PHYSICAL REVIEW C VOLUME 23, NUMBER 4 APRIL 1981

Relativistic generalizations of simple pion-nucleon models

R. J. McLeod

Physics Department, Texas A & M University, College Station, Texas 77843
*and Nuclear Physics Laboratory, University of Colorado, Boulder, Colorado 80309**

D. J. Ernst

Physics Department, Texas A & M University, College Station, Texas 77843†
*and Institute for Nuclear Theory, Physics Department, University of Washington, Seattle, Washington 98195**

(Received 27 May 1980)

A relativistic, partial wave N/D dispersion theory is developed for low energy pion-nucleon elastic scattering. The theory is simplified by treating crossing symmetry only to lowest order in the inverse nucleon mass. The coupling of elastic scattering to inelastic channels is included by taking the necessary inelasticity from experimental data. Three models are examined: pseudoscalar coupling of pions and nucleons, pseudovector coupling, and a model in which all intermediate antinucleons are projected out of the amplitude. The phase shifts in the dominant P_{33} channel are quantitatively reproduced for $P_{lab} \leq 1.2$ GeV/c with a pion-nucleon vertex of range 1110 MeV/c. We find that there are large (not of the order of the inverse nucleon mass) kinematic corrections to Chew-Low models, and that the Chew-Low model is successful because a reduction in the pion-nucleon cutoff provides a remarkable compensation for the large kinematic corrections. The intermediate antinucleon states are found to provide a significant fraction of the interaction in both S and P waves, and the model which explicitly removes them is incompatible with the P_{33} phase shifts. Thus a model of the pion-nucleon interaction which does not include antinucleon degrees of freedom is found to be unphysical.

[NUCLEAR REACTIONS Pion-nucleon scattering, kinematic corrections, role of intermediate antinucleon states.]

I. INTRODUCTION

The construction of pion facilities has generated a renewed interest in the pion-nucleon interaction at low energies. This interest has centered about constructing pion-nucleon models[1-8] which could be used as input to pion-nucleus studies. In order to be useful these models must satisfy several criteria. First, they must be consistent with the known on-shell data. Second, if they are to be used in a many-body problem, a prescription of how to continue the model off-shell must be provided. Third, the model should be motivated by what is understood to be the underlying physics of the interaction. Finally, for pragmatic reasons, the model should be as simple as possible.

The first of these models was the separable potential model of Landau and Tabakin.[1] This model was generalized in Ref. 2 to include the effect of the coupling of the inelastic channels to the elastic channels. In Ref. 3, this model was shown to arise from quite general arguments. This model provides an off-shell extension of the pion-nucleon amplitude which is consistent with off-shell unitarity. The separable potential model does not, however, derive from any underlying physical mechanism for the interaction of a pion with a nucleon.

In the P-wave channels, the elastic scattering of the pion is dominated, near threshold, by the absorption and emission of the pion. This basic physics is incorporated in the Chew-Low[4] model. The original model could reproduce only *qualitatively*[5] the dominant P_{33} channel at low energies. Extending the model[6,7] beyond the "one-meson approximation" to include the effects of the coupling of the inelastic channels to the elastic channels produced excellent *quantitative*[7] agreement in the P_{33} channel for pion laboratory energies below 1.2 GeV.

Recently a model which combines the Chew-Low model with the separable potential model has been proposed.[8] The model uses the separable potential of Ref. 3 in the S, D, and F-wave channels. Simple analytic form factors are found which are capable of reproducing the elastic scattering data for laboratory pion kinetic energies below 1.2 GeV. In the P_{13}, P_{31}, and P_{33} channels, the Chew-Low model in the no-crossing approximation is used. In order to fit the data in each channel and to approximately account for the neglect of the crossing cut, the pion-nucleon coupling constant and form factor are varied independently in each of these channels. In the P_{11} channel, the phase shift changes sign at low energy. The model combines an attractive separable potential with the repulsive Chew-Low interaction to reproduce the data in this channel.

These models are capable of reproducing the pion-nucleon elastic scattering data and also provide methods for continuing the amplitudes off the energy shell. The validity of the off-shell behavior of the amplitudes remains a question.

One of the purposes of this work is to investigate the validity of these models. Specifically, we generalize these models to include relativistic kinematics and pion-nucleon couplings. We are thus able to study the size and nature of relativistic corrections to these models. We also investigate the possibility of describing the low energy pion-nucleon interaction in terms of pion and nucleon degrees of freedom only; we do this by suppressing explicitly the contribution of intermediate antinucleon states. None of the models examined here are sufficiently general to reproduce quantitatively the low energy pion-nucleon data. However, the basic theoretical framework developed here is sufficiently general that it can hopefully be extended to produce a dynamical relativistically invariant model consistent with pion-nucleon data.

The relativistic generalization of the Chew-Low model is not unique; there are many models which, in the limit of an infinite nucleon mass, reduce to the Chew-Low model. We examine here three of these models. Each uses partial wave N/D dispersion theory to generate unitary amplitudes and to include the effects of the coupling of the inelastic channels to the elastic channels. We treat crossing symmetry approximately; we retain the crossing relation to lowest order in the inverse nucleon mass M but retain relativistic kinematics. The models differ in the choice of the underlying basic interaction. The first model assumes pseudoscalar coupling between the pion and nucleon. This model has the advantage that the coupling is relativistically invariant and yields a renormalizable[9] field theory. We also consider the pseudovector coupling of pions to nucleons. Although this coupling does not yield a field theory which can be renormalized, it is sometimes used because it, in some sense, suppresses the contribution of the intermediate nucleon-antinucleon pairs. Finally, we examine the pseudoscalar coupling in a model in which we completely remove the contribution of intermediate nucleon-antinucleon pairs. This is an extreme version of pair suppression and is motivated by the Chew-Low model. The Chew-Low model, besides beginning with an interaction which assumes an infinite nucleon mass, contains no coupling of antinucleons to pions and nucleons. A consequence of these assumptions is that the Chew-Low model produces no scattering in S waves. A model which assumes pseudoscalar coupling but removes the contribution of intermediate antinucleons is thus a natural relativistic generalization of the Chew-Low model.

We find that our version of the partial wave N/D approach and either the pseudoscalar or pseudovector coupling can reproduce quantitatively the scattering in the dominant P_{33} channel for pion laboratory kinetic energies of $T_\pi \leq 1.2$ GeV. This was not true of earlier versions[10] of partial wave N/D approaches which yielded very poor agreement in the dominant P_{33} channel. This success here is not surprising because the nonrelativistic version[7,8] of our approach was found to work quite well.

Contradictory claims exist in the literature concerning the size and nature of kinematic corrections to the Chew-Low model. Several authors[11-13] have argued that the infinite mass model was a reasonable approximation to a relativistic model, particularly in the P-wave channels. In Ref. 14, however, it was shown that there are substantial corrections to the nonrelativistic theory, at least in Born approximation. In Ref. 15 it was shown that the static theory can be derived as an expansion in inverse power of the nucleon mass (M^{-1}) of a relativistic dispersion theory. There, they reach two conclusions: first, the corrections to the static limit are found to be small; and second, the static theory reproduces the P_{33} resonance essentially because the dispersion relation is prinicipally an internally consistent expression—i.e., if one inserts a resonating amplitude under the dispersion integral, it will generate a resonating amplitude.

