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Polarization in intermediate energy inelastic scattering

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The simple relationship between the elastic polarization and elastic cross section derived previously is generalized to include inelastic processes. The resulting data-to-data relations express the inelastic polarization in terms of the corresponding unpolarized scattering distribution and two spin "lengths" related to the relative strength and range of the spin orbit to central fundamental amplitudes. The success of these relations in describing inelastic analyzing powers indicates that little new nuclear structure information can easily be obtained from such experiments.

[NUCLEAR REACTIONS Intermediate energy inelastic polarization, data-to-data relationship between polarization and unpolarized angular distribution.]

I. INTRODUCTION

In a previous paper[1] we showed how the elastic polarization is simply related to the corresponding unpolarized scattering distribution using the analytic methods of Amado, Dedonder, and Lenz (ADL).[2] Similar relations between inelastic and elastic angular distrinutions were given in Ref. 3 by Amado, Lenz, McNeil, and Sparrow (ALMS). We call these relations data-to-data formulas in that they use the experimental elastic distribution as the nuclear structure input. In this paper we extend those results to inelastic polarization processes.

In Ref. 1 the polarization P was shown to be comprised of three terms. The first is a linear rise of P with momentum transfer q and is target independent. The second involved tangent-like oscillations reminiscent of the log-derivative phenomenology, while the last term oscillates like the inverse of the cross section and is new to polarization phenomenology. We find each of these terms present in inelastic polarization as well. However, since the envelopes for inelastic cross sections are not purely exponential as in the elastic case,[3,4] the data-to-data relations require slight modification for application to inelastic processes. In this paper we present the generalized data-to-data formulas and compare their results to the data for 800 MeV inelastic p-^{54}Fe scattering.[5] The excellent agreement suggests that the nuclear structure enters the unpolarized distributions in the same way as it enters the polarizations; so little new structure information can be extracted from such inelastic polarization experiments.

In the next section we derive the analytic polarization formula by a simple synthesis of two earlier papers and generalize the data-to-data relations. In the third section we compare the generalized data-to-data formula to 800 MeV analyzing power data on three low-lying collective excitations in ^{54}Fe. We discuss these results and contrast their basic features to that of direct unnatural parity excitations. A summary is provided in the last section.

II. THEORY

The spin-dependent amplitude for the natural parity excitation of a spinless target by a spin $\frac{1}{2}$ projectile in the eikonal formulation is $[\vec{r} = (\vec{b}, z)]$

$$\mathfrak{F}^{LM} = -\frac{ik}{2\pi}\int d^2b\, e^{i\vec{q}\cdot\vec{b}} e^{i\chi(\vec{b})} \times \int_{-\infty}^{\infty} dz\, \langle \vec{r}LM | V_t | 0^+ \vec{r}\rangle\,, \qquad (1)$$

where q is the momentum transfer, k is the incident wave number, and $|LM\rangle$ is the final nuclear state. The eikonal phase is given by

$$i\chi(\vec{b}) = -\gamma\int_{-\infty}^{\infty} dz\,[\rho_c(r) - w\vec{\sigma}\cdot(\vec{b}\times\vec{k})\rho_s'(r)]\,, \qquad (2)$$

with ρ_c and ρ_s referring to the central and spin orbit interaction densities and the prime denoting differentiation with respect to b. The remaining parameters are defined in terms of the fundamental nucleon-nucleon amplitude,

$$t_{NN}(q) = A(q) + iq\vec{\sigma}\cdot\hat{n}C(q)\,, \qquad (3)$$

where $\hat{n}$ is normal to the scattering plane. The parameters γ and w appropriate to a short range approximation are defined by t_{NN};

$$\gamma = \tfrac{1}{2}\sigma_T(1 - ir)\,,\quad r = \mathrm{Re}[A(0)]/\mathrm{Im}[A(0)]\,,\quad w = C(0)/A(0)\,, \qquad (4)$$

where σ_T is the total nucleon-nucleon cross section.

