PHYSICAL REVIEW C VOLUME 23, NUMBER 5 MAY 1981

Spin dependence in intermediate energy *p*-nucleus scattering

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(Received 15 October 1980)

We obtain closed form approximations for the spin dependent eikonal p-nucleus scattering amplitudes. The polarization P and spin rotation function Q can be written in terms of the unpolarized elastic scattering cross section and two complex parameters related to the nucleon-nucleon spin orbit strength and range. These forms agree well with the available data and illuminate the dynamical content of P and Q.

NUCLEAR REACTIONS Closed form eikonal amplitude for spin dependent p-nucleus scattering. Data-to-data forms relating spin observables to elastic scattering.

I. INTRODUCTION

The study of spin dependent effects in proton-nucleus scattering forms an important part of the intermediate energy program. At these energies polarization has a rich structure that is closely related to the diffraction structure in the corresponding elastic scattering process; nevertheless, attempts to develop that relationship have not proceeded far beyond *ad hoc* phenomenology.[1] In this paper we extend our method for obtaining analytic expressions for diffraction scattering to include spin. We obtain closed form expressions for the spin observables that accurately reproduce the numerically evaluated eikonal results and make explicit what new features of nucleon-nucleon and nucleon-nucleus dynamics are involved. Best of all, our closed forms can be expressed in terms of the elastic scattering cross section directly (data-to-data relationship) and thus provide both a derivation and a realization of the heretofore phenomenological connection between spin observables and the corresponding elastic scattering.

Diffraction theory has enjoyed much success in accurately describing a wide body of intermediate energy hadron-nucleus scattering phenomena. Until recently, however, it has been necessary to evaluate the underlying eikonal integral numerically in realistic applications. With the high speed of modern computers this has proved no more than an inconvenience, but an inconvenience which nevertheless obscures an essentially very simple theory. A recent paper by Amado, Dedonder, and Lenz (hereafter referred to as ADL)[2] shows how the principal scattering features can be extracted analytically from the eikonal integral. Exploiting the dominance of the nuclear radius in setting the length scale, they obtain closed form expressions for the (spinless) central scattering amplitude. We used the same methods to obtain closed form amplitudes for the low lying collective excitations as well.[3] In each case the closed form is found to reproduce the direct numerical evaluation of the eikonal amplitude as well as the principal features of the data except in the extreme forward direction where the approximation methods employed are not valid. Furthermore, having closed forms for both the elastic and inelastic cross sections allows one to obtain relations between them.[3] These data-to-data formulas in which the elastic cross section is used as input are found to be even more successful in describing the inelastic data than their parent closed form expressions. Processes left out of the theory or not well approximated are automatically restored or corrected when the elastic data itself is used as input. For a full discussion of the elastic-inelastic relationship see Ref. 3 and a forthcoming paper on two-step processes.[4]

In this paper we extend this approach to include the projectile spin degree of freedom within the eikonal framework. Several studies of polarization at intermediate energies already exist.[5] Besides Ref. 1 there are the recent works of Auger and Lombard[6] and Glauber and Osland.[7] These later works share with us an eikonal theory starting point, but the methods and results of this paper will differ considerably since we use the methods of ADL to derive closed form spin-channel amplitudes. We use these, in turn, to obtain for the spin-dependent observables a new set of data-to-data formulas which are the primary focus of this work. These are formulas for the polarization (analyzing power) P and spin rotation function[8] Q expressing each in terms of the elastic cross section and two complex length parameters which characterize the underlying nucleon-nucleon spin dynamics. The first of these, w, is a measure of the spin-orbit to central strength and can be taken directly from nucleon-nucleon phase shifts. The second, δ, characterizes the difference in geometry (radius and diffusivity) of the central and spin-orbit parts of the nucleon-nucleus effective potential. In a simple first order optical model δ is related to the differences in range of central and spin-orbit nucleon-nucleon forces. In particular we find that if this difference is neglected, the resulting polarization will display negligible structure. The well known and strikingly evident structure in polarization is therefore a direct consequence of this geometric difference. Though implicit in the empirical need for such a difference in optical model fits, this point and its generality seem not to be well known. The origins of this geometric difference merit further investigation which may perhaps best be carried out in an analytic framework where the influence of geometry is manifest.

In Sec. II we start with the spin dependent eikonal amplitude, explicitly allowing for different central and spin orbit densities, and derive closed forms for the spin channel amplitudes, Eq. (25). We compare 800 MeV proton-^{208}Pb elastic polarizations computed from the closed form amplitudes with the numerically evaluated full eikonal amplitude for the cases of identical and different spin-orbit and central geometries. In Sec. III we present the data-to-data relations for the spin observables in terms of the elastic cross section. These results are summarized in Eqs. (31), (35), and (40). We then compare 800 MeV p-^{208}Pb elastic polarizations computed from both the analytic closed form and the data-to-data relation with the data. The agreement is excellent. We also present a prediction for Q, the spin rotation function, for the same process. In Sec. IV we discuss our results, present our conclusions, and point to avenues for future development. We present some details in the Appendix.

