Reduced $M1$, $E1$, $E2$, and $E3$ transition probabilities for transitions in $^{156-160}$Gd and $^{160-164}$Dy

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Direct $E2$ and $E3$ Coulomb excitation of 2^+ and 3^- states with 13.5-MeV ^{4}He ions on isotopically enriched targets of $^{156-160}$Gd and $^{160-164}$Dy has been measured by means of γ-ray spectroscopy. Vibrational-like 2^+ and 3^- states were identified in each nucleus. The $B(E\lambda, 0 \rightarrow J = \lambda)$ is obtained for excitation of each state, and information is given on the reduced transition probabilities for the different decay modes of these states. The experimental results are compared with theoretical predictions of nuclear models describing these states. A reasonable account of the reduced $E2$ transition probabilities is provided by the interacting boson model.

[NUCLEAR REACTIONS $^{156-160}$Gd$(\alpha,\alpha'\gamma)$, $^{160-164}$Dy$(\alpha,\alpha'\gamma)$, $E=13.5$ MeV; measured I_γ, E_γ, $\gamma(\theta)$, Doppler broadening. $^{156-160}$Gd, $^{160-164}$Dy levels deduced $B(E2)$, $B(M1)$, $B(E3)$, $B(E1)$, $T_{1/2}(3^-)$, J^π, δ. Enriched targets.]

I. INTRODUCTION

In contrast to the generally systematic behavior of the $K=2$ vibrational states in the rare-earth nuclei, the $K=0$ vibrational states are highly variable. Only a few cases are known which exhibit appreciable $E2$ strength. The usual situation is that one simply cannot find $K=0$ states with appreciable $E2$ strength. One of the motivations for the present investigation was the hope of locating more $B(E2)$ strength associated with other higher lying 2^+ states. We have identified additional Coulomb excitation of 2^+ states, but the $B(E2)$ values are all an order of magnitude smaller than the $B(E2, 0 \rightarrow 2)$ for exciting the 2^+ member of the $K=2$ vibrational band.

In earlier investigations,[1,2] the inelastic scattering of 12-MeV deuterons from the even-A Gd and Dy isotopes was used to identify collective vibrational states. As a rule, several collective octupole states, collective γ vibrational-like states, and in some cases β vibrational-like states have been identified in each nucleus. The $B(E2)$ and $B(E3)$ values were deduced from the inelastic cross sections measured at 90° and 125° which were assumed to be proportional to the transition probabilities. Domingos *et al.*[3] have observed the 2^+ and 4^+ members of the γ-vibrational band in ^{162}Dy by multiple Coulomb excitation with 60-MeV ^{16}O ions using the backscattered particle-gamma coincidence method. Also by this method, Oehlberg *et al.*[4] obtained the Coulomb excitation probabilities for exciting the 2^+, 4^+, and 6^+ members of the γ-vibrational band in $^{160-164}$Dy with 57.5- to 60-MeV ^{16}O ions. They also observed excitation of several octupole states in each nucleus. With 13.5- to 15.0-MeV ^{4}He ions, the $B(E2)$ for excitation of the $K, J^\pi = 2, 2^+$ state in each nucleus was determined by the backscattered particle-gamma coincidence method. Precision Coulomb-excitation measurements via the (α, α') reaction have been performed to determine the $B(E2)$ values for the $K, J^\pi = 2, 2^+$ state in ^{158,160}Gd and ^{162,164}Dy by Baktash *et al.*[5] and the $B(E\lambda)$ values for vibrational-like states in $^{156-160}$Gd by Ronningen *et al.*[6] The Coulomb excitation reaction induced by ^{4}He ions selectively excites 2^+ and 3^- states by direct $E2$ and $E3$ Coulomb excitation. In this paper, we present results from γ-ray spectroscopy with 13.5-MeV ^{4}He ions on isotopically enriched targets of $^{156-160}$Gd and $^{160-164}$Dy.

II. EXPERIMENTAL METHOD

Gamma-ray spectra were observed at $\theta_\gamma = 0°$, 55°, and 90° with respect to the beam direction with a 91-cm^3 Ge(Li) detector [energy resolution full width at half maximum (FWHM) = 2.15 keV at an energy of 1173 keV] at 10 cm from the target. The metallic targets (93.58% ^{156}Gd, 92% ^{158}Gd, 97.86% ^{160}Gd, 78.93% ^{160}Dy, 96.26% ^{162}Dy, and 83.23% ^{164}Dy) were between 42 and 49 mg/cm^2 thick. An example of a section from a 8192-channel γ-ray spectrum for ^{156}Gd is shown in Fig. 1. Above each peak are given the energy of the γ ray in keV and the transition assignment in ^{156}Gd. The assignment of the J values is based on γ-ray angular distribution measurements. Many of the contaminant γ rays are identified in Fig. 1. Those due to the ^{56}Fe$(\alpha, n)^{59}$Ni result from target impurity which undoubtedly was introduced by the rolling technique to produce the foils. Other light element target impurities and the reactions contributing contaminant γ rays to the spectra are ^{14}N(α, p), ^{16}O(α, α'), ^{16}O(α, p), ^{16}O(α, pn), ^{18}O(α, n), ^{19}F(α, α'), ^{19}F(α, n), ^{19}F(α, p), ^{27}Al(α, α'),

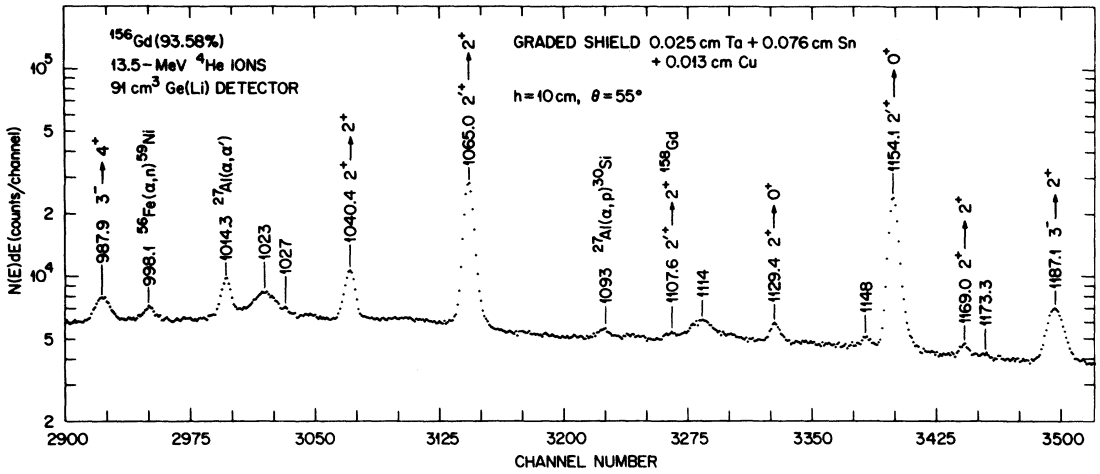

FIG. 1. A section from a 8192-channel pulse-height spectrum of the γ rays from ^{156}Gd bombarded with 13.5-MeV ^{4}He ions.

^{27}Al(α,n), ^{27}Al(α,p), ^{28}Si(α,α'), ^{28}Si(α,p), ^{35}Cl(α,n), and ^{35}Cl(α,p).

