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Inclusive breakup reactions

N. Austern and C. M. Vincent
Department of Physics, University of Pittsburgh, Pittsburgh, Pennsylvania 15260
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Inclusive breakup reactions of the type $d + A \rightarrow p +$ anything require sums over excited states of the unobserved $n + A$ system. We show that the summed distorted-wave Born approximation cross section can be expressed in closed form as an elastic distorted-waves expectation of an optical model propagator for the unobserved system. The contribution of fluctuations to the energy-averaged cross section is automatically contained in the derived identity. Previous results can be recovered from this identity if corresponding surface approximations are introduced. A new approximation simplifies the identity by omitting the imaginary part of the optical potential. This approximation selects the energy-shell part of the propagator and reduces the inclusive cross section to the same form obtained for the ground state reaction $A(d,pn)A$, with a modified (real) optical potential for the unobserved neutron.

[NUCLEAR REACTIONS Scattering theory, closed form sum rule for inclusive breakup, DWBA, approximations extracted.]

I. INTRODUCTION

Recent studies of breakup by Baur and collaborators[1] emphasize inclusive reactions, such as

$$d + A \rightarrow p + \text{anything} ,$$

which are summed over the states of the unobserved particle + residual nucleus system. The technique of calculation[2,3] is based on unitarity and a surface approximation of the form factors of excited states; by this means the sum over excited states of the residual nucleus is reduced to a correction factor that multiplies the cross section for the ground state (or elastic) breakup reaction $A(d, pn)A$. The correction for excited states substantially increases the inclusive (d,p) cross section and allows good fits to experiment.[1]

The present article describes a new procedure for summing the distorted-wave Born approximation (DWBA) (d,p) cross section over the excited states of the residual nucleus, without making explicit use of the properties of those states. An identity is derived which expresses the inclusive breakup cross section in closed form as an expectation of an optical model Green's function for the unobserved system. Two derivations are given. A wave function analysis in Sec. II leads to an equation of flux conservation and from this to an expression for the total breakup flux. The same result is obtained in Sec. III by a formal sum over excited states, which leads to a ground state expectation of a projected propagator. A further analysis in the Appendix shows that the summed cross section expressed by our identity correctly incorporates fluctuations of the inclusive cross section as a function of energy transfer.

An analysis of the formal identity and suggestions for practical evaluations are given in Sec. IV. It is shown how the previous result can be recovered if corresponding approximations are introduced. A new approximation is suggested, whereby the identity is simplified by omitting the imaginary part of the optical potential. This approximation selects the energy-shell part of the propagator and reduces the inclusive cross section to the same form obtained for the ground state reaction $A(d,pn)A$, with a modified (real) optical potential for the unobserved neutron. An application of this approximation is presented.

II. WAVE FUNCTION DERIVATION

We begin with a simplified Lippman-Schwinger equation written in terms of the Green's function for breakup channels,

$$\Psi(\xi, \vec{r}_n, \vec{r}_p) = [E^+ - H_p - H_B(\xi,n)]^{-1} V_{pn} \Psi, \quad (1)$$

where $E^+ \equiv E + i\eta$, $\eta \rightarrow 0(+)$. In our model H_p describes independent-particle motion of the proton (it includes a distorting potential), and

$$H_B(\xi,n) = H_A(\xi) + K_n + V(\xi,n) \quad (2)$$

is the Hamiltonian for the interacting neutron-nucleus system, with K_n the neutron kinetic energy operator. The notation of Pampus *et al.*[3] is adopted as far as possible. The final-state interaction V_{pn} in Eq. (1) is the residual interaction not otherwise used in the Green's function. [Although the homogeneous Lippmann-Schwinger equation for rearrangement channels does not have unique solutions,[4] it is suitable for our present application which is based on an explicit approximation for Ψ on the right hand side of Eq.

(1).]

