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Energy and width of Σ hyperon in nuclear matter

J. Dąbrowski and J. Rożynek
Institute of Nuclear Research, Hoża 69, PL-00-681 Warsaw, Poland
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The width Γ and the potential well depth $-V_\Sigma$ of the Σ hyperon in nuclear matter are calculated with the help of the Brueckner theory. Results obtained for Γ and V_Σ with model D of the Nijmegen baryon-baryon interaction and simple estimates of Γ in terms of the cross section for the $\Sigma^- p \to \Lambda n$ process are consistent with empirical estimates. It is shown that the exclusion principle and dispersive effects strongly suppress the $\Sigma N \to \Lambda N$ process in nuclear matter and are capable of reducing Γ to values suggested by recent (K,π) experiments.

[NUCLEAR STRUCTURE Energy and width of Σ hyperon in nuclear matter calculated; ΣN interaction with $\Sigma\Lambda$ coupling.]

I. INTRODUCTION

Hypernuclei with Σ particles have been recently observed at CERN in the (K,π) reaction.[1] The shape and position of the observed pion spectrum suggest that the width of the Σ states $\Gamma \lesssim 8$ MeV, and that the nuclear potential well depth of Σ, $-V_\Sigma$, is approximately the same as that of Λ, $-V_\Lambda$, i.e., ~20–30 MeV. Most surprising is the narrow width Γ of these states which are expected to undergo a fast decay via the strong conversion process $N\Sigma \to N\Lambda$.

In the present paper, we estimate the energy $E_\Sigma = V_\Sigma - i\Gamma/2$ of the ground state of Σ in nuclear matter (NM). We apply the Brueckner reaction matrix method, applied previously[2] in calculating the energy E_Λ of the ground state of Λ in NM. We show that the exclusion principle and dispersive effects strongly suppress the $N\Sigma \to N\Lambda$ process in NM, and are capable of reducing Γ to values suggested by the (K,π) experiments.

The sensitivity of Γ to the dispersive effects, i.e., to the momentum dependence of the single particle (s.p.) potentials in NM, makes a precise calculation of Γ difficult. Namely, the proper form of s.p. potentials remains an open problem of the Brueckner theory. In this situation, we apply two limiting forms of s.p. potentials, and obtain upper and lower bounds for Γ and V_Σ.

The plan of the paper is as follows. Section II outlines our theoretical scheme of applying the Brueckner theory in calculating V_Σ and Γ. Optical theorem type identities satisfied by the reaction matrix lead to a connection between Γ and the cross section σ for the $\Sigma^- p \to \Lambda n$ process. Section III discusses the reduction of the phase space available to nucleons emerging from the $\Sigma N \to \Lambda N$ process in NM, caused by the exclusion principle. This reduction is strongly enhanced by dispersive effects, estimated for two limiting choices of the s.p. potentials. Section IV gives simple estimates of Γ in terms of σ. Section V outlines a calculation of V_Σ and Γ with the Nijmegen baryon-baryon interaction. Section VI summarizes our results and discusses previous estimates of Γ.

II. FORMALISM

To describe the YN interaction $(Y=\Sigma,\Lambda)$ we use the two channel approach[3] with a 2×2 potential matrix

$$\hat{v} = \begin{pmatrix} v(\Sigma N \to \Sigma N) & v(\Lambda N \to \Sigma N) \\ v(\Sigma N \to \Lambda N) & v(\Lambda N \to \Lambda N)\end{pmatrix} = \begin{pmatrix} v_{\Sigma\Sigma} & v_{\Sigma\Lambda} \\ v_{\Lambda\Sigma} & v_{\Lambda\Lambda}\end{pmatrix}. \tag{2.1}$$

The $\Sigma\Lambda$ conversion occurs only in the $T=\frac{1}{2}$ state, and only in this isospin state is the two-channel approach necessary. In the $T=\frac{3}{2}$ state, the only nonvanishing component of $\hat{v}$ is $v_{\Sigma\Sigma}$, and only the ΣN channel exists.

To calculate the ground state energy E_Σ of Σ in NM, we apply the Brueckner reaction method (see Ref. 2, and references quoted therein). In the case of two channels $(T=\frac{1}{2})$, the reaction matrix $\hat{K}^{(+)}$ for YN interaction in NM is a 2×2 matrix, with four components denoted by $K^{(+)}_{\Sigma\Sigma}$, $K^{(+)}_{\Sigma\Lambda}$, $K^{(+)}_{\Lambda\Sigma}$, $K^{(+)}_{\Lambda\Lambda}$, similar to the four components of $\hat{v}$, Eq. (2.1). In the $T=\frac{3}{2}$ state, the only nonvanishing component of $\hat{K}^{(+)}$ is $K^{(+)}_{\Sigma\Sigma}$.