A summary of the γ-ray yields from the decay of states in $^{156-160}$Gd and $^{160-164}$Dy is presented in Table I. The last column also gives the anistropies $R \equiv W(0°)/W(90°)$ from the γ-ray angular distributions. For the conditions of these experiments, the use of first-order treatment of the Coulomb excitation process is adequate for the analysis of thick-target γ-ray yields. The procedure needed to extract $B(E\lambda)$ values from thick-target γ-ray yields has been described previously.[7,8] The 2^+ states and 3^- states are observed to be directly populated by $E2$ and $E3$ Coulomb excitation, respectively. The remaining states are excited by multiple Coulomb excitation or are populated by the γ-ray decay of the directly excited states. For instance, the yield of the 4^+ state at 1148 keV in ^{160}Gd is due to the excitation processes $E2E2$ via the 2^+ state at 75.3 keV and $E2E2$ via the 2^+ state at 988.2 keV. On the other hand, the yield of the 3^+ state at 828.2 keV in ^{164}Dy is due mostly to the γ-ray decay of the 3^- state.

In Fig. 1 there is obvious Doppler broadening of the 987.9- and 1187.1-keV γ rays from decay of the 3^- state at 1276.1 keV during the slowing down of the recoiling nucleus in the target. Actually, the 1065.0- and 1154.1-keV γ rays from decay of the 2^+ γ-vibrational state contain some Doppler broadening. The maximum kinetic energy imparted to a ^{156}Gd nucleus is 1.25 MeV which corresponds to $v/c = 0.41\%$. The stopping power $S(E)$ for these recoil velocities is mostly nuclear stopping power. The DBLS (Doppler broadened line shape) analysis depends on many quantities besides the lifetime of the γ-emitting state, viz., the incident projectile mass and energy, the mass of the target nucleus, the γ-ray energy, the angular distribution of the excited recoiling nuclei, the angular distribution of the γ rays emitted by the recoiling nuclei, and the size, energy resolution, and location of the Ge(Li) detector. Fortunately, the theory of Coulomb excitation tells us the angular distribution of the recoiling nuclei and the variation of the cross section with decreasing energy of the incident projectile.

The central problem in the DBLS analysis is the time dependence of v which requires a detailed knowledge of $S(E)$ for the recoiling nuclei in the target. Also, at these low velocities a knowledge of the average cosine of the scattering angle resulting from multiple nuclear collisions (Blaugrund factor[9]) is required. We used nuclear stopping power $S_n(E)$ taken from the theory of Linhard *et al.*[10] in $S(E)$ for the DBLS analysis. This choice of stopping powers reproduced the known mean lifetime, deduced from $B(E2, 0 \rightarrow 2)$ for excitation of the $K, J^\pi = 2, 2^+$ state in ^{156}Gd and ^{158}Gd, viz., $\tau = (0.85 \pm 0.04)$ and (0.87 ± 0.04) ps, respectively. Omitting the Blaugrund factor produces a $\Delta\tau(2^+) \simeq 0.4$ ps in the extracted mean lifetime. Decreasing $S_n(E)$ by 20% produces a $\Delta\tau(2^+) = 0.2$ ps. Our choice of $S_n(E)$ is in marked contrast to the nuclear stopping power used in the DBLS analysis[11,12] of data where the initial recoil velocities are large, viz., nuclear stopping power $\simeq$10 to 100 $S_n(E)$. Apparently, there must be some physics of the slowing down process which is not fully understood in the DBLS analysis of data where the initial recoil velocities are large.

TABLE I. Summary of γ-ray yields from the decay of states in $^{156-160}$Gd and $^{160-164}$Dy. The column headed yield denotes the thick-target yield in excitations per nC (6.24×10^9 ions).

Nucleus	Initial state J^π	Initial state energy (keV)	Final state J^π	Final state energy (keV)	E_γ (keV)	Yield (exc per nC)	$W(0°)/W(90°)$
^{156}Gd	2^+	1129.4	4^+	288.2	841.3	13.3	1.16 ± 0.06
			2^+	89.0	1040.3	46.1	0.98 ± 0.03
			0^+	0.0	1129.4	10.7	1.21 ± 0.15
	2^+	1154.1	4^+	288.2	865.9	7.5	1.02 ± 0.13
			2^+	89.0	1065.1	234	0.859 ± 0.013
			0^+	0.0	1154.1	219	1.47 ± 0.03
	2^+	1258.0	4^+	288.2	969.8	13.1	1.04 ± 0.11
			2^+	89.0	1169.1	5.3	1.96 ± 0.29
			0^+	0.0	1258.0		
	3^-	1276.1	4^+	288.2	987.9	26.9	0.78 ± 0.06
			2^+	89.0	1187.1	53.7	0.77 ± 0.03
^{158}Gd	3^-	1041.6	4^+	261.4	780.2	27.3	0.91 ± 0.02
			2^+	79.5	962.1	63.1	0.70 ± 0.02
	2^+	1187.1	4^+	261.4	925.7	2.5	1.19 ± 0.24
			2^+	79.5	1107.6	183	0.816 ± 0.015
			0^+	0.0	1187.1	137	1.48 ± 0.03
	2^+	1259.8	4^+	261.4	998.4	11.2	1.17 ± 0.13
			2^+	79.5	1180.4	2.9	
			0^+	0.0	1259.8	9.3	
	3^-	1402.9	4^+	261.4	1141.5	3.8	
			2^+	79.5	1323.5		
	2^+	1517.4	4^+	261.4	1256.0		
			2^+	79.5	1438.0	5.9	0.55 ± 0.09
			0^+	0.0	1517.4	5.2	1.62 ± 0.20
^{160}Gd	2^+	988.2	4^+	248.2	740.0	5.6	1.23 ± 0.10
			2^+	75.3	912.9	300	0.909 ± 0.009
			0^+	0.0	988.2	264	1.44 ± 0.03
	3^-	1289.3	4^+	248.2	1041.1	24.1	1.07 ± 0.14
			2^+	75.3	1214.0	30.0	0.73 ± 0.04
	4^+	1148.0	4^+	248.2	899.8	2.6	
^{160}Dy	2^+	966.2	4^+	283.8	682.4	7.5	
			2^+	86.8	879.4	398	0.884 ± 0.009
			0^+	0.0	966.2	348	1.44 ± 0.03
	3^-	1286.7	4^+	283.8	1002.9	23.4	0.90 ± 0.06
			2^+	86.8	1199.9	40.9	0.70 ± 0.03
	2^+	1349.5	4^+	283.8	1065.7	10.4	1.13 ± 0.06
			2^+	86.8	1262.7	14.4	0.57 ± 0.14
			0^+	0.0	1349.5	14.0	1.34 ± 0.05
	3^-	1642	4^+	1156	486	6.7	0.92 ± 0.06
			3^+	1049	593	1.1	
			2^+	966	676	3.2	0.70 ± 0.13
	4	1156	4^+	283.8	872.2	4.6	
^{162}Dy	2^+	888.2	2^+	80.7	807.5	457	0.912 ± 0.009
			0^+	0.0	888.2	391	1.43 ± 0.04
	3^-	1210.2	3^+	963	247	4.4	1.67 ± 0.20
			2^+	888.2	322	6.4	0.70 ± 0.13
			4^+	265.7	944.5	12.3	0.77 ± 0.04
			2^+	80.7	1129.5	22.8	0.76 ± 0.02
	3^+	963	2^+	80.7	882.3	4.5	
	4^+	1061	4^+	265.7	795.3	7.6	
^{164}Dy	2^+	761.8	4^+	242.2	519.6	16.2	
			2^+	73.4	688.4	585	0.913 ± 0.009
			0^+	0.0	761.8	538	1.40 ± 0.02
	3^-	1039.3	3^+	828.2	211.1	21.2	1.34 ± 0.07
			2^+	761.8	277.5	22.5	0.76 ± 0.03
			4^+	242.4	769.9	3.5	
			2^+	73.4	965.9	2.6	

TABLE I. (*Continued*).