Projection onto a definite energy eigenstate $\chi_p^{(-)}$ for the outgoing proton gives a corresponding neutron-nucleus outgoing wave function

$$Z_p(\xi,\vec{r}_n)=(E^+-E_p-H_B)^{-1}(\chi_p^{(-)}|V_{pn}|\Psi\rangle\,, \qquad (3)$$

which still retains the entire set of excited states of the residual nucleus and their associated outgoing neutron wave functions. The round bracket on the matrix element indicates integration only over the coordinates of the enclosed wave function.

The wave function Z_p for the unobserved particles has a spectral expansion

$$Z_p(\xi,\vec{r}_n)=\sum_c \frac{|\Phi_{Bc}^{(-)}\rangle\langle\Phi_{Bc}^{(-)}\chi_p^{(-)}|V_{pn}|\Psi\rangle}{E^+-E_p-E_{Bc}}\,, \qquad (4)$$

in terms of energy eigenstates $\Phi_{Bc}^{(-)}$ of the neutron-nucleus Hamiltonian H_B. The amplitudes

$$T_{d,pc}=\langle\Phi_{Bc}^{(-)}\chi_p^{(-)}|V_{pn}|\Psi\rangle\,, \qquad (5)$$

together with a DWBA replacement of Ψ by the entrance-channel wave function,

$$\Psi\approx\Phi_A\chi_d^{(+)}\phi_d\,, \qquad (6)$$

are the starting point of previous analyses.[2,3,5] A complicated sequence of approximations is needed to sum the associated cross sections over the excited states c, to reach a calculable result.

However, the outgoing neutron-nucleus wave function Z_p already contains a sum over c. We only need to compute the inclusive cross section directly from Z_p without any expansion. We go first to the Schrödinger equivalent of Eq. (3),

$$(E-E_p-H_B)Z_p=(\chi_p^{(-)}|V_{pn}|\Psi\rangle\,. \qquad (7)$$

In DWBA approximation this reduces to

$$(E-E_p-H_B)Z_p=\Phi_A(\xi)\rho_p(\vec{r}_n)\,, \qquad (8)$$

with

$$\rho_p(\vec{r}_n)\equiv\int d^3r_p\,\chi_p^{(-)*}V_{pn}\chi_d^{(+)}\phi_d\,. \qquad (9)$$

Equation (8) resembles the "source term method" of Ascuitto and Glendenning.[6] It expresses the entire multichannel breakup wave function $Z_p(\xi,\vec{r}_n)$ in DWBA in terms of a single, known inhomogeneity in the ground state channel, $c=A$.

Equation (8) is converted to coupled equations by use of projection operators P, Q that select the ground state or excited states of H_A, respectively. The ground-state projector selects the neutron relative wave function

$$\psi(\vec{r}_n)\equiv(\Phi_A|Z_p\rangle\,, \qquad (10)$$

which is of primary interest. The excited-state projection of Eq. (8) is solved formally in the usual fashion and the solution is substituted in the ground state projected equation to yield a reduced equation for $\psi(\vec{r}_n)$,

$$[E_n-K_n-\mathfrak{U}]\psi(\vec{r}_n)=\rho_p(\vec{r}_n)\,, \qquad (11)$$

in which

$$E_n=E-E_p-\epsilon_A\,, \qquad (12)$$

$$\mathfrak{U}_A\psi(\vec{r}_n)\equiv(\Phi_A|\,V(\xi,n)+V(\xi,n)(E^+-E_p-H_{QQ})^{-1}QV(\xi,n)|\Phi_A\psi(\vec{r}_n)\rangle, \qquad (13)$$

and H_{QQ} is the Q-space projection of H_B.