II. THEORY

Consider the elastic scattering of a spin $\frac{1}{2}$ projectile (proton) from a spinless target nucleus. The scattering amplitude can be written

$$F(q)=F_1(q)+\vec{\sigma}\cdot\hat{n}F_2(q)\,, \tag{1}$$

where q is the momentum transfer, $\vec{\sigma}$ is the projectile spin operator, and $\hat{n}$ is a unit vector normal to the scattering plane. In the eikonal formulation one has[6]

$$\begin{aligned} F_1(q)&=ik\int db\, bJ_0(qb)\\ &\quad\times\{1-e^{-\gamma t_c(b)}\cosh[w\gamma t_s'(b)]\}\,,\\ F_2(q)&=-k\int db\, bJ_1(qb)e^{-\gamma t_c(b)}\sinh[w\gamma t_s'(b)]\,. \end{aligned} \tag{2}$$

In the above expression k is the incident wave number and t is the thickness function defined in terms of the nuclear interaction density ρ by

$$t(b)=\int_{-\infty}^{\infty}dz\,\rho[(b^2+z^2)^{1/2}]. \tag{3}$$

The subscripts c and s on t of (2) refer to the central and spin-orbit interaction densities, respectively. In the limit of zero range forces and a first order theory the interaction density would equal to the underlying nuclear matter density. This designation enables us to choose these densities differently. The primes on t_s of (2) denote differentiation with respect to b. The parameters w and γ in (2) are complex strengths which in a simple first order optical model are related to the fundamental nucleon-nucleon amplitude t_{NN} suitably averaged over neutrons and protons. We ignore double spin-flip contributions in considering p-nucleus applications. From symmetry considerations the nucleon-nucleon spin-orbit amplitude must vanish as $q\to 0$ like q. Extracting this dependence explicitly, we may write for the fundamental amplitude

$$t_{NN}(q)=A(q)+iq\vec{\sigma}\cdot\hat{n}C(q)\,. \tag{4}$$

In the context of a first order optical model only the short range parts of t_{NN} are needed and we define

$$\begin{aligned} \gamma&=\frac{\sigma_T}{2}(1-ir)\,,\\ r&=\mathrm{Re}A(0)/\mathrm{Im}A(0)\,,\\ w&=C(0)/A(0)\,, \end{aligned} \tag{5}$$

where σ_T is the nucleon-nucleon total cross section. Note that w is complex and has dimensions of length as it is associated with q to give the dimensionless relative strength of the spin orbit to central amplitude. In practice, for intermediate energy applications $|w|$ is much smaller than the nuclear radius. Additional q dependence of A and C is implicitly included via the freedom to take ρ_c and ρ_s to be different.

To evaluate (1) we shall lean heavily on the methods of ADL. The first step is to write the Bessel functions in (2) as Hankel functions.

$$J_n(x)=\tfrac{1}{2}[H_n^{(1)}(x)+H_n^{(1)*}(x)]\,. \tag{6}$$

This division manifests the two terms that interfere to produce the diffraction pattern. It is analogous to the classic treatment of diffraction from a slit where the slit is divided into interfering halves. Writing the sinh and cosh terms of (2) in exponential form and recalling that γ and w are the only complex quantities under the integrals of (2), we have

$$F_1=\frac{-ik}{2}[G_0(q,\gamma,w)+G_0(q,\gamma,-w)+G_0^*(q,\gamma^*,w^*)+G_0^*(q,\gamma^*,-w^*)],$$
$$F_2=\frac{-k}{2}[G_1(q,\gamma,w)-G_1(q,\gamma,-w)+G_1^*(q,\gamma^*,w^*)-G_1^*(q,\gamma^*,-w^*)], \quad (7)$$

where

$$G_n(q,\gamma,w)=\tfrac{1}{2}\int db\, bH_n^{(1)}(qb)\times\exp[-\gamma t_c(b)+w\gamma t_s'(b)]. \quad (8)$$

The 1 term of (2) has been dropped since it contributes only at $q=0$. As shown in ADL, the scattering is dominated by the nuclear radius so that, except for the extreme forward angles, $qb\gg 1$ and we may approximate the Hankel functions by their asymptotic forms

$$H_0^{(1)}\to\left(\frac{2}{\pi qb}\right)^{1/2}e^{-i\pi/4}e^{iqb}$$

and

$$H_1^{(1)}\to -iH_0^{(1)}. \quad (9)$$

From this last relation we get $G_1\to -iG_0$, and we need evaluate only G_0,

$$G_0(q,\gamma,w)\simeq\frac{e^{-i\pi/4}}{(2\pi q)^{1/2}}\times\int db\sqrt{b}\,\exp[iqb-\gamma t_c(b)+w\gamma t_s'(b)]. \quad (10)$$

To evaluate this integral we will use the method of steepest descent, or stationary phase, taking for the density a Fermi distribution normalized to A nucleons,

$$\rho=\frac{\rho_0}{1+e^{(r-c)/\beta}} \quad (11)$$

in the thickness function integral. Differences in central and spin-orbit geometry are reflected in different values of the half density radius c and diffusivity β. In ADL it is shown that the complex density parameter $b_0=c+i\pi\beta$ (the nearest pole of the Fermi distribution) controls the amplitude. We call this parameter b_0 for the central density as before and

$$b_{0\delta}=b_0+\delta \quad (12)$$

for the spin-orbit density. The complex length parameter δ characterizes the effective shape differences between the central and spin-orbit densities. We assume $|\delta/b_0|\ll 1$ as well as $|w/b_0|\ll 1$ so that we may calculate the stationary point perturbatively in w and δ.