Nucleus	Initial state J^π	Initial state energy (keV)	Final state J^π	Final state energy (keV)	E_γ (keV)	Yield (exc per nC)	$W(0°)/W(90°)$
	3^+	828.2	2^+	73.4	754.8	13.4	
	4^+	916.0	4^+	242.4	673.6	9.0	

Figure 2 shows detailed comparison of the observed Doppler-broadened 1187.1-keV γ ray from the $3^- \rightarrow 2^+$ transition in ^{156}Gd with calculated shapes for different assumed values for the total mean lifetime of the Coulomb-excited 3^- state. Spectra for $\theta_\gamma = 90°$ are essential to establish the unshifted peak position. The mean lifetimes, obtained from a least square fit of the calculated line shapes to the experimental data for the $3^- \rightarrow 2^+$ and $3^- \rightarrow 4^+$ transitions of the 3^- states in ^{156,158,160}Gd and ^{160}Dy are (0.11 ± 0.03), (0.78 ± 0.21), (0.074 ± 0.020), and (0.32 ± 0.09) ps, respectively. The uncertainties include the statistical error of the data and uncertainties of the stopping power.

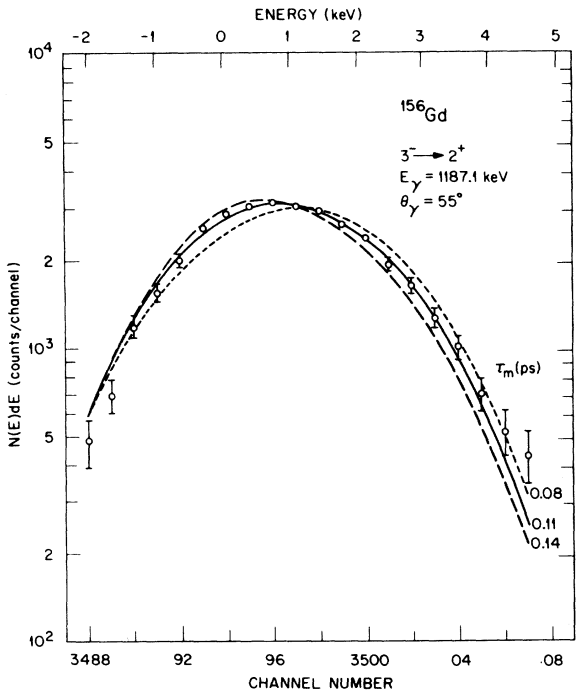

FIG. 2. Detailed comparison of the observed Doppler-broadened shape of the 1187.1-keV γ-ray peak from the $3^- \rightarrow 2^+$ transition in ^{156}Gd with calculated shapes for different assumed values of the total mean lifetime of the Coulomb-excited 3^- state.

III. EXPERIMENTAL RESULTS AND DISCUSSION

The experimental results for the reduced transition probabilities, $B(E\lambda, 0 \rightarrow J = \lambda)$, are summarized in Table II. The K, J^π assignments were deduced primarily from our $\gamma(\theta)$ measurements, ratios of reduced transition probabilities, and the results from other nuclear spectroscopy studies which have been summarized[13-17] for $A = 156$–164.

The experimental results for the reduced transition probabilities, $B(E1, J_i \rightarrow J_f)$, deduced from the mean lifetimes of the 3^- states, are summarized in Table III. In addition, we have extracted $B(E1)$ values for $E1$ transitions from the decay of the $K, J^\pi = 0, 2^+$ states at 1259.8 and 1517.4 keV to states of the $K = 0$ and 1 octupole bands in ^{158}Gd. For this purpose we used our measured $B(E2, 0 \rightarrow 2)$ values and the γ-ray branching data[18] from the (n, γ) reaction.

A. ^{156}Gd

The $B(E2, 0 \rightarrow 2)$ for the $K = 0$ β-vibrational-like state at 1129.4 keV is in fair agreement with the value $(1.3 \pm 0.4) \times 10^{-2}\ e^2b^2$ from the Coulomb excitation measurement by Ronningen *et al.*[6] The value obtained for $\delta = (E2/M1)^{1/2}$ from the angular distribution of the 1040.2-keV γ rays for the sequence $0(E2)2'(E2+M1)2$ is $\delta = 11^{+7}_{-2.5}$. The phase convention adopted for δ throughout this paper is that of Krane and Steffen,[19] viz., $\delta = \langle J_f || E2 || J_i \rangle / \langle J_f || M1 || J_i \rangle$. Our value of δ does not agree with the value $\delta = -(66^{+\infty}_{-42})$ by Collins *et al.*[20] The $B(M1, 2' \rightarrow 2)$ from the branching data is

$$B(M1, 2' \rightarrow 2) = (1.3 \pm 0.8) \times 10^{-4}\ \mu_N{}^2 .$$

and the mean lifetime $\tau_m(2') = 2.14 \pm 0.11$ ps.

The $B(E2, 0 \rightarrow 2)$ for the $K = 2$ γ-vibrational-like state at 1154.1 keV is in good agreement with the Coulomb excitation measurement,[6] viz., $(1.20 \pm 0.04) \times 10^{-1}\ e^2b^2$. The value obtained for δ from the angular distribution of the 1065.1-keV γ rays for the $2' \rightarrow 2$ transition is $-(20 \pm 4)$ and the $B(M1, 2' \rightarrow 2)$ from this branching data is

$$B(M1, 2' \rightarrow 2) = (7^{+4}_{-2.1}) \times 10^{-5}\ \mu_N{}^2 .$$

TABLE II. Experimental results for $B(E\lambda, 0 \rightarrow J = \lambda)$.

Nucleus	Level (keV)	K, J^π	$E\lambda$	$B(E\lambda, 0 \rightarrow J)$ (e^2b^λ)	$\frac{B(E\lambda)}{B(E\lambda)_{sp}}$ [a]	Theory $B(E\lambda, 0 \rightarrow J)$ (e^2b^λ)
^{156}Gd	1129.4	$0, 2^+$	$E2$	$(1.58 \pm 0.09) \times 10^{-2}$	0.63	2.0×10^{-2}
	1154.1	$2, 2^+$	$E2$	$(1.11 \pm 0.06) \times 10^{-1}$	4.46	1.43×10^{-1}
	1258.0	$0, 2^+$	$E2$	$(7.7 \pm 0.7) \times 10^{-3}$	0.31	2×10^{-3}
	1276.1	$1, 3^-$	$E3$	$(17.1 \pm 0.7) \times 10^{-2}$	16.9	13.5×10^{-2}
^{158}Gd	1041.6	$1, 3^-$	$E3$	$(12.4 \pm 0.7) \times 10^{-2}$	11.9	6.5×10^{-2}
	1187.1	$2, 2^+$	$E2$	$(8.48 \pm 0.49) \times 10^{-2}$	3.34	8.3×10^{-2}
	1259.8	$0, 2^+$	$E2$	$(8.01 \pm 0.56) \times 10^{-3}$	0.32	6.7×10^{-2}
	1402.9	$0, 3^-$	$E3$	$(2.28 \pm 0.26) \times 10^{-3}$	2.2	1.1×10^{-2}
	1517.4	$0, 2^+$	$E2$	$(9.33 \pm 0.93) \times 10^{-3}$	0.36	1×10^{-3}
^{160}Gd	988.2	$2, 2^+$	$E2$	$(8.82 \times 0.44) \times 10^{-2}$	3.42	10.3×10^{-2}
	1289.3	$1, 3^-$	$E3$	$(11.8 \pm 0.7) \times 10^{-2}$	11.0	10.3×10^{-2}
^{160}Dy	966.2	$2, 2^+$	$E2$	$(1.22 \pm 0.06) \times 10^{-1}$	4.73	
	1286.7	$1, 3^-$	$E3$	$(17.1 \pm 1.0) \times 10^{-2}$	16	11.4×10^{-2}
	1349.5	$0, 2^+$	$E2$	$(1.84 \pm 0.15) \times 10^{-2}$	0.71	
	1642	$2, 3^-$	$E3$	$(6.5 \pm 1.0) \times 10^{-2}$	6.1	0.80×10^{-2}
^{162}Dy	888.2	$2, 2^+$	$E2$	$(1.18 \pm 0.06) \times 10^{-1}$	4.50	
	1210.2	$2, 3^-$	$E3$	$(10.4 \pm 0.7) \times 10^{-2}$	9.6	7.9×10^{-2}
^{164}Dy	761.8	$2, 2^+$	$E2$	$(1.14 \pm 0.06) \times 10^{-1}$	4.26	
	1039.3	$2, 3^-$	$E3$	$(8.8 \pm 0.6) \times 10^{-2}$	7.9	5.9×10^{-2}