The reaction matrix equation is

$$\hat{K}^{(+)}(z) = \hat{v} + \hat{v}\hat{G}^{(+)}(z)\hat{K}^{(+)}(z)\,, \tag{2.2}$$

where $\hat{G}^{(+)}$ is a diagonal matrix with the diagonal components

$$\begin{aligned} G^{(+)}(z)_{\Sigma\Sigma} &= Q_\Sigma/(z - h_N - h_\Sigma + i\epsilon)\,, \\ G^{(+)}(z)_{\Lambda\Lambda} &= Q_\Lambda/(z + \Delta - h_N - h_\Lambda + i\epsilon)\,. \end{aligned} \tag{2.3}$$

We denote by Q_Y the exclusion principle operator in the YN channel (a projection operator onto nucleon states above the Fermi sea); $\Delta = (M_\Sigma - M_\Lambda)c^2$ and h_X $(X=N,Y)$ are s.p. Hamiltonians in NM

$$h_X = (\hbar^2/2M_X)\Delta_X + V_X^*\,, \tag{2.4}$$

where V_X^* is the s.p. potential of X in NM (in an intermediate excited state).

The parameter z, which determines the "starting energy" of the interacting $N\Sigma$ pair, is negative. However, $z+\Delta$ ($\Delta\approx 80$ MeV) may become positive, and singularities are expected to appear in $G_{\Lambda\Lambda}^{(+)}$. In other words, real energy conserving transitions $\Sigma N\to\Lambda N$ are expected to occur. The infinitesimal parameter $+i\epsilon$ guarantees that only outgoing waves appear in the ΛN channel. The appearance of real $\Sigma N\to\Lambda N$ transitions leads to a complex value of E_Σ. We identify its imaginary part with $-\Gamma/2$.

For E_Σ, we have

$$E_\Sigma=\tfrac{1}{3}E_\Sigma{}^{1/2}+\tfrac{2}{3}E_\Sigma{}^{3/2}, \tag{2.5}$$

where

$$E_\Sigma^T=\sum_{\mathbf{k}_N}^{k_F}(\vec{\mathbf{k}}_N\vec{\mathbf{k}}_\Sigma=0\,|K_{\Sigma\Sigma}^{(+)}(z(k_N))^T|\,\vec{\mathbf{k}}_N\vec{\mathbf{k}}_\Sigma=0)\,. \tag{2.6}$$

To simplify the notation, spins are suppressed here. Notice that in the ground state of the noninteracting Σ + NM system, the Σ hyperon has zero momentum, $\vec{\mathbf{k}}_\Sigma=0$. (All momenta are in units of $\hbar$.) The Fermi momentum of symmetric NM ($N=Z$) is denoted by k_F, and

$$z(k_N)=\hbar^2k_N{}^2/2M_N+V_N(k_N)+V_\Sigma\,, \tag{2.7}$$

where $V_N(k_N)$ is the nucleon s.p. potential in NM for $k_N<k_F$, and V_Σ is the s.p. potential of zero momentum Σ in NM. Notice that we distinguish by an asterisk s.p. potentials in excited states, Eq. (2.4), from those in occupied states.

So far, we have not specified the s.p. potentials V_X. We only assume that V_X are real. Notice that with $V_\Sigma=\mathrm{Re}E_\Sigma$, the denominators in the propagators, Eqs. (2.3), are real (except for the parameter $+i\epsilon$).

To simplify the calculation of E_Σ, we apply the approximate procedure used by Shaw[4] in the problem of the nucleon optical potential. We introduce the propagators $G_{YY}^{(P)}$ which differ from the propagators $G_{YY}^{(+)}$, Eqs. (2.3), by the prescription of taking the principal value (instead of the $+i\epsilon$). We denote the corresponding reaction matrix by $\hat{K}^{(P)}$. Equation (2.2) and the corresponding equation for $\hat{K}^{(P)}$ imply the following equation for $\hat{K}^{(+)}$ in terms of $\hat{K}^{(P)}$:

$$\hat{K}^{(+)}(z)=\hat{K}^{(P)}(z)+\hat{K}^{(P)}(z)[\hat{G}^{(+)}(z)-\hat{G}^{(P)}(z)]\hat{K}^{(+)}(z)\,. \tag{2.8}$$

If we approximate $\hat{K}^{(+)}(z)$ on the right-hand side of (2.8) by $K^{(P)}(z)$, we get, in the first iteration of (2.8),

$$\hat{K}^{(+)}(z)\approx\hat{K}^{(P)}(z)+\hat{K}^{(P)}(z)[\hat{G}^{(+)}(z)-\hat{G}^{(P)}(z)]\hat{K}^{(P)}(z)\,. \tag{2.9}$$

