ΔE_l with l, it is not unreasonable to take these parameters as channel independent.

For isospin nonzero targets such as ^{7}Li, ^{13}C, ^{18}O, etc., we have shown in an earlier work[30] that elastic scattering cross sections are not changed very significantly by allowing the parameters to be isospin dependent. On the other hand, charge exchange scattering, which is determined by differences of elastic scattering amplitudes in different (pion-nucleus) isospin channels, is very sensitive to the isospin dependence of the parameters. Thus in making a choice between channel independent and channel dependent parametrization, one should look at the channel decomposition of the amplitude for the process under consideration. If the amplitudes in different channels interfere destructively, one has to include the channel dependence of the parameters.

Finally we give a justification for the factorization approximation made in writing Eq. (11). Using a separable form of the pion nucleon scattering amplitude, Eq. (8) can be written as

$$V^\dagger_{\pi N\Delta}(\vec{\kappa}\,')V_{\pi N\Delta}(\vec{\kappa})=(E-M_\Delta+i\Gamma_\Delta/2)\vec{\kappa}\cdot\vec{\kappa}\,'\frac{g(\kappa)g(\kappa')}{g^2(\kappa_0)}t_{33}(\kappa_0)\,, \tag{21}$$

where $g(\kappa)$ are the off-shell extrapolation factors and we have separated out the threshold depen dence explicitly. Since the pole in $t_{33}(\kappa_0)$ is exactly canceled by the factor $(E-M_\Delta+i\Gamma_\Delta/2)$, and since g varies with κ smoothly, we can factorize $[g(\kappa)g(\kappa')]/g^2(\kappa_0)$ and $(E-M_\Delta+i\Gamma_\Delta/2)t_{33}(\kappa_0)$ out of the integral on the right hand side of Eq. (9) without any appreciable error. Under the assumptions

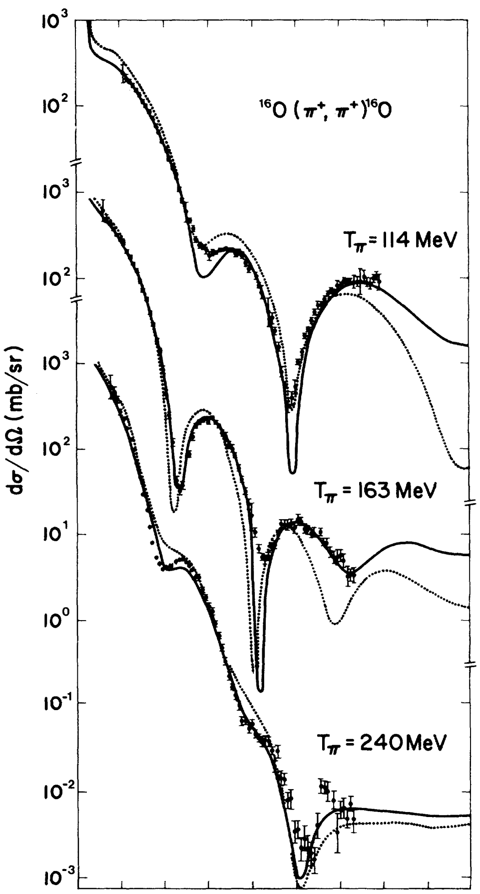

FIG. 2. π^+-^{16}O elastic scattering angular distributions at pion kinetic energies of 114, 163, and 240 MeV. Solid curve: fit obtained by using channel-independent parametrization of the isobar-doorway model. Dotted curve: prediction of the first order optical potential. The data are from Ref. 36.

made earlier we then have to evaluate the integral

$$I=\sum_N\int \vec{\kappa}\cdot\vec{\kappa}'\phi_N^*(\vec{p}')\rho_\Delta(\vec{p}'+\vec{k}',\vec{p}+\vec{k})\phi_N(\vec{p})d^3p\,d^3p' , \tag{22}$$

where $\vec{\kappa}$ and $\vec{\kappa}'$ are given by Eq. (10). With Gaussian wave functions and a momentum conserving form of nonlocality, this reduces to

$$I=\frac{M_N{}^2}{S_{\pi N}}\left[\vec{k}\cdot\vec{k}'\left(1+\frac{a}{2}+\frac{b}{2}\right)-\frac{ak^2}{2}-\frac{bk'^2}{2}\right]F(\lambda;\vec{k}',\vec{k}) . \tag{23}$$