[a] $B(E\lambda)_{sp} = \frac{2\lambda+1}{4\pi}\left(\frac{3}{\lambda+3}\right)^2 (0.12A^{1/3})^{2\lambda} e^2b^\lambda$ for $J_i = 0$, $J_f = \lambda$.

For the excitation of the $K, J^\pi = 0, 2^+$ at 1258.0 keV, the $2' \rightarrow 0$ transition was too weak in our spectra to obtain a useful intensity for this decay branch. The branching data[13] from ^{156}Eu and ^{156}Tb decay and from the (n, γ) reaction was used to deduce the $B(E2, 0 \rightarrow 2)$ in Table II. The value of δ from an analysis of $\gamma(\theta)$ for the 1169.1-keV γ rays for the sequence $0(E2)2'(E2+M1)2$ is

$$0.34 < \delta < 1.07 .$$

This large spread in the experimental result for δ introduces large errors in the extracted values for $B(E2, 2' \rightarrow 2)$ (see Table VI) and $B(M1, 2' \rightarrow 2) = (4.3 \pm 1.4) \times 10^{-3}\ \mu_N{}^2$. Our result for δ is consistent with $\delta = 0.34^{+0.06}_{-0.05}$ from a $\gamma\gamma(\theta)$ measurement.[20] The mean lifetime of this 2^+ state is 2.11 ± 0.34 ps.

The state at 1276.1 keV is assigned $J^\pi = 3^-$ based on our $\gamma(\theta)$ measurements for the $3^- \rightarrow 4^+$ and $3^- \rightarrow 2^+$ transitions and probably corresponds to the 3^- state observed at 1277 keV by the (d, d') reaction.[1] The branching data for decay of the 3^-

TABLE III. Experimental results for $B(E1, J_i \rightarrow J_f)$.

Nucleus	Initial state K_i, J_i^π	Initial state E (keV)	Final state K_f, J_f^π	Final state E (keV)	E_γ (keV)	$B(E1, J_i \rightarrow J_f)$ $(e^2 \times 10^{-30}\ \mathrm{cm}^2)$	$\frac{B(E1)}{B(E1)_{sp}}$
^{156}Gd	$1, 3^-$	1276.1	$0, 2^+$	89.0	1187.1	22.7 ± 6.1	1.2×10^{-3}
			$0, 4^+$	288.2	987.9	19.8 ± 5.3	1.1×10^{-3}
^{158}Gd	$1, 3^-$	1041.6	$0, 2^+$	79.5	962.1	6.3 ± 1.7	3.3×10^{-4}
			$0, 4^+$	261.5	780.1	5.1 ± 1.4	2.7×10^{-4}
	$0, 2$	1259.8	$1, 1^-$	977.0	282.8	1.23 ± 0.14	6.5×10^{-5}
			$1, 2^-$	1023.6	236.2	0.24 ± 0.03	1.2×10^{-5}
			$1, 3^-$	1041.6	218.2	3.6 ± 0.4	1.9×10^{-4}
	$0, 2$	1517.4	$1, 1^-$	977.0	540.2	0.051 ± 0.025	2.7×10^{-6}
			$1, 2^-$	1023.6	493.8	0.115 ± 0.016	6.1×10^{-6}
			$1, 3^-$	1041.6	475.8	0.70 ± 0.10	3.7×10^{-5}
			$0, 1^-$	1263.5	253.9	2.8 ± 0.4	1.5×10^{-4}
			$0, 3^-$	1402.9	114.5	4.6 ± 0.8	2.4×10^{-4}
^{160}Gd	$1, 3^-$	1289.3	$0, 2^+$	75.3	1214.0	26.1 ± 7.0	1.4×10^{-3}
			$0, 4^+$	248.2	1041.1	33.8 ± 9.1	1.8×10^{-3}
^{160}Dy	$1, 3^-$	1286.7	$0, 2^+$	86.8	1199.9	7.3 ± 2.0	3.8×10^{-4}
			$0, 4^+$	283.8	1002.9	7.1 ± 1.9	3.7×10^{-4}

state are also in good agreement with branching data[13] from the decay of ^{156}Tb and from the (n,γ) reaction.[21] Our $B(E3, 0 \rightarrow 3)$ agrees with the value $(16 \pm 4) \times 10^{-2}\ e^2b^3$ from the Coulomb excitation measurement by Ronningen *et al.*[6] A 3^- state at 1538.9 keV has been observed in the (n,γ) reaction.[13,21] We did not observe the $3^- \rightarrow 4^+$ or the $3^- \rightarrow 2^+$ transitions at 1250.7 and 1449.9 keV. Upper limits to these γ-ray yields from our spectra correspond to the value

$$B(E3, 0 \rightarrow 3) < 1.3 \times 10^{-2}\ e^2b^3 = 1.2 B(E3)_{sp}.$$

B. ^{158}Gd

The $B(E2, 0 \rightarrow 2)$ for the $K, J^\pi = 2, 2^+$ state at 1187.1 keV is slightly smaller than the values $(10.6 \pm 1.5) \times 10^{-2}\ e^2b^2$ and $(9.0 \pm 1.0) \times 10^{-2}\ e^2b^2$ from other Coulomb excitation measurements[5,6] via the (α, α') reaction. The value of δ deduced from the angular distribution of the 1107.6-keV γ rays for the $2' \rightarrow 2$ transition is $-(9.0 \pm 1.5)$ and the $B(M1, 2' \rightarrow 2)$ from this branching data is

$$B(M1, 2' \rightarrow 2) = (3.3^{+1.5}_{-0.9}) \times 10^{-4}\ \mu_N{}^2.$$

The assignment of $K, J^\pi = 0, 2^+$ for the weak excitation of the state at 1259.8 keV is based on $\gamma(\theta)$ for the $J \rightarrow 4$ transition and on the branching ratio data[18] for the decay mode of this level. The mean lifetime deduced from these data for this state is 5.2 ± 0.4 ps. The weak excitation of the state at 1517.4 keV is assigned $K, J^\pi = 0, 2^+$ on the basis of $\gamma(\theta)$ for the $J \rightarrow 0$ transition and of branching data[18] for the decay mode. A substantial $E0$ component[18] in the $2' \rightarrow 2$ transition confirms that the state has $K = 0$. The value of δ deduced from $\gamma(\theta)$ of the 1438.0-keV γ rays for the $2' \rightarrow 2$ transition is $-(1.6 \pm 0.5)$ and the $B(M1, 2' \rightarrow 2)$ from this branching data is

$$B(M1, 2' \rightarrow 2) = (1.1^{+1.3}_{-0.4}) \times 10^{-3}\ \mu_N{}^2.$$

The branching data from the (n,γ) reaction suggest possible decay of this 2^+ state to the 2^+, 3^+, and 4^+ members of the $K = 2$ band. Based on our $B(E2, 0 \rightarrow 2)$ for excitation of the 1517.4-keV state, the $B(E2)$ values are 0.41, 8.1, and 11.4 $\times 10^{-2}\ e^2b^2$, respectively. The latter two values are rather large and far too large to be explained by a mutual mixing[18] of the two bands. Greenwood *et al.*[18] conclude that the placement of these transitions in the decay mode of the 2^+ state at 1517.4 keV is uncertain. The mean lifetime deduced for this 2^+ state is 2.0 ± 0.2 ps.