Following ADL, we separate t into a nonsingular piece $\tilde{t}$ and a singular piece t_0, which carries the singular dependence of the thickness integral near the pole b_0. For $qb\gg 1$ it is the singular piece which is needed to guarantee a stationary phase solution. The nonsingular part is simply evaluated at the singular point b_0. We write

$$G_0(q,\gamma,w)\simeq\exp[-\gamma\tilde{t}_c(b_0)+w\gamma\tilde{t}_s'(b_0)]\times\left(\frac{b_0}{2\pi q}\right)^{1/2}\int_0^\infty db\, e^{g_s(b)}, \quad (13)$$

where

$$g_s(b)=iqb-\gamma t_0(b,b_0)+w\gamma t_0'(b,b_{0\delta}) \quad (14)$$

and we have used (11) and (12). In ADL the singular part t_0 is explicitly evaluated to be

$$\gamma t_0(b,b_0)=\frac{-i\alpha b_0}{(b_0^2-b^2)^{1/2}}, \quad (15)$$

where $\alpha=2\pi\beta\rho_0\gamma$ is a dimensionless strength parameter. To first order in δ we have

$$t_0'(b,b_{0\delta})\simeq\left(1+\delta\frac{\partial}{\partial b_0}\right)t_0'(b,b_0)=\mu(b)\gamma t_0'(b,b_0), \quad (16)$$

where

$$\mu(b)=1+\frac{\delta}{b_0}\left(1-\frac{3b_0^2}{b_0^2-b^2}\right). \quad (17)$$

To first order in w we can use Taylor's theorem in reverse to obtain

$$g_s(b)=iqb-\gamma t_0(b,b_0)+w\mu(b)\gamma t_0'(b,b_0)\cong iqb-\gamma t_0(b-w\mu(b),b_0). \quad (18)$$

This is precisely the form of the spinless case considered in ADL. We may therefore use the ADL result at the shifted argument. Note that this will *not* change t evaluated at the stationary point; the shift will appear only in the iqb term. That is, if b_c is the (central potential only) stationary point of ADL, then the stationary point of (18) is $b_s=b_c+w\mu(b_c)$ to the desired order. The phase evaluated at the stationary point becomes simply

$$g_s(b_s)=g_c(b_c)+iqw\mu(b_c). \quad (19)$$

From ADL we have the stationary phase solution

$$b_c = b_0(1-\Delta/2)\,,$$
$$\Delta = \left(-\frac{\alpha}{qb_0}\right)^{2/3} \ll 1\,. \tag{20}$$

Thus to lowest order

$$\mu(b_c) = 1 + \frac{\delta}{b_0}\left(1 - \frac{3}{\Delta}\right). \tag{21}$$

Note also that since the nonsingular parts of G will be the same as those of ADL when the shift from b_0 to b_s is used, we have

$$G_0(q,\gamma,w) \simeq G_c(q,\gamma)e^{iqw\mu(b_c)}\,, \tag{22}$$

where G_c is the central only result of ADL

$$G_c(q,\gamma) = b_0{}^2\frac{\alpha^{1/3}}{\sqrt{3}(qb_0)^{4/3}} \times\exp[i\tfrac{5}{6}\pi - \gamma\tilde{t} + \tfrac{3}{2}(\alpha^2 qb_0)^{1/3}e^{i\pi/6}]e^{iqb_0}\,, \tag{23}$$

with

$$\gamma\tilde{t} \simeq 1.46\rho_0(2\pi\beta b_0)^{1/2}\,.$$

We may summarize the approximations made in obtaining our solution by describing the hierarchy of lengths involved. The dominant length characterizing the nuclear geometry is $b_0 = c + i\pi\beta$, the nearest pole of the Fermi distribution. For the spinless case we may ignore the other poles when the stationary phase point (b_c) is much closer to b_0 than the pole separation ($2\pi\beta$), that is, $b_c - b_0 = \frac{1}{2}\Delta b_0 \ll 2\pi\beta$. When including spin we may treat the additional shift in the stationary phase point as a perturbation of the spinless solution when $w\mu(b_0) \ll \frac{1}{2}\Delta b_0$. In addition, to treat the δ dependence of μ perturbatively, we need $\delta \ll \frac{1}{3}\Delta b_0$, but in practice this combination is redundant since $w > \delta$ and we already have $w \ll \frac{1}{2}\Delta b_0$. Summarizing these conditions we have

$$\left|\frac{w}{b_0}\mu(b_0)\right| \ll \tfrac{1}{2}|\Delta| \ll \left|\frac{2\pi\beta}{b_0}\right|.$$

For 800 MeV p-^{208}Pb scattering applications at $qc = 4\pi$, this relation yields $0.05 \ll 0.15 \ll 1$. The left (right) inequality is worsened (improved) by increasing q.

It is convenient at this stage to introduce spin channel amplitudes

$$F_\pm = F_1 \pm F_2\,, \tag{24}$$

with

$$F_\pm = -ik[G_c(q,\gamma)e^{\mp iqw\mu} + G_c^*(q,\gamma^*)e^{\mp iqw\mu^*}]\,. \tag{25}$$

From (23) we note that the asymptotic q dependence is given by the simple phase factor $e^{iqb_0} = e^{-\pi\beta q}e^{iqc}$. For $\gamma = \gamma^*$ in (25) this gives the oscillating and exponentially falling cross sections characteristic of diffractive scattering.

We have achieved our goal of obtaining closed form expressions for the spin channel amplitudes by reducing the spin dependence to a trivial geometric shift of the previously solved central case. Since the thickness function dependent part of the phase is unchanged when evaluated at the stationary point even when its argument is shifted, the the final result can be written in terms of the ADL G function. This function appears in an analysis of inelastic processes as well and is important in deriving the data-to-data relations appropriate to those processes. In the present case we see that the spin amplitudes are controlled in a simple way by four complex parameters which describe (1) the nuclear geometry (b_0), (2) the central nucleon-nucleon strength (γ), (3) the relative nucleon-nucleon spin-orbit to central strength (w), and (4) the central/spin-orbit geometry difference (δ). The first two of these are familiar from the spinless case and the second two are the new dynamical parameters relevant to spin dependent scattering.