This not only provides a justification for the factorization approximation used in writing Eq. (9), but also provides a definition of the pion nucleon amplitude in the pion nucleus center of mass frame:

$$\langle\vec{k}'|\tilde{t}^{R}_{\pi\pi}|\vec{k}\rangle\simeq\xi\frac{M_N{}^2}{S_{\pi N}}\left[\vec{k}\cdot\vec{k}'\left(1+\frac{a}{2}+\frac{b}{2}\right)-\frac{ak^2}{2}-\frac{bk'^2}{2}\right]\times\frac{g(\kappa)g(\kappa')}{g^2(\kappa_0)}t_{33}(\kappa_0) , \tag{24}$$

where

$$\xi=\left(\frac{E_\pi(\kappa)E_N(\kappa)E_\pi(\kappa')E_N(\kappa')}{E_N(k)E_N(p)E_N(k')E_N(p')}\right)^{1/2} \tag{25}$$

takes into account the noncovariant normalization of the plane waves.[31] ξ, a, b, and κ's in the above equation can now be evaluated using a frozen nucleus approximation.

IV. NUMERICAL RESULTS AND COMPARISON WITH DATA

The calculations were done using the momentum space code PIPIT.[28] The code is modified so that the background potential V^{NR} was calculated with the amplitude in the pion-nucleon 3-3 channel excluded. The contribution of the 3-3 channel is then calculated by replacing the single particle form factor $\rho(\vec{k}'-\vec{k})$ by the modified form factor $F(\lambda;\vec{k}',\vec{k})$ and multiplying the resulting expression