As in ALMS we take a local surface peaked transition density of Tassie form appropriate for low-lying collective excitations. We write for the (projectile) spin dependent transition density

$$\langle \vec{r}\, LM | V_t | 0^+ \vec{r}\rangle = \lambda_L P_{LM}(\theta) e^{im\varphi} \frac{r^L d}{r\, dr} \times [\rho_c(r) - w\vec{\sigma}(\hat{b}\times\hat{k})\rho_s'(r)]\,, \quad (5)$$

where $P_{LM}(\theta)$ is an associated Legendre polynomial and λ_L is the excitation strength which can be related to the electromagnetic $B(EL)$'s.[3] Following ALMS we write P_{LM} as a homogeneous polynomial in b and z, $P_{LM} = r^{-L}\sum_{n=0}^{L} a_n^{LM} b^{L-n} z^n$. For large (qb) the highest power of b dominates the sum; terms with additional z^2 factors are relatively smaller by $(qb)^{-2/3}$ in the final amplitude (see Appendix A of ALMS). Using (5) and (2) and retaining only the highest power of b in the expansion of P_{LM}, we have

$$\mathfrak{F}^{LM} = -\frac{ik\lambda_L a_0^{LM}}{2\pi} \times \int d^2b\, e^{i\vec{q}\cdot\vec{b}} e^{im\varphi} e^{i\chi(\vec{b})} b^{L-1} \frac{d}{db}\left[\frac{-i}{\gamma}\chi(\vec{b})\right] = \frac{ik\lambda_L a_0^{LM}}{2\pi\gamma} \int d^2b\, b^{L-1} e^{i\vec{q}\cdot\vec{b}} e^{im\varphi} \frac{d}{db} e^{i\chi(\vec{b})}\,. \quad (6)$$

Integrating by parts and retaining only the asymptotically dominant term, we find

$$\mathfrak{F}^{LM} = \frac{ia_0\lambda_L k}{2\pi\gamma} q \frac{d}{dq} \int d^2b\, b^{L-2} e^{i\vec{q}\cdot\vec{b}} e^{im\varphi} e^{i\chi(\vec{b})}\,. \quad (7)$$

We define the following phases:

$$i\chi_c(b) = -\gamma\, t_c(b)\,, \qquad i\chi_s(b) = w\gamma\, t_s'(b)\,, \quad (8)$$

where

$$t(b) = \int_{-\infty}^{\infty} dz\, \rho[(b^2+z^2)^{1/2}] \quad (9)$$

is the thickness function. Using (8), the spin dependent exponential is expanded to give

$$\mathfrak{F}^{LM} = -\frac{ia_0^{LM}\lambda_L k}{2\pi\gamma} q \frac{d}{dq} \times \int d^2b\, b^{L-2} e^{im\varphi} e^{i\chi_c(b)} \times \left[\cos\chi_s(b) + \frac{\vec{\sigma}\cdot(\vec{\nabla}_q\times\vec{k})}{b}\sin\chi_s(b)\right] e^{i\vec{q}\cdot\vec{b}}\,, \quad (10)$$

where $\hat{b}e^{i\vec{q}\cdot\vec{b}} = -ib^{-1}\vec{\nabla}_q e^{i\vec{q}\cdot\vec{b}}$ has been used. Performing the angle integration and the indicated differentiations gives

$$\mathfrak{F}^{LM} = F_1^{LM} + \vec{\sigma}\cdot\hat{n} F_2^{LM} \quad (11)$$

with

$$F_1^{LM} = i^{-M-1} \frac{a_0^{LM}\lambda_L kq}{\gamma} \times \int_0^\infty db\, b^L e^{-\gamma t_c(b)} \times \cosh[w\gamma t_s'(b)] \frac{d}{d(qb)} J_M(qb)\,, \quad (11a)$$

$$F_2^{LM} = i^{-M} \frac{a_0^{LM}\lambda_L kq}{\gamma} \times \int_0^\infty db\, b^L e^{-\gamma t_c(b)} \times \sinh[w\gamma t_s'(b)] \frac{d^2}{d(qb)^2} J_M(qb)\,. \quad (11b)$$