The state at 1041.6 keV is assigned $J^\pi = 3^-$ based on our $\gamma(\theta)$ measurements for the $3^- \rightarrow 4^+$ and $3^- \rightarrow 2^+$ transitions. The branching data[18] from the (n,γ) reaction for the 1^-, 2^-, 3^-, and 5^- states are consistent only with a $K = 1$ rotational band. The 3^- state at 1402.9 keV is assigned $K = 0$. This conclusion is based on the branching data[18] for $J = 1^-$ and 3^- states from (n,γ) reaction and the absence of even-spin members.

C. ^{160}Gd

Our $B(E2, 0 \rightarrow 2)$ for the $K, J^\pi = 2, 2^+$ state at 988.2 keV is slightly smaller than the values $(10.4 \pm 0.4) \times 10^{-2}\ e^2b^2$ and $(10.1 \pm 0.3) \times 10^{-2}\ e^2b^2$ from other Coulomb excitation measurements.[5,6] The value obtained for δ from the angular distribution of the 912.9-keV γ rays for the $2' \rightarrow 2$ transition is $|\delta| \geq 100$. Although our data do not allow a determination of the sign of δ, the choice of a negative sign would be consistent with the results for the decay of $K, J^\pi = 2, 2^+$ γ-vibrational-like states of deformed nuclei in the rare-earth region. The limit for $B(M1, 2' \rightarrow 2)$ from this branching data is

$$B(M1, 2' \rightarrow 2) \leq 1.7 \times 10^{-6}\ \mu_N{}^2,$$

and the mean lifetime of this 2^+ state is 2.27 ± 0.09 ps.

The state at 1289.3 keV is assigned $J^\pi = 3^-$ based on our $\gamma(\theta)$ measurements for the $3^- \rightarrow 2^+$ transition and probably corresponds to the 3^- state observed at 1289 keV by the (d, d') reaction.[1] Although Neergård and Vogel[22] predict the lowest 3^- state and the largest $B(E3, 0 \rightarrow 3)$ for $K = 2$ as the largest component in the wave function, an assignment of $K = 0$ would be more consistent with the branching data for decay of the 3^- state. The Coriolis-coupled wave function does contain appreciable components from $K = 0$ and 1. Our $B(E3, 0 \rightarrow 3)$ is in excellent agreement with the value $(12.7 \pm 1.4) \times 10^{-2}\ e^2b^3$ from the (α, α') reaction.[6]

Several additional states between 1070 and 1688 keV were weakly populated in the (d, d') reaction.[1] The limits of $B(E\lambda, 0 \rightarrow J = \lambda)$ for excitation of these states deduced from upper limits of the γ-ray yields from our spectra are

1070 keV: $B(E2, 0 \rightarrow 2) < 0.001\ e^2b^2 = 0.04\ B(E2)_{sp}$,

1462 keV: $B(E3, 0 \rightarrow 3) < 0.012\ e^2b^3 = 1.1\ B(E3)_{sp}$,

1688 keV: $B(E3, 0 \rightarrow 3) < 0.022\ e^2b^3 = 2.1\ B(E3)_{sp}$.

The $B(E2)$ limit for the 1070-keV state is a factor of 2 smaller than the value deduced from the (α, α') reaction by Ronningen *et al.*[6]

D. ^{160}Dy

Our $B(E2, 0 \rightarrow 2)$ for the $K, J^\pi = 2, 2^+$ state at 966.2 keV is slightly larger than the value $(1.05 \pm 0.08) \times 10^{-1}\ e^2b^2$ from the $(\alpha, \alpha'\gamma)$ reaction by Oehlberg *et al.*[4] The value obtained for δ from the angular distribution of the 879.4-keV γ rays

for the $2' \rightarrow 2$ transition is $-(30^{+20}_{-7})$ and the $B(M1, 2' \rightarrow 2)$ from this branching data is

$$B(M1, 2' \rightarrow 2) = 2.5 \pm 1.7 \times 10^{-5}\ \mu_N{}^2,$$

and the mean lifetime of this 2^+ state is 1.84 ± 0.04 ps which agrees with the value 1.95 ± 0.04 ps from a DBLS analysis[23] of the γ rays from multiple Coulomb excitation with 348-MeV ^{84}Kr ions.

The weak excitation of the 1349.5-keV state is assigned $K, J^\pi = 0, 2^+$ on the basis of $\gamma(\theta)$ for $J \rightarrow 0$ and $J \rightarrow 4$ transitions and of branching data for the decay mode. A substantial $E0$ component[15] in the $2' \rightarrow 2$ transition confirms the $K = 0$ assignment. The value of δ deduced from $\gamma(\theta)$ of the 1262.7-keV γ rays for the $2' \rightarrow 2$ transition is $-(1.5^{+2}_{-0.65})$ and the $B(M1, 2' \rightarrow 2)$ is

$$B(M1, 2' \rightarrow 2) = (1.8^{+3.9}_{-1.5}) \times 10^{-3}\ \mu_N{}^2.$$

The mean lifetime of this 2^+ state is 1.8 ± 0.3 ps.

The state at 1286.7 keV is assigned $J^\pi = 3^-$ based on our $\gamma(\theta)$ measurements for the $3^- \rightarrow 2^+$ transitions. The branching data are consistent with a $K = 1$ assignment. Likewise, the state at 1642 keV is assigned $J^\pi = 3^-$ based on our $\gamma(\theta)$ measurements for the $3^- \rightarrow 4'^+$ and $3^- \rightarrow 2'^+$ transitions. An assignment of $K = 2$ would be consistent with the fact that we did not observe the $3^- \rightarrow 4^+$ or $3^- \rightarrow 2^+$ transitions.

Several additional states between 1386 and 1670 keV have been assigned[15] either $J^\pi = 2^+$ or $J^\pi = 3^-$. The limits of $B(E\lambda, 0 \rightarrow J = \lambda)$ for excitation of these states deduced from upper limits of the γ-ray yields from our spectra are

1386 keV: $B(E3, 0 \rightarrow 3) < 0.005\ e^2b^3 = 0.5\, B(E3)_{sp}$,

1399 keV: $B(E3, 0 \rightarrow 3) < 0.008\ e^2b^3 = 0.8\, B(E3)_{sp}$,

1431 keV: $B(E2, 0 \rightarrow 2) < 0.0011\ e^2b^2 = 0.04\, B(E2)_{sp}$,

1670 keV: $B(E2, 0 \rightarrow 2) < 0.0016\ e^2b^2 = 0.06\, B(E2)_{sp}$.