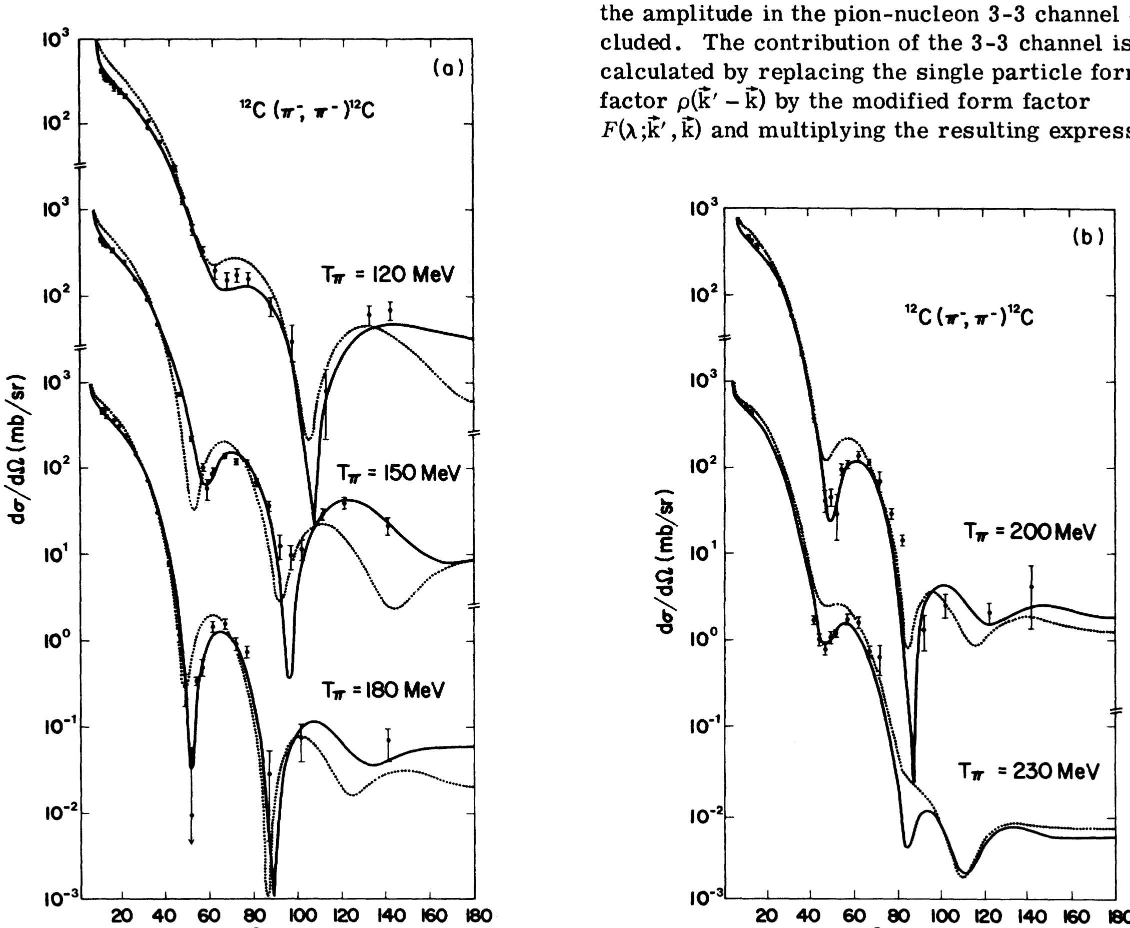

FIG. 3. (a) π^--^{12}C elastic scattering angular distributions at pion kinetic energies of 120, 150, and 180 MeV. Solid curve: fit obtained by using channel-independent parametrization of the isobar-doorway model. Dotted curve: prediction of the first order optical potential. The data are from Ref. 37. (b) Same as (a) except at pion kinetic energies of 200 and 230 MeV.

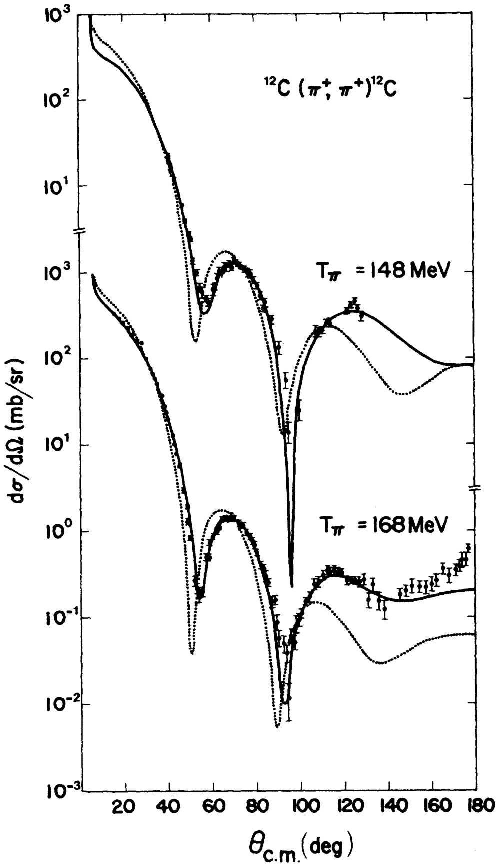

FIG. 4. π^+-^{12}C elastic scattering angular distributions at pion kinetic energies of 148 and 162 MeV. Solid curve: fit obtained by using channel-independent parametrization of the isobar-doorway model. Dotted curve: prediction of the first order optical potential. The data are from Ref. 38.

