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Isospin effects in pion-nuclear scattering

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A simple isospin model of inelastic pion-nucleus scattering is discussed and quantitative results compared with recent experiments on light nuclei.

[NUCLEAR REACTIONS ^{13}C, ^{16}O, $^{28}Si(\pi\pi')$ (3,3) resonance energy to stretched configuration states; ratios of π^-/π^+ and isoscalar-isovector transitions estimated; effects of isospin $\frac{1}{2}$ reaction channel ascertained.]

I. INTRODUCTION

Intermediate energy pion beams, especially with incident energies near the (3, 3) resonance, are now frequently used as sensitive probes of nuclear transition densities.[1-4] The (3, 3) resonance energy region is particularly interesting, since by assuming (3, 3) dominance and ignoring any charge splitting in the pion-nucleus T matrix,[5] the cross section ratio $R[\sigma(\pi^-)/\sigma(\pi^+)]$ to a specific nuclear level tends to values of $\frac{1}{9}$ and 9 for scattering from bound protons and neutrons, respectively. Therefore pion reaction data could be used to differentiate between proton and neutron transition densities in nuclei and to a higher degree than has been achieved by analyses of electromagnetic decay of (or inelastic hadron scattering to) discrete states of nuclei.[6] With this possibility in mind, Lee and Lawson[7] used a general pion-nucleus distorted-wave impulse approximation (DWIA) scattering theory and analyzed inelastic pion scattering data from the excitation of discrete, collective 2^+ states of ^{18}O. As with direct reaction hadron scattering to (and electromagnetic excitation of) such collective states, the effective charge concept had to be employed in the restricted basis used, to explain the magnitudes of observed pion cross sections. However, strong state dependent effective charges[7] change the ratio R, from that expected with a pure pion-nucleon t matrix, in a complex manner. Clearly, therefore, transitions which are better determined by the usual model prescriptions are required to make good use of the pion reaction sensitivity. Transitions to states of stretched, unique, particle-hole excitation are candidates, especially if they are of unnatural parity, and therefore do not support a collective enhancement. Data from inelastic pion scattering to such states has been[8-10] recently reported and we analyze this data using a simple isospin reaction model avoiding the complex numerics of a full DWIA analysis but nevertheless providing a gross test of the pion nuclear scattering reaction mechanism near the (3, 3) resonance.

II. DESCRIPTION OF THE MODEL

The isospin dependence of pion-nuclear scattering matrices, T_{if}, is displayed by

$$T_{if} \simeq \langle 1M_{T_3};T_4M_{T_4}|t|1M_{T_1};T_2M_{T_2}\rangle \ , \qquad (1)$$

where M_{T_1} and M_{T_3} are the initial and final state pion isospin projections, respectively, and $T_2(M_{T_2})$ and $T_4(M_{T_4})$ are the isospin quantum numbers of the initial and final nuclear states. Clearly, for inelastic (non-charge-exchange) pion scattering M_{T_1} and M_{T_2} equate to M_{T_3} and M_{T_4}, respectively. Then, if the pion-nucleus interaction is of one body form, by introducing the total isospin T_0 we can reexpress Eq. (1) as

$$\begin{aligned} T_{if} = A \sum_{\alpha\beta\gamma T' T_0} & \langle 1T_4M_{T_3}M_{T_4}|T_0M_0\rangle\langle 1T_2M_{T_1}M_{T_2}|T_0M_0\rangle \\ & \times\langle\alpha\tfrac{1}{2};\gamma T'|\}T_4\rangle\langle\beta\tfrac{1}{2};\gamma T'|\}T_2\rangle \\ & \times\langle 1,(\alpha\tfrac{1}{2}xT')T_4;T_0M_0|t|1,(\beta\tfrac{1}{2}xT')T_2;T_0M_0\rangle \qquad (2)\end{aligned}$$

in which $\langle\cdots|\}\cdots\rangle$ are the nuclear, one body coefficients of fractional parentage (cfp) connecting the initial (final) mass A nuclear states to an intermediate, mass A-1, nuclear state with isospin T'. Assuming that the pion-nucleon interaction is independent of the angular momentum states and recoupling in isospin, we obtain

