Induced tensor and ft asymmetries

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Looked at from a model-independent point of view, the existence of a non-zero first class induced tensor form factor appears to provide a substantial contribution to the observed ft asymmetry in mirror Gamow-Teller beta decays. However, we demonstrate that, when analyzed in terms of the impulse approximation, this is only a negligibly small effect.

[RADIOACTIVITY Calculated contribution to ft asymmetry from first class induced tensor.]

In several discussions concerning mirror Gamow-Teller transitions, the question has arisen: Since the existence of an induced tensor has been confirmed for several systems, shouldn't this contribute several percent to the observed ft asymmetry? The purpose of this note is to provide an answer to this question.

First we define notation. Consider a nuclear beta decay of the Gamow-Teller type. We define

$$T=\frac{G}{\sqrt{2}}\cos\theta_c l_\mu \langle\beta|A^\mu|\alpha\rangle, \tag{1}$$

where $G\cong 10^{-5}m_p{}^2$ is the weak coupling constant, $\theta_c\approx 13°$ is the Cabibbo angle, and

$$l_\mu=\begin{cases}\bar{u}_e\gamma_\mu(1+\gamma_5)v_\nu, & e^- \text{ decay}\\ \bar{v}_e\gamma_\mu(1+\gamma_5)u_\nu, & e^+ \text{ decay}\end{cases} \tag{2}$$

is the lepton current. Let the parent (daughter) nucleus $\alpha(\beta)$ have four-momentum $p_1(p_2)$ spin $J(J')$ and spin component $M(M')$ along some quantization axis, and define the auxiliary variables

$$q=p_1-p_2,\quad P=p_1+p_2,\quad M=\tfrac{1}{2}(M_1+M_2). \tag{3}$$

Then for the matrix element of the axial current A_μ we write

$$\langle\beta_{p_2}|A_\mu|\alpha_{p_1}\rangle\equiv C^{M';k;M}_{s';1;J}\,\epsilon_{ijk}\epsilon_{ij\mu\eta}\times\frac{1}{4M}\left[c(q^2)P^\eta-d(q^2)q^\eta\right]. \tag{4}$$

Here $c(q^2)$ is the usual Gamow-Teller form factor while $d(q^2)$ is the induced tensor. We shall assume the absence of second class currents,[1] so that $d(q^2)$ must vanish for a transition between members of a common isotopic multiplet.[2] However, for a nonanalog transition $d(q^2)$ can be, and generally is, non-zero. In fact, via delayed particle-correlation experiments $d(q^2)$ has been measured for at least three systems:

$A=8$ $\quad {}^8\mathrm{B}\to{}^8\mathrm{Be}^*\,(2^+,\ 2.90\ \mathrm{MeV})+e^++\nu_e,$

${}^8\mathrm{Li}\to{}^8\mathrm{Be}^*\,(2^+,\ 2.90\ \mathrm{MeV})+e^-+\bar{\nu}_e,$

$d/Ac=0.5\pm0.5$

(see Ref. 3),

$A=12$ $\quad {}^{12}\mathrm{N}\to{}^{12}\mathrm{C}+e^++\nu_e,$

${}^{12}\mathrm{B}\to{}^{12}\mathrm{C}+e^-+\bar{\nu}_e,$

$d/Ac=3.9\pm0.5$

(see Ref. 4),

$A=20$ $\quad {}^{20}\mathrm{Na}\to{}^{20}\mathrm{Ne}^*\,(2^+,\ 1.63\ \mathrm{MeV})+e^++\nu_e,$

${}^{20}\mathrm{F}\to{}^{20}\mathrm{Ne}^*\,(2^+,\ 1.63\ \mathrm{MeV})+e^-+\bar{\nu}_e,$

$d/Ac=6.6\pm2.0$

(see Ref. 5), where A is the atomic mass number. In the cases $A=8$ and $A=20$ the "experimental" values of d/Ac are somewhat dependent on impulse approximation predictions of higher order form factors.[6] However, for the $A=12$ case this measurement is model independent.

Non-zero values of the induced tensor can in principle affect the ft asymmetry due to the differing end point energies for the e^+, e^- decays, and due to induced Coulomb effects[7]

$$\frac{ft^+}{ft^-}=\left(\frac{c_-}{c_+}\right)^2+\frac{2}{3M}\frac{d}{Ac}\left(E_0^+-E_0^-+\frac{3}{2}\frac{\alpha Z}{R}\right), \tag{5}$$

where here c_+, c_- are the respective Gamow-Teller form factors for the e^+, e^- transition. In general $c_+/c_-\neq 1$ due to binding energy differences and other Coulombic effects, and the size of the ft asymmetry arising from this difference has been estimated by Wilkinson, Towner, and others.[8] However, comparison with experiment has not included the effects of the induced tensor, and, using the experimentally determined values given previously, this effect appears to be sizable:

A	$\frac{2}{3M}\frac{d}{Ac}\left(E_0^- - E_0^- + \frac{3}{2}\frac{\alpha Z}{R}\right)$	$\left(\frac{ft^-}{ft^+} - 1\right)$expt.[9]
8	$+0.2\pm 0.2\%$	$10.6\pm 1.1\%$
12	$+2.2\pm 0.3\%$	$11.6\pm 0.1\%$
20	$+5.3\pm 1.6\%$	$4.1\pm 1.1\%$

Thus for $A=20$ this effect appears comparable to the entire experimental asymmetry, while for $A=12$ it constitutes a considerable fraction of the experimental effect. This would seem then to impact previous theoretical estimates for c_+/c_-.