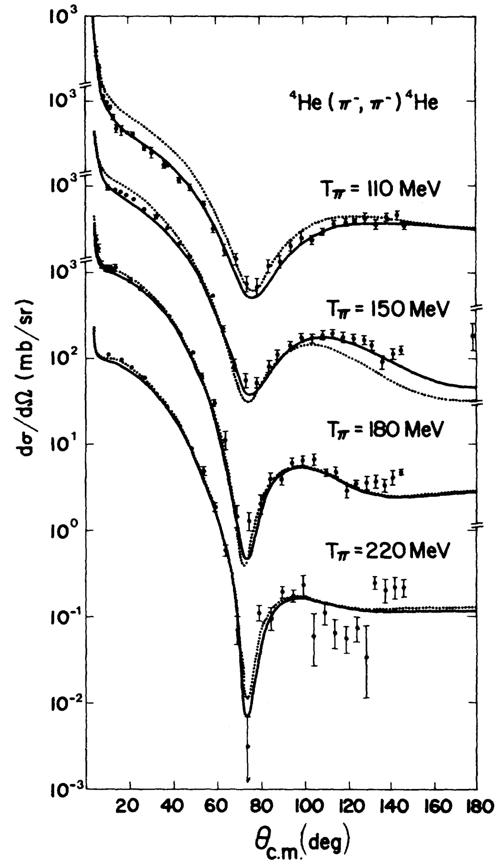

FIG. 5. π^--^{4}He elastic scattering angular distributions at pion kinetic energies of 110, 150, 180, and 220 MeV. Solid curve: fit obtained by using channel-independent parametrization of the isobar-doorway model. Dotted curve: prediction of the first order optical potential. The data are from Ref. 39.

with the complex energy factor $(E-M_\Delta+i\Gamma_\Delta/2)/(E-M_\Delta+i\beta\Gamma_\Delta/2)$. The resulting optical potential is used in the Lippman-Schwinger equation to generate the pion-nucleus scattering amplitude. The parameters ΔE, β, and λ were determined at each energy by fitting the elastic scattering angular distribution. The fitting is done by doing a chi-square search[32] for each energy and target using the CERN minimization routine MINUIT.[33] The harmonic oscillator parameter c is taken as 1.36, 1.64, and 1.77 fm for ^{4}He, ^{12}C, and ^{16}O, respectively, as determined by electron scattering experiments.[34,35]

Our fits to the pion elastic scattering angular distributions in the resonance energy region using the channel-independent parametrization of the isobar-doorway model are given in Figs. 2–5. Also given for comparison are the predictions of the first order optical potential.[28] (The latter are slightly different than the published results due to inclusion of the effect of center-of-mass motion of the target which increases the cross section at large angles.) The parameters of the isobar-doorway model so determined are given in Tables I–III. As can be seen from the figures the agreement between theory and experiments is very good at all angles except in the region of the second

TABLE I. Parameters of the isobar-doorway model (IDM) for π-^{16}O scattering. Quantities in parentheses correspond to the channel-dependent IDM.

T_π (MeV)	ΔE (MeV)	β	λ
114	4.07	1.264	0.38
	(0.023)	(1.538)	(0.373)
163	0.201	1.12	0.329
	(−7.7)	(1.333)	(0.327)
240	4.26	1.2	0.176
	(11.99)	(1.168)	(0.135)

TABLE II. Parameters of the IDM for π-^{12}C scattering.

T_π (MeV)	ΔE (MeV)	β	λ
120	19.9	1.26	0.368
148	16.59	1.094	0.400
150	19.87	1.065	0.45
162	−3.7	1.146	0.462
180	12.59	1.186	0.336
200	16.14	1.263	0.270
230	19.9	1.160	0.112

minimum, with χ^2 in the range 0.3–1.5 at the minimum. The positions of the minima and the slope of the diffraction peak are reproduced very well. Although the parameters of the model are strongly correlated at the minimum of χ^2, in general it was found that the width parameter β is sensitive to the forward angle cross section, and the nonlocality parameter is sensitive to the shape of the angular distribution. The parameter ΔE is positive in most cases, except around 160 MeV. The parameter β was found to be larger than the impulse approximation value of 1 by 5–30%. In most cases we found that a finite nonlocality is required to fit the data. As $\lambda = 0$ corresponds to the fixed-scatterer approximation ($V \propto t\rho$), we may interpret our results as an indication of the importance of including isobar dynamics in the pion-nucleus interaction. By introducing a complex energy shift in the subenergy at which pion-nucleon scattering amplitude is evaluated, these dynamical effects can only be included partially in conventional optical potentials.