$$\begin{aligned} T_{if} = A \sum_{\alpha\beta\gamma T' T_0 s} & \langle\alpha\tfrac{1}{2};\gamma T'|\}T_4\rangle\langle\beta\tfrac{1}{2};\gamma T'|\}T_2\rangle \\ & \times\langle 1T_4M_{T_3}M_{T_4}|T_0M_0\rangle\langle 1T_2M_{T_1}M_{T_2}|T_0M_0\rangle\,\hat{s}\hat{T}_2\hat{T}_4 \\ & \times\begin{pmatrix}1 & \frac{1}{2} & s\\ T' & T_0 & T_4\end{pmatrix}\begin{pmatrix}1 & \frac{1}{2} & s\\ T' & T_0 & T_2\end{pmatrix}t^{(s)} \ , \qquad (3)\end{aligned}$$

with $\hat{S}=(2S+1)^{1/2}$ and in which $t^{(s)}$ is the reduced pion-nucleon scattering amplitude for isospin $s(\frac{1}{2}$ or $\frac{3}{2})$, namely

$$t^{(s)} = \langle 1\tfrac{1}{2};s\|t\|1\tfrac{1}{2};s\rangle \ . \tag{4}$$

Clearly in this model we have suppressed all specific angular momentum dependence of the reaction, whereas in a full analysis the complete angular momentum dependence of the nuclear states, the nucleons, and the pions (via a partial wave expansion) must be specified.[7]
Thus

$$T_{if} = A \sum_{T' T_0 s} \langle 1 T_4 M_{T_4} | T_0 M_0\rangle \times \langle 1 T_2 M_{T_1} M_{T_2} | T_0 M_0\rangle \hat{s}\hat{T}_2\hat{T}_4 \times \begin{pmatrix} 1 & \frac{1}{2} & s \\ T' & T_0 & T_4 \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{2} & s \\ T' & T_0 & T_2 \end{pmatrix} t^{(s)} C(T';T_2T_4) \ , \tag{5}$$

with

$$C(T';T_2T_4) = \sum_{\alpha\beta\gamma} \langle \alpha\tfrac{1}{2};\gamma T' | \} T_4\rangle \langle \beta\tfrac{1}{2};\gamma T' | \} T_2\rangle \tag{6}$$

being the isospin parentage overlap of the initial and final nuclear states. This overlap may be formally evaluated by using standard shell model techniques and can be expressed in terms of the isoscalar and isovector transition densities between the two nuclear states.[7]

In factoring out the parentage overlap we have neglected the specific angular momentum dependence (α,β) of the scattering amplitude, $t^{(s)}$. However if the transition density is specified (or dominated) by a single (α,β) pair, as is usually the case in the excitation of high spin, stretched particle-hole configuration, then this factorization and therefore Eq. (5) is "exact" to within an overall normalization. Thus our model is particularly suited for an analysis of recent inelastic pion scattering experiments, on ^{13}C,[5,8] ^{16}O,[9] and ^{28}Si,[10] involving scattering to the stretched particle-hole states $\frac{9}{2}^+$ (9.5 MeV), 4^- (circa 18 MeV), 6^-(11.58 and 14.36 MeV), respectively. The unnatural parity transitions are of particular interest in that hadron scattering analyses[11] indicate that collective enhancement (effective charge) of transition strengths are not required and therefore the ratio R should be defined via Eq. (5) by isospin arguments alone.

The form of Eq. (5) is such that the scattering matrix for inelastic scattering from selfconjugate nuclei $(T_2=0)$, assuming (3,3) dominance, is

$$T_{if} = t^{(3/2)}/3(\delta_{T_4 0} + M_\pi/2\delta_{T_4 1}) \ . \tag{7}$$

For excitation of states with pure isospin, therefore, the $\pi^\pm$ cross sections are the same with the isoscalar transitions being fourfold the isovector ones. The experimental ratios from both the 4^- states in ^{16}O and from the 6^- states ^{28}Si are, however, smaller than 4, with the reduction in the ^{16}O data being interpreted by Holtkamp *et al.*[12] as a reflection of isospin mixing amongst the 4^- states. Barker *et al.*[13] have shown, however, that a fully consistent description of these 4^- excitations requires not only isospin mixing of the states but also non-negligible $s=\frac{1}{2}$ admixtures in the transition amplitudes for pion nucleon scattering. For selfconjugate nuclei, the intermediate isospin in our model, T', has the value of $\frac{1}{2}$ implying, in the absence of isospin mixing in the final nuclear states, all transitions are of purely isoscalar or of isovector type. For nonselfconjugate nuclei T_2 and T_4 are nonzero hence a sum over T' values of $T_2 \pm \frac{1}{2}$ must be made in general. The precise values of the cfp factor $C(T'=0,1)$ then affect results. For the case of ^{13}C T' is 0 or 1 and with the incoming (and outgoing) pion isospin projection now defined as M_π, the scattering amplitudes to $T_4=\frac{1}{2}$ final states are

