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(p-n) interacting boson approximation model in the O(6) limit and the spectra of ^{196,198,202}Hg

I. Morrison

Department of Theoretical Physics, Research School of Physical Sciences, The Australian National University, Canberra, A.C.T., 2600, Australia

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The interacting boson approximation is extended to include equivalent proton and neutron bosons and the spectra of ^{196,198,202}Hg are discussed in terms of the O(6) group appropriate to this mass region.

[NUCLEAR STRUCTURE IBA model O(6) limit with equivalent p-n bosons, ^{196,198,202}Hg calculated levels, J,π.]

The interacting boson approximation (IBA) of Arima and Iachello[1-3] has been applied with success to nuclei which are vibrators,[4] axial rotors,[5] and γ-unstable rotors.[6] In addition in the Os-Pt region, the transition from a γ-unstable rotor to axial-rotor description can be interpreted[7] in terms of the underlying boson Hamiltonian changing from the O(6) (Ref. 3) to SU(3) (Ref. 5) limits.

The above IBA models suffer from the objection that only one type of boson has been considered, whereas both proton and neutron bosons should be required, in general, to describe experimental data. Previous considerations of active proton-neutron (p-n) boson systems have discussed relevance in terms of the underlying Hamiltonian,[8] relevance in terms of the underlying fermion shell model,[9] and have performed numerical calculations.[10]

In this paper, we present some analytical results in the (p-n) O(6) limits and apply them to ^{196,198,202}Hg.

Using the concept of F spin[8] and assuming that the underlying boson Hamiltonian is F-spin (charge) independent, the N boson wave function, carrying the totally symmetric irreducible representation $[N]$ of the group U(12) [dimension 2 for F spin $\times$ 6 for the (s-d) boson space], can be decomposed[11]:

$$\mathrm{U}(12)\supset \mathrm{U}(2)\times \mathrm{U}(6)\,, \tag{1a}$$

$$[N]\Rightarrow \sum_{\lambda}[\lambda]\otimes[\lambda]\,, \tag{1b}$$

where the representation $[\lambda]$ of U(2), having at most two rows, can be written $[N-a,a]$. Denoting the number of proton (neutron)bosons as $N_p(N_n)$, respectively,

$$\begin{aligned} F &= (N-2a)/2\,,\\ M_F &= (N_p-N_n)/2\,, \end{aligned} \tag{2}$$

where $N=N_p+N_n$; the proton (neutron) boson has F-spin projection $+\frac{1}{2}(-\frac{1}{2})$.

The standard IBA models have $a=0$, $F\equiv M_F=N/2$ and hence derive from the irreducible representation $[N]$ of U(6). In general, however, the two-rowed irreducible representation $[N-a,a]$ of U(6) will introduce new states into any spectrum, their importance being dependent on the choice of boson Hamiltonian in the U(6) subgroup chain, e.g., (i) vibrator $\mathrm{U}(6)\supset\mathrm{U}(1)\times\mathrm{U}(5)$, (ii) axial rotor $\mathrm{U}(6)\supset\mathrm{SU}(3)$, and, in particular, (iii) γ-unstable rotor:

$$\begin{array}{cccccccc} \mathrm{U}(6) & \supset & \mathrm{R}(6) & \supset & \mathrm{R}(6) & \supset & \mathrm{R}(3) & \quad (3\mathrm{a})\\ \downarrow & & \downarrow & & \downarrow & & \downarrow & \\ [N-a,a] & & (\omega_1\omega_2\omega_3) & & (\tau_1\tau_2) & & L\,, & \quad (3\mathrm{b}) \end{array}$$

where the irreducible representation labels are given for the various groups. Assuming, as usual,[3,5] that the boson Hamiltonian H_B can be written in terms of the Casimir operators of the group chain, we have

$$\begin{aligned} H_B \equiv\; & C_1\mathcal{G}(\mathrm{U}(6))+C_2\mathcal{G}(\mathrm{R}(6))\\ & +C_3\mathcal{G}(\mathrm{R}(5))+C_4\mathcal{G}(\mathrm{R}(3))\,. \end{aligned} \tag{4}$$

The two-rowed irreducible representations $[N-a,a]$ of U(6) generate irreducible representations $(\omega_1\omega_2\omega_3)$ of R(6) with $\omega_3=0$ [hence $\mathrm{R}(6)\equiv\mathrm{O}(6)$], and on expanding the Casimir operators[12] we find the eigenstate $|[N-a,a](\omega_1\omega_2 0)(\tau_1\tau_2)L\rangle$ of H_B has energy

$$\begin{aligned} E = & A[(N-a)(N-a-1)+a(a-3)]\\ & +B[\omega_1(\omega_1+4)+\omega_2(\omega_2+2)]\\ & +C[\tau_1(\tau_1+3)+\tau_2(\tau_2+1)]\\ & +DL(L+1)\,. \end{aligned} \tag{5}$$

The spectra generated in the identical boson O(6) limit[3] is a subset of (5), obtained on the restriction $a=0$ (hence $\omega_2=\tau_2=0$). The parameters B,C,D have the same physical interpretation,[3] namely pairing strength, d phonon energy, and angular momentum shift. The parameter A serves to split those irreducible representations $(\omega_1\omega_2 0)(\tau_1\tau_2)$ which occur in different irreducible representations of U(6) and hence can be viewed as an F-spin splitting as the first term in (5) can be written

$$A[\tfrac{1}{2}N(N-4)+2F(F+1)]\,. \tag{6}$$

This expression is equivalently generated by assuming a residual Majorana type space exchange interaction P_{12}^{S} (and hence $+P_{12}^{F}$) between the bosons.