The effective interaction $\mathfrak{U}_A$ is the familiar formal operator whose energy average with respect to E or E_p gives the ground-state neutron-nucleus optical potential,

$$\bar{\mathfrak{U}}_A=U(r_n)=-V(r_n)-iW(r_n)\,. \qquad (14)$$

We take advantage of this identification to develop a theory of *energy-averaged inclusive breakup* by going over to the simplified equation

$$[E_n-K_n-U(r_n)]\bar{\psi}(\vec{r}_n)=\rho_p(\vec{r}_n)\,, \qquad (15)$$

in which $\bar{\psi}(\vec{r}_n)$ is defined as the scattering solution of Eq. (15), based on the slowly varying potential $U(r_n)$. Obviously, loss of flux to the open-channel excited state parts of $Z_p(\xi,\vec{r}_n)$ contributes to the imaginary part of $U(r_n)$; by this means *the ground state optical potential in Eq. (15) implicitly contains the required sum over breakup to excited states*. We show in the Appendix that the energy averaging of $\mathfrak{U}_A$ also implies that the breakup cross section calculated from Eq. (15) correctly includes fluctuation contributions.

Equation (15) leads to an equation of flux conservation by the usual procedure in which (15) is combined with its complex conjugate by cross multiplication and subtraction. This yields

$$\frac{\hbar}{2m}\int d\vec{S}\cdot(\bar{\psi}^*\nabla\bar{\psi}-\bar{\psi}\nabla\bar{\psi}^*)=-2i\int d^3r_n(\mathrm{Im}\rho_p^*\bar{\psi}+W|\bar{\psi}|^2). \qquad (16)$$

Here the left hand side is a surface integral over a large sphere that encloses the interaction volume and the right hand side is an integral over the interior of this sphere. If the sphere is chosen sufficiently large, the outgoing wave property of $\bar{\psi}(\vec{r}_n)$ allows a reduction of Eq. (16) to the form

$$\frac{\hbar^2k_n}{2m}\int dS\,|\bar{\psi}(\vec{r}_n)|^2+\int d^3r_n W(r_n)|\,\bar{\psi}\,|^2=-\mathrm{Im}\int d^3r_n\rho_p^*\bar{\psi}. \qquad (17)$$

By construction the surface integral in Eq. (17) is

the outgoing flux in the elastic breakup channel and the integral over $W(r_n)$ is the flux transferred to excited breakup channels by optical absorption from the elastic channel. We therefore recognize the quantity

$$N \equiv -\operatorname{Im}\int d^3r_n \rho_p^*(\vec{r}_n)\bar{\psi}(\vec{r}_n) \tag{18}$$

as the *total neutron flux transferred from the deuteron channel into all varieties of breakup.* The inclusive breakup cross section is a multiple of N. Equation (17) resembles the optical theorem for scattering cross sections.

The source term $\rho_p(\vec{r}_n)$ used above is not short ranged, therefore the convergence of mathematical expressions based on ρ_p must be questioned. Indeed ρ_p is defined in Eq. (9) as an overlap of proton and deuteron distorted waves, hence it is approximately proportional to the product $\chi_p^{(-)*}(\vec{r}_n)\chi_d^{(+)}(\vec{r}_n)$, which oscillates with constant magnitude as $r_n \to \infty$. Fortunately the long range of ρ_p is not a problem for the derivation of Eq. (17), which is based on integration over a large, but finite sphere; however, the long range does complicate the subsequent analysis of the integral expression for N (see Appendix and Sec. IV). Indeed it is seen in Sec. IV that the real part of the $\rho_p^*\bar{\psi}$ integral diverges strongly as the volume of integration becomes infinite. The imaginary part, which we require, is evaluated in terms of energy-shell, scattering wave functions whose asymptotic momenta never sum to zero; therefore it does converge (in the Césaro sense). This behavior of the imaginary integral is familiar from previous analyses of breakup[7] and stripping to unbound states.[8] Techniques to improve the convergence of such oscillatory integrals are available.

Although ρ_p also appears as a long-range source in Eq. (15), that equation is correctly solved by using an outgoing Green's function, as already noted in connection with Eq. (1).