$$T_{if}^{(0)} = AC(0;\tfrac{1}{2}\tfrac{1}{2})[(1+M_\pi)t^{(1/2)}/3\sqrt{2} + (2-M_\pi)t^{(3/2)}/6] \tag{8}$$

for $T'=0$ and

$$T_{if}^{(1)} = AC(1;\tfrac{1}{2}\tfrac{1}{2})[(1+M_\pi)(\sqrt{2}\,t^{(1/2)} + 8t^{(3/2)})/54 + (2-M_\pi)(4\sqrt{2}\,t^{(1/2)} + 5t^{(3/2)})/54] \tag{9}$$

for $T'=1$. These simplify if (3,3) dominance is assumed to give $(T_2=T_4=\frac{1}{2})$

$$T_{if} = A\{C(0;\tfrac{1}{2}\tfrac{1}{2})(2-M_\pi)/6 + C(1;\tfrac{1}{2}\tfrac{1}{2})[(1+M_\pi)8 + (2-M_\pi)5]/54\} \times t^{(3/2)} \ . \tag{10}$$

The difference between $\pi^\pm$ scattering strength to a given level depends not only upon the pion charge but also upon the cfp overlaps, $C(T')$. Specifically, the asymmetry in pion scattering defined by

$$A = [\sigma(\pi^-) - \sigma(\pi^+)]/[\sigma(\pi^-) + \sigma(\pi^+)] \tag{11}$$

in which $\sigma(\pi)$ are proportional to $|T_{fi}|^2$, give the results

$$\begin{aligned} A &= +0.80, \quad C(0)=1, \quad C(1)=0, \\ A &= -0.324, \quad C(0)=0, \quad C(1)=1, \end{aligned} \tag{12}$$

with the value of A being very sensitive to relative changes in the values of $C(T')$ and biased towards any admixtures of $C(1)$.

Finally we reiterate that if many $(\alpha\beta)$ pairs contribute to scattering amplitudes for the excitation of low lying collective states, our model is approximate, as it neglects the details of the relative contribution of these pairs. Our model should therefore be used with a "collective" state only as a qualitative guide to explain any isospin dependence. Nevertheless this may be a useful technique as the more complex DWIA analyses have problems due to the collectivity of such transitions.[7]

III. RESULTS

The asymmetries, defined by Eq. (11), for transitions to low lying states in ^{13}C have been recently measured[5,8] and are reproduced in Fig. 1. The limiting values of our model assuming (3,3) dominance are also shown. Clearly positive parity excitations are synonomous with positive pion asymmetries and vice versa. This property is anticipated as the pertinent shell model calculations[14] reveal significant parentage $T'=1$ for the negative parity states in ^{13}C below 12 MeV excitation, while the positive parity states result from the weak coupling of an extra core (s-d shell) neutron with the low lying $T=0$ levels in ^{12}C. Hence the positive parity levels have dominant $T'=0$ parentage and therefore positive asymmetries, and vice versa for negative parity states with large $T'=1$ parentage amplitudes.

Of all the transitions, only the $\frac{9}{2}^+$ (9.5 MeV)

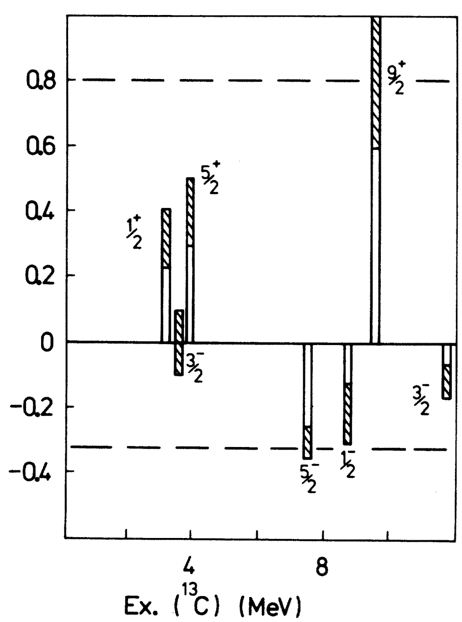

FIG. 1. The measured asymmetries A from pion inelastic scattering to low lying, discrete states in ^{13}C. The (3,3) dominance model predictions for positive parity (isoscalar parentage) and negative parity (isovector parentage) are shown for comparison.

excitation agrees with our simple model prediction. Of all the states given, this state is the only example of a stretched (single α,β) transition. The transitions to other levels involve multiple α,β pairs, for which our model is only an approximation. Nevertheless the qualitative agreement is good in view of the sensitivity of the asymmetries to small changes in the $\pi^\pm$ scattering cross sections.