The relevance of the group chain (3a) has been discussed in the fermion (s-d) shell by Elliott,[13] where tables of the decomposition (3b) are given for $N\leq 4$. The decomposition for $N>4$ can be obtained using the techniques given in Wybourne.[14] Generalization to F-spin conserving H_B (charge independent) which break the symmetry of the $R(6)\supset R(5)$ chain has been discussed, in the atomic fermion case, in terms of the group generators by Feneuille,[12] but for application to the Hg isotopes the symmetry conserving form (5) was used.

The first "new" state from the $[N-1,1]$ irreducible representation will be $|[N-1,1](N-1,1,0)(10)2\rangle$, having a separation from the 2_1^+ state $|[N](N00)(10)2\rangle$ of

$$-2N(A+B)\,, \tag{7}$$

and hence for physical choices of A, B [$A<0$, $B<0$: irreducible representation $[N]$ of U(6) (standard model) forming the lowest states[8]], states with $a\neq 0$ will linearly increase in excitation energy with boson number N. Hence attempts to find these states in systems with a large boson number will fail.[10]

The Hg isotopes would offer an ideal opportunity, having small $N\approx 2$–6, to look for these "Majorana"[10] states in an area of γ instability where the O(6) limit is appropriate. The model predictions for the spectra of ^{196,198,202}Hg are compared with experiment[15-17] in Figs. 1–3. In each nucleus, the parameter set (B,C,D) was obtained from the

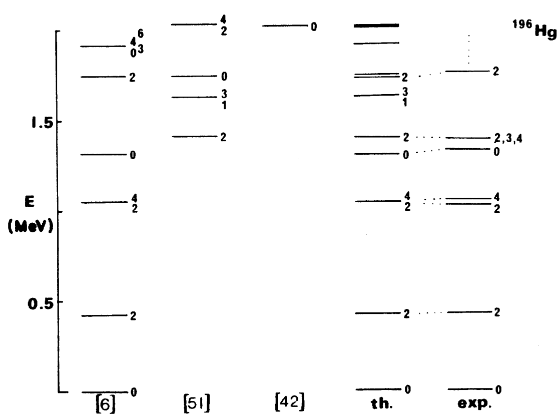

FIG. 1. Level spectra for ^{196}Hg. $[A,B,C,D]=[-0.034, -0.047, 0.106, 0.0]$.

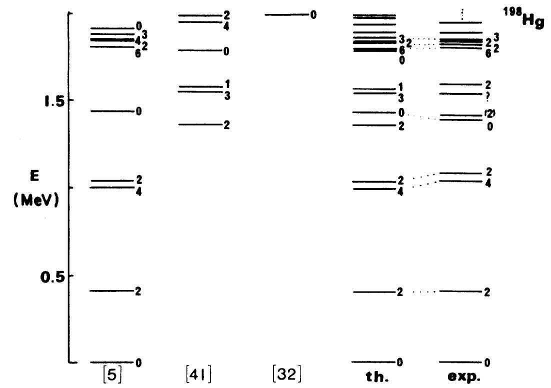

FIG. 2. Level spectra for ^{198}Hg. $[A,B,C,D]=[-0.034, -0.060, 0.106, -0.0025]$.

ground state band (gsb, irreducible representation $[N]$) excitation energies, and in each case A was small. In all cases, it is possible to make level assignments not only for the gsb$[N]$, but also for Majorana states $[N-1,1]$, particularly for the $J=(1,2,3)$ cluster around 1.5 MeV excitation. The mapping is not complete due to ambiguities in experimental spin assignments but three average features are "explained" by inclusion of the following $[N-1,1]$ states:

(i) extra states in 1–1.5 MeV region,

(ii) very high density of states $\gtrsim 1.6$–1.7 MeV,

(iii) three 0^+ states at ~ 1.6 MeV.