In Fig. 6 we give a comparison of the fits obtained by using the channel-independent and the channel-dependent parametrizations of the energy shift and width parameters [cf. Eq. (20)]. By introducing an l dependence in the parameters, the second minimum becomes less sharp. However, the overall quality of the fit for the two cases is the same. We note that the simple l dependence given by Eq. (20) may be adequate only if we are considering processes such as elastic scattering or coherent π^0 photoproduction. If we study other reactions, e.g., asymmetries for a spin nonzero

TABLE III. Parameters of the IDM for π-^{4}He scattering.

T_π (MeV)	ΔE (MeV)	β	λ
110	14.42	1.183	0.32
150	14.42	1.0417	0.32
180	0.87	1.055	0.113
220	0.87	1.055	0.079

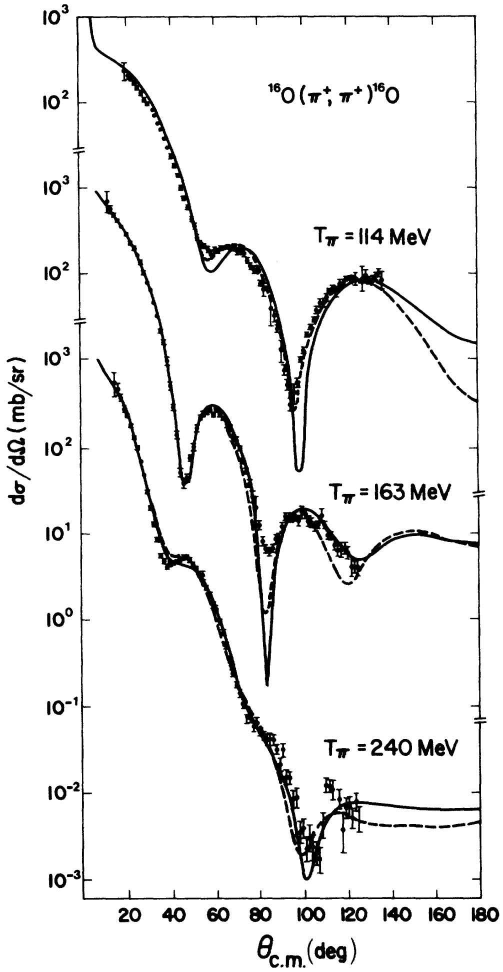

FIG. 6. Comparison of the fits obtained by using channel-independent parametrization (solid curve) and channel-dependent parametrization (dashed curve) of the isobar-doorway model to π^+-^{16}O elastic scattering angular distribution.

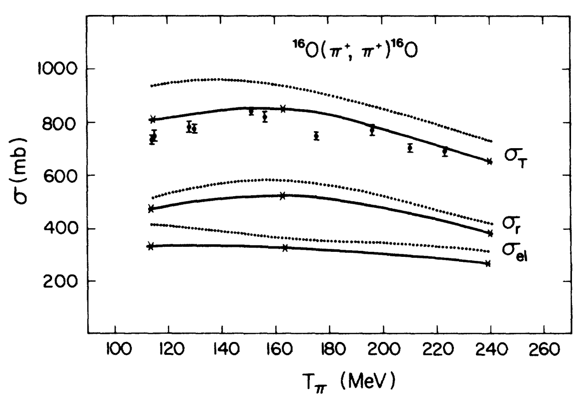

FIG. 7. Comparison of total cross sections for π^+-^{16}O given by the isobar-doorway model (crosses joined by solid line) and the first order optical potential (dotted curve) with the data of Refs. 40 and 41.