The excitation of high-spin unnatural parity states in selfconjugate nuclei, such as in the recent experiments on ^{16}O (Ref. 9) and ^{28}Si,[10] is of more direct interest in that our model is "exact" so that analyses of the relevant data will test the (3,3) dominance assumption, namely that for good isospin the isovector to isoscalar transition ratios are

$$R_\pm=\sigma(\pi^\pm,\Delta T=0)/\sigma(\pi^\pm,\Delta T=1)=4\ . \qquad (13)$$

The 4^- states in ^{16}O lie close together near 18 MeV excitation and some isospin mixing occurs to affect their use in such a test,[13] but the two 6^- states in ^{28}Si are separated by some 2.4 MeV and therefore are expected to be very pure. As the measured values[10] for $R_\pm$ for the 6^- states (11.58 MeV isoscalar, 14.36 MeV isovector) are 1.5 and 1.7 ($\pm$0.4) for R_+ and R_-, respectively, we anticipate a significant breaking of the (3,3) dominance assumption.

The specific forms of the ^{28}Si 6^- scattering amplitudes are

$$T_{if}=A/3[(\sqrt{2}\,t^{(1/2)}+2t^{(3/2)})\delta_{T_40} + M_\pi(-\sqrt{2}\,t^{(1/2)}+t^{(3/2)})\delta_{T_41}]\ . \qquad (14)$$

If we assume that the two 6^- states have pure isospin and allow a breaking of the (3,3) dominance assumption by including

$$t^{(1/2)}=\alpha e^{i\theta}t^{(3/2)}, \qquad (15)$$

then the ratios are predicted by

$$R_\pm=|\sqrt{2}\alpha e^{i\theta}+2|^2/|-\sqrt{2}\alpha e^{i\theta}+1|^2\ , \qquad (16)$$

hence

$$R_+=R_- = R=(4+2\alpha^2+4\sqrt{2}\alpha\cos\theta)/(1+2\alpha^2-2\sqrt{2}\alpha\cos\theta). \qquad (17)$$

We note that the result $R_+=R_-$ is independent of the specific values of (α,θ) which is supported within experimental error. But the magnitude R is very dependent on (α,θ) and a ratio of 4 is obtained, not only with $\alpha=0$ [the (3,3) dominance assumption] but also if α has the value $2\sqrt{2}\cos\theta$. However, using an "empirical" value of $R\simeq1.5$, the mixing amplitude α is then determined by the quadratic equation

$$2\alpha^2-14\sqrt{2}\alpha\cos\theta-5=0$$

which, for the physical constraint $|\alpha| < 1$, has allowed solutions that are shown graphically in Fig. 2. For zero phase ($\theta = 0$), α is -0.246 so that the $t^{(1/2)}$ amplitudes are required to be at least 25% of those assumed in a (3,3) dominance picture. A similar result was found in the analysis of the excitation data for pion scattering to the 4^- states in ^{16}O allowing for isospin (and configuration) mixing in the state description.[13]

We now consider isospin mixing in the ^{28}Si 6^- states by using

$$|\psi_6 - (12\text{ MeV})\rangle = \cos(\epsilon)|6^-; T=0\rangle + \sin(\epsilon)|6^-; T=1\rangle,$$
$$|\psi_6 - (14\text{ MeV})\rangle = -\sin(\epsilon)|6^-; T=0\rangle + \cos(\epsilon)|6^-; T=1\rangle, \quad (18)$$

implying

$$T_\pm(12\text{ MeV}) = \cos(\epsilon) T_\pm(T=0) + \sin(\epsilon) T_\pm(T=1)$$

and

$$T_\pm(14\text{ MeV}) = -\sin(\epsilon) T_\pm(T=0) + \cos(\epsilon) T_\pm(T=1), \quad (19)$$

using Eq. (14)

$$T_\pm(12) = \sqrt{2}[\cos(\epsilon) \mp \sin(\epsilon)] t^{(1/2)} + [2\cos(\epsilon) \pm \sin(\epsilon)] t^{(3/2)}, \quad (20)$$

and

$$T_\pm(14) = \sqrt{2}[-\sin(\epsilon) \mp \cos(\epsilon)] t^{(1/2)} + [-2\sin(\epsilon) \pm \cos(\epsilon)] t^{(3/2)}. \quad (21)$$