Analogous to the inelastic case we considered in Ref. 3, the closed form expressions will allow simple relations between the spin dependent observables and the elastic cross section to be uncovered. The spin-dependent observables are the polarization (or analyzing power)

$$P = \frac{2\,\mathrm{Re}F_1F_2^*}{\sigma} = \tfrac{1}{2}(|F_+|^2 - |F_-|^2)/\sigma \tag{26}$$

and the spin rotation function[8]

$$Q = \frac{2\,\mathrm{Im}F_1F_2^*}{\sigma} = \tfrac{1}{2}(F_+^*F_- - F_-^*F_+)/\sigma\,, \tag{27}$$

where

$$\sigma = |F_1|^2 + |F_2|^2 = \tfrac{1}{2}(|F_+|^2 + |F_-|^2) \tag{28}$$

is the unpolarized cross section. For convenience we define the following quantities:

$$\begin{aligned} u &= \mathrm{Re}(w)\,, \\ v &= \mathrm{Im}(w)\,, \\ x &= \mathrm{Re}[\mu(b_c)]\,, \\ y &= \mathrm{Im}[\mu(b_c)]\,, \\ C &= \tfrac{1}{2}[G_c(q,\gamma) + G_c^*(q,\gamma^*)]\,, \\ S &= \frac{1}{2i}[G_c(q,\gamma) - G_c^*(q,\gamma^*)]\,. \end{aligned} \tag{29}$$

From the dominance in G_c of the phase factor e^{iqb_0}, these last two quantities are recognized as cosine- and sine-like in behavior, respectively (modulated by an exponentially falling envelope).

Note also that for $|\delta/b_0| \ll 1$, $x \sim 1+O(\delta/b_0)$ while $y \sim O(\delta/b_0)$.

To first order in w the cross section is simply

$$\sigma = 4k^2 C^* C \tag{30}$$

and to the same order the polarization and spin rotation function are, respectively,

$$P = 2qvx - 2qvy\,\frac{\mathrm{Re}C^*S}{C^*C} + 2quy\,\frac{\mathrm{Im}CS^*}{C^*C}\,,$$
$$Q = 2qux - 2quy\,\frac{\mathrm{Re}C^*S}{C^*C} - 2qvy\,\frac{\mathrm{Im}CS^*}{C^*C}\,. \tag{31}$$

We see that P and Q are essentially identical in structure but with the roles of u and v interchanged. Note that these linearized expressions are not properly bounded, i.e., they do not in general satisfy $P^2+Q^2 \leq 1$, since they are only valid to first order in qw and μ. The properly normalized expressions based on (25) are provided in the Appendix.

The dependence of P or Q on momentum transfer q is carried in the overall kinematic factor q and in the nuclear structure quantities C and S. The first term has only the linear kinematic q dependence. Since $x \to 1$ and $y \to 0$ as $\delta \to 0$, only this linear term survives when the spin-orbit and central geometries are identical. No nuclear structure terms are present. This is true to higher orders in w as well in the analytic treatment. The spin observables computed from (25) take a particularly convenient and illuminating form when $\delta = 0$ (see Appendix).

$$P + iQ = \tanh 2qv + i \sin 2qu/\cosh 2qv\,. \tag{32}$$

Without a geometric shape difference between the spin-orbit and central densities, there are no oscillations in either P or Q, and no nuclear structure dependence. Furthermore, we see that asymptotically $P \to 1$ and $Q \to 0$ while $P^2+Q^2 \leq 1$ throughout. We will return to this point later.

The second term in P or Q of (31) involves $\mathrm{Re}C^*S/C^*C$. Recalling that C and S are, respectively, cosine- and sine-like, $\mathrm{Re}C^*S/C^*C$ is tangent-like in q. It is these tangent-like oscillations that are so characteristic of medium energy polarizations and have been the focus of much phenomenological study. We stress again that the magnitude of the oscillations in P or Q is governed by y, which depends on a central and spin-orbit geometry difference.

The numerator of the last term in (31) depends on $\mathrm{Im}CS^*$ which is proportional to $|G_c(q,\gamma)|^2 - |G_c(q,\gamma^*)|^2 \sim O(r)$. This combination does not oscillate since it is simply the difference of magnitude with no interference term. It is dependent on the imaginary part of γ [real part of the forward nucleon-nucleon amplitude; see Eq. (5)] which is responsible for minima filling in elastic scattering. Because of the C^*C term in the denominator of the last term in P or Q, the ratio will oscillate, but simply like the inverse of the elastic scattering cross section.

It is also interesting to note that Eq. (25) admits different minimum filling behavior in the spin channel cross sections. To lowest order in w and the fundamental forward real to imaginary ratio r the spin channel minimum to maximum envelope ratio is given by

$$\left(\frac{\sigma_{\min}}{\sigma_{\max}}\right)_{\pm} = \frac{\eta_s^2}{1+\eta_c^2}\left\{1 \mp \frac{2qy}{\eta_s^2(1+\eta_c^2)}\left[u(1+\eta_c^2-\eta_s^2) + v\eta_c(1+\eta_c^2+\eta_s^2)\right]\right\}, \tag{33}$$

where $\eta_s = 2r(\alpha^2 q B_0)^{1/2}\sin(\phi/3+\pi/6)$, η_c is the same expression but with cosine replacing the sine, and B_0 and ϕ are the magnitude and phase of b_0. The factor outside the curly brackets of (33) is just the unpolarized cross section minimum to maximum ratio. The term in square brackets provides for different minimum filling of the spin channels. This difference is also proportional to y and thus to the spin-orbit/central geometry difference. Other properties of the spinless closed form scattering amplitude and further details including a treatment of Coulomb corrections are provided in ADL.