Our new calculation of the energy-averaged inclusive breakup cross section is now based on the expression N of Eq. (18), which contains only ground state functions $\bar{\psi}$ and ρ_p. This expression becomes more symmetrical if we solve Eq. (15) for $\bar{\psi}$ in terms of ρ_p and substitute in N. Then

$$N = -\int d^3r_n \rho_p^* \operatorname{Im}[E_n^+ - K_n - U(r_n)]^{-1}\rho_p \ , \tag{19}$$

an expectation of the anti-Hermitian part of the Green's function for the unobserved neutron, with respect to the source function for ground state breakup. Further discussions of the properties of this expression and suggestions for practical approximations are given in Section IV and in the Appendix.

III. OPERATOR DERIVATION

This alternative approach to the derivation of the inclusive breakup cross section begins with a general formal expression[9] for breakup to a range of outgoing proton momenta $\Delta\vec{p}$,

$$\Delta\sigma = \frac{(2\pi)^4}{v_d}\sum_c |T_{d,pc}|^2 \delta(E - E_p - E_{Bc})\Delta\vec{p} \ , \tag{20}$$

in which $T_{d,pc}$ is the amplitude defined previously in Eq. (5), and v_d is the velocity of the incident deuteron in the center of mass coordinate system. The sum (integral) over c extends over all the outgoing states of the neutron + target nucleus system. However, the energy-conserving δ function restricts the sum over states and relates it to the observed range of proton momenta.

The δ function is expressed in terms of an energy denominator, so that

$$\frac{\Delta\sigma}{\Delta\vec{p}} = \frac{-(2\pi)^4}{\pi v_d}\operatorname{Im}\sum_c \frac{|T_{d,pc}|^2}{E^+ - E_p - E_{Bc}} \ . \tag{21}$$

It is now straightforward to insert Eq. (5) for $T_{d,pc}$ in Eq. (21), to replace the energy denominator by its operator equivalent, and to employ completeness for the sum on c. As a result,

$$\frac{\Delta\sigma}{\Delta\vec{p}} = \frac{-(2\pi)^4}{\pi v_d}\operatorname{Im}\Big[\langle \chi_d^{(+)}\phi_d\Phi_A|V_{np}|\chi_p^{(-)}\rangle \times \frac{1}{E^+ - E_p - H_B(\xi,n)} \times \langle\chi_p^{(-)}|V_{np}|\chi_d^{(+)}\phi_d\Phi_A\rangle\Big] . \tag{22}$$

Taking the ground state expectation of the full final state Green's function in Eq. (22) and averaging with respect to energy recovers the optical Green's function of Sec. II, so that

$$\frac{\Delta\sigma}{\Delta\vec{p}} = \frac{-(2\pi)^4}{\pi v_d}\operatorname{Im}\langle\rho_p|(E_n^+ - K_n - U_n)^{-1}|\rho_p\rangle \ , \tag{23}$$

as before. The coefficients needed to relate the previous quantity N to the observed inclusive cross section are now apparent.

IV. APPROXIMATIONS AND APPLICATIONS

Let us define M as the entire integral, say, from Eq. (19), whose imaginary part is N, thus

$$M \equiv \int d^3r_n \rho_p^*(E_n^+ - K_n - U)^{-1}\rho_p \ . \tag{24}$$

Because the convergence difficulties of M arise only at asymptotic radii, where U is negligible, they are very easily isolated by separating the Green's function into a free part plus a part that contains U, using the identity

$$M=\int d^3r_n\rho_p^*(E_n^+-K_n)^{-1}\rho_p + \int d^3r_n\rho_p^*(E_n^+-K_n)^{-1}U(E_n^+-K_n-U)^{-1}\rho_p . \tag{25}$$