Following ALMS again, we write the Bessel functions as Hankel functions which are subsequently approximated by their asymptotic form. We have

$$F_1^{LM} = -\frac{ik}{2}[G_{LM}(q,\gamma,w) + G_{LM}(q,\gamma,-w) + G_{LM}^*(q,\gamma^*,w^*) + G_{LM}^*(q,\gamma^*,-w^*)]\,, \quad (12)$$

$$F_2^{LM} = -\frac{ik}{2}[G_{LM}(q,\gamma,w) - G_{LM}(q,\gamma,-w) + G_{LM}^*(q,\gamma^*,w^*) - G_{LM}^*(q,\gamma^*,-w^*)]\,,$$

where

$$G_{LM}(q,\gamma,w) = -iq \frac{a_0^{LM}\lambda_L}{2\gamma} e^{-i\pi(M+1/4)} \times \int_0^\infty db\, b^L e^{-\gamma t_c(b) + w\gamma t_s'(b)} \left(\frac{2}{\pi qb}\right)^{1/2} e^{iqb} \quad (13)$$

and again only the asymptotically dominant terms are retained.

The problem is to evaluate (13) using the analytic methods of ADL. Following ADL and ALMS, we take a Fermi distribution for the density normalized to A nucleons,

$$\rho(r) = \rho_0/(1+e^{(r-c)/\beta})\,, \quad (14)$$

where c is the half density radius and β the surface diffusivity. These parameters are united in the single complex parameter $b_0 \equiv c + i\pi\beta$ which is the nearest pole of (14) and which plays an important role in our analysis. To represent a different geometry for the spin-orbit thickness function we use a different pole position $b_{0\delta}$:

$$b_{0\delta} = b_0 + \delta\,. \quad (15)$$

The length parameter δ represents the difference in central and spin orbit geometries and was seen in Ref. 1 to be crucial to an understanding of the structure of polarization. In practice, its magnitude is about what one would expect from the different central and spin orbit fundamental force ranges. The central and spin orbit thickness functions have identical functional forms but with different poles; so we denote them by $t(b, b_0)$ and $t(b, b_{0\delta})$, respectively.

Using the stationary phase methods of ADL, we can reduce (13) to the previously solved central potential only case of ALMS by absorbing the w dependence as a shift in the argument of the central thickness function. For asymptotic (qb) we note that the singular parts of the thickness function t_0 will determine the stationary phase condition. Using the explicit forms of ADL, one finds

$$t_0'(b, b_{0\delta}) \simeq \mu t_0'(b, b_0), \tag{16}$$

where

$$\mu = 1 + \frac{\delta}{b_0}\left(1 - \frac{3}{\Delta}\right) \tag{17}$$

and we have neglected the effect of δ on the density normalization ρ_0. In (17) Δ is the deviation of the central only stationary phase point b_c from the pole b_0; that is,

$$\Delta = 2(b_0 - b_c) = \left(-\frac{\alpha}{qb_0}\right)^{2/3}, \tag{18}$$

where $\alpha = 2\pi\beta\rho_0\gamma$ is a dimensionless strength parameter. Calling the central potential only result of ALMS $G_{LM}^{(0)}$, we have for (12)

$$\begin{aligned} F_1^{LM} &= -ik[G_{LM}^{(0)}(q,\gamma)\cos(qw\mu) \\ &\quad + G_{LM}^{(0)*}(q,\gamma^*)\cos(qw\mu^*)] , \\ F_2^{LM} &= k[G_{LM}^{(0)}(q,\gamma)\sin(qw\mu) \\ &\quad + G_{LM}^{(0)*}(q,\gamma^*)\sin(qw\mu^*)] . \end{aligned} \tag{19}$$