The $B(E3)$ limit for the 1399-keV state is a factor of 8 times smaller than the value deduced from the (^{16}O, ^{16}O$'\gamma$) reaction by Oehlberg *et al.*[4]

E. ^{162}Dy

Our $B(E2, 0 \rightarrow 2)$ for the $K, J^\pi = 2, 2^+$ state at 888.2 keV is to be compared with the values $(1.52 \pm 1.1) \times 10^{-1}\ e^2b^2$, $(1.05 \pm 0.08) \times 10^{-1}\ e^2b^2$, and $(1.30 \pm 0.05) \times 10^{-1}\ e^2b^2$ from other Coulomb excitation measurements.[3,4,5] The value of δ deduced from the angular distribution of the 807.5-keV γ rays for the $2' \rightarrow 2$ transition is $|\delta| > 90$ and again the choice of a negative sign would be consistent with the results for other nuclei in this region. The limit for $B(M1, 2' \rightarrow 2)$ from this branching data is

$$B(M1, 2' \rightarrow 2) < 2.5 \times 10^{-6}\ \mu_N{}^2$$

and the mean lifetime of this 2^+ state is 2.83 ± 0.11 ps.

The state at 1210.2 keV is assigned $J^\pi = 3^-$ based on our $\gamma(\theta)$ measurements for the $3^- \rightarrow 2'^+$, $3^- \rightarrow 4^+$, and $3^- \rightarrow 2^+$ transitions. The branching data for decay of the 3^- state to members of the $K = 2$ γ-vibrational-like band are consistent with the assignment $K = 2$. However, the decay of the 3^- state to the 4^+ and 2^+ members of the ground-state band would require appreciable $K = 0$ or 1 components in the makeup of the 3^- wave function.

Two additional states at 1357.9 and 1453 keV have been assigned[16] $J^\pi = 3^-$ and 2^+, respectively. The limits of $B(E\lambda, 0 \rightarrow J = \lambda)$ for excitation of these two states deduced from upper limits of the γ-ray yields from our spectra are

1357.9 keV: $B(E3, 0 \rightarrow 3) < 0.015\ e^2b^3 = 1.4\, B(E3)_{sp}$,

1453 keV: $B(E2, 0 \rightarrow 2) < 0.0024\ e^2b^2 = 0.09\, B(E2)_{sp}$.

This $B(E3)$ limit is a factor of 3 times smaller than the value deduced from the (^{16}O, ^{16}O$'\gamma$) reaction by Oehlberg *et al.*[4]

F. ^{164}Dy

The $B(E2, 0 \rightarrow 2)$ for the $K, J^\pi = 2, 2^+$ state at 761.8 keV agrees with the values.[4,5] $(1.01 \pm 0.09) \times 10^{-1}\ e^2b^2$ and $(1.21 \pm 0.05) \times 10^{-1}\ e^2b^2$ from other Coulomb excitation measurements. The value of δ deduced from the angular distribution of the 688.4-keV γ rays for the $2' \rightarrow 2$ transition is $\delta = -(80^{+\infty}_{-35})$ and the $B(M1, 2' \rightarrow 2)$ from this branching data is

$$B(M1, 2' \rightarrow 2) \leq 6.7 \times 10^{-6}\ \mu_N{}^2.$$

The mean lifetime of this 2^+ state is 6.6 ± 0.3 ps.

The state at 1039.3 keV is assigned $J^\pi = 3^-$ based on our $\gamma(\theta)$ measurements for the $3^- \rightarrow 3'^+$ and $3^- \rightarrow 2'^+$ transitions. The branching data for decay of the 3^- state to members of the $K = 2$ γ-vibrational-like band from ^{164}Tb, (n, γ) reaction, and Coulomb excitation are consistent with the assignment $K = 2$. Again, the decay of the 3^- state to the 4^+ and 2^+ members of the ground-state band would require appreciable $K = 0$ or 1 components in the makeup of the 3^- wave function.

A state at 1394 keV has been tentatively assigned $J^\pi = 2^+$. The limit of $B(E2, 0 \rightarrow 2)$ for excitation of this state deduced from upper limits of the γ-ray yields from our spectra is

$$B(E2, 0 \rightarrow 2) < 0.0035\ e^2b^2 = 0.13\, B(E2)_{sp}.$$

G. Octupole-vibrational states

A vibrational interpretation of the negative parity states provides an interesting framework with which to compare experimental information. The microscopic calculations for the $B(E3, 0 \rightarrow 3)$ values for the 3^- members of the one-phonon

octupole quadruplet by Neergård and Vogel,[22] which also included the influence of the Coriolis coupling between states with K and $K \pm 1$, have reproduced the general features of early experimental information for the rare-earth nuclei. The inclusion of the Coriolis interaction in the calculations is required to explain satisfactorily the distribution of the $B(E3)$ strength among the one-phonon octupole vibrational states, viz., most of the $B(E3)$ strength is in the excitation of the lowest 3^- state. In Table II our results for $B(E3, 0 \rightarrow 3)$ are compared with the theoretical calculations of Neergård and Vogel. We modified the results of these calculations by adjusting the unperturbed band-head energies and the rotational inertial parameter $A \equiv \hbar^2/2\mathcal{I}$ slightly in order to improve the fit to the observed energies of the members of the $K^\pi = 0^-$, 1^-, 2^-, and 3^- rotational bands for the one-phonon octupole quadruplet. The rotor Hamiltonian is diagonalized to obtain the energies and the components of the Coriolis-coupled wave functions. Although the general features of the experimental information are reproduced by the modified calculations of Neergård and Vogel, the experimental values of $B(E3, 0 \rightarrow 3)$ tend to be somewhat larger than the predictions. The modified calculations did shift the $B(E3)$ strength in ^{160}Gd and ^{160}Dy from $K = 2$ to $K = 1$ as the largest component in the Coriolis-coupled wave function.

The reduced $E1$ transition probabilities for transitions from members of the octupole quadruplet to the ground-state band $K^\pi = 0^+$ provide critical information concerning the Coriolis interaction. The Coriolis-coupled octupole vibrational-state wave function is of the form[24]

$$|JM\alpha\rangle = \sum_{K=0}^{3} C(JK;\alpha)\,|JMK\rangle ,$$

where K denotes the projection of the angular momentum J on the nuclear symmetry axis and α is an additional label, which usually can be chosen as the K corresponding to the largest component in the wave function. The reduced $E1$ transition probability can be expressed in the form

$$\begin{aligned}[B(E1, J_i \rightarrow J_f)]^{1/2} = {} & C(J_i 0;\alpha)\langle J_i 010|J_f 0\rangle \\ & \times \langle 0^+|M'(E1;0)|0^-\rangle \\ & + \sqrt{2}\,C(J_i 1;\alpha)\langle J_i 11-1|J_f 0\rangle \\ & \times \langle 0^+|M'(E1;-1)|1^-\rangle .\end{aligned}$$

Of particular interest is the fact that it is possible to extract values of the $E1$ matrix elements M' between the octupole-vibrational bands and the ground-state band. The admixed amplitudes are available from the microscopic calculations of Neergård and Vogel. The deduced $E1$ matrix elements from the results in Table III are summarized in Table IV. Of course, the absolute phases of the $E1$ matrix elements are not known. In this analysis, we have arbitrarily chosen that of the matrix element with $\Delta K = 0$ to be positive. The $\Delta K = 0$ matrix elements are found to be nearly equal for ^{156,158}Gd and ^{160}Dy. In contrast, the $\Delta K = -1$ matrix elements are opposite in sign and much smaller. The values range from 0.065 to 0.88×10^{-15} e cm. Hindrance factors for $E1$ transitions between the unmixed $K = 0^-$ and 1^- octupole states and the ground-state band in ^{160}Dy are $\simeq 370$ and 3×10^5, respectively. Although the largest component in the wave function for ^{160}Gd is $K = 1$, the branching ratio, $B(E1, 3^- \rightarrow 4^+)/B(E1, 3^- \rightarrow 2^+) = 1.30 \pm 0.08$, is dominated by the $K = 0$ admixed amplitude because the ratio of the $E1$ matrix element M' with $\Delta K = 0$ to that with $\Delta K = -1$ is very large (428).