The (3,3) dominance assumption then gives the ratios

$$R_\pm = [2\cos(\epsilon) \pm \sin(\epsilon)]^2 / [2\sin(\epsilon) \mp \cos(\epsilon)]^2, \quad (22)$$

and $R_\pm = 4$ for pure isospin ($\epsilon = 0$) as known from our earlier discussion. If we consider the difference between the ratios, independent of their actual individual values, by using the fraction difference

$$f = 2(R_- - R_+)/(R_- + R_+)$$
$$= -40\sin(\epsilon)\cos(\epsilon)/[25\cos^2(\epsilon)\sin^2(\epsilon) + 4], \quad (23)$$

a variation with isospin mixing angle results that is shown in Fig. 3. The experimental fraction difference lies in the range -0.33 to 0.47 which coincides with an isospin mixing angle range of 1.92 to -2.72 degrees. Thus the possible difference in the ratios $R_\pm$ under the (3,3) dominance assumption needs little isospin mixing between the two 6^- states in ^{28}Si, although the the individual ratios will still be in the region of 4.

If we do not make the (3,3) dominance assumption but again use the relation [Eq. (15)] between the $t^{(1/2)}$ and $t^{(3/2)}$ amplitudes, the isospin mixing transition amplitudes are

$$T_\pm(12) = \{\sqrt{2}[\cos(\epsilon) \mp \sin(\epsilon)]\alpha e^{i\theta} + [2\cos(\epsilon) \pm \sin(\epsilon)]\} X,$$

and

$$T_\pm(14) = \{\sqrt{2}[-\sin(\epsilon) \mp \cos(\epsilon)]\alpha e^{i\theta} + [-2\sin(\epsilon) \pm \cos(\epsilon)]\} X. \quad (24)$$

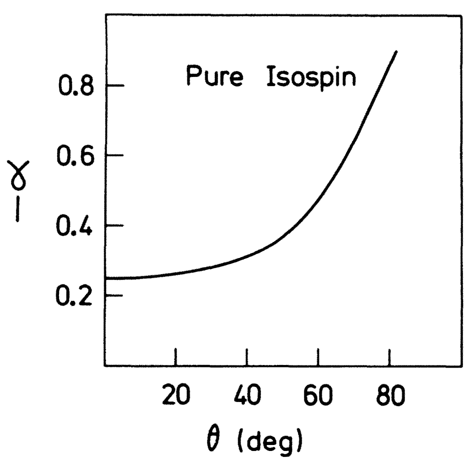

FIG. 2. The $t^{(1/2)}$ magnitude (α) and phase (θ) values (relative to the $t^{(3/2)}$ amplitudes) required to predict an isoscalar to isovector transition ratio of 1.5 for both $\pi^\pm$ inelastic scattering to the 6^- states in ^{28}Si.

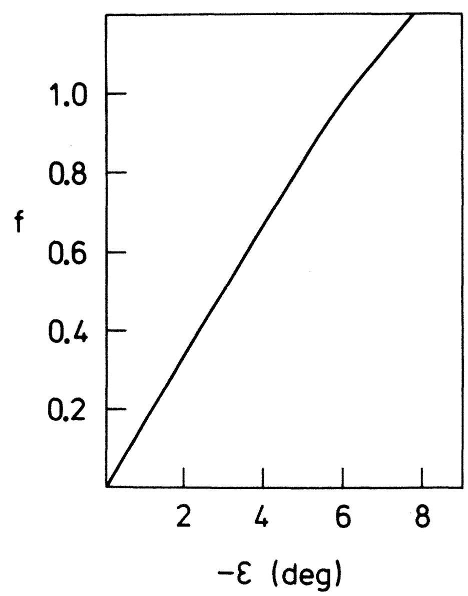

FIG. 3. The fraction difference f in $\pi^\pm$ scattering ratios to the 6^- states in ^{28}Si, assuming (3,3) dominance and allowing isospin mixing, as a function of the isospin mixing angle ϵ.