The integration indicated in the second term of Eq. (25) has no convergence problems because U limits the volume of space that contributes to the integral. The first term of Eq. (25) separates uniquely, under the decomposition

$$(E_n^+-K_n)^{-1}=\frac{P}{E_n-K_n}-i\pi\delta(E_n-K_n) , \tag{26}$$

into a principal value integral that is real and divergent plus an on-energy-shell imaginary integral that converges. The principal value integral diverges because it contains a mixture of momenta, which in part match the asymptotic momentum of ρ_p; this introduces a nonoscillatory part in the integrand. This part of M is avoided when the imaginary part is taken. It is thus clear that approximations of N can be more trustworthy if they are based on the expression

$$N=\pi\int d^3r_n\rho_p^*\delta(E_n-K_n)\rho_p - \mathrm{Im}\int d^3r_n\rho_p^*(E_n^+-K_n)^{-1}U(E_n^+-K_n-U)^{-1}\rho_p , \tag{27}$$

from which the real part of the free Green's function has been entirely excluded.

Use of a plane wave expansion for the δ function in Eq. (27) gives

$$N=\frac{2\pi mk_n}{\hbar^2 v_d}\int d\hat{k}_n|\langle e^{i\vec{k}_n\cdot\vec{r}_n}|\rho_p\rangle|^2 - \mathrm{Im}\int d^3r_n\rho_p^*(E_n^+-K_n)^{-1}U(E_n^+-K_n-U)^{-1}\rho_p . \tag{28}$$

In the limit where the distorted waves in ρ_p also are plane waves, the integrand of the first term in Eq. (28) becomes zero by orthogonality because energy conservation prevents momentum matching, as noted in Sec. II. Evidently the first term of Eq. (28) tends to be small; it is omitted in some of the subsequent analysis.

An exact numerical evaluation of Eq. (28) in partial waves expansion would be straightforward and not prohibitively time consuming. Values of such functions as

$$\psi(\vec{r}_n)=(E_n^+-K_n-U)^{-1}\rho_p=\int d^3r_nG(\vec{r}_n,\vec{r}_n')\rho_p(\vec{r}_n')$$

are needed. For a given $\vec{r}_n$, this integral has the same convergence properties as the distorted waves breakup matrix element.

A. Surface approximation

Previous approximate summations[2,3] over target nucleus excited states rely on surface approximations for projectile wave functions. We can now see that use of corresponding approximations in our closed form expression for the inclusive sum restores the previous result. We omit the first term[2] of Eq. (28) and we insert partial wave expansions for the two Green's functions in the second term to obtain[10]

$$G(\vec{r},\vec{r}')=\langle\vec{r}|(E_n^+-K_n-U)^{-1}|\vec{r}'\rangle = -\frac{2m}{\hbar^2}\sum_{lm}\frac{f_l(r_<)h_l(r_>)}{k_nrr'}Y_{lm}(\hat{r})Y_{lm}^*(\hat{r}') . \tag{29}$$

Here f_l and h_l are regular and outgoing radial wave functions, respectively. A surface approximation[11] is introduced in Eq. (28) by the stipulation that $\rho_p(\vec{r}_n)\neq 0$ only if r_n is external to the interaction U. This stipulation selects one arrangement of the two radial arguments in Eq. (29), with the result that the remaining radial Green's function factors. The second term of Eq. (28) then reduces to

$$N\approx+\mathrm{Im}\frac{2mk_n}{\hbar^2}\sum_{lm}(T_{0lm}^{(-)F})^*t_l(T_{lm}^{(+)F}) , \tag{30}$$

in which

$$T_{lm}^{(+)F}\equiv\int d^3r_n[h_l(r_n)/k_nr_n]Y_{lm}^*\rho_p(\vec{r}_n) , \tag{31}$$

while $(T_{0lm}^{(-)F})^*$ is calculated from the corresponding free Green's function, and

$$t_l\equiv-\frac{\hbar^2k_n}{2m}\int dr_nf_{0l}^*(r_n)U(r_n)f_l(r_n) \tag{32}$$

is the amplitude for neutron scattering by the potential $U(r_n)$. Finally we recognize that the contributions of the radial Hankel functions $h_l(r_n)$ and $h_{0l}(r_n)$ in the amplitudes $T_l^{(+)F}$ and $T_{0l}^{(-)F}$ tend to be dominated[2] by the irregular functions $n_l(r_n)$, and under the surface approximation these functions are identical and real. Therefore Eq. (30) reduces to