We find, as was found in Ref. 14, that the kinematic corrections to the static model are large and not of order M^{-1}. This comes about because the expansion in inverse power of M is, at best, slowly convergent. Thus the arguments[11-13,15] which were based on comparing terms of lowest order to terms of first order in M^{-1} were misleading. We find the statement of Ref. 15—that the dispersion integral determines the width of the resonance independent of the model, if the position of the resonance is fixed—to be valid within certain limits. However, only if one includes the coupling to the inelastic channels, as was done in Ref. 7, does one obtain the experimentally measured width. In our models, we allow ourselves the freedom to adjust the range of the pion-nucleon form factor in order to reproduce the position of the resonance. For the Chew-Low model, the pseudoscalar coupling, or the pseudovector coupling one finds excellent agreement with the data in the P_{33} channel for pion laboratory kinetic energies of less than 1.2 GeV. The form factor required for the pseudoscalar coupling is found to have a cutoff of 1110 MeV, while the Chew-Low model was found to require a cutoff of 764 MeV. The large kinematical differences between the static model and the relativistic models can be remarkably well compensated for by an adjustment of the form factors.

We have found that this ability of the theory to compensate for changes in the basic theory by a readjustment of the form factor is true over a large but limited range. The model in which we explicitly remove the contribution of intermediate antinucleons produces a much weaker interaction than the other models. In order to have the position of the resonance correct, the range of the form factor must be extended to 2550 MeV. With this large cutoff, the resulting resonance is too narrow.

Another interesting question which we are able to investigate is the size and nature of the contribution of the coupling of the pion to nucleon-antinucleon pairs. One might infer from the success of static models in P-wave channels that a reasonably consistent theory could be developed which would include only pions and nucleons. A similar inference could be drawn from Refs. 11–13, 15, and 16, while in Ref. 14 it is pointed out that the elimination of nucleon-antinucleon pairs greatly reduces the P-wave Born amplitude. We remove the intermediate antinucleons by using projection operators[17] to project intermediate states onto the subspace of on-mass shell nucleons and eliminating completely the contributions which arise from the other half of the space which corresponds to on-mass shell antinucleons. This approach is preferable to using a Foldy-Wolthuysen transformation to generate a model which eliminates coupling to antinucleons. The transformation will generate a model which is an expansion in M^{-1} and we have found that such expansions are not very rapidly convergent.

We find, in agreement with Ref. 14, that the removal of intermediate nucleon-antinucleon pairs greatly weakens the interaction in the P_{13}, P_{31}, and P_{33} channels. The reduction in the P_{33} channel is so dramatic that one can no longer adjust the form factor and quantitatively produce a model which is consistent with the data. From this, we conclude (in contradiction with what one would infer from Refs. 11–13, 15, and 16) that a model of pion-nucleon scattering which includes only pions and nucleons, but not antinucleons, is unphysical. The success of the Chew-Low model lies in the fact that it is not a totally unreasonable approximation to a relativistic dispersion theory and that, although there are large kinematic corrections, these may be compensated for by an adjustment of the pion-nucleon form factor.

By examining the various models in S wave in a no-crossing approximation we can arrive at the following conclusions. The well-known difficulty of pseudoscalar coupling (without some form of pair suppression) producing much too large S-wave phase shifts is found. The pseudovector coupling will suppress the intermediate nucleon-antinucleon pairs only in the vague sense that it produces significantly smaller S-wave phase shifts. It cannot, however, form the basis of a physical model for the pion-nucleon interaction as it produces phase shifts in the S-wave channels of the incorrect sign. Complete pair suppression, however, produces S waves which are too small in magnitude, although the two channels do possess the correct sign.

None of the models—the pseudoscalar coupling, the pseudovector coupling, the pseudoscalar coupling with removal of intermediate antinucleons, or Chew-Low—is sufficiently general to form a dynamic model of the pion-nucleus interaction. The sigma model, as developed by Banerjee and Cammarata,[18] is capable of producing good results in S waves but no results for the P waves have been presented. Other, less dynamical models[19] are capable of reproducing the measured amplitudes either very near or below the elastic scattering threshold. We are currently examining the possibility of generalizing the work done here, with guidance taken from these other approaches, to build a model of the pion-nucleon interaction which is reasonably consistent with the experimental data.

This paper is structured as follows. In Sec. II, the relativistic partial wave N/D approach[20] is generalized to include the coupling of the inelastic channels to the elastic channel following the approach used in Refs. 3 and 7. Crossing symmetry is maintained only to lowest order in M^{-1}. In Sec. III each of the models is developed in detail, while in Sec. IV the results are reviewed. In Sec. V, we summarize the conclusions which we are able to draw from this work.

II. PARTIAL WAVE N/D

For all of the models considered we will use partial wave N/D dispersion theory to generate unitary amplitudes. The technique of including the inelastic, pion-production cut we shall take from Ref. 7 and, in this sense, we are using a generalization of the approach of Chew and Mandlestam.[20] The analytic structure of the partial wave amplitudes is quite intricate[10]; we simplify the situation by treating the nucleon mass as a large parameter in treating singularities other than the elastic and inelastic cuts.

The invariant scattering matrix $\mathfrak{M}$ for the elastic scattering of a pion of initial momentum q_1 and final momentum q_2 from a nucleon of initial momentum p_1 and final momentum p_2 can be written in terms of two independent amplitudes A and B in the usual way:

$$\mathfrak{M}^\alpha = i\bar{u}(p_2, s_2)[A^\alpha + \not{Q}B^\alpha]u(p_1, s_1), \tag{2.1}$$

where the superscript α denotes the isotropic spin, $\alpha = \frac{1}{2}$ or $\frac{3}{2}$, the $u(p, s)$ and $\bar{u}(p, s)$ are the usual nucleon spinors, and $\not{Q} = \not{q}_1 = \not{q}_2 = \not{q}$ in the center-of-mass system. The notation, metric, and normalizations are from Ref. 16. From the invariant amplitudes A and B we may form the helicity amplitudes f_1 and f_2 according to

$$f_1^\alpha = (E+M)[A^\alpha + (W-M)B^\alpha]/(8\pi W),$$
$$f_2^\alpha = (E-M)[-A^\alpha + (W+M)B^\alpha]/(8\pi W), \tag{2.2}$$

where E is the nucleon energy in the center-of-mass frame, W is the total energy in the center-of-mass frame, and M is the nucleon mass. The helicity amplitudes may be angular momentum decomposed according to

$$f_{l\pm}^\alpha = \frac{1}{2}\int_{-1}^{+1} dx[f_1^\alpha(x)P_l(x) + f_2^\alpha(x)P_{l\pm 1}(x)]dx, \tag{2.3}$$

where $x = \cos\theta$, θ is the scattering angle in the center-of-mass frame, l is the orbital angular momentum, and the $\pm$ is determined by the value of the total angular momentum j,

$$j = l \pm \tfrac{1}{2}. \tag{2.4}$$

We write partial wave dispersion relations for the amplitudes $f_{l\pm}^\alpha$.