With the help of the relation

$$1/(x+i\epsilon)=P(1/x)-i\pi\delta(x)\,, \tag{2.10}$$

we obtain from Eqs. (2.9) and (2.6)

$$\begin{aligned}
V_\Sigma^T&=\mathrm{Re}E_\Sigma^T\approx\int_{<k_F}\frac{d\vec{\mathbf{k}}_N}{(2\pi)^3}\,\langle\vec{\mathbf{k}}_{N\Sigma}|K_{\Sigma\Sigma}^{(P)}(z(k_N))^T|\vec{\mathbf{k}}_{N\Sigma}\rangle\,,\\
-\Gamma/2&=\mathrm{Im}E_\Sigma=\mathrm{Im}E_\Sigma{}^{1/2}/3\\
&\approx(\tfrac{1}{3})\int_{<k_F}\frac{d\vec{\mathbf{k}}_N}{(2\pi)^3}\Big[-(1/8\pi^2\hbar^2)\mu_{N\Lambda}k_{N\Lambda}Q_\Lambda(k_Nk_{N\Lambda})\\
&\qquad\times\int d\hat{k}_{N\Lambda}|\langle\vec{\mathbf{k}}_{N\Lambda}|K_{\Lambda\Sigma}^{(P)}(z(k_N))|\vec{\mathbf{k}}_{N\Sigma}\rangle|^2\Big],
\end{aligned} \tag{2.12}$$

where $k_{N\Lambda}$ is determined by the energy equation

$$\begin{aligned}\hbar^2k_{N\Lambda}{}^2/2\mu_{N\Lambda}&=\hbar^2k_{N\Sigma}{}^2/2\mu_{N\Sigma}+\Delta+V_N(k_N)\\
&\quad+V_\Sigma-V_N^*-V_\Lambda^*\,.\end{aligned} \tag{2.13}$$

[In our nonrelativistic treatment, the term $-[\hbar^2k_N{}^2/2(M_N+M_\Sigma)]\Delta/(M_N+M_\Lambda)c^2$ is neglected on the right hand side of Eq. (2.13).]

Here we have introduced relative momenta

$$\vec{\mathbf{k}}_{N\Sigma}/\mu_{N\Sigma}=\vec{\mathbf{k}}_N/M_N-\vec{\mathbf{k}}_\Sigma/M_\Sigma=\vec{\mathbf{k}}_N/M_N\,, \tag{2.14}$$

$$\vec{\mathbf{k}}_{N\Lambda}/\mu_{N\Lambda}=\vec{\mathbf{k}}'_N/M_N-\vec{\mathbf{k}}_\Lambda/M_\Lambda\,, \tag{2.15}$$

where $\mu_{NY}=M_NM_Y/(M_N+M_Y)$. The conserved total momentum is $\vec{\mathbf{k}}_N$ because ($\vec{\mathbf{k}}_\Sigma=0$), and the final momentum $\vec{\mathbf{k}}'_N$ of the nucleon in the ΛN channel (after the $\Sigma\Lambda$ conversion) is

$$\vec{\mathbf{k}}'_N=\mu_{N\Lambda}\vec{\mathbf{k}}_N/M_\Lambda+\vec{\mathbf{k}}_{N\Lambda}\,. \tag{2.16}$$

The exclusion principle operator Q_Λ depends on the total ΛN momentum k_N, the relative momentum $k_{N\Lambda}$, and the angle between $\vec{\mathbf{k}}_N$ and $\vec{\mathbf{k}}_{N\Lambda}$. Here, we approximate Q_Λ by the angle averaged operator[5]:

$$\begin{aligned}Q_\Lambda(k_Nk_{N\Lambda})&=0\text{ for }k_{N\Lambda}<k_F-\mu_{N\Lambda}k_N/M_\Lambda\\
&=1\text{ for }k_{N\Lambda}>k_F+\mu_{N\Lambda}k_N/M_\Lambda\\
&=[(k_{N\Lambda}+\mu_{N\Lambda}k_N/M_\Lambda)^2-k_F{}^2]/(4\mu_{N\Lambda}k_Nk_{N\Lambda}/M_\Lambda)\\
&\qquad\text{otherwise}\,.\end{aligned} \tag{2.17}$$

In the following sections, we assume constant values of the s.p. potentials V_N^* and V_Λ^*. For this reason, the dependence of these potentials on k'_N and k_Λ has been neglected in (2.12).

Notice that real elastic scattering in the entrance ΣN channel is prohibited by the exclusion principle (the Q_Σ operator), and no terms with $K_{\Sigma\Sigma}$ appear in expression (2.12) for Γ. Consequently, only the interaction in the $T=\frac{1}{2}$ state contributes to Γ. The obvious index $T=\frac{1}{2}$ at $K_{\Lambda\Sigma}$ is not indicated in (2.15).