$$N\approx\left(\frac{2mk_n}{\hbar^2}\right)\sum_{lm}\mathrm{Im}t_l|T_{lm}^{(+)F}|^2 , \tag{33}$$

in agreement with the previous analysis.[2] It is of course easy to repair the neglect of regular-function contributions, and so recover the result of Ref. 3.

We note that Eq. (33) is a linear function of t_l. Surface approximation would lead to this property even if the energy averaging of Eq. (14) had not been introduced. For this reason, surface ap-

proximation allows a very easy alternative treatment of fluctuations, as already noted in Ref. (2).

The ease with which the surface approximation result is recovered from Eq. (28) suggests the use of Eq. (28) to generate corrections to this approximation. This might be done by using a modified version of Eq. (28), in which $K+U$ is split into $(K+U_R)+(U-U_R)$, where U_R is an auxiliary real potential and only $(U-U_R)$ is treated by the surface approximation.

B. Use of real potential

An alternative approximation for N is derived by simply omitting the imaginary part of the optical potential from the Green's function for the unobserved neutron. The imaginary part of the Green's function then simplifies, so that from Eq. (23), say, we obtain

$$\frac{\Delta\sigma}{\Delta\vec{p}} \approx \frac{(2\pi)^4}{v_d} \int d^3r_n \rho_p^* \delta(E_n - K_n + V)\rho_p \tag{34}$$

$$\approx \frac{2\pi m k_n}{\hbar^2 v_d} \int d\hat{k}_n |\langle \chi_{\vec{k}_n}^- | \rho_p \rangle|^2 \ . \tag{35}$$

Here the δ function has been expanded in terms of scattering eignfunctions $\chi_{\vec{k}_n}^-$ that are normalized asymptotically to modulus unity and are governed by the equation

$$[E_n - K_n + V(r_n)]\chi_{\vec{k}_n}^- = 0 \ . \tag{36}$$

We recognize Eq. (35) as having the form of the familiar angle-integrated DWBA cross section for ground state (elastic) breakup, except that in the present discussion the $\chi_{\vec{k}_n}^-$ are calculated with a real neutron optical potential.

A principal motivation for the above approximation emerges from the observation that in the original flux conservation analysis of Eq. (17) the imaginary potential for the unobserved neutron accounted for transitions to excited channels. But the consequence of that analysis was to establish the meaning of the quantity N as a measure of the *total flux* into all breakup channels, irrespective of their individual physical nature. Because the neutron imaginary potential primarily rearranges some of this flux among different kinds of breakup channels, it should not have much effect on the total amount, and it should be reasonable to omit this potential from the calculation of the total breakup flux. Of course, our flux argument is rough—the channels through which the breakup flux emerges must have some influence on its creation.

Although the real potential approximation is not equivalent to the surface approximation discussed in A, it is to some extent supported by the previous discussion. Thus, under the surface approximation, Eq. (33) expresses N as a weighted sum of partial wave total cross sections $\mathrm{Im}t_l$. We find by direct calculation with the Becchetti-Greenlees neutron optical potential[12] that the distribution of these partial wave cross sections with respect to l is rather stable against omission of the imaginary potential. Therefore N should be stable against omission of this potential.

Thus it is plausible that under some conditions of practical interest the omission of the imaginary potential for the unobserved particles can produce simplifications of useful accuracy. In any case the simplifications obtained by this approximation are substantial; therefore, the approximation must be of interest for this reason alone.