The analytic structure of $f_{l\pm}^\alpha$ may be summarized[10] in the following way. We may write

$$f_{l\pm}^\alpha(W) = P_{l\pm}^\alpha(W) + B_{l\pm}^\alpha(W) + \frac{1}{\pi}\int_{M+\mu}^{\infty} \frac{\mathrm{Im} f_{l\pm}^\alpha(W')\,dW'}{W' - W - i\epsilon}, \tag{2.5}$$

where $P_{l\pm}^\alpha(W)$ is the nucleon pole term (including the crossed U-channel pole). The integral over the imaginary part of $f_{l\pm}^\alpha$ is taken along the physical cut which runs from $M+\mu$ to infinity. The term $B_{l\pm}^\alpha(W)$ is then the contribution to $f_{l\pm}^\alpha(W)$ which results from all other singularities and thus contains, in practice, a multitude of approximations.

We take guidance from Refs. 7 and 8 in deciding which singularities are dominant and must be retained. In addition to the nucleon pole terms which arise from the processes in Fig. 1 and

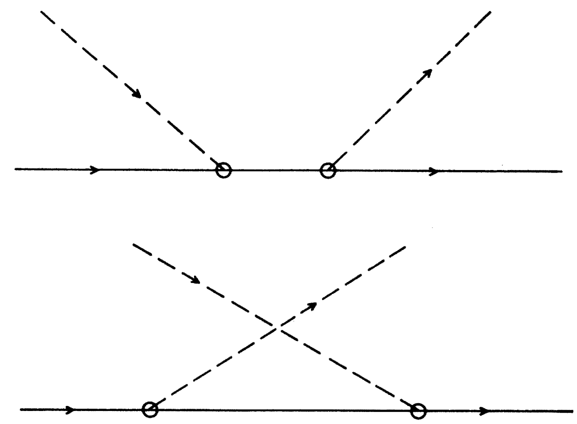

FIG. 1. The uncrossed and crossed nucleon pole diagrams. The dashed lines represent a pion, the solid lines represent a nucleon.

have explicitly been included in the term $P_{l\pm}^\alpha(W)$, we keep in $B_{l\pm}^\alpha(W)$ the crossing cut. This cut arises from a fundamental symmetry, and thus we feel it is important. It is the only singularity in $B_{l\pm}^\alpha(W)$ which survives in the limit of infinite nucleon mass and it was found in Ref. 7 that as much as twenty percent of the attraction in the low energy, resonance region arises from the crossing cut.

Fully relativistically, the singularities in the complex W plane which arise from crossing symmetry are not simple. We thus generate the crossing cut from the right-hand physical cut utilizing the infinite nucleon mass crossing relations. We feel that this is an adequate procedure because, although important, the crossing cut is not a dominant contribution to the amplitude along the physical cut.

We shall generate the N/D dispersion relation not for $f_{l\pm}^\alpha(W)$ but for a quantity $h_{l\pm}^\alpha(W)$ defined by

$$h_{l\pm}^\alpha(W) = f_{l\pm}^\alpha(W)/\rho_{l\pm}^\alpha(W), \tag{2.6}$$

where $\rho_{l\pm}^\alpha(W)$ is an arbitrary function. It can be chosen, as in Refs. 4, 7, 10, and 20, to cancel some of the singularities in $f_{l\pm}^\alpha(W)$ and thus render $h_{l\pm}^\alpha(W)$ a function with simpler analytic structure than $f_{l\pm}^\alpha(W)$.

In order that $h_{l\pm}^\alpha(W)$ have simple properties under crossing symmetry, we chose, as in Ref. 4, $\rho_{l\pm}^\alpha(W)$ to be real and invariant under crossing symmetry. This can be done in the limit of infinite nucleon mass, where crossing symmetry will then become

$$h_\alpha(-W) = \sum_\beta A_{\alpha\beta} h_\beta(W). \tag{2.7}$$

Here we have simplified the notation by using the single subscript α (or β) to represent both isospin and angular momentum quantum numbers with $\alpha = 1$ to 4 for the states P_{11}, P_{13}, P_{31}, and P_{33}, respectively. The matrix A is given by

$$A = \frac{1}{9}\begin{pmatrix} 1 & -4 & -4 & 16 \\ -2 & -1 & 8 & 4 \\ -2 & 8 & -1 & 4 \\ 4 & 2 & 2 & 1 \end{pmatrix}. \tag{2.8}$$

We limit ourselves to only qualitative discussions in S waves and thus neglect $B^{\alpha}_{l\pm}(W)$ altogether in these partial waves.

In this infinite nucleon mass limit, the crossing cut becomes a reflection of the elastic scattering cut onto the left-hand real axis by the matrix relation Eq. (2.7). Cauchy's theorem allows us to write for the contribution to $h_\alpha(W)$ from the crossing cut, $h_\alpha^{CR}(W) \equiv B_\alpha(W)/\rho_\alpha(W)$, which has only a left-hand cut from $M-\mu$ to $-\infty$,

$$h_\alpha^{CR}(W) = -\frac{1}{\pi}\int_{-M+\mu}^{+\infty} \frac{\operatorname{Im} h_\alpha(-W')}{W+W'}\, dW'. \tag{2.9}$$

With this approximation, the analytic structure of $h_\alpha(W)$ is as pictured in Fig. 2. The discontinuity of $h_\alpha(W)$ across the right-hand cut is proportional to $\operatorname{Im} h_\alpha(W)$. Unitarity can then be used to replace $\operatorname{Im} h_\alpha(W)$ by $|h_\alpha(W)|^2$ to generate a nonlinear Low equation for $h_\alpha(W)$. The N/D dispersion theory is an attempt to generate a solution to this equation. We shall take as our numerator function the contribution from the nucleon pole term and the crossing-cut

$$N_\alpha(W) = P_\alpha(W)/\rho_\alpha(W) + h_\alpha^{CR}(W). \tag{2.10}$$

It has historically been the practice[5,7] to include the crossing cut in the denominator function. For that approach, the iteration of the resulting nonlinear equation, although quite stable, converges to something[5,7] which is not a solution of the Low equation, Eq. (2.5). Here, we leave the crossing cut in the numerator in order to gain some additional insight into the possible origin of this difficulty. A denominator function $D_\alpha(W)$ is defined by