The $E1$ branching ratios[16,17] for decay of the 2^-, 3^-, and 4^- states in ^{162,164}Dy to levels of the γ-vibrational-like band agree with the Alaga-rule predictions. This conclusion is consistent with the microscopic calculations of Neergård and Vogel. The Coriolis-coupled wave function for ^{162,164}Dy is nearly a pure $K = 2$ wave function; i.e., the admixed amplitudes with $K = 2$ are 0.974 and 0.997, respectively.

Finally, the $E1$ matrix elements M' deduced from the $B(E1)$ values for $E1$ transitions from decay of the $K, J^\pi = 0, 2^+$ states at 1259.8 and 1517.4 keV to the $K = 0$ and 1 octupole bands in

TABLE IV. E1 matrix elements.

Nucleus	Admixed amplitudes $K=0$	Admixed amplitudes $K=1$	$\langle 0^+\|M'(E1;0)\|0^-\rangle$ ($e\times10^{-15}$ cm)	$\langle 0^+\|M'(E1;-1)\|1^-\rangle$ ($e\times10^{-15}$ cm)
^{156}Gd	0.610	0.782	10.6 ± 1.3	$-(0.88 \pm 0.06)$
^{158}Gd	0.328	0.937	10.2 ± 0.6	$-(0.45 \pm 0.05)$
^{160}Gd	0.278	0.863	27.8 ± 3.6	$-(0.065 \pm 0.006)$
^{160}Dy	0.368	0.803	10.3 ± 1.3	$-(0.36 \pm 0.05)$

^{158}Gd are summarized in Table V. Note the position of the positive- and negative-parity states in the matrix elements have been interchanged in order to make a direct comparison with those in Table IV, viz., the octupole states occur at the right in the matrix elements. The $\Delta K=-1$ matrix elements are opposite in sign to those with $\Delta K=0$ for the ground-state and the second excited 0^+ bands and have the same sign for the first excited 0^+ band in ^{158}Gd. The magnitudes of the $\Delta K=0$ and $\Delta K=-1$ matrix elements for the ground-state band and the first excited 0^+ band are nearly equal, while the $\Delta K=0$ matrix for the second excited 0^+ band is nearly a factor of 4 smaller. Similar conclusions have been reached by Greenwood *et al.*[18] from less direct, but plausible, arguments.

H. Reduced $E2$ transition probabilities

In Table II, our experimental $B(E2,0\rightarrow 2)$ for $^{156\text{-}160}$Gd are compared with recent theoretical calculations of Kumar and Gupta[25] based on a dynamic deformation theory combined with the pairing-plus-quadrupole model. In the case of the $K,J^\pi=2,2^+$ γ-vibrational-like state, the $B(E2)$ values are reproduced reasonably well by these predictions. However, for the $K,J^\pi=0,2^+$ states, the $B(E2)$ values are not reproduced very satisfactorily by these calculations. In fact, our upper limit of $B(E2)$ for excitation of a 1070-keV state in ^{160}Gd is 60 times smaller than their prediction.

The $B(E2)$ values for the different decay modes of the 2^+ states for $^{156\text{-}160}$Gd and $^{160\text{-}164}$Dy are compared with theoretical predictions from the interacting boson model (IBA) in Table VI. These calculations were done using the IBA computer code[26] PHINT. The interaction parameters in the IBA Hamiltonian were adjusted to fit typically 12 state energies in each nucleus. The energies and eigenvectors are obtained from a diagonalization of the IBA Hamiltonian. In the calculations of the transition probabilities, the parameters $E2SD$ and $E2DD$ in the $E2$ transition operator were adjusted to reproduce the experimental $B(E2,0\rightarrow 2)$ values for excitation of the 2^+ members of the ground-state and γ-vibrational-like bands. Values of the parameters used in the IBA calculations are given in Table VII. For a typical fit to 12 state energies in ^{158}Gd, the average deviation between experimental and theoretical energies is 8 keV. The pairing term of the IBA Hamiltonian removes the degeneracy which exists in an energy level spectrum with SU(3) symmetry, viz., the degeneracy of the states with a given J in the γ- and β-vibrational-like bands. It is evident from Table VI that the $B(E2)$ values for the $2\rightarrow 2$ and $2\rightarrow 4$ transitions from the decay of the $K,J^\pi=2,2^+$ state are reproduced reasonably well by the IBA predictions. The breaking of the SU(3) symmetry by the pairing term plays a crucial role in giving a much better account of the $B(E2)$ values for decay of the excited $K=0$ band. The experimental $B(E2)$ values for excitation of the 2^+ state in this band are an order of magnitude smaller than the $B(E2)$ values for excitation of the $K,J^\pi=2,2^+$ state and thus provide a comprehensive test of the IBA predictions.

Our upper limits of the $B(E2,0\rightarrow 2)$ for excitation of $K,J^\pi=0,2^+$ states at 1070, 1453, and 1394 keV in ^{160}Gd, ^{162}Dy, and ^{164}Dy are 0.001, 0.0014, and 0.0035 e^2b^2. For ^{160}Gd this limit is 10 times smaller than the IBA prediction and for ^{162}Dy and ^{164}Dy these limits are 2 and 90 times larger than the IBA predictions. The $B(E2)$ values from the IBA calculations are indeed small and give a reasonable account of the experimental results.

Finally, the $K,J^\pi=0,2^+$ states at 1258 keV in ^{156}Gd and 1517 keV in ^{158}Gd cannot be reproduced by the IBA calculations of the energy level spectra which include the 1049- and 1129-keV states in ^{156}Gd and the 1196- and 1260-keV states in ^{158}Gd. These intruder bands could possibly come from a subshell closure[27] at proton number $Z=64$, giving rise to new degrees of freedom in addition to those included in the description of the

TABLE V. $E1$ matrix elements M' deduced from the $B(E1)$ values for $E1$ transitions from decay of the $K,J^\pi=0,2^+$ states at 1259.8 and 1517 keV in ^{158}Gd.

Initial state		Final state		Admixed amplitudes		$\langle 0^+\|M'(E1;0)\|0^-\rangle$	$\langle 0^+\|M'(E1;-1)\|1^-\rangle$
K_i,J_i^π	E (keV)	K_f,J_f^π	E (keV)	$K=0$	$K=1$	($e\times10^{-15}$ cm)	
$0,2^+$	1259.8	$1,1^-$	977.1	0.164	0.986		
		$1,2^-$	1023.6		0.977	8.85 ± 0.38	0.491 ± 0.029
		$1,3^-$	1041.6	0.328	0.937		
$0,2^+$	1517.4	$1,1^-$	977.1	0.164	0.986		
		$1,2^-$	1023.6		0.997		
		$1,3^-$	1041.6	0.328	0.937	2.63 ± 0.21	−(0.340 ± 0.024)
		$0,1^-$	1263.5	0.986	−0.164		
		$0,3^-$	1402.9	0.944	−0.320		

TABLE VI. Experimental and IBA predicted $B(E2)$ values.