In terms of the elastic G_c of ADL, $G_{LM}^{(0)}$ is given by (ALMS)

$$G_{LM}^{(0)} = -\frac{\lambda_L}{\gamma} a_0^{LM} e^{-i(\pi/2)M} b_0^{L-1} q G_c(q,\gamma), \tag{20}$$

where

$$G_c(q,\gamma) = \frac{b_0{}^2\alpha^{1/3}}{\sqrt{3}\,(qb_0)^{4/3}} \exp\left[\frac{i5\pi}{6} - \gamma\bar{t} + \tfrac{3}{2}(\alpha^2 qb_0)^{1/3} e^{i\pi/6}\right] e^{iqb_0}, \tag{21}$$

with the nonsingular part of the thickness function $\bar{t}$ evaluated at the stationary phase point given by

$$\gamma\bar{t} \simeq 1.46\ldots\, \rho_0(2\pi\beta b_0)^{1/2} . \tag{22}$$

The key features of (21) are the extra power of q and the asymptotically dominant e^{iqb_0} phase dependence [Eq. (21)] which gives rise to the oscillating and exponentially falling cross sections so characteristic of intermediate energy (diffractive) scattering.

The form of the spin-dependent amplitudes [Eq. (19)] is identical to that of Ref. 1; so we simply carry over the spin observables to lowest order in w from there. We have the polarization or analyzing power

$$P = \frac{2\,\mathrm{Re}\,F_1F_2^*}{\sigma} \cong 2qvx - 2qvy\frac{\mathrm{Re}\,CS^*}{C^*C} + 2quy\frac{\mathrm{Im}\,CS^*}{C^*C}, \tag{23}$$

the spin rotation function

$$Q = \frac{2\,\mathrm{Im}\,F_1F_2^*}{\sigma} \cong 2qux - 2quy\frac{\mathrm{Re}\,CS^*}{C^*C} - 2qvy\frac{\mathrm{Im}\,CS^*}{C^*C}, \tag{24}$$

and unpolarized cross section

$$\sigma \cong 4k^2C^*C , \tag{25}$$

where

$$\begin{aligned} w &= u + iv , \\ \mu &= x + iy , \\ C &= \tfrac{1}{2}[G_{LM}^{(0)}(q,\gamma) + G_{LM}^{(0)*}(q,\gamma^*)] , \\ S &= \frac{1}{2i}[G_{LM}^{(0)}(q,\gamma) - G_{LM}^{(0)*}(q,\gamma^*)] . \end{aligned} \tag{26}$$

This is identical to the result obtained for elastic scattering except now one uses the appropriate inelastic G's. Due to the $\sim e^{iqb_0}$ asymptotic behavior of G [Eq. (17)], C and S of (26) are cosine- and sine-like. Thus $\mathrm{Re}\,C^*S/C^*C$ is tangent-like, and, since $\mathrm{Im}\,CS^*$ is flat, $\mathrm{Im}\,CS^*/C^*C$ goes like $1/\sigma$ [Eq. (25)]. The three terms contributing to the polarization in (23) are (1) a linear rise in q proportional to $v = \mathrm{Im}(w)$ and $x = \mathrm{Re}(\mu)$; (2) a tangent-like term $\mathrm{Re}\,C^*S/C^*C$, proportional to $v = \mathrm{Im}(w)$ and $y = \mathrm{Im}(\mu)$, and (3) a $1/\sigma$-like term proportional to $u = \mathrm{Re}(w)$ and $y = \mathrm{Im}(\mu)$. Since $x \to 1$ and $y \to 0$ as $\delta \to 0$, the significant structure seen in polarization is principally due to the different central and spin orbit geometries, since with $\delta = 0$ only the linearly rising term survives.

To obtain a data-to-data formula we note that the relationship of the nuclear structure terms, $\mathrm{Re}\,CS^*/C^*C \sim \tan qc$ and $\mathrm{Im}\,CS^*/C^*C \sim 1/\sigma$, to the appropriate angular distribution will be the same as that given in Ref. 1 except for the slight modifications due to the extra power of q modulating the envelope. The modified expressions are

$$\frac{\mathrm{Re}\,CS^*}{C^*C} \simeq \frac{1}{2}\,\frac{F_I(\delta q)\sigma(q-\delta q)-F_I(\delta q)\sigma(q+\delta q)}{\sigma(q)}\,, \tag{27}$$

where $\delta q = \pi/4c$ and $F_I(\delta q) = [1+(\delta q/q)]^2 e^{-2\pi\beta\delta q}$, and