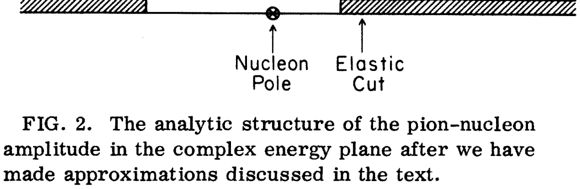

FIG. 2. The analytic structure of the pion-nucleon amplitude in the complex energy plane after we have made approximations discussed in the text.

$$h_\alpha(W) = \frac{f_\alpha(W)}{\rho_\alpha(W)} = \frac{N_\alpha(W)}{D_\alpha(W)}. \tag{2.11}$$

This denominator function $D_\alpha(W)$ must contain the elastic and inelastic cuts. In addition, if $f_\alpha(W)$ contains any zero not in N, then $D_\alpha(W)$ must contain poles[21] which correspond to these zeros. We shall make the usual assumption that no such singularities contribute significantly to the physical elastic scattering. With this assumption, $D_\alpha(W)$ contains only the elastic and inelastic cuts, and thus can be written as

$$D_\alpha(W) = D_\alpha(\infty) + \frac{1}{\pi}\int_{M+\mu}^{\infty} \frac{\operatorname{Im}[D_\alpha(W')]}{W'-W-i\epsilon}\, dW'. \tag{2.12}$$

The constant $D_\alpha(\infty)$ is a renormalization factor which we shall determine in the next section. Unitarity requires that $f_\alpha(W)$ can be written in the form

$$f_\alpha(W) = \hat{\eta}_\alpha e^{i\hat{\delta}_\alpha} \sin\hat{\delta}_\alpha / q, \tag{2.13}$$

where $\hat{\eta}_\alpha$ is the ratio of the elastic to the total cross section in the channel α, and q is the pion momentum in the center-of-mass frame. This implies a generalized optical theorem

$$\operatorname{Im}[f_\alpha(W)] = \frac{q}{\hat{\eta}_\alpha}|f_\alpha(W)|^2, \tag{2.14}$$

which gives for the imaginary part of $D_\alpha(W)$

$$\operatorname{Im}[D_\alpha(W)] = -\frac{q}{\hat{\eta}_\alpha(W)} N_\alpha(W)\rho(W). \tag{2.15}$$

Substituting this into Eq. (2.12) yields

$$D_\alpha(W) = D_\alpha(\infty) - \frac{1}{\pi}\int_{M+\mu}^{\infty} \frac{qN_\alpha(W')}{\hat{\eta}_\alpha(W')}\frac{\rho_\alpha(W)}{W'-W-i\epsilon}\, dW'. \tag{2.16}$$

In the next section we derive models for $P_\alpha(W)$. These expressions for $P_\alpha(W)$, together with Eqs. (2.7), (2.9), (2.11), and (2.16), allow us to solve for $f_\alpha(W)$ if we take $\hat{\eta}_\alpha(W)$ from experiment. The solution is found in the following manner. We take as a first approximation $N_\alpha(W)$ to be simply $P_\alpha(W)/\rho_\alpha(W)$. This is used in Eq. (2.6) to generate $D_\alpha(W)$ and thus an $h_\alpha(W)$. Crossing symmetry, Eq. (2.7) then gives $h_\alpha(-W)$ to be used in Eq. (2.9), which when inserted in Eq. (2.10) generates a new estimate for $N_\alpha(W)$. The process is repeated until one reaches a stable result.

III. MODELS

In this section we derive three relativistic models for the nucleon-pole term, each of which reduces, for P waves, to the Chew-Low model in the infinite nucleon mass limit. These nucleon-pole terms are then used in the dispersion relations derived in the previous section to generate elastic scattering amplitudes.

The first model we examine assumes pseudoscalar coupling of the pion to the nucleon

$$\mathcal{L}_{\rm int} = -ig_0\bar{\Psi}\gamma_5\vec{\tau}\cdot\vec{\phi}\Psi\,. \tag{3.1}$$

The usual Feynman diagram techniques then lead to the following amplitudes:

$$A_{\rm NP}{}^{1/2} = A_{\rm NP}{}^{3/2} = 0\,,$$

$$B_{\rm NP}{}^{1/2} = -4\pi f^2\left(\frac{2M}{\mu}\right)^2\left(\frac{3}{S-M^2}+\frac{1}{U-M^2}\right)v^2(q)\,,$$

and

$$B_{\rm NP}{}^{3/2} = 4\pi f^2\left(\frac{2M}{\mu}\right)^2\left(\frac{2}{U-M^2}\right)v^2(q)\,. \tag{3.2}$$

The variables are the usual variables with $S=W^2$ and

$$U = M^2+\mu^2-2E\omega-2q^2\cos\theta\,. \tag{3.3}$$

The nucleon pole partial wave helicity amplitudes can be derived using Eqs. (2.2) and (2.3). The angular integral in Eq. (2.3) is easily evaluated and explicit formulas for $P^{\alpha}_{l\pm}(W)$ are given in Ref. 22.

We also examine the case where the pion couples to the nucleon via pseudovector coupling,

$$\mathcal{L}_{\rm int} = -\frac{ig_0}{2M}\bar{\Psi}\not{q}\gamma_5\vec{\tau}\cdot\vec{\phi}\Psi\,. \tag{3.4}$$

Although pseudovector coupling yields a field theory which is not renormalizable, it is interesting for several reasons. First, it can be taken as an approximation[23] to a more general sigma model and thus, in a semiquantitative way, can be considered the lowest order approximation to a more realistic, renormalizable theory. Second, in a quark bag model[24] where the pion couples to the axial vector current at the surface of the bag in a chiral invariant manner, the pion coupling is of a derivative form.

The contribution to the A and B amplitudes from the nucleon pole then became

$$A_{\rm NP}{}^{1/2} = A^{3/2} = 4\pi f^2/M\left(\frac{2M}{\mu}\right)^2 v^2(q)\,,$$

$$B_{\rm NP}{}^{1/2} = -\frac{4\pi f^2}{M}\left(\frac{2M}{\mu}\right)^2\left(\frac{3}{S-M^2}+\frac{1}{U-M^2}+\frac{1}{M^2}\right)v^2(q)\,, \tag{3.5}$$

and

$$B_{\rm NP}{}^{3/2} = \frac{4\pi f^2}{M}\left(\frac{2M}{\mu}\right)^2\left(\frac{2}{U-M^2}-\frac{1}{2M^2}\right)v^2(q)\,.$$

In Eqs. (3.1) to (3.6) we have assumed the pion interacts with a nucleon of finite extent and thus the form factor $v(q)$ appears. One can derive[25] the form factor from the interaction of a pion with a nucleon which is a quark bag. One need not, however, ascribe such a meaning to $v(q)$. In a much less fundamental view (as in Ref. 15), one might view $v(q)$ merely as a method (perhaps artificial) of separating the pion-nucleon interaction into a low energy region [where $v(q)$ is finite] from an assumed independent, high energy region.