We have seen how the difference in the central and spin-orbit geometries plays a crucial role in providing structure in the spin-dependent observables. Without this difference P and Q are smooth functions independent of the nuclear structure. This point has been previously noted, at least implicitly, in the eikonal treatment of Auger and Lombard.[6] It is not difficult to prove in the high energy limit of potential scattering that, to first order in w, the nucleon-nucleus spin flip amplitude is proportional to the central amplitude, and hence provides no oscillations to P or Q. Since both P and Q must be odd in w (the sign of the nucleon-nucleus polarization must reverse if the spin is reversed), corrections to the linear form must be of order w^3. To this order there may be oscillations in P or Q even with $\delta = 0$, but as Glauber and Osland point out, the oscillations are too small to account for those observed in polarization. This is clearly seen in Fig. 1(a), which shows the polarization of 800 MeV protons on ^{208}Pb with $\delta = 0$ for the full eikonal amplitude (2) evaluated numerically and the analytic result (25). For all the figures of this paper the parameters were chosen as follows. The nuclear geometry parameters were taken from electron scattering[9]: $c = 6.60$ fm and $\beta = 0.63$ fm. The strength parameter $\gamma = 2.10\,(1 + i0.18)$ fm^2 and spin-length parameter $w = 0.16 + i0.2$ fm for ^{208}Pb were determined by suitably averaging the nucleon-nucleon phase shifts of Arndt.[10] These parameters remain fixed throughout the paper. We see in Fig. 1(a) that the analytic result tracks the full

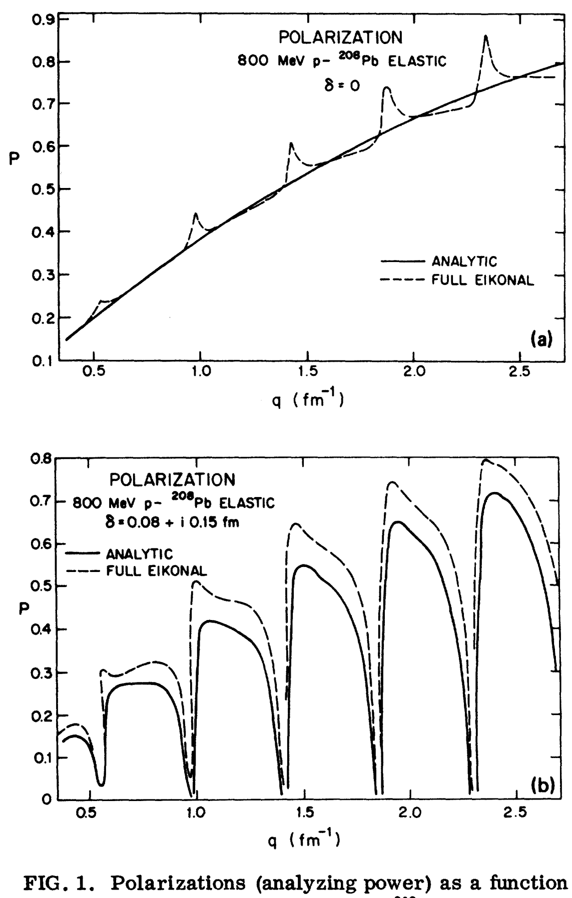

FIG. 1. Polarizations (analyzing power) as a function of momentum transfer for 800 MeV p-^{208}Pb scattering computed from the eikonal amplitude, Eq. (2), and the analytic expression, Eq. (25), are compared. The spin-orbit strength parameter $w = 0.16 + i0.2$ fm was taken from a phase shift analysis of Arndt and is used throughout; the remaining scattering parameters are provided in the text. In (a) the spin-orbit and central geometries are identical, $\delta = 0$. The absence of structure is evident. In (b) the spin-orbit and central complex radii differ by $\delta = 0.08 + i0.15$ fm (see text). Note the dramatic difference in the magnitude of the oscillations between (a) and (b).

eikonal result quite well. The very small oscillations in the full eikonal polarization are due to w^3 contributions and are clearly much too small compared with the observed oscillations. From finite range effects we expect $|\delta| \sim 0.1$ fm. In Fig. 1(b) we compare the polarization calculated with the same two equations, (2) and (25), but with $\delta = 0.08 + i0.15$ fm, a value fit in Sec. III. The oscillations grow dramatically to the observed magnitude. We also see that the analytic result reproduces the full eikonal numerical calculation fairly well.

III. DATA-TO-DATA RELATIONS

In Ref. 3 we showed that once closed forms are obtained for inelastic as well as elastic processes, it is possible to relate the two cross sections directly, thus stepping over much of the intermediate theoretical framework. One uses the elastic scattering distributions as input, exploits the dominance of nuclear geometry, and obtains directly an expression for the inelastic cross section. We show here that similar data-to-data relations can be obtained for the spin observables. This is not new to polarization phenomenology; the familiar log derivative relationship between the cross section and polarization is probably the first such formula. Our goal is to relate the structure of each of the three terms of Eq. (31) to the elastic cross section and momentum transfer in an explicit way. The coefficients of these three terms depend on w and δ. These two complex lengths represent the additional dynamical content of P and Q over the elastic scattering.

The first term in (31) is the trivial rise in q which is proportional to v and x in the case of P, and u and x in the case of Q. There is no added nuclear structure contained in this term.