$$\frac{\mathrm{Im}\,CS^*}{C^*C} \simeq \tfrac{1}{2}(\mathrm{sgn}\,r)\left(\frac{\sigma_{\min}}{\sigma_{\gtrless}}\right)^{1/2} e^{\pm\pi^2\beta/2c}\left(1 \pm \frac{\pi}{2cq_{\gtrless}}\right) \times \left(\frac{\sigma_< q_>^{\,2}}{\sigma_> q_<^{\,2}}\right)^{(q-q_<)/(q_>-q_<)} \frac{\sigma_> q^2}{\sigma(q) q_>^{\,2}}\,, \tag{28}$$

where $\sigma_{><}$ are the nearest cross section maxima located at $q_{><}$ and $\sigma_{\min}$ is the intervening cross section minimum. The main structure to this term is the $1/\sigma$ which provides an oscillatory behavior of a fixed sign determined by r, Eq. (4). A complete discussion of the relationship of these terms to the spin observables is given in Ref. 1.

The data-to-data relations easily generalize to n (natural parity) step transitions by noting that the envelopes for such transitions have, relative to the elastic cross section, an extra power of q^2 for each step. For an n (natural parity) step excitation coupled to an unnatural parity final state, the envelope shows an extra $q^{2n-2/3}$ power dependence (relative to the elastic cross section). These effects are so minor that multistep excitations of both natural and unnatural parity states will have the same qualitative features as the direct excitations studied in the next section.

III. COMPARISON WITH EXPERIMENT

Using (23), (27), and (28) one can write the polarization for a given process in terms of the unpolarized cross section and the spin parameters w and δ. Similar formulas relating inelastic to elastic processes are presented in ALMS.

We now consider 800 MeV protons on ^{54}Fe exciting three low-lying natural parity collective 2^+(1.408 MeV), 3^-(4.872 MeV), and 4^+(2.538 MeV) states. We use the angular distributions of Adams *et al.*[5] as input for the nuclear structure terms (27) and (28). We obtain $w = 0.2 - i0.15$ fm from a phase shift analysis of Arndt[6] suitably averaging over the nuclear constituents. The central to spin-orbit radius difference $\delta = 0.07 + i0.09$ fm is fit to the 2^+ case. Presumably, if available, we could have determined δ from the polarized elastic angular distributions. The nuclear geometry parameters $c = 4.01$ fm and $\beta = 0.54$ fm were the same as those used in ALMS. In Fig. 1 the data-to-data formula is compared with the analyzing powers of Adams *et al.*[5] The agreement is excellent. Since only the nuclear structure manifest in the unpolarized angular distribution is used, these results suggest that there is little new structure information to be obtained from such analyzing power measurements of inelastic excitations.

These calculations have been carried out using the Tassie model to relate the coupling and distorting potentials.[7] The conclusions should have the same range of applicability as the Tassie model, and may even have wider validity at 800 MeV for two reasons. First, the Tassie part of excitation form factors dominates the transitions at large q. Second, in practice, the collective model is used only to obtain a connection between the cross section and analyzing power which probably has much wider applicability than just to Tassie-type transitions. Experiments are cur-