The third model we examine is a model which assumes pseudoscalar coupling but which expressly projects out all intermediate antinucleon states. We find this model interesting for several reasons. First, the Chew-Low model is derived assuming a nonrelativistic coupling of the pions to only nucleons. Its success,[7,8] then, tends to indicate that, at least qualitatively, a model which includes only the pion and nucleon degrees of freedom and suppresses the antinucleon degrees of freedom might be possible. The success of the Chew-Low model is *not* sufficient evidence to require this conclusion, however. This is because pseudoscalar coupling (which contains a substantial contribution from intermediate antinucleons), pseudovector coupling (which contains a partial suppression of the antinucleon states), and the complete suppression of antinucleon states all yield identical P-wave amplitudes in the infinite nucleon mass limit. Second, this model will allow us to investigate the contribution of intermediate antinucleon states beyond Born approximation and in more detail than has been done previously.

To derive this model, we must first define what we mean by an antinucleon state when a particle is off its mass shell. Such a definition is not unique; we will follow the choice often made in the nucleon-nucleon problem[17] of using the completeness of on-shell nucleon and antinucleon states to yield a definition. We define a projection operator onto on-shell nucleons by

$$\Lambda_+(p) = \frac{\gamma_0 E_p+\vec{\gamma}\cdot\vec{p}+M}{2M}\,, \tag{3.6}$$

and the projector onto antinucleons by

$$\Lambda_-(p) = \frac{-\gamma_0 E_p+\vec{\gamma}\cdot\vec{p}+M}{2M}\,, \tag{3.7}$$

where E_p is *restricted* to have the value $E_p=(p^2+M^2)^{1/2}$. These are clearly projection operators, as $\Lambda_+{}^2(p)=\Lambda_-{}^2(p)=1$, $\Lambda_+(p)+\Lambda_-(p)=1$, and

$\Lambda_+(p)\Lambda_-(p) = 0.$

The Feynmann propagator for a fermion, either on or off its mass shell, can be written

$$S_F(y'-y) = \int \frac{d^4p}{(2\pi)^4} \exp[-ip\cdot(y'-y)] \frac{\not p + M}{p^2 - M^2 + i\epsilon}$$
$$= \int \frac{d^4p}{(2\pi)^4}\left(\frac{M}{E_p}\right) e^{-ip\cdot(y'-y)} \frac{\Lambda_+(p)}{P_0 - E_p + i\epsilon}$$
$$+ \int \frac{d^4p}{(2\pi)^4}\left(\frac{M}{E_p}\right) e^{+ip\cdot(y'-y)} \frac{\Lambda_-(p)}{-P_0 + E_p - i\epsilon}\,. \tag{3.8}$$

Keeping only nucleon states in intermediate states corresponds to keeping only the first term on the right-hand side of Eq. (3.8).

The use of the nucleon-only propagator in evaluating the nucleon-pole diagrams, Fig. 1, yields, for the invariant amplitude $\mathfrak{M}$, Eq. (2.1),

$$\mathfrak{M} = i\bar{u}\left\{\frac{\vec{\tau}\cdot\hat{\phi}_2^*\vec{\tau}\cdot\hat{\phi}_1[\not Q - \gamma_0(E+\omega-M)]}{2M(E+\omega-M)} + \frac{\vec{\tau}\cdot\hat{\phi}_1\vec{\tau}\cdot\hat{\phi}_2^*[\not Q + \gamma_0(E-\omega-E_q^*)]}{2E_q^*(E_q^*-E+\omega)}\right\}u\,, \tag{3.9}$$

where

$$E_q^* = [M^2 + 2q^2(1+\cos\theta)]^{1/2}\,. \tag{3.10}$$

This amplitude is clearly not of the invariant form, Eq. (2.1), as it contains a term proportional to γ_0. This is because the projection onto nucleon states only is not an invariant operation. In order to generate partial wave helicity amplitudes we must generalize Eq. (2.2) to the case where

$$\mathfrak{M}^\alpha = \bar{u}(p_2, s_2)(A^\alpha + \gamma_0 C^\alpha + \not Q B^\alpha)u(p_1, s_1)\,. \tag{3.11}$$

The necessary generalization of Eq. (2.2) is derived in Ref. 22 and yields

$$f_1^\alpha = \frac{E+M}{8\pi W}[A^\alpha + C^\alpha + (W-M)B^\alpha],$$
$$f_2^\alpha = \frac{E-M}{8\pi W}[-A^\alpha + C^\alpha + (W+M)B^\alpha]. \tag{3.12}$$

Extracting the amplitudes A^α, B^α, and C^α from Eq. (3.9) yields

$$A_{NP}^{1/2} = A_{NP}^{3/2} = 0\,,$$
$$B_{NP}^{1/2} = -\left[\frac{3}{2M(E+\omega-M)} - \frac{1}{2E_q^*(E_q^*-E+\omega)}\right],$$
$$B_{NP}^{3/2} = -\frac{1}{E_q^*(E_q^*-E+\omega)}\,, \tag{3.13}$$
$$C_{NP}^{1/2} = -\left[-\frac{3}{2M} - \frac{E-\omega-E_q^*}{2E_q^*(E_q^*-E+\omega)}\right],$$

and

$$C_{NP}^{3/2} = -\left[\frac{E-\omega-E_q^*}{E_q^*(E_q^*-E+\omega)}\right].$$

For this model, one is not able to evaluate the angular integrals in Eq. (2.3) analytically. We are thus forced to expand the square root in Eq. (3.10). The natural expansion parameter would be $2q^2/M^2$; this expansion gives the Chew-Low result as the lowest order approximation. We have found this expansion to be much too slowly, if at all convergent. Instead, we choose as our expansion parameter

$$\kappa = \frac{2q^2}{M^2 + 2q^2}\,. \tag{3.14}$$

For small q^2 we have $\kappa \sim 2q^2/M^2$ but for large q we now have $\kappa \lesssim 1$ and the expansion remains convergent. This is a simple variation of Padé approximates. We thus write E_q^* in the form

$$E_q^* = (M^2 + 2q^2)^{1/2}(1 + \kappa\cos\theta)^{1/2}\,, \tag{3.15}$$

and expand the square root in a Taylor series. This is then used in Eq. (3.13) where the B and C amplitudes are expanded in a Taylor series. The final algebraic expressions for the partial wave helicity amplitudes are quite lengthy; an explicit example is given in Ref. 22.

IV. RESULTS

Certain qualitative features of the amplitudes can be understood by examining the nucleon-pole term $P_\alpha(W)$ alone. In Fig. 3, we present $\bar{N}(W) \equiv P_\alpha(W)/v^2(q)$ for α equal to the S_{11} channel. These curves are obtained from the amplitudes Eq. (3.2) for pseudoscalar coupling, Eq. (3.5) for pseudovector coupling, and Eq. (3.13) for the case in which we remove the contribution of intermediate antinucleons. For this last case the amplitude is expanded in a power series in κ, defined in Eq. (3.14), and the curves are labeled by n, the largest power of κ kept in the expansion. One can immediately conclude that this expansion is nicely convergent over the range pictured.