Figure 1 shows an application of Eq. (35) by Baur[13] for the breakup reaction ^{62}Ni(α, ^{3}He) at the energy $E_\alpha = 172.5$ MeV, c.m. The inclusive cross section for ejectiles of energy $E_{^3\mathrm{He}} = 120$ MeV, c.m., is plotted as a function of the angular momentum of the incident projectile. (It is summed over l_n). The cross sections obtained from Eq. (35) are seen to agree well with previous results obtained by use of the surface approximation to sum explicit inelastic DWBA cross sections.

Other authors have suggested omitting the imaginary potentials for unobserved particles in inclusive reactions. For example, an application of real optical potentials in inclusive (p, p') reac-

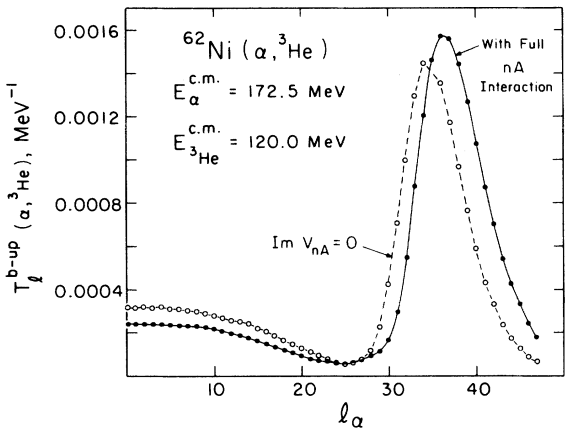

FIG. 1. Breakup probability (Ref. 13) for the reaction ^{62}Ni(α, ^{3}He) as a function of the α angular momentum. The solid line is calculated with the full Becchetti-Greenlees potential for the n-^{62}Ni system, with breakup to excited states of ^{62}Ni taken into account by surface approximation (Ref. 3). The calculation of the dashed line follows from Eq. (35); the ground-state breakup probability is evaluated *without* the imaginary part of the optical potential.

tions was discussed by Kroll and Wall.[14] A Green's function formula like Eq. (23) has also been developed for inclusive (e,e') reactions; however, the use of real optical potentials is not considered sufficiently accurate for this case.[15] Statistical reaction theories that deal with inclusive breakup from different starting points have also been developed recently.[16]

One condition under which the omission of $W(r_n)$ may be a questionable approximation is that the real potential $V(r_n)$ may have single-particle resonances[17] at low values of E_n. Such resonances, for example, would interfere with the stability of the $\mathrm{Im}t_i$ distribution when W is omitted. Although it might be possible to use Eq. (35) even in the presence of resonances, and to energy average the calculated cross sections, it would be difficult to claim much value for such a procedure. The omission of W may also cause incorrect threshold behavior[13] at $E_n=0$.

V. SUMMARY

An identity is derived which expresses the DWBA inclusive breakup cross section in closed form as an expectation of an optical model Green's function for the unobserved system. The optical potential automatically takes account of transitions to excited states of the target nucleus, without any need for surface approximations or explicit analyses of form factors. The energy averaging implied by an optical potential analysis also takes account of any fluctuations of the breakup cross section as a function of the energy transfer.

The exact identity is put into a form that is suitable for explicit numerical evaluation. It is also seen to be simple to recover the previous result derived by surface approximation. In an alternative approximation the imaginary part of the optical potential is omitted; only the imaginary part of the Green's function then contributes, and the basic identity reduces to a simple modification of the cross section for inelastic breakup.

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APPENDIX: ENERGY AVERAGING

An inclusive cross section $N(E_n)$ that fluctuates with respect to bombarding energy or observed breakup energy is defined if we omit the energy-averaging step of Eq. (14). Then

$$N(E_n)=-\,\mathrm{Im}\langle\rho|\,(E_n^+-K_n-\mathfrak{U}_A)^{-1}\,|\rho\rangle\ , \tag{A1}$$

where $\mathfrak{U}_A$ is the exact formal effective interaction of Eq. (13). Equation (A1) is also equivalent to Eq. (23) of the main text. We now examine the detailed procedure by which $N(E_n)$ goes over to $\bar{N}$, the energy average.