Expression (2.12) for Γ involves approximation (2.9). To avoid this approximation, we may introduce $\hat{G}^{(-)}(z)_{YY}$ by Eqs. (2.3) with $+i\epsilon$, replaced by $-i\epsilon$, and the corresponding reaction matrix $\hat{K}^{(-)} = [K^{(+)}]^{\dagger}$. Proceeding similarly as before, we get

$$\Gamma = \frac{1}{3}\int_{<k_F} \frac{d\vec{k}_N}{(2\pi)^3}\Big[(1/2\pi\hbar)^2 \mu_{N\Lambda} k_{N\Lambda} Q_\Lambda(k_N k_{N\Lambda}) \times \int d\hat{k}_{N\Lambda} |\langle \vec{k}_{N\Lambda}|K^{(+)}_{\Lambda\Sigma}(z(k_N))|\vec{k}_{N\Sigma}\rangle|^2\Big]. \tag{2.18}$$

Let us consider the situation in which we completely disregard the effect of NM on the YN interaction. That means, we disregard the exclusion principle ($Q_Y = 1$) and dispersion effects ($V_X = V_X^* = 0$) in the reaction matrix equation, which then becomes the scattering matrix for an isolated YN pair, denoted here by $\hat{K}_0^{(+)}$. In particular, we have for the $\Sigma^- p \to \Lambda n$ total cross section σ.

$$\sigma = \tfrac{2}{3}\sigma^{T=1/2} = \tfrac{2}{3}(\mu_{\Sigma N} k_{N\Lambda}/\mu_{\Lambda N} k_{N\Sigma})(\mu_{\Lambda N}/2\pi\hbar^2)^2 \times \int d\hat{k}_{N\Lambda} |\langle \vec{k}_{N\Lambda}|K^{(+)}_{\Lambda\Sigma,0}|\vec{k}_{N\Sigma}\rangle|^2 . \tag{2.19}$$

In this simplified situation, Eq. (2.18) takes the form

$$\Gamma_0 = \tfrac{1}{2}\rho(\hbar^2/M_N)\langle k_N \sigma\rangle , \tag{2.20}$$

where $\rho = 2k_F^2/3\pi^2$ is the density of NM, and $\langle\ \rangle$ denotes the average value over the nucleon momenta in the Fermi sea. Since the Σ hyperon is at rest in NM, one should use in (2.20) the cross section σ for Σ^- laboratory momentum $k_{\Sigma^-,\mathrm{lab}} = M_\Sigma k_N/M_N$.

Now let us consider the case when we take into account the exclusion principle while disregarding the dispersive effects. We denote the corresponding width by Γ_Q. If, in this case, we approximate $K^{(+)}_{\Lambda\Sigma}$ in (2.18) by $K^{(+)}_{\Lambda\Sigma,0}$, we get

$$\Gamma_Q \approx \tfrac{1}{2}\rho(\hbar^2/M_N)\langle Q_\Lambda(k_N k_{N\Lambda}) k_N \sigma\rangle , \tag{2.21}$$

where [see Eqs. (2.13) and (2.14)]

$$k_{N\Lambda} = k_{N\Lambda}(k_N) = (\mu_{N\Lambda}\mu_{N\Sigma} k_N^2/M_N^2 + k_0^2)^{1/2} , \tag{2.22}$$

$$k_0 = (2\mu_{N\Lambda}\Delta/\hbar^2)^{1/2} . \tag{2.23}$$

III. KINEMATICS OF $\Sigma\Lambda$ CONVERSION IN NM

Here, and throughout this paper, we use for M_Σ the average value $M_\Sigma = 1192.5$ MeV/c^2, and the corresponding value of $\Delta = 76.9$ MeV. For k_F, we use the value $k_F = 1.35$ fm^{-1} ($\rho = 0.166$ fm^{-3}).

First, let us consider the $\Sigma N \to \Lambda N$ process for an isolated ΣN pair in the relevant case when $k_\Sigma = 0$. The relative momentum $k_{N\Lambda}$ is given by Eq. (2.22), with $k_0 = 1.42$ fm^{-1}. The magnitude of the final nucleon momentum, k_N', depends on the angle between $\vec{k}_N$ and $\vec{k}_{N\Lambda}$. The upper and lower limits of k_N', $k'_{N,U}$, and $k'_{N,L}$ follow from Eq. (2.16),

$$\left.\begin{matrix} k'_{N,U} \\ k'_{N,L} \end{matrix}\right\} = k_{N\Lambda} \pm \mu_{N\Lambda} k_N/M_\Lambda , \tag{3.1}$$

and are shown as functions of the initial nucleon momentum k_N in Fig. 1.

Now let us consider the exclusion principle effect of NM on the $\Sigma\Lambda$ conversion. For $k_N = 0$, $k_N' = k_0 = 1.42$ fm^{-1} is slightly bigger than $k_F = 1.35$ fm^{-1}. However, as is seen in Fig. 1, for $k_N \gtrsim 0.1 k_F$, an increasing part of the final nucleon momenta k_N' is smaller than k_F, and is thus prohibited by the exclusion principle. The situation for an average value of k_N, $\bar{k}_N = \sqrt{0.6}\,k_F$, is shown in Fig. 2. The value of Q_Λ here is $Q_\Lambda[\bar{k}_N, k_{N\Lambda}(\bar{k}_N)] = 0.76$.