The well-known features of large S-wave amplitudes for pseudoscalar coupling and partial "pair suppression" for the pseudovector coupling are evident. The experimental phase shifts in the S_{11} channel are attractive [which corresponds to a positive $P(W)$], and we see that simple pseudoscalar or pseudovector coupling produces amplitudes of the incorrect sign in this channel. The complete suppression of the intermediate antinucleons is seen to greatly reduce the S-wave amplitude and also produce an amplitude of the correct sign.

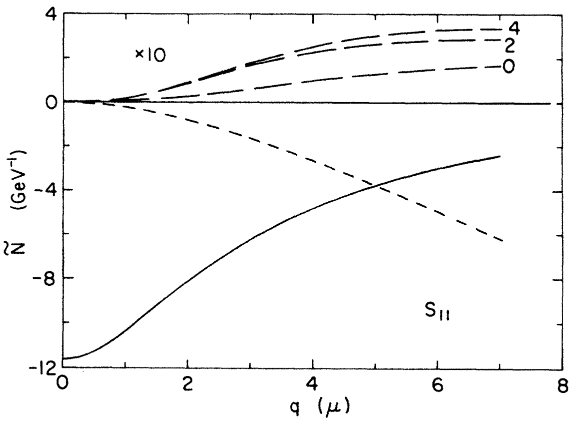

FIG. 3. The contribution to the S_{11} pion-nucleon amplitude of the nucleon-pole diagrams. $\tilde{N}=P/v^2(q)$ is plotted as a function of pion center-of-mass momentum q for three models; the solid line assumes pseudoscalar coupling, the short dash is pseudovector coupling, and the long dash is the model in which intermediate antinucleon states have been removed. For this last case, each curve is labeled by the highest power of κ, Eq. (3.14), kept in the expansion. Notice that the long dash curves have been multiplied by 10.

Perhaps more interesting is the nucleon pole amplitude in the P_{33} channel which is pictured in Fig. 4. Here we also include the Chew-Low amplitude as defined in Ref. 7. This shows immediately one of the main conclusions of this work: Any of the relativistic models differ substantially from the Chew-Low model. Previous discussions[11-13,15] have not found this large

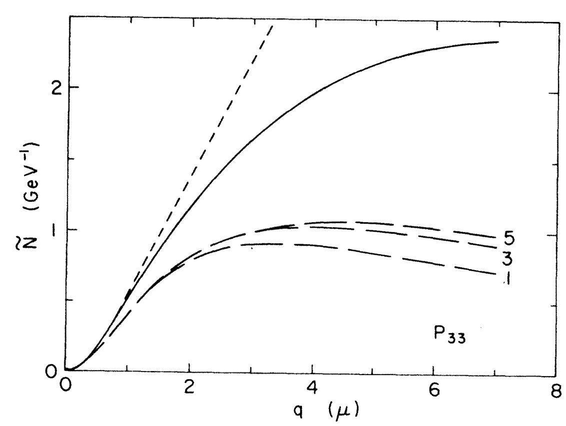

FIG. 4. The contribution to the P_{33} pion-nucleon amplitude of the nucleon-pole diagrams. $\tilde{N}=P/v^2(q)$ is plotted as a function of pion center-of-mass momentum q for four models; the short dash is for Chew-Low, the solid line is pseudoscalar coupling, which is the same as the pseudovector coupling, and the long dash is the model in which intermediate antinucleon states have been removed. For this last case, each curve is labeled by the highest power of κ, Eq. (3.14), kept in the expansion.

difference (with the exception of Ref. 14) because they dealt with expansions in M^{-1}. The conclusions of these previous works were misleading because they examined only the lowest order term in M^{-1} of a series which is not at all convergent. Another important conclusion to be drawn is that the intermediate antinucleon states contribute significantly to the P-wave channels.

Conclusions drawn from $P(W)$ are at best qualitiative. Approximating the full amplitude by $P(W)$ is, in the partial wave N/D approach, a form of the Born approximation. Since in many partial waves and for some of our models the interaction is very strong, the Born approximation is not at all valid. We thus use the partial wave N/D approach developed in Sec. II to sum a large class of Feynmann diagrams and to generate unitary amplitudes.

Before we proceed, we must determine the constant $D_\alpha(\infty)$ which appears in Eq. (2.12). This constant in the Chew-Low theory serves to renormalize the pion-nucleon coupling constant; there it may be chosen to fix the value of the residue of the nucleon pole. For the more general case which we are considering, the crossed nucleon pole graph in Fig. 1 yields a U-channel pole, which as a function of W is a short cut. One is thus unable to choose $D_\alpha(\infty)$ to fix the value of the pole term everywhere along this short cut.

We choose to fix the value of the amplitude at the point

$$W_U^0 = M - \frac{\mu^2}{2M}\,. \tag{4.1}$$

This should be compared to the S channel, uncrossed nucleon pole, which occurs at

$$W_S^0 = M\,. \tag{4.2}$$

We choose the point in Eq. (4.1) as it is the center of the U-channel cut and this cut plays a significant role in all the P-wave channels. Choosing $D_\alpha(\infty)$ such that $D_\alpha(W=W_U^0)=1$ then allows us to use f^2 as the renormalized pion nucleon coupling constant and fix it at its experimentally determined value of 0.082. The corrections to this procedure are of order $(\mu/M)\kappa\cos^2\theta$ and are small for our purpose.

With this choice of $D_\alpha(\infty)$, we take the nucleon pole terms and iterate in the partial-wave N/D equations derived earlier. We choose the function $\rho_\alpha(W)$, as was done in the Chew-Low model,

$$\rho_\alpha(W) = \frac{v^2(q)\,q^{2l}}{\omega}\,. \tag{4.3}$$

The results for the P_{33} channel are shown in Fig. 5 together with data from Refs. 26–29. For each model we assume a Gaussian form factor of the

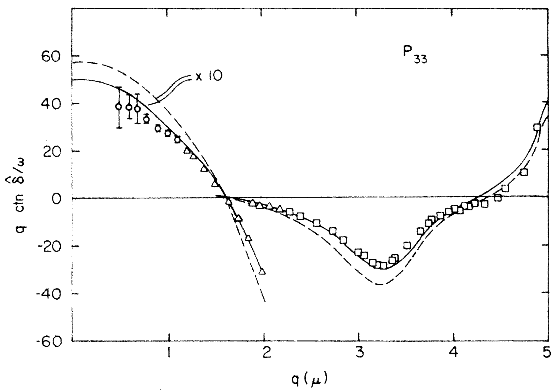

FIG. 5. The factor $q^3 \cot\delta/\omega$ versus center-of-mass momentum q in the P_{33} channel. The circles are data from Ref. 26, the triangles from Ref. 28, and the squares from Ref. 29. The solid line is the result of our model assuming pseudoscalar coupling; the dashed line is the intermediate antinucleons removed.

form

$$v(p) = e^{-p^2/\beta^2}, \tag{4.4}$$

and treat β as an adjustable parameter. We find that for pseudoscalar coupling and a value of $\beta = 1100$ MeV, we are in detailed quantitative agreement with the measured phase shifts. Similar results are obtained for pseudovector coupling which differs from the pseudoscalar coupling in the P_{33} channel only by small changes in the crossing term. This is considerably better agreement than was found by earlier work[10]; the improvement is due to our technique for including the inelastic cut. The phase shifts predicted by this model are almost identical to those predicted by the Chew-Low model with a cutoff of $\beta = 764$ MeV. This is the type of compensation discussed in Ref. 15. There it was noted that the dispersion relation is simply a formula which, if one allows an adjustable parameter to position the resonance, fixes the width of the resonance. We have found that if the inelastic cut is included then the dispersion relation yields the correct physical width, that this width is indeed independent of the details of the model, and that not only the width of the resonance, but also the quantitative details of the phase shifts are independent of the details of the model.