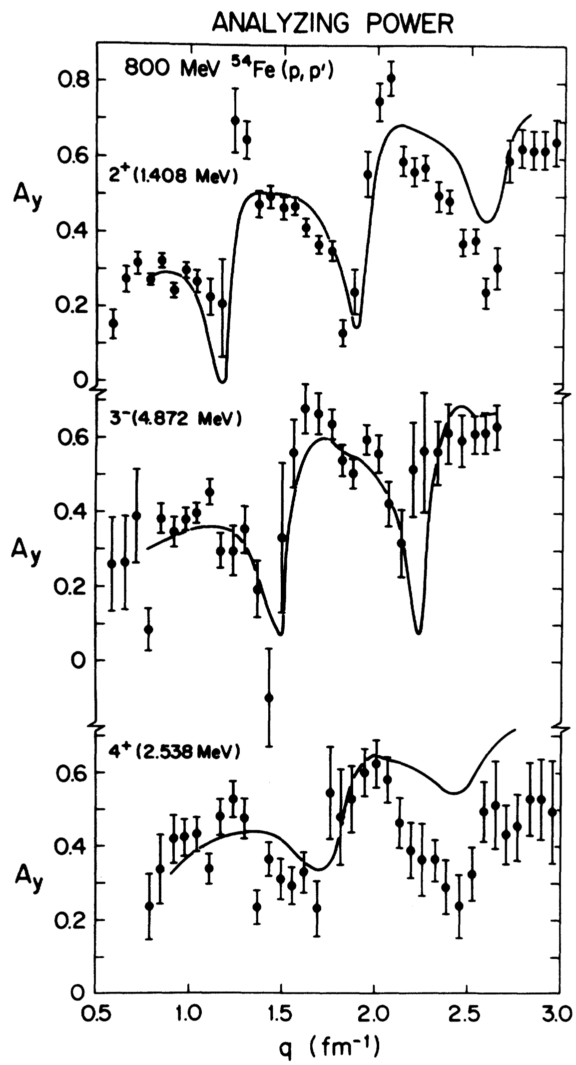

FIG. 1. The 800 MeV p-^{54}Fe analyzing powers of Adams *et al.* are compared with the data-to-data relations, Eqs. (23), (27), and (28), using the unpolarized angular distribution of Adams *et al.* as input.

rently underway which will measure the analyzing powers of states populated by two step processes induced by 800 MeV protons.[8] We expect this approach to have comparable success in these cases. In short, we expect that most measurements of medium energy inelastic analyzing powers will provide no information not already present in the elastic analyzing power and inelastic angular distribution.

Let us now point out two exceptions to this somewhat discouraging conclusion. The first concerns lower energy (~200 MeV) proton excitation of nearly pure shell model transitions. Here the possibility exists for departures from the Tassie model and from our asymptotic ($qc \to \infty$) limit, permitting some independent nuclear structure information to be carried by the analyzing power. A recent study of this sort focuses on the $(\vec{p}, p')$ excitation of the ^{90}Zr 8^+ state by 160 MeV protons.[9] An attempt to devise a data-to-data formula for this transition would require, at least, inclusion of additional terms of order $1/qc$ and possibly consideration of double spin flip fundamental amplitudes as well.

We may strike an even more positive note concerning the generality of our conclusions for unnatural parity two-step excitations. Such processes, we predict, will universally have rising, oscillating analyzing powers. However, unnatural parity excitations may also proceed directly via the double spin-flip terms in the nucleon-nucleon amplitude which include target spin operators. It turns out that the analyzing power in such transitions is expected to be radically different. The measured analyzing power for 800 MeV proton excitation of the 1^+, $T = 0$ state in ^{12}C is negative,[10] as predicted by plane-wave impulse approximation (PWIA) arguments based on the Arndt phase shifts. Hence we have a signature of spin-spin versus two-step population of unnatural parity states. This should prove useful in interpreting the planned measurements of the analyzing powers in $^{20}Ne(\vec{p}, p')2^-$ and $^{24}Mg(\vec{p}, p')3^+, 5^+$.[8]

IV. SUMMARY

We have constructed a data-to-data formula relating the inelastic polarization to the angular distribution. This relation relies on the Tassie model and our approximate analytic expression for the scattering amplitudes. Analyzing powers calculated for 800 MeV $^{54}Fe(\vec{p}, p')$ with this relation show excellent agreement with the data. We conclude that nuclear structure information not contained in medium energy angular distributions will only rarely be contained in analyzing powers. Exceptions may be found at lower energies for transitions of nearly pure shell model character. By happy circumstance, our very general predictions for the polarizations for unnatural parity states populated by two-step mechanisms are radically different from the polarizations expected in $\Delta T = 0$ one-step unnatural parity excitations. We hope that both this qualitative result, and our more quantitative data-to-data formula will prove useful in the analysis of analyzing power measurements currently underway or just completed. Unfortunately, there is little hope that these measurements will provide any new information about either nuclear structure or nuclear reactions.

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