To estimate the dispersive effect of NM on the $\Sigma\Lambda$ conversion, we have to specify the s.p. potentials which appear in the energy equation, Eq. (2.13). We replace the nucleon s.p. potential $V_N(k_N)$ by its average value in the Fermi sea, $\langle V_N\rangle$, and assume that it leads to the empirical energy per nucleon in NM, $-\epsilon_{\mathrm{vol}}$:

$$\tfrac{3}{5}\epsilon_F + \tfrac{1}{2}\langle V_N\rangle = -\epsilon_{\mathrm{vol}} , \tag{3.2}$$

where $\epsilon_F = \hbar^2 k_F^2/2M_N$ is the Fermi energy. For $\epsilon_{\mathrm{vol}} = 15.8$ MeV, we get $\langle V_N\rangle = -77$ MeV. For V_Σ, we assume the value of $V_\Sigma \sim -(20\text{–}30)$ MeV which follows from the interpretation of (K,π) experiments in Ref. 1, and which is consistent with the

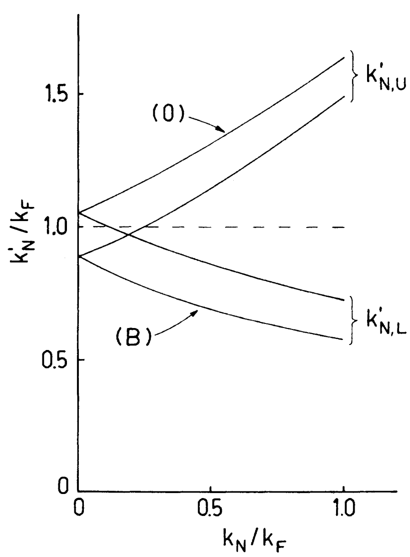

FIG. 1. Ranges of final nucleon momenta in $\Sigma\Lambda$ conversion in NM, k_N', as functions of initial nucleon momenta k_N without (0) and with dispersive effects (B).

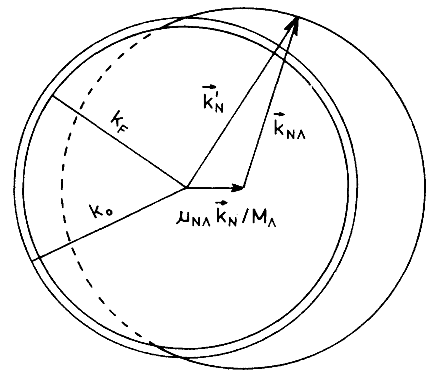

FIG. 2. Final nucleon momentum in ΣΛ conversion in NM, $\vec{k}_N'$ for an average initial nucleon momentum $k_N=\bar{k}_N$ without dispersive effects (k_0 is the length of $\vec{k}_N'$ for $k_N=0$).

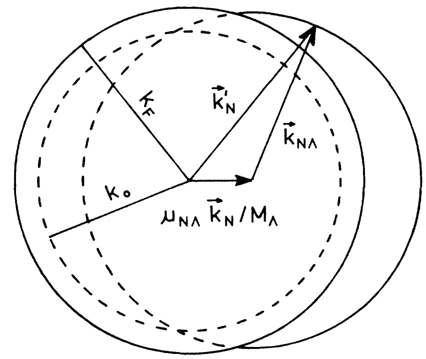

FIG. 3. Same as Fig. 2 except that dispersive effects are taken into account with $W=W_B$.

Σ^- atomic-data analysis[6] and also with our own estimate in Sec. V.

The most difficult problem is the specification of the s.p. potentials V_Y^* in excited states. First, let us consider the nucleon s.p. potential $V_N^*(k_N')$, $k_N'>k_F$. Since in calculating Γ we deal with real energy conserving transitions, a continuous s.p. potential, used by Mahaux and his collaborators[7] in the problem of the nucleon optical potential, appears to be the proper one. The continuous choice of $V_N^*(k_N')$, in the relevant range of k_N', is intermediate between two extreme choices: (A) $V_N^*(k_N')=0$ and (B) $V_N^*(k_N')=V_N(k_N)$. Both of them are calculationally simple. By applying them, we may obtain lower and upper limits for Γ and $-V_\Sigma$. Choice (A) is usually referred to as the standard or reference spectrum choice.

We fix the value of $V_N(k_F)$ by the condition that at the saturation density the s.p. energy at the Fermi surface is equal to the energy per nucleon in NM (Ref. 8),

$$\epsilon_F+V_N(k_F)=-\epsilon_{\rm vol}\,, \tag{3.3}$$

which leads to the value of $V_N(k_F)=-54$ MeV.

We apply the same two limiting choices for the hyperon s.p. potentials: (A) $V_\Sigma^*=V_\Lambda^*=0$ and (B) $V_\Sigma^*=V_\Sigma$, $V_\Lambda^*=V_\Lambda$. A continuous s.p. potential of Λ in NM, examined by Chong, Nogami, and Satoh,[9] is again intermediate between our two limiting choices. The behavior of a continuous s.p. potentail of Σ is expected to be similar.