Thus the cross-section ratios provide a pair of constraint equations from which two of the three variables α, θ, and ϵ can be defined from the third. Using ratios $R_\pm$ of 1.5 and 1.7, respectively, and demanding $|\alpha|$ to be less than 1 for θ varying from 0° to 60°, we find α can vary between −0.23 and −0.43 while the isospin mixing angle ϵ is only −1° to −2°. Fixing θ as zero and taking extreme limits for the ratios $R_\pm$ of 1.3 and 2.0, α remains at a value of −0.23 while ϵ increases to a value of −3°. Therefore the pion scattering data to the ^{28}Si 6^- states are consistent with very pure isospin states, in concurrence with the hadron scattering data analyses.[11] The reduction in R (from the value 4) and differences between the ratios $R_\pm$ are attributed in the model to substantial $t^{(1/2)}$ amplitudes.

We now correlate these results with the ^{13}C reaction analyses. No longer using the (3,3) dominance assumption in an analysis of the $\pi^\pm$ asymmetry of scattering to the $\frac{9}{2}^{(+)}$ (9.5 MeV) state and choosing an isoscalar ($T'=0$) cfp factor only, we find

$$T_\pm \sim [\{(1+m_\pi)/3\sqrt{2}\}\alpha e^{i\theta} + (2-m_\pi)/6]$$

from which it may be deduced that

$$\begin{aligned} A &= [\sigma(\pi^-)-\sigma(\pi^+)]/[\sigma(\pi^-)+\sigma(\pi^+)] \\ &= (9-|2\sqrt{2}\alpha e^{i\theta}+1|^2)/(9+|2\sqrt{2}\alpha e^{i\theta}+1|^2) \\ &= (8-8\alpha^2-4\sqrt{2}\alpha\cos\theta)/(10+8\alpha^2+4\sqrt{2}\alpha\cos\theta)\ . \end{aligned} \tag{25}$$

Clearly, an asymmetry of +0.8 results if

$$8\alpha^2+4\sqrt{2}\alpha\cos\theta=0$$

or, equivalently, that α vanishes [(3,3) dominance] or equals $\cos\theta\sqrt{2}$. In the latter case, the ^{28}Si and ^{13}C results are consistent if $\alpha \simeq -0.4$ and $\theta \simeq 55°$. For other values of A allowed by the experimental uncertainty[5] for the $\frac{9}{2}^+$ transition, a spectrum of values is possible and is shown in Table I for asymmetries in the range 0.7 to 1.0.

TABLE I. Asymmetries for ^{13}C($\pi^\pm,\pi^\pm$); $\frac{9}{2}^+$ (9.5 MeV).

θ / α	0°	30°	45°	60°
−0.5	0.96	0.88	0.80	0.70
−0.4	1.00	0.93	0.86	0.77
−0.3	0.99	0.95	0.89	0.89
−0.2	0.95	0.93	0.89	0.84
−0.1	0.89	0.88	0.86	0.82
0	0.80	0.80	0.80	0.80
0.1	0.69	0.70	0.72	0.74
0.2	0.57	0.59	0.62	0.65

Using the +0.8 value for ^{13}C, and the ^{28}Si isovector to isoscalar 6^- ratio, a $t^{(1/2)}$ amplitude being $-0.4\exp(i55°)$ of the $t^{(3/2)}$ amplitude is the result which is also consistent with the ^{16}O, 4^- transition analyses[13] where a real scaling of −0.24 was deduced.

IV. CONCLUSIONS

A simple isospin model of the pion-nucleus T matrix has been defined from which the asymmetries in inelastic pion scattering from ^{13}C and isovector to isoscalar ratios of pion inelastic scattering to the 4^- states in ^{16}O and to the 6^- states in ^{28}Si can be explained in a simple but consistent manner. All the measured asymmetries in the ^{13}C scattering are explained qualitatively by this model whilst quantitative results were obtained for the special case of excitation of the stretched particle hole $\frac{9}{2}^+$ (9.5 MeV) state.

To obtain the consistent agreement between an asymmetry +0.8 in the $\frac{9}{2}^+$ excitation in ^{13}C and the measured isoscalar to isovector transition ratio of 1.5 in ^{28}Si, a significant component of isospin $\frac{1}{2}$ effective pion-nucleon scattering amplitude is required in the reaction analyses.

The required ratio $[t^{(1/2)}/t^{(3/2)}]$ is larger than would be obtained from a partial wave analysis of free pion-nucleon scattering[15] and such enhancement we expect is due in part to off-shell effects in the pion-nucleus[16] system and in part to kinematic-partial-wave mixing in reaction amplitudes when the pion-nucleon t matrixes are folded into the pion-nucleus frame.[17]

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