We begin by removing the real part of the free Green's function from Eq. (A1), as in Eqs. (25)–(28), to prevent the averaging process from accidentally mingling divergent and convergent expressions. Then $N(E_n)$ becomes

$$N(E_n)=-\langle\rho_p|\,\mathrm{Im}(E_n^+-K_n)^{-1}\,|\rho_p\rangle - \mathrm{Im}\langle\rho_p|\,(E_n^+-K_n)^{-1}\mathfrak{U}_A(E_n^+-K_n-\mathfrak{U}_A)^{-1}\,|\rho_p\rangle\ . \tag{A2}$$

We see that the first term of Eq. (A2) is a slowly varying function of E_n. Therefore it is not affected when $N(E_n)$ is averaged over the compound nucleus resonances contained in $\mathfrak{U}_A$.

It is not obvious how the second term of Eq. (A2) should be averaged, because it depends nonlinearly on the fluctuating quantity $\mathfrak{U}_A$. Fortunately, any causal analytic function of E_n, suitably bounded at infinity, can be averaged over a Lorentzian of width I by simply replacing E_n by E_n+iI. For this type of averaging, it is easy to see that averaging over E_n is equivalent to averaging first over the energy on which $\mathfrak{U}_A$ depends and then over the E_n that appears explicitly in Eq. (A2). The first averaging gives

$$-\,\mathrm{Im}\langle\rho_p|\,(E_n^+-K_n)^{-1}\,\bar{\mathfrak{U}}_A(E_n^+-K_n-\bar{\mathfrak{U}}_A)^{-1}\,|\rho_p\rangle\ , \tag{A3}$$

where

$$\bar{\mathfrak{U}}_A(E_n)=\mathfrak{U}_A(E_n+iI)\ . \tag{A4}$$

Because $\bar{\mathfrak{U}}_A$ has an appreciable imaginary part, Eq. (A3) cannot have resonances, and because energy conservation prohibits momentum matching, Eq. (A3) has no infinities. Therefore Eq. (A3) is a smooth function, so that the effect of the second averaging can consistently be neglected. Thus the average of Eq. (A2) can be taken as

$$\bar{N}=-\langle\rho_p|\,\mathrm{Im}[E_n^+-K_n]^{-1}\,|\rho_p\rangle - \mathrm{Im}\langle\rho_p|\,(E_n^+-K_n)^{-1}\bar{\mathfrak{U}}_A(E_n^+-K_n-\bar{\mathfrak{U}}_A)^{-1}\,|\rho_p\rangle\ . \tag{A5}$$

This may be regarded as the definition of the formally equivalent but more compact expression

$$\bar{N} = -\,\mathrm{Im}\langle \rho_\phi \,|\, [E_n^+ - K_n - \bar{u}_A]^{-1} \,|\, \rho_\phi \rangle \qquad \text{(A6)}$$

used in the main text.

We still have to verify the familiar result that $\bar{u}_A$ can be identified with the optical potential U, as in Eq. (14) of the text. This is done by showing that $\bar{u}_A$ generates the energy-averaged T matrix for neutron nucleus scattering. The Lorentzian average of the transition operator generated by u_A is

$$\bar{T}_A = \bar{u}_A + \bar{u}_A (E_n + i\Gamma - K_n - \bar{u}_A)^{-1} \bar{u}_A \ . \qquad \text{(A7)}$$

As before, this can be replaced by

$$\bar{T}_A = \bar{u}_A + \bar{u}_A (E_n^+ - K_n - \bar{u}_A)^{-1} \bar{u}_A \ . \qquad \text{(A8)}$$

Finally, taking momentum-space matrix elements of Eq. (A8) establishes that $\bar{u}_A$ generates the energy-averaged T matrix as required. An energy-averaged wave function can be constructed from $\bar{T}_A$ in the usual way.

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