Altogether, we may write energy equation (2.13) as

$$\hbar^2k_{N\Lambda}{}^2/2\mu_{N\Lambda}=\hbar^2k_{N\Sigma}{}^2/2\mu_{N\Sigma}+\Delta-W\,, \tag{3.4}$$

where for the choice (A)

$$W=W_A=-[\langle V_N\rangle+V_\Sigma]\sim 100\text{ MeV}\,, \tag{3.5}$$

and for choice (B)

$$\begin{aligned}W=W_B&=-[\langle V_N\rangle-V_N(k_F)+V_\Sigma-V_\Lambda]\\&\approx-[\langle V_N\rangle-V_N(k_F)]=23\text{ MeV}\,.\end{aligned} \tag{3.6}$$

Now, for $\Delta-W<0$, the final nucleon momentum $k_N'<k_F$ for all initial nucleon momenta k_N in the Fermi sea [it may be seen easily from (2.16) and (3.4)], and the $\Sigma N\to\Lambda N$ process is prohibited by the exclusion principle. Hence, choice (A) leads to zero width $\Gamma_A=0$.

The final relative momentum $k_{N\Lambda}$, which follows from Eq. (3.4), is given by Eq. (2.22) with

$$k_0=k_0(W)=[2\mu_{N\Lambda}(\Delta-W)/\hbar^2]^{1/2}\,. \tag{3.7}$$

With the help of Eq. (3.1), we may obtain the range of final nucleon momenta k_N' as a function of k_N. The result for choice (B), $W=W_B$, is shown in Fig. 1. For $k_N=0$, $k_N'=k_0(W_B)=1.18\text{ fm}^{-1}$ is smaller than k_F. Only for $k_N\gtrsim 0.2k_F$, is a part of the final nucleon momenta k_N' bigger than k_F, and thus allowed by the exclusion principle. The situation for the average value of k_N, $\bar{k}_N$, is shown in Fig. 3. The value of Q_Λ is here $Q_\Lambda[\bar{k}_N, k_{N\Lambda}(\bar{k}_N)]=0.56$.

The value of $\bar{Q}_\Lambda=Q_\Lambda[\bar{k}_N,k_{N\Lambda}(\bar{k}_N)]$ is the measure of the suppression of the $\Sigma N\to\Lambda N$ process in NM. Without dispersive effects, $\bar{Q}_\Lambda=0.76$. An estimate of the lower limit of the dispersive effects [our choice (B)] leads to $\bar{Q}_\Lambda=\bar{Q}_{\Lambda B}=0.56$, i.e., to an almost 50% reduction in the conversion rate in NM. This big reduction is due to an essential decrease in the final nucleon momentum k_N', because part of the available energy Δ is used to excite NM, as visualized by the momentum dependence of the s.p. potentials. And with decreasing values of k_N', a decreasing part of the final nucleon states is allowed by the exclusion principle.

IV. ESTIMATE OF Γ BASED ON σ

Values of the total $\Sigma^-p\to\Lambda n$ cross section σ fitted by Nagels, Rijken, and de Swart[11] to the experimental data are given in Table X of Ref. 10. By inserting these values into expressions (2.20) and (2.21), we get

$$\Gamma_0 = 22.5 \text{ MeV}, \quad \Gamma_Q = 17.3 \text{ MeV}. \tag{4.1}$$

If we approximate $\langle k_N \sigma \rangle$ in (2.20) by $\bar{k}_N \sigma(\bar{k}_N)$, we get $\Gamma_0 \approx \bar{\Gamma}_0 = 20.9$ MeV. A similar approximation applied to (2.21) gives $\Gamma_Q \approx \bar{\Gamma}_Q = \bar{Q}_\Lambda \bar{\Gamma}_0 = 15.8$ MeV.

If we take into account exclusion principle and dispersive effects, we may still write expression (2.21) for Γ, provided we replace σ by an effective cross section in NM, σ_{NM}. For σ_{NM}, we should then use expression (2.19) with the scattering matrix $K^{(+)}_{\Lambda\Sigma,0}$ replaced by the NM reaction matrix $K^{(+)}_{\Lambda\Sigma}$. However, determining $K^{(+)}_{\Lambda\Sigma}$ requires solving the NM reaction matrix equation. This is a hard computational problem with which we deal in Sec. V.

Here, let us assume that $\sigma_{\text{NM}} \approx \sigma$, i.e., that the main factor in reducing Γ is the decrease in Q_Λ due to the dispersive effects. This assumption appears justified by the results of Sec. V. In this way, for choice (B) of s.p. potentials we get

$$\Gamma_B \approx \bar{Q}_{\Lambda B} \bar{\Gamma}_0 = 11.6 \text{ MeV}. \tag{4.2}$$

V. V_Σ AND Λ CALCULATED WITH THE NIJMEGEN INTERACTION

A realistic form of the YN interaction $\hat{v}$ has been worked out by the Nijmegen group. Two recent forms of the Nijmegen interactions are models D (Refs. 10 and 11) and F (Ref. 12). We apply model D, because model F—within our calculational scheme—leads to a repulsive V_Σ [mainly due to the big repulsive $(T=\frac{3}{2},\ S=1)$ contribution].