There are limitations on the models for which this type of compensation holds. The model in which we suppress completely the intermediate antinucleon states produce a very weak interaction as can be seen in Fig. 4. In order to produce the correct position of the resonance, the cutoff must be increased to over 2000 MeV, and the dispersion theory then produces a resonance which is too narrow. The dashed curve in Fig. 5 presents the results of this model for $\beta = 2550$ MeV. For this value of β the resonance is too narrow.

The fit to the phase shifts remains qualitatively reasonable and is as good as several fits in the literature which have been termed satisfactory. The fit is obviously inferior to that which results from our other models or from the Chew-Low model. We are thus led to conclude that, even in P waves, the coupling to the intermediate antinucleons provides a substantial part of the interaction and is necessary to produce precise quantitative agreement with the data. The possibility of building a model which includes only the pion and nucleon, without antinucleons, is thus dubious. The Chew-Low model, which appears to be such a model, is better condidered as an approximation to a fully relativistic model in which intermediate antinucleons are present.

For completeness we present the results of our N/D approach for the remaining S and P waves in Figs. 6–10. The results in the P_{13} and P_{31} channels are reasonable. The results for the S-wave channels bear out the conclusions drawn from examining the nucleon-pole terms alone. In some of the P-wave channels we see an interesting phenomenon: the contribution from crossing becomes sufficiently large so as to cancel the pole terms and produce a zero in our numerator function, and by Eq. (2.11), a zero in our amplitude. The original dispersion integral for the amplitude, Eq. (2.5), will not have a zero at that particular energy. Thus the N/D approach is *not* generating a solution to the original equation. It is not clear how to generate a crossing symmetric solution to the nonlinear Low equation. The occurrence of a zero in our approach, where we leave the crossing cut in the numerator, indicates that a Castillejo-Dalitz-Dyson (CDD) pole[21] is required

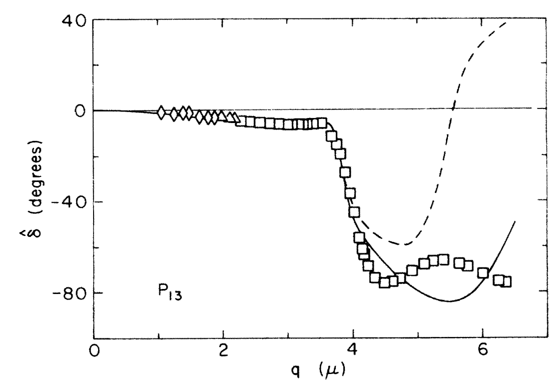

FIG. 6. The phase shifts versus center-of-mass momentum of q in the P_{13} channel. The diamonds are data from Ref. 27.

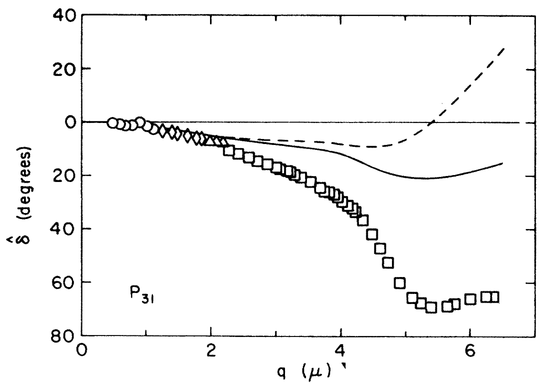

FIG. 7. The same as Fig. 6 except the P_{31} channel is shown.

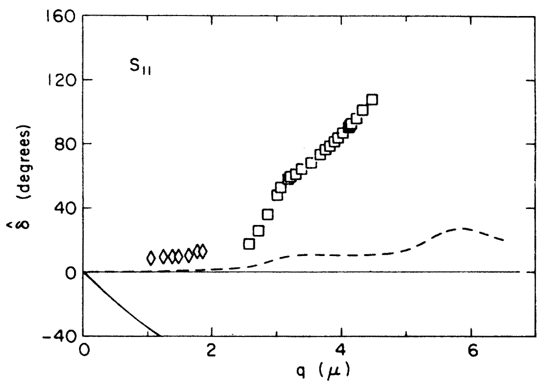

FIG. 9. The same as Fig. 6 except the S_{11} channel is shown.

(as suggested in Ref. 30) in the denominator function of Refs. 5 and 7 where the crossing cut is put into the denominator.

V. CONCLUSIONS

The ability to use pions as a probe of finite nuclei has generated a renewed interest in the understanding of the pion-nucleon interaction. This interest has in turn generated a number of models. We have examined several models, and although these models are not sufficiently general to reproduce the experimental phase shifts in each partial wave channel, the models are sufficiently physical that we are able to reach several important conclusions. First, we find that by extending the N/D approach to include that inelastic cut the approach is in excellent quantitative agreement with the measured P_{33} phase shifts with a pion-nucleon vertex of a range of 1100 MeV. We have found that relativity produces significant changes in the amplitude, changes which are not of order M^{-1}. However, the theory remains in detailed quantitative agreement with the P_{33} channel phase shifts by readjusting the cutoff of the pion-nucleon vertex. Since the cutoffs for both cases, 764 MeV for Chew-Low and 1110 MeV for the relativistic model, are large in comparison with typical nuclear momenta, this difference is probably not significant for our understanding of the interaction of pions with nuclei. This ability of the theory to compensate for quite different interactions by an adjustment of the cutoff was found to have only a limited range of validity. For the considerably weaker interaction in which we suppressed totally the intermediate antinucleon states, the resulting phase shifts are no longer in precise agreement with the data. Thus a model which is based on pion and nucleon degrees of freedom only seems unphysical; the Chew-Low model works well because it is an approximation to the relativistic model which includes antinucleons and because, although the relativistic corrections are large, the readjustment of the form factor is able to compensate for the large kine-

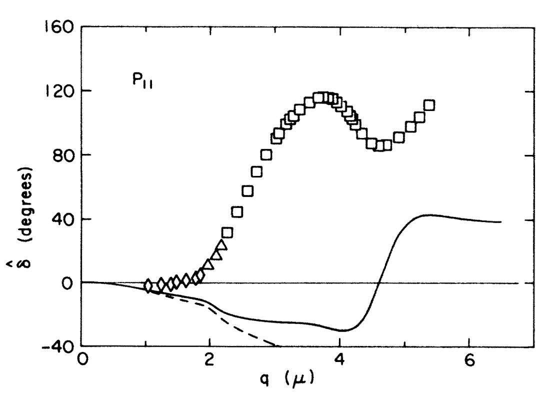

FIG. 8. The same as Fig. 6 except the P_{11} channel is shown.