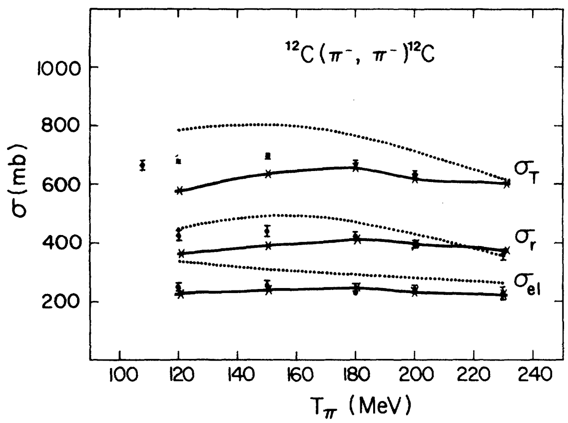

FIG. 8. Comparison of total cross sections for π^--^{12}C given by the isobar-doorway model (crosses joined by solid line) and the first order optical potential (dotted curve) with the data of Ref. 37.

target, we might have to construct a little more sophisticated model for the channel dependence of the parameters.

Finally, in Figs. 7–9 we give the comparisons of the total cross sections in the resonance energy region for the isobar-doorway model and the first-order optical potential with the experimental data. Whereas the first-order optical potential gives total and integrated elastic cross sections which are too large, the isobar-doorway model gives a reasonable fit to both. This is hardly surprising, as we fit the forward angle cross section as well as the slope of the diffraction peak very well.

V. CONCLUSIONS

In this work we have used the isobar-doorway model to obtain a phenomenological pion-nucleus optical potential which incorporates the effects of isobar-nuclear dynamics in terms of a small number of parameters. The parameters of the model are obtained by fitting the elastic scattering data. It is found that to obtain a reasonable description of the experimental data we have to include the nonlocality associated with isobar propagation in the nucleus—a feature completely missing in the models based on impulse approximation. Although it is hard to disentangle various many-body effects leading to the inelastic energy shift and width, we find that the cumulative effect of coupling to inelastic and reaction channels is to make the Δ isobar less bound than a nucleon. This is indicated by the positive value of ΔE found in most cases and an increase of the width by 10–30%. It is interesting to note that this result is consistent with the results for $\Lambda(1405)$ and $\Lambda(1520)$ isobars recently obtained with an isobar-doorway study of kaon-nucleus scattering.[42]

At present we have analyzed data for light nuclei where we could use harmonic oscillator wave functions. To extend this model to heavier nuclei, one could use numerical methods to generate a modified nuclear form factor in terms of the isobar density matrix $\rho_\Delta(\vec{r}_1, \vec{r}_2)$, or look for some other convenient parametrization to include the nonlocal nature of the isobar propagator.

Finally, we emphasize that in view of the significant deviations from the impulse approximation for elastic scattering found in this work, it is important to include these many-body effects in the transition operator for other pionic reactions where isobar production contributes to the elementary process. Using the model described here we have shown that this is indeed the case for coherent π^0 photoproduction.[20] We find that cross sections calculated using the impulse approximation and the one given by the isobar-doorway differ substantially, even when the pion optical potential is the same. On the other hand, different optical potentials which fit elastic scattering data give very similar cross section, provided we modify the transition operator correspondingly.

Although there are still a number of theoretical problems to be solved, we conclude that it is possible to extract information about the interaction of the Δ's with nuclei and at the same time obtain an improved optical potential for use in the analyses of pionic scattering and reactions on nuclei.

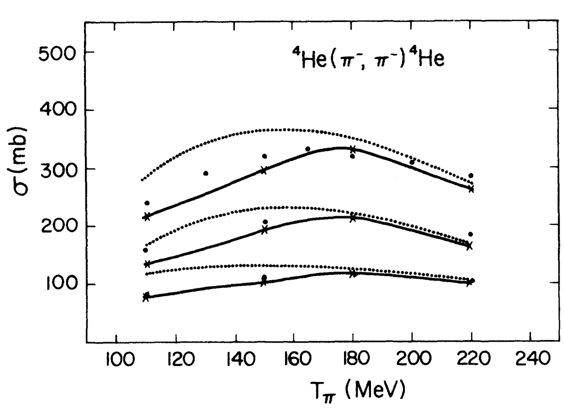

FIG. 9. Comparison of total cross sections for π^--^4He given by the isobar-doorway model (crosses joined by solid line) and the first order optical potential (dotted curve) with the data of Ref. 39.

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*Present address: Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627.

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