With both models, we face the following situation. The $(T=\frac{1}{2},\ S=1)$, $(T=\frac{3}{2},\ S=0)$ contributions to V_Σ are attractive, and the $(T=\frac{1}{2},\ S=0)$, $(T=\frac{3}{2},\ S=1)$ contributions are repulsive. The value of V_Σ is the result of an appreciable cancellation between the attractive and repulsive contributions. Consequently, V_Σ is sensitive to small changes in the YN interaction $\hat{v}$, allowed by the existing YN scattering data. The value of $V_\Sigma \sim -(20\text{–}30)$ MeV suggested by the recent (K, π) experiments,[1] and consistent with the atomic level shifts,[6] indicates that model D is more realistic than model F. Also, let us mention that one obtains a slightly worse fit to the YN data with model F than with model D.

In our calculation of V^T_Σ, we use expression (2.11), and for Γ, we use expression (2.12). These expressions require the knowledge of $K^{(P)}_{\Sigma\Sigma}$ and $K^{(P)}_{\Lambda\Sigma}$ which are determined by Eqs. (2.2), with (+) replaced by (P). In solving these equations, we apply the same method as in calculating V_Λ in Ref. 2 (see also Ref. 6). There is only one difference: in calculating the Green's functions, we take the principal value of the integrals whenever singularities occur (in the ΛN channel).

The energy denominator of $G^{(P)}[z_N(k_N)]_{YY}$ contains the potential part

$$-W = V_N(k_N) + V_\Sigma - V^*_N - V^*_Y, \tag{5.1}$$

in which we approximate $V_N(k_N)$ by $\langle V_N \rangle$. If for V^*_N and V^*_Y we apply one of the two limiting choices (A) and (B) described in Sec. III, W becomes a constant (the same in both channels) whose magnitude for the two choices is given in Eqs. (3.5) and (3.6). In our computations of V_Σ and Γ, we have treated W as a variable parameter.

Our results for V_Σ and Γ as functions of W are shown in Fig. 4. Separate partial-wave contributions to V^T_Σ and to Γ for $W = W_B$ are shown in Table I. Notice that Γ is strongly dominated by the $\Sigma\Lambda$ conversion in the ${}^3S_1 + {}^3D_1$ state.

The results for V_Σ in Fig. 4 are approximately consistent with our estimates of W_A and W_B in Eqs. (3.5) and (3.6). For $W = W_A \sim 100$ MeV, we get from Fig. 4 $V_\Sigma \sim 10$ MeV which, inserted into (3.5), gives $W_A \sim 90$ MeV. For $W = W_B = 23$ MeV, we have from Fig. 4 $V_\Sigma \approx -35$ MeV, which roughly agrees with the equality $V_\Sigma \approx V_\Lambda$ assumed in (3.6).

Our results obtained with model D of the Nijmegen interaction, shown in Fig. 4, and our estimates of W_A and W_B, Eqs. (3.5) and (3.6), lead to the following estimates:

$$-35 \text{ MeV} \lesssim V_\Sigma \lesssim -10 \text{ MeV}, \tag{5.2}$$

$$0 < \Gamma \lesssim \Gamma_B = 11.6 \text{ MeV}. \tag{5.3}$$

The large range of V_Σ values in (5.2) and Γ values in (5.3) reflects our schematic treatment of the s.p. potentials.

Notice that the results for Γ in Fig. 4 agree nicely with the simple estimates of Sec. IV. This justifies approximation (2.12) [i.e., the approximation $K^{(+)}_{\Lambda\Sigma} \approx K^{(P)}_{\Lambda\Sigma}$ in Eq. (2.18)] as well as the approximate estimate in Eq. (4.2).

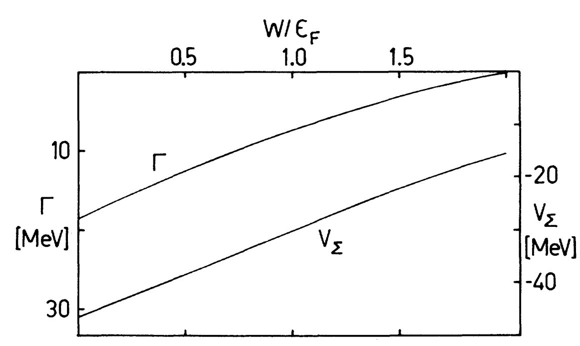

FIG. 4. V_Σ and Γ calculated with model D of the Nijmegen interaction as functions of W (in units of $\epsilon_F = 37.8$ MeV).

TABLE I. V_Σ and Γ (in MeV) for $W=W_B$ calculated with model D of the Nijmegen interaction.