From a model-independent standpoint this is as far as we can go. However, within the context of the impulse approximation we shall see that this effect is much reduced and can be neglected in attempting to understand the ft asymmetry. The impulse approximation predictions for c, d are[10]

$$c \cong g_A\left(\mathcal{M}_\sigma + A\mathcal{M}_{\sigma L}\frac{E_0}{2M} + \frac{E_0}{2}\mathcal{M}_{\sigma rp}\right),$$
$$d \cong g_A(-\mathcal{M}_\sigma + A\mathcal{M}_{\sigma L} + M\mathcal{M}_{\sigma rp}), \tag{6}$$

where

$$\mathcal{M}_\sigma = \left\langle\beta\left|\left|\sum_i \tau_i\vec{\sigma}_i\right|\right|\alpha\right\rangle,$$
$$\mathcal{M}_{\sigma L} = i\left\langle\beta\left|\left|\sum_i \tau_i\vec{\sigma}_i\times\vec{L}_i\right|\right|\alpha\right\rangle, \tag{7}$$
$$\mathcal{M}_{\sigma rp} = \frac{i}{2m}\left\langle\beta\left|\left|\sum_i \tau_i[\{\vec{\sigma}\cdot\vec{r}_i,\vec{p}_i\}+\{\vec{\sigma}\cdot\vec{p}_i,\vec{r}_i\}]\right|\right|\alpha\right\rangle$$

are reduced matrix elements. Hence there is a piece of the Gamow-Teller form factor c which depends on the end point energy and affects the ft asymmetry. We find[10]

$$\left(\frac{c_+}{c_-}\right)^2 = \left[\frac{\mathcal{M}_\sigma^{(+)}}{\mathcal{M}_\sigma^{(-)}}\right]^2 + \frac{(E_0^+ - E_0^-)}{M}\left(A\frac{\mathcal{M}_{\sigma L}}{\mathcal{M}_\sigma} + M\frac{\mathcal{M}_{\sigma rp}}{\mathcal{M}_\sigma}\right)$$
$$\cong \left[\frac{\mathcal{M}_\sigma^{(+)}}{\mathcal{M}_\sigma^{(-)}}\right]^2 + \frac{(E_0^+ - E_0^-)}{m}\frac{d}{Ac} \tag{8}$$

$$\left(\frac{ft^-}{ft^+} - 1\right)^{\text{theory}} = \left[\frac{\mathcal{M}_\sigma^{(+)}}{\mathcal{M}_\sigma^{(-)}}\right]^2 - \frac{1}{3M}\frac{d}{Ac}\left(E_0^+ - E_0^- - 3\frac{\alpha Z}{R}\right) - 1. \tag{9}$$

Using the Coulomb energy difference estimate

$$E_0^+ - E_0^- \cong \frac{12}{5}\frac{\alpha Z}{R}, \tag{10}$$

we find then

$$\left(\frac{ft^+}{ft^-} - 1\right)^{\text{theory}} = \left[\frac{\mathcal{M}_\sigma^{(+)}}{\mathcal{M}_\sigma^{(-)}}\right]^2 + \frac{1}{5M}\frac{d}{Ac}\frac{\alpha Z}{R} - 1. \tag{11}$$

Looked at in this way, the induced Coulomb distribution is reduced by more than a factor of 10 from the naive estimate given previously and can be included simply as part of the rather considerable error associated with the theoretical estimates of $(\mathcal{M}_\sigma^+/\mathcal{M}_\sigma^-)^2$.[9]

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[1]S. Weinberg, Phys. Rev. 112, 1375 (1958).
[2]B. R. Holstein and S. B. Treiman, Phys. Rev. C 3, 1921 (1971); S. P. Rosen, Phys. Rev. D 5, 760 (1972).
[3]R. E. Tribble and G. T. Garvey, Phys. Rev. C 12, 967 (1975); R. McKeown, Princeton University Ph.D. thesis, 1979 (unpublished).
[4]W-Y. P. Hwang, Phys. Rev. C 20, 805 (1975); 20, 814 (1979).
[5]R. E. Tribble and D. P. May, Phys. Rev. C 18, 2704 (1978).
[6]B. R. Holstein, Phys. Rev. C 5, 1947 (1972).
[7]A. Bottino, G. Ciochetti, and C. W. Kim, Phys. Rev. C 9, 2052 (1974); B. Holstein, *ibid.* 10, 1215 (1974).
[8]D. H. Wilkinson, Phys. Rev. Lett. 27, 1018 (1971); I. S. Towner, Nucl. Phys. A216, 589 (1973).
[9]D. H. Wilkinson, Phys. Lett. 48B, 169 (1974).
[10]B. R. Holstein, Rev. Mod. Phys. 46, 789 (1974). The derivation of Eqs. (6) and (8) is outlined in Appendix A.