Nucleus	Initial state K_i, J_i^π	Initial state E (keV)	Final state K_f, J_f^π	Final state E (keV)	$B(E2, J_i \to J_f)$ (10^{-50} e^2cm^4) Exp	IBA	IBA[a]
^{156}Gd	$0, 2^+$	1129.4	$0, 0^+$	0	0.32 ± 0.02	0.31	
			$0, 2^+$	89.0	2.04 ± 0.14	0.43	
			$0, 4^+$	288.2	1.71 ± 0.16	1.12	
	$2, 2^+$	1154.1	$0, 0^+$	0	2.22 ± 0.11	2.22	2.22
			$0, 2^+$	89.0	3.55 ± 0.19	3.87	3.50
			$0, 4^+$	288.2	0.32 ± 0.03	0.29	0.23
	$0, 2^+$	1258	$0, 0^+$	0	0.15 ± 0.01		0.25
			$0, 2^+$	89.0	0.21 ± 0.14		0.40
			$0, 4^+$	288.2	2.27 ± 0.27		0.94
^{158}Gd	$2, 2^+$	1187.1	$0, 0^+$	0	1.70 ± 0.10	1.70	1.69
			$0, 2^+$	79.5	3.16 ± 0.07	2.66	2.61
			$0, 4^+$	261.4	0.11 ± 0.02	0.18	0.16
	$0, 2^+$	1259.8	$0, 0^+$	0	0.16 ± 0.01	0.20	
			$0, 2^+$	79.5	0.123 ± 0.015	0.33	
			$0, 4^+$	261.4	0.71 ± 0.06	0.76	
	$0, 2^+$	1517.4	$0, 0^+$	0	0.19 ± 0.02		0.08
			$0, 2^+$	79.5	0.20 ± 0.04		0.13
			$0, 4^+$	261.4	0.19 ± 0.03		0.29
^{160}Gd	$2, 2^+$	988.2	$0, 0^+$	0	1.76 ± 0.09	1.76	
			$0, 2^+$	75.3	2.98 ± 0.18	2.75	
			$0, 4^+$	248.2	0.16 ± 0.01	0.18	
^{160}Dy	$2, 2^+$	966.2	$0, 0^+$	0	2.44 ± 0.13	2.44	
			$0, 2^+$	86.8	4.45 ± 0.28	3.75	
			$0, 4^+$	283.8	0.30 ± 0.03	0.23	
	$0, 2^+$	1349.5	$0, 0^+$	0	0.37 ± 0.03	0.10	
			$0, 2^+$	86.8	0.37 ± 0.19	0.16	
			$0, 4^+$	283.8	0.89 ± 0.09	0.36	
^{162}Dy	$2, 2^+$	888.2	$0, 0^+$	0	2.37 ± 0.12	2.36	
			$0, 2^+$	80.7	4.46 ± 0.27	3.55	
			$0, 4^+$	265.7	0.47 ± 0.05	0.22	
^{164}Dy	$2, 2^+$	761.8	$0, 0^+$	0	2.27 ± 0.11	2.28	
			$0, 2^+$	73.4	4.11 ± 0.25	3.34	
			$0, 4^+$	242.2	0.46 ± 0.04	0.18	

[a] See text for a discussion of the IBA predictions in this column.

TABLE VII. Values of the parameters used in the IBA calculations. These parameters correspond to variable names in program PHINT for the IBA Hamiltonian expressed in terms of the "multipole expansion."

Nucleus	Pair (keV)	ELL (keV)	QQ (keV)	OCT (keV)	$E2SD$ (eb)	$E2DD$ (eb)
^{156}Gd	0.0	12.40	−29.17	2.46	0.1537	−0.1279
^{156}Gd[a]	4.71	14.97	−31.62	0.0	0.1538	−0.1230
^{158}Gd	3.08	14.39	−29.67	0.0	0.1449	−0.1484
^{158}Gd[a]	14.85	14.70	−29.82	0.0	0.1420	−0.1577
^{160}Gd	3.39	15.21	−22.75	0.0	0.1385	−0.1358
^{160}Dy	15.93	19.60	−22.32	0.0	0.1390	−0.1061
^{162}Dy[b]	30.60	18.54	−20.63	0.0	0.1396	−0.1305
^{164}Dy[b]	85.00	17.79	−18.16	0.0	0.1391	−0.1597

[a] See text for a discussion of the IBA predictions for these values of the parameters.

[b] Total number of bosons truncated to 14.

bands built on the ground-state configuration.

For completeness, we have done IBA calculations treating the 0^+ and 2^+ states at 1168 and 1258 keV in ^{156}Gd and at 1452 and 1517 keV in ^{158}Gd as the members of the $K=0$ β-vibrational-like band. This means omitting the 1049- and 1129-keV states in ^{156}Gd and the 1196- and 1260-keV states in ^{158}Gd in the fit to the energy level spectra. In this case, these latter states would be identified as possible intruder states. There are strong arguments for assigning the $K=0$ band at 1452 keV in ^{158}Gd as being the β-vibrational-like band. First, there is substantial $E0$ decay between this band and the ground-state band. Second, there is strong population of this band in the ^{160}Gd(p,t) reaction.[28,29] The results of the IBA calculations are given in the right-hand column of Table VI.

Again, the IBA predictions give a good account of the general features of the $B(E2)$ values.

The IBA predictions of the $B(E2)$ values for decay of the 2^+ state at 1517 keV to the 3^+ and 4^+ members of the $K=2$ band are 0.91 and 2.49 $\times 10^{-2}\ e^2b^2$, respectively. These two values are approximately 10 times smaller than the values inferred from the possible placement of the 252- and 159-keV transitions in the decay mode of the 2^+ state at 1517 keV. This lends support to the conclusion that the placement of these γ rays from the (n,γ) reaction is uncertain.

IV. CONCLUSIONS

In this paper we made use of the direct $E2$ and $E3$ Coulomb excitation reaction to investigate the nuclear structure properties of states up to excitation energies $\lesssim 1.7$ MeV. The occurrence of a rather low-lying $K=2$ γ-vibrational-like band is a systematic feature of the even-A deformed nuclei. The observed nuclear structure properties of the $2'^+$ member in $^{156-160}$Gd and $^{160-164}$Dy are remarkably similar, viz., the $B(E2, 0 \rightarrow 2')$, the large value for δ for the $2'^+ \rightarrow 2^+$ transition, and the branching data for decay of the $2'^+$ state. On the other hand, the $K=0$ vibrational-like states are highly variable and usually the $B(E2, 0 \rightarrow 2)$ values are at least an order of magnitude less than the $B(E2, 0 \rightarrow 2')$ values. The pairing interaction of the IBA Hamiltonian provides a good description of these general features. The displacement of the $K, J^\pi = 0, 2^+$ state above the $K, J^\pi = 2, 2^+$ increases with increasing pairing interaction and the $B(E2, 0 \rightarrow 2)$ for excitation of the $K, J^\pi = 0, 2^+$ state decreases. This behavior seems to be a systematic feature of the even-A deformed nuclei in the rare-earth region. This is true of the $^{166-170}$Er nuclei[30] and of six nuclei studied in this paper. In general, the IBA model provides a detailed quantitative description of the level energies and of the $E2$ transition probabilities. In ^{156,158}Gd the properties of the two excited $K=0^+$ bands cannot be reproduced simultaneously by the present IBA calculations. These intruder states are presumably associated with other low-lying degrees of freedom. Work in the direction of extending the IBA model to include other modes of excitation is in progress.[27]

A microscopic description of octupole vibrational states explains satisfactorily the general features of the distribution of the $B(E3)$ strength and the energy intervals within rotational bands. In particular, the agreement between experimental data and the calculations provides further evidence of the strong Coriolis coupling between the states of the octupole quadruplet in the deformed nuclei. The Coriolis coupling is of major importance for interpretation of the reduced $E1$ transition probabilities for transitions connecting members of the octupole quadruplet with $K^\pi = 0^+$ bands. The reduced $E1$ matrix elements $\langle 0^+ | M'(E1; -K) | K^- \rangle$ with $K=0$ and 1 extracted from our data have a characteristic feature, viz., the ratio of the $E1$ matrix element M' with $\Delta K = -1$ to that with $\Delta K = 0$ is very small (10^{-1} to 10^{-3}). Extreme caution must be exercised in making K assignments for octupole states based on experimental $B(E1)$ branching ratios and the Alaga-rule prediction.

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