	Partial-wave contributions									
	$^3S_1+^3D_1$	$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$	$^1P_1$	$^3D_2$	$^3D_3$	$^1D_2$	Total
$V_\Sigma^{1/2}$	−79.7	11.8	7.9	−24.2	−7.5	−1.2	−6.0	0.0	2.3	−96.8
$V_\Sigma^{3/2}$	31.5	−18.0	−3.8	7.8	−5.6	−17.7	2.5	−0.8	−1.9	−6.1
V_Σ	−5.6	−8.0	0.1	−2.9	−6.2	−12.2	−0.3	−0.5	−0.5	−36.2
Γ	9.5	0.4	0.1	0.7	0.0	0.0	0.1	0.0	0.0	11.6

VI. CONCLUDING REMARKS

Our calculated values of Γ decrease with increasing values of W. Changing the s.p. potentials in our choice (B) to more realistic continuous potentials would be equivalent to decreasing W, and would lower the value of Γ. Consequently, we expect Γ to be well below the upper limit Γ_B, in agreement with the experimental finding of Ref. 1.

Our main conclusion is that the Σ width in NM is substantially reduced, because the $\Sigma N \to \Lambda N$ process is strongly suppressed in NM. Namely, the $\Sigma N \to \Lambda N$ process in NM is accompanied by the excitation of NM, which uses part of the released energy. This, in turn, diminishes the final nucleon momenta to such a degree that an essential part of them are smaller than the Fermi momentum, and are excluded by the Pauli principle.

The striking feature of the Σ width in NM, Γ, is that Γ is extremely sensitive to the choice of the s.p. potentials. This makes the problem of Γ particularly interesting from the point of view of the theory of NM.

Recently, Gal and Dover[13,14] gave an estimate of Γ in NM with the result 20 MeV $\lesssim \Gamma \lesssim$ 50 MeV. As long as they use the experimental value of σ in their estimate, their result, $\Gamma \approx 22$ MeV,[14] agrees with our value of Γ_0, Eq. (4.1). The point, however, is that in Γ_0 the important exclusion principle and dispersive effects are not taken into account, and they drastically reduce the width. Since σ increases fast with decreasing Σ^- laboratory momentum, ignoring the nucleon Fermi motion leads to a serious overestimate of Γ_0, as is illustrated by the value of $\Gamma = 47$ MeV in Ref. 13. The value of $\Gamma \approx 28$ MeV (Refs. 13 and 14) is the result of "renormalizing" σ by a factor of $\frac{18}{14}$ to get a fit to the imaginary part of the Σ^- nuclear potential, $\mathrm{Im}U = -\Gamma/2$, determined by Batty[6] from the measured Σ^- atomic level shifts. What appears incorrect in this renormalization is the assumption that $\mathrm{Im}U \sim \rho$. Now, the decreasing role of exclusion principle and dispersive effects at low density (at which Σ^- – nucleus interaction in Σ^- atoms predominantly occurs) as well as the fast increase of σ with decreasing Σ^- laboratory momentum (connected with a resonance near the Σ threshold) make $-\mathrm{Im}U/\rho$ increase with decreasing ρ.[15]

A different estimate of Γ was given recently by Kisslinger.[16] In his model of plane wave final ΛN states, he estimates the width of a ΣN cluster in the lowest, second order in $v_{\Lambda\Sigma}$. For $v_{\Lambda\Sigma}$, he assumes the one-pion-exchange (OPE) potential. He does not specify $v_{\Sigma\Sigma}$. Instead, he assumes the existence of a bound ΣN state described by a wave function $R_{\Sigma N}(r) \sim \exp(-\alpha r) - \exp(-\beta r)$, where α is determined by the assumed binding energy of 4 MeV, and β takes care of the short range repulsion (not present in the OPE interaction). His result for Γ, one to a few MeV, is also obtained in his model of broken SU(3) symmetry,[17,18] in which again the function $R_{\Sigma N}$ is used. It would be desirable to confirm this result by a more quantitative calculation with a realistic interaction $\hat{v}$, and with the function $R_{\Sigma N}$ determined by this interaction. However, the recent realistic interactions $\hat{v}$ (Refs. 10 and 12) do not lead to a ΣN bound state. The small result for Γ is interpreted in Ref. 16 as the result of short range correlations, represented by $R_{\Sigma N}$. Now, the short-range correlations are taken into account in our complete calculation with the Nijmegen interaction $\hat{v}$, outlined in Sec. V. There, we have calculated the relative ΣN wave function in NM, which has the property of healing, and is quite different from $R_{\Sigma N}$ used in Ref. 16. Consequently, the connection between the width of an hypothetical ΣN bound state and the width in NM is not clear. [Let us notice that the simplest model for the ΣN function in NM with the healing property is $R_{\Sigma N} = \Theta(r-c)$, where Θ = step function, c = hard core radius. This function, inserted into Eq. (4) of Ref. 16, would increase the resulting value of Γ by at least an order of magnitude.]

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