The second term is proportional to the tangent-like factor $\mathrm{Re}C^*S/C^*C$. Using the trigonometric identity

$$\tan x = \frac{1}{2}\,\frac{\cos^2(x+\pi/4) - \cos^2(x-\pi/4)}{\cos^2 x} \tag{34}$$

and noting that for $\gamma = \gamma^*$, σ varies like $e^{-2\pi\beta q}\cos^2 qc$ by (30), we may write

$$\frac{\mathrm{Re}C^*S}{C^*C} \simeq \frac{1}{2}\,\frac{F(\delta q)\sigma(q+\delta q) - F(-\delta q)\sigma(q-\delta q)}{\sigma(q)}, \tag{35}$$

where $\delta q = \pi/4c$ and $F(\delta q) = e^{2\pi\beta\delta q}$ reflects the scale change necessary to compensate for the δq shift due to the exponential envelope. Thus to evaluate $\mathrm{Re}C^*S/C^*C$ we need only the elastic cross section and the nuclear shape parameters c and β (b_0).

The last term of (31) is proportional to the real part of the forward amplitude γ which is responsible for filling minima. Using (23) and (30) and calculating to first order in γ, we find

$$\mathrm{Im}CS^* \simeq \tfrac{1}{2}\eta_s |G_c(q,\gamma)|^2, \tag{36}$$

where only the asymptotic qc term is retained. Using (30) again with $\gamma = \gamma^*$, we have

$$\frac{\mathrm{Im}CS^*}{C^*C} \simeq \tfrac{1}{2}\eta_s/\cos^2(qc). \tag{37}$$

We recognize the $\cos^2 qc$ term which is easily extracted from the cross section. Using the exponential envelope dependence, we have

$$\frac{1}{\cos^2 qc} \simeq \sigma_> \left(\frac{\sigma_<}{\sigma_>}\right)^{(q-q_<)/(q_>-q_<)} \frac{1}{\sigma(q)}, \tag{38}$$

where $\sigma_>$ and $\sigma_<$ are the two nearest cross section maxima (which determine the scale), and the exponential q dependence follows from $\sigma(q) \sim e^{-2\pi\beta q}$. The γ-dependent coefficient can be extracted from the minimum filling. Again using (30) to first order in γ and retaining only the asymptotically dominant term, we compute the ratio of cross section minima to maxima. ($\sigma_>$ or $\sigma_<$ refer to the maximum at the larger or smaller momentum transfer, respectively.) Since neighboring minima and maxima are separated by $\pi/2c$, we find

$$\sigma_{\min}/\sigma_{\gtrless} \simeq \eta_s{}^2 e^{\pm\pi^2\beta/c}. \tag{39}$$

Solving for η_s and combining with (37) and (38) gives

$$\frac{\mathrm{Im}CS^*}{C^*C} \simeq \frac{(\mathrm{sign}\,\gamma)}{2}\left(\frac{\sigma_{\min}}{\sigma_{\gtrless}}\right)^{1/2} e^{\mp\pi^2\beta/2c} \times \left(\frac{\sigma_<}{\sigma_>}\right)^{(q-q_<)/(q_>-q_<)} \frac{\sigma_>}{\sigma(q)}. \tag{40}$$

Besides the sign of γ, the only residual model dependence in this expression is in the complex nuclear radius parameter b_0. Combining Eq. (31) [or (A5) and (A6) for proper normalization] with (35) and (40) gives data-to-data formulas for the polarization or spin rotation function in terms of just the complex lengths w and δ, the nuclear radius b_0, and the elastic distribution. The data-to-data formulas are consolidated and summarized in the Appendix. In Fig. 2 we compare the analytic expression for the polarization with the normalized data-to-data formulas and the experimental data of elastically scattered 800 MeV protons on ^{208}Pb.[11] The agreement is remarkable. The spin-orbit central geometric difference is adjusted to $\delta = 0.08 + i0.15$ fm. The analytic form, like the full eikonal result of Fig. 1(b), has minima that are too deep. It would also lead to overly deep minima for the elastic scattering. Rather than try to adjust γ to fill these minima we need only use the data-to-data form which by construction uses the correct elastic scattering, and thus agrees with the polarization data.

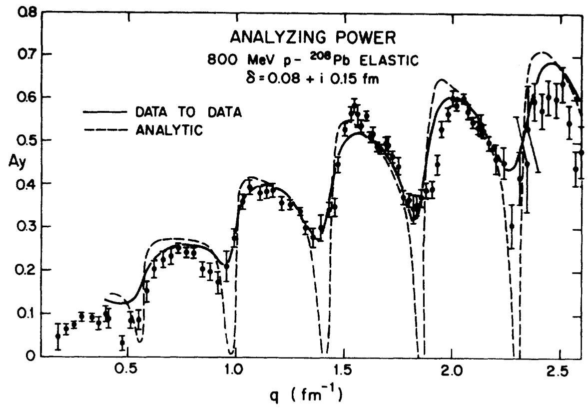

FIG. 2. The analyzing powers as a function of momentum transfer for 800 MeV p-^{208}Pb scattering computed from the closed form analytic amplitudes, Eq. (25), and the data-to-data expressions, Eqs. (A5), (35), and (40), are compared with experiment (Ref. 11). The elastic scattering data of Hoffman *et al.* was (Ref. 11) used as input for the data-to-data expression. Again $w = 0.16 + i0.2$ fm and δ was adjusted to the value $0.08 + i0.15$ fm. The data-to-data form agrees well with the data, automatically filling the deep minima present in the analytic result.