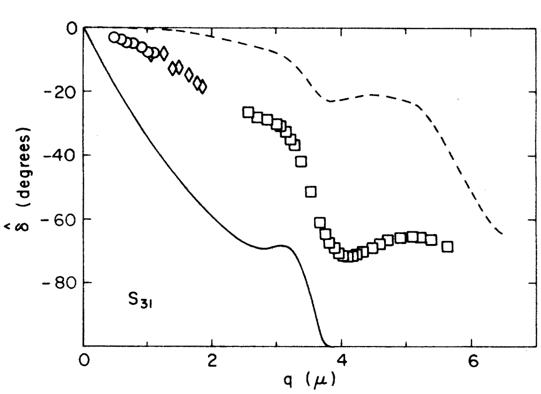

FIG. 10. The same as Fig. 6 except the S_{31} channel is shown.

matic differences.

There clearly remains much work to be done. A dynamic relativistic model of the pion-nucleon interaction which is in agreement with measured phase shifts is still nonexistent. The dynamic model of Cammarata and Banerjee[18] works well in the S wave but P-wave results are lacking. The model of Liu and Shakin[31] is, like the model of Ref. 8, only semidynamical in the sense that it involves potentials as a general replacement or approximation to whatever the underlying dynamics may be. Their model also incorporates a background term whose off-shell moment dependence is not determined. With our present limitations of the understanding of the interaction of a pion with the many-body nuclear target, a dynamic model may not be immediately necessary in order to gain quantitative information from pion-nucleus scattering. Such a model, however, would certainly be useful in building a microscopic theory of the pion-nucleus interaction and could certainly help remove some of the ambiguities in the information that is extracted from pion-nucleus experiments.

Note added in proof. We have received recently the extension of the model of Ref. 18 to p waves. These results have appeared in N-C. Wei and M. K. Banerjee, Phys. Rev. C 22, 2052 (1980); 22, 2061 (1980).

ACKNOWLEDGMENTS

This work is based, in part, on the Texas A &*M* Ph.D. thesis of R.J.M. and was supported, in part, by the National Science Foundation and the U.S. Department of Energy. The authors are grateful discussions with M. Johnson, B. Ver West, and C. Dominguez.

*Present address.

†Permanent address (on leave).

[1]R. Landau and F. Tabakin, Phys. Rev. D 5, 2746 (1972).

[2]J. T. Londergan, K. W. McVoy, and E. J. Moniz, Ann. Phys. (N.Y.) 86, 147 (1974).

[3]D. J. Ernst, J. T. Londergan, E. J. Moniz, and R. M. Thaler, Phys. Rev. C 10, 1708 (1974).

[4]G. F. Chew and F. E. Low, Phys. Rev. 101, 1570 (1956); F. E. Low, *ibid.* 97, 1392 (1955).

[5]G. Salzman and F. Salzman, Phys. Rev. 108, 1619 (1957).

[6]C. B. Dover, D. J. Ernst, R. A. Friedenberg, and R. M. Thaler, Phys. Rev. Lett. 33, 728 (1974); K. K. Bajaj and Y. Nogami, *ibid.* 34, 701 (1975).

[7]D. J. Ernst and M. B. Johnson, Phys. Rev. C 17, 247 (1978).

[8]D. J. Ernst and M. B. Johnson, Phys. Rev. C 22, 651 (1980).

[9]P. T. Matthews and A. Salam, Rev. Mod. Phys. 23, 311 (1951).

[10]G. L. Shaw, in *Pion-Nucleon Scattering*, edited by G. L. Shaw and D. Y. Wong (Wiley-Interscience, New York, 1969).

[11]F. J. Dyson, Phys. Rev. 95, 1644 (1954).

[12]G. Wentzel, Phys. Rev. 92, 173 (1953).

[13]E. M. Henley and W. Thirring, *Elementary Quantum Field Theory* (McGraw-Hill, New York, 1962).

[14]H. W. Wyld, Phys. Rev. 96, 1661 (1954).

[15]G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. 106, 1337 (1957).

[16]J. Bjorken and S. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[17]G. E. Brown and A. D. Jackson, *The Nucleon-Nucleon Interaction* (North-Holland, New York, 1976); S. A. Coon, M. D. Scadron, P. C. McNamee, B. R. Barrett, D. W. E. Blatt, and B. H. J. McKellar, Nucl. Phys. A317, 242 (1979).

[18]J. B. Cammarata and M. K. Banarjee, Phys. Rev. D 16, 1334 (1977); Phys. Rev. C 17, 1125 (1978).

[19]M. G. Olsson and E. M. Osypowski, Nucl. Phys. B101, 136 (1975); M. D. Scadron, in *Few Body Dynamics*, edited by E. N. Mitra *et al.* (North-Holland, Amsterdam, 1976).

[20]G. Chew and S. Mandlestam, Phys. Rev. 119, 467 (1960).

[21]L. Castillejo, R. H. Dalitz, and F. J. Dyson, Phys. Rev. 101, 453 (1956).

[22]R. J. McLeod, Texas A & M Ph.D. thesis, 1979 (unpublished).

[23]G. E. Brown, in *Mesons in Nuclei*, edited by M. Rho and D. H. Wilkinson (North-Holland, New York, 1979).

[24]G. E. Brown, M. Rho, and V. Vento, Phys. Lett. 84B, 383 (1979); G. E. Brown and M. Rho, *ibid.* 82B, 177 (1979).

[25]G. A. Miller, A. W. Thomas, and S. Theberge, Phys. Lett. 91B, 192 (1980).

[26]P. Y. Bertin, B. Coupat, A. Hivernat, D. B. Isabella, J. Duclos, A. Gerard, J. Miller, J. Morgenstern, J. Picard, P. Vernin, and R. Powers, Nucl. Phys. B106, 341 (1976).

[27]H. Zimmerman, Helv. Phys. Acta 48, 191 (1975).

[28]J. R. Carter, D. V. Bugg, and A. A. Carter, Nucl. Phys. B58, 378 (1978).

[29]CERN theoretical in D. J. Herndon, A. Barbarao-Galtieri, and A. H. Rosenfeld, LRL Report No. UCRL-20030 πN (1970).

[30]D. J. Ernst, R. A. Friedenberg, and M. B. Johnson, Z. Phys. A 287, 363 (1978).

[31]L. C. Liu and C. M. Shakin, Phys. Rev. C 18, 604 (1978).