However, by themselves, level spectra are not conclusive and it is possible to also study $E2$ decay patterns. Writing the $E2$ operator, in general, as

$$T(E2)=e_p(s^+d+d^+s)_p+e_n(s^+d+d^+s)_n+e_p'(d^+d)_p+e_n'(d^+d)_n\,, \tag{8}$$

$e_p(e_n)$ being effective proton (neutron) transition charges, the very weak cross-over decay $2_2^+\to 0_1^+$,

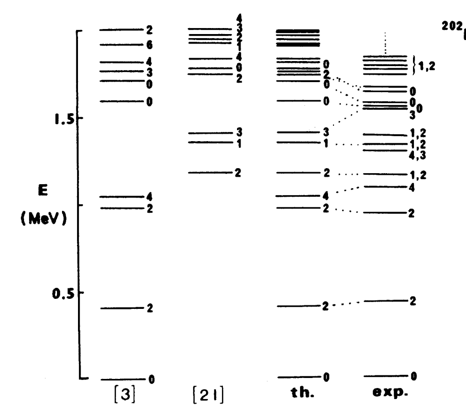

FIG. 3. Level spectra for ^{202}Hg. $[A,B,C,D]=[-0.030, -0.100, 0.095, 0.005]$.

which could proceed via the $\Delta\tau = 0, \pm 2$ (d^+d) operator (where $\tau = \tau_1 + \tau_2$), forces $e_p' \approx e_n' \approx 0$.

Then

$$T(E2) = e_p(s^+d + d^+s)_p + e_n(s^+d + d^+s)_n \qquad (9)$$

and, in the case $e_p = e_n$, this expression reverts to that of Arima[3] with the $E2$ selection rules (a) $T(E2)$ will not couple different U(6) and R(6) irreducible representations; (b) $\Delta\tau = \pm 1$; and hence the Majorana states cannot decay to the $[N]$ gsb.

However, it is unreasonable to expect $e_p = e_n$ even if H_B is charge independent (cf., fermion case). If $e_p \neq e_n$ then only condition (b) above holds and the Majorana states can decay to the gsb. Values for both e_p and e_n can be unambiguously extracted from $B(E2, 2_1 \to 0_1)$ as the following relationship holds for the gsb:

$$S \equiv \left[\frac{B(E2, 0_1 \to 2_1)}{N(N+4)}\right]^{1/2} = e_s + e_v \frac{M_F}{F}, \qquad (10)$$

where $e_s(e_v)$ is the F-scalar (-vector) transition charge. A plot of $S : M_F/F$ over a range of N values will give e_s (intercept), e_v (slope), and hence e_p, e_n. Using the results of Bockisch *et al.*[18] for ^{204,202,200}Hg, a linear dependence is indeed obtained giving $e_p = 23.86\ e\,\mathrm{fm}^2$, $e_n = 13.82\ e\,\mathrm{fm}^2$. The ratio e_n/e_p (0.58) is identical to that obtained by Ma and True[19] in fermion shell model calculations in this mass region. Extension of the above technique to lighter Hg and Pt isotopes gives an empirical indication of phase transitions in ^{190}Pt and ^{200}Hg.[20]

Using the above values for e_p, e_n in ^{202}Hg, the transitions from the Majorana levels to gsb remain weak $[B(E2) \approx 30\ e^2\,\mathrm{fm}^4]$ in comparison with the $B(E2, 2_1 \to 0_1)$ of 1233 $e^2\,\mathrm{fm}^4$ and hence do not differ significantly from the $e_p = e_n$ value of 0. The general $E2$ decay pattern is in fair agreement with the experimental work of Breitig[17] but it should be noted that the experiment assigned $B(E2)$ rates, normalized to a theoretical model, to tentative level spins and therefore detailed comparison is not possible. However, for the Majorana states, the model predicts

(i) $2_3 \to 2_2, 0_1, 4_1$ (weak);
(ii) $1_1 \to 2_3$ (strong), 2_1 (weak);
(iii) $3_1 \to 2_3$ (strong), 2_1 (weak);

and these features can generally be seen. The decays of the 2_2, 4_1, 0_2 states are adequately explained by normal selection rules applied to the $[N]$ irreducible representation. There is an obvious need for more experimental work on $E2$ rates in the Hg isotopes, particularly for states in the 1–2 MeV region, if the $(p\text{-}n)$ O(6) model is to be tested even in its symmetry conserving limit.

The extension to discuss symmetry breaking in H_B can best be divided into two areas: (i) F-spin conserving but R(6) breaking, e.g., via $\vec{Q}\cdot\vec{Q}$ quadrupole force. This is necessary for $Q_{2_1^+}$ predictions. (ii) F-spin breaking.

From the fact that the model appears to work in the Hg isotopes ($N_p = 1$, $N_n = 1$–5) *and* that the spectra of ^{196,198}Pt ($N_p = 2$) is *not* exactly identical to ^{196,198}Hg, we conclude that the p-n and n-n boson force is identical but that the p-p force differs. Symmetry breaking via (ii) would therefore appear more appropriate although again more experimental work would be required in the even-even 196 $< A < 204$ mass region.

In conclusion, the O(6) limit of the IBA model has been extended to equivalent $(p\text{-}n)$ bosons. The irreducible representation $[N]$, as in the standard model, produces the gsb and the "intruder" Majorana states by having an excitation energy which increases with boson number N, and would usually only be seen in nuclei with N small. Current experimental data in the Hg isotopes suggest that the model is a useful extension, but more experimental data is required, not only for electromagnetic transitions but especially for the location of 1^+ states which are the earmark of non-totally symmetric $[N-a, a]$ representations. Collection of this data would test model predictions particularly in investigating F-spin breaking in Pt and Hg.

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