In Fig. 3 we show the spin rotation function Q calculated using the full eikonal, Eq. (2), the analytic, Eq. (25), and data-to-data formulas, Eqs. (A6), (35), and (40). This figure is a prediction, as there are no measurements of this observable yet.

The relationship between P and Q is clearly

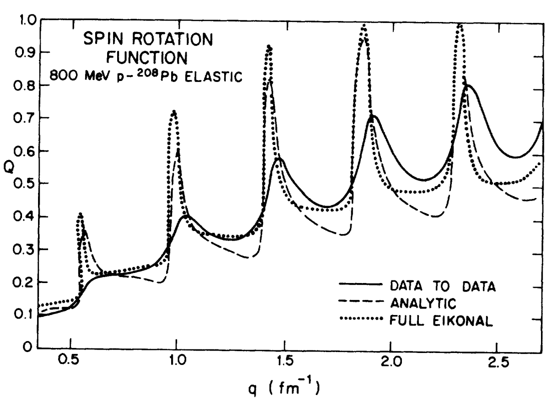

FIG. 3. The spin-rotation function Q for 800 MeV p-^{208}Pb elastic scattering as a function of momentum transfer is predicted using the data-to-data expressions, Eqs. (A6), (35), and (40), closed form (25), and eikonal amplitudes (2). Again the elastic scattering data of Hoffman *et al.* (Ref. 11) was used as input for the data-to-data expressions. The other scattering parameters are the same as in Fig. 2.

illustrated in Figs. 2 and 3. Consider the three terms of Eq. (31) and how each contributes to the polarization in Fig. 2. The linear term is immediately seen to be positive since the polarization rises with q. Since $x \simeq 1 > 0$, the slope of this rise fixes the sign and approximate magnitude of v. The polarization data appears to have a negative tangent-like component, which by (31) and the previous analysis fixes the sign of y as positive. The $1/\sigma$ term enters negatively and since $\text{Im}CS^* \propto r$, the sign of ur is fixed as negative. If we choose to take the nuclear constituent weighted fundamental phase shift value for r ($r < 0$ for 800 MeV p-^{208}Pb applications), the sign of u is thereby fixed as positive. These assignments agree with Arndt's independent nucleon-nucleon phase shift analysis. The signs and approximate magnitudes of all the relevant parameters are determined by the polarization data alone (modulus the sign of r) enabling a prediction to be made of Q. In particular the gross features are given solely by the signs just determined. The spin rotation function for 800 MeV p-^{208}Pb will rise with q, with negative tangent-like and positive $1/\sigma$ oscillations. These features are clearly seen in Fig. 3.

IV. DISCUSSION

It has long been known that the structure seen in the polarization of protons scattered from nuclei in the diffractive region is closely related to the oscillations seen in the scattering cross section. In this paper we show how such a relationship arises from the nuclear geometry which dominates the diffractive structure. Starting from an eikonal formulation of the spin-dependent amplitudes, we derive accurate closed form approximations, Eq. (25), using the analytic methods of ADL. Beside the geometric and dynamical variables which enter a discussion of spinless scattering, the spin-dependent amplitudes depend on only two new dynamical quantities—the complex lengths w and δ. The first, w, is the ratio of central to spin-orbit scattering in the fundamental nucleon-nucleon amplitude. The other, δ, is a measure of the range and diffusivity difference of the central and spin-orbit nucleon-nucleus potentials. One immediate result of our closed form expression is that to first order in the spin-orbit strength w there are no oscillations in the polarization when $\delta = 0$. Furthermore, higher order terms either normalize the expression as in Eq. (32) or are too small to account for the observed oscillations in polarization. Of the origins of the central/spin-orbit geometric difference δ we are uncertain. The different ranges of the fundamental central and spin-orbit forces upon folding over the nuclear density will induce an effective shift in the central and spin-orbit densities of approximately the correct size needed to explain the polarization structure, but more study is needed before we can conclude that δ arises solely from such fundamental differences.

Perhaps the most important use of the closed form spin-dependent amplitudes is to display explicitly the close relationship between the unpolarized cross section and the spin observables, P and Q, using only w and δ as the spin-dynamic input. This relationship becomes manifest in the data-to-data expressions for the spin observables in terms of the cross section and these parameters.

We find that P and Q each contain three terms. The first two are familiar to spin dynamic phenomenology. The first is the simple linear rise related to the symmetry condition that polarizations vanish like q as $q \to 0$. Only this term survives if $\delta = 0$. The second term is the tangent-like term familiar to polarization phenomenologists as the log derivative of the cross section. Indeed, since from ADL we know that the oscillations in the cross section come from $\sigma \sim \cos^2 qc$, we have $d\ln\sigma/dq \sim -\tan qc$, which recovers the old phenomenology. We use a related trigonometric identity to derive the relationship used in the data-to-data form, Eq. (35).

The last term in P or Q is new to spin phenomenology. It is proportional to the reciprocal of the cross section $\sim 1/\sigma$, and thus it oscillates but does not change sign as the tangent term does. Furthermore, it is proportional to r, the forward real to imaginary ratio, and will thus be relatively more important in energy regions where r is large. The structure of P and Q are symmetric (modulus signs) in the three terms with the roles of Re(w) and Im(w) reversed. In principle, one can determine both the real and imaginary parts of w from P alone (modulus the sign of r) and thereby predict Q. This is, in practice, difficult since the sensitivity of P to the real part of w is weak. On the other hand, the double scattering experiments needed to determine Q are technically difficult. In any event the simple relationship of P and Q manifested in (31) should aid in the planning and analysis of experiments to determine Q both by giving an indication of the structure to be expected and by showing what few quantities determine that structure.

We plan to extend these ideas to polarization in inelastic scattering as well. Extensions to lower energies will require some caution. In deriving the closed form expressions here, a simple perturbation of the earlier ADL result was used. For 800 MeV applications, as we have seen, this is perfectly satisfactory. For lower energy applications where the spin strength w grows and in particular the forward real to imaginary ratio approaches unity, the solution given here will presumably fail. At 180 MeV, for example, $w=-0.27+i0.41$ fm and $r\simeq 1$.[10] This is not to say that the analytic methods will not work in this region, only that the algebraic solution to the stationary phase condition is no longer given by a trivial perturbation of the ADL result; a more careful approach is necessary.

In summary we have obtained closed form expressions for the spin dependent p-nucleus scattering amplitudes at intermediate energies. These forms can be cast into data-to-data relationships for the spin observables, that is, into relations for these observables directly in terms of the elastic scattering and two fundamental lengths, w and δ. These forms fit the data well and illuminate the dynamical origin of its rich structure.

ACKNOWLEDGMENTS

We are grateful to J.-P. Dedonder for calling our attention to Ref. 1 and for interesting correspondence. We thank R. Arndt for providing his nucleon-nucleon analysis results. This work was supported in part by a grant from the National Science Foundation.

APPENDIX

For reasons of space economy the properly normalized equations for P, Q, and σ computed from (25) along with the data-to-data expressions for the nuclear structure factors $\mathrm{Re}CS^*/C^*C$ and $\mathrm{Im}CS^*/C^*C$ are presented in this appendix.

First we write (25) as

$$F_\pm = -2ike^{\mp iqwx}(C\cosh qwy \pm iS\sinh qwy). \qquad (A1)$$

The spin channel cross sections are given by

$$\sigma_\pm = |F_\pm|^2 = 4k^2 e^{\pm 2qxv} C^*C|\cosh qyw|^2 \xi_\pm, \qquad (A2)$$

with

$$\xi_\pm = 1 + \frac{S^*S}{C^*C}\frac{|\sinh qyw|^2}{|\cosh qyw|^2} \mp \frac{\mathrm{Re}C^*S}{C^*C}\frac{\sin 2qvy}{|\cosh qyw|^2} \pm \frac{\mathrm{Im}CS^*}{C^*C}\frac{\sinh 2quy}{|\cosh qyw|^2},$$

$$\xi_\pm \xrightarrow[\delta\to 0]{} 1. \qquad (A3)$$

The cross section and polarization are then given by

$$\sigma = \tfrac{1}{2}(|F_+|^2 + |F_-|^2), \qquad (A4)$$

$$P = \frac{e^{2qxv}\xi_+ - e^{-2qxv}\xi_-}{e^{2qxv}\xi_+ + e^{-2qxv}\xi_-} \xrightarrow[\delta\to 0]{} \tanh 2qv. \qquad (A5)$$

The spin rotation is given by

$$Q = \tfrac{1}{2}(F_+^*F_- - F_+F_-^*)/\sigma$$

$$= \left[\sin 2qxu\left(1 - \frac{S^*S}{C^*C}\frac{|\sinh qyw|^2}{|\cosh qyw|^2}\right) - \frac{\mathrm{Re}CS^*}{C^*C}\frac{\cos qxu \sinh 2qyu}{|\cosh qyw|^2} - \frac{\mathrm{Im}CS^*}{C^*C}\frac{\cos qxu \sin 2qyv}{|\cosh qyw|^2}\right]\frac{2}{e^{2qxv}\xi_+ + e^{-2qxv}\xi_-},$$

$$Q \xrightarrow[\delta\to 0]{} \frac{\sin 2qu}{\cosh 2qv}. \qquad (A6)$$

In the identical spin-orbit and central geometry limit we have

$$P + iQ \to \tanh 2qv + i\frac{\sin 2qu}{\cosh 2qv}. \qquad (A7)$$

These expressions for P and Q are properly normalized. Since P and Q measure phase differences, they obey the inequality

$$P^2 + Q^2 \le 1. \qquad (A8)$$

The nuclear structure information of P and Q is contained in the factors $\mathrm{Re}C^*S/C^*C$ and $\mathrm{Im}CS^*/C^*C$. For completeness we summarize here the formulas for computing these factors directly from the elastic cross section data. Using these expressions in (A3)–(A5) and (A6) gives the data-to-data relations

$$\frac{\mathrm{Re}C^*S}{C^*C} \simeq \tfrac{1}{2}[e^{-\pi^2\beta/2c}\sigma(q-\pi/4c) - e^{\pi^2\beta/2c}\sigma(q+\pi/4c)]\frac{1}{\sigma(q)}, \qquad (A9)$$

$$\frac{\mathrm{Im}CS^*}{C^*C} = \frac{(\mathrm{sign}\,r)}{2}\left(\frac{\sigma_{\min}}{\sigma_\gtrless}\right)^{1/2} e^{\pm\pi^2\beta/2c}\left(\frac{\sigma_<}{\sigma_>}\right)^{(q-q_<)/(q_>-q_<)}\frac{\sigma_>}{\sigma(q)}, \qquad (A10)$$

where $\sigma_{\min}$ is the nearest (in q) cross section minimum, $\sigma_>$ ($\sigma_<$) are the nearest cross section maxima located at the larger ($q_>$) and smaller ($q_<$) values of q, and $\sigma(q)$ is the elastic cross section. In both (A9) and (A10), c is the half density radius and β the diffusivity.

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