Effect of exchange-energy corrections on self-consistent meson masses and on the equation of state of nuclear matter

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An estimate is made of the effect of expanding about the full nucleon and meson propagators as well as the mean meson fields in certain model renormalizable relativistic field theories of nuclear matter. Meson masses are determined self-consistently by satisfying, at zero momentum only, the Schwinger-Dyson equations which result when the energy functional is truncated at the two-loop level. The tachyonic pion problem of the standard mean field expansion is eliminated but the opposite problem of a far too massive pion in normal nuclear matter arises.

[NUCLEAR STRUCTURE Relativistic mean field theories of nuclear matter, tachyonic pions. Approximate solutions of Schwinger-Dyson equations.]

I. INTRODUCTION

The relativistic mean field model of high density nuclear matter as proposed by Walecka[1] and elaborated on by Chin[2] has many virtues. The model is relatively easy to solve, it seems to describe the bulk properties of nuclei including the spin-orbit interaction,[3] and the exchange energies calculated perturbatively about the mean field result[2] and certain higher order direct energies[4] were shown not to be of major importance.

However, questions still remain about the mean field as the basic solution about which to do perturbative corrections. One must always be cautious when the coupling constants are as large as $g^2/4\pi \cong 10$. For instance, a naive application of perturbation theory results in vacuum polarization causing the effective coupling constant to diverge at a Fermi momentum on the order of the nucleon mass,[2] $k_F \cong m_N$. A second worry is that in fact correlation effects may be large at normal nuclear matter densities.[5] Since the parameters of the model are determined by fitting to the properties of nuclear matter at normal density, one's extrapolation to high density may be very uncertain.

Recently an extension of the model was proposed which includes the pi and rho mesons in a renormalizable way.[6] Unfortunately, as we show below, the mean field approximation predicts that the pion goes tachyonic at a density lower than normal nuclear density. This reemphasizes the importance of understanding corrections to the mean field theory.

In fact this latter difficulty seems to be a rather general result; that is, if a renormalizable Lagrangian has cubic meson couplings, then at high enough density at least one of these mesons will go tachyonic in a strictly mean field treatment. This problem was noticed by Lee and Margulies[7] in the context of the sigma model without pions. The solution to the problem was also given by those authors: namely, one must expand about not only the mean meson fields but also about the full renormalized propagators.

The aim of this paper is to estimate numerically the effect of expanding about the full propagators on the mean field results of the Walecka model. Since the exact propagators are not known, we will employ a particular approximation to the problem, to be discussed later.

II. TACHYONS IN THE MEAN FIELD APPROXIMATION

The basic Walecka Lagrangian is

$$\mathcal{L}_W = \bar{\psi}(i\not\partial - m_N)\psi + \tfrac{1}{2}(\partial_\mu\phi\,\partial^\mu\phi - m_s{}^2\phi^2) - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}m_v{}^2\omega^\mu\omega_\mu + \bar{\psi}(g_s\phi - g_v\not\omega)\psi\,, \quad (1)$$

where ψ, ϕ, and ω^μ are the nucleon, scalar, and vector meson fields. Serot has included the pion by adding a term

$$\mathcal{L}_\pi = \tfrac{1}{2}(\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - m_\pi{}^2\vec{\pi}^2) - ig_\pi\bar{\psi}\gamma_5\vec{\tau}\cdot\vec{\pi}\psi + \tfrac{1}{2}g_{\phi\pi}m_s\phi\,\vec{\pi}^2\,. \quad (2)$$

The rho meson may also be included but for a mean field treatment of symmetric nuclear matter it will not affect our result.

A mean field treatment may be given based on $\mathcal{L}_W + \mathcal{L}_\pi$. The resulting equations are derived and discussed in Appendix A, where it is shown that the effective pion mass squared is given by

$$m_\pi^{*2} = m_\pi{}^2 - \frac{g_s g_{\phi\pi}}{m_s}n_s\,. \quad (3)$$

Here $n_s = \langle\bar{\psi}\psi\rangle$ is the scalar number density, equal to the baryon number density $n = \langle\psi^\dagger\psi\rangle$ at low density. With $m_s/g_s = 57.5$ MeV as determined by fitting the saturation point of nuclear matter, one has $g_{\phi\pi} \cong 10$ in order to get the correct πN s-wave scattering lengths.[6] Thus $m_\pi^{*2} < 0$ when the density is greater than 0.015 fm^{-3}, which is roughly 10% of nuclear density.

The reason for this behavior is not hard to find. In the tree approximation to πN scattering in the above Lagrangian there are *two* contributions, which are shown in Fig. 1(a). For s-wave scattering these two contributions have opposite signs and cancel each other to 1 part in 50 in order to yield the very small s-wave scattering length $a_0^{(+)}$ observed experimentally. (For a simple explicit calculation see Campbell.[8]) This was the prime motivation for introducing the $\phi\vec{\pi}^2$ coupling in Eq. (2).[6] These two contributions then lead to contributions to the pion self energy in nuclear matter as shown in Fig. 1(b). Only the *attractive* tadpole diagram is included in the mean field solution, so it is no wonder that the pion goes tachyonic at such a low density. The consequence would be an s-wave pion condensate, which is known not to occur from other studies.

A similar behavior occurs in the sigma model with or without a massive vector meson. In fact this behavior should occur in any mean field treatment of a model Lagrangian which has a pseudoscalar πN coupling and a scalar meson exchange mechanism to give the correct $a_0^{(+)}$ value. A pseudovector coupling would avoid this problem, but then the theory is not renormalizable and we would be playing under different rules.

As noticed by Lee and Margulies, any model which has a cubic meson coupling has the possible defect of generating a tachyon pole in a mean field treatment. Thus this is a problem of rather general significance and is not restricted to the pion problem.

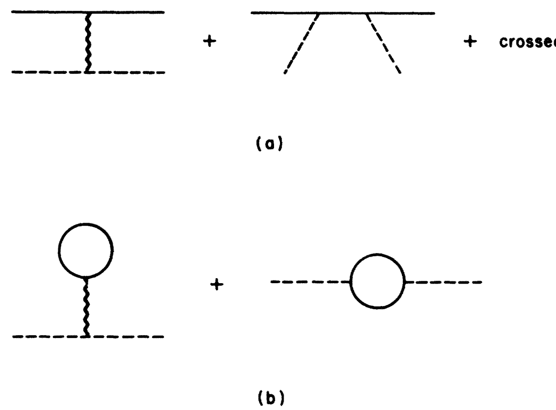

FIG. 1. The tree approximation to πN scattering in a model field theory is shown in (a). The solid line is a nucleon, the dashed line a pion, and the squiggly line a scalar meson. The corresponding contributions to the pion self-energy are shown in (b).

III. VARIATIONAL FORMALISM

The technique for handling unphysical tachyon poles is to write the energy density as a functional of not only the mean meson fields but also of the full propagators. For derivations and a more complete discussion, see Lee and Margulies[7] and also Norton and Cornwall.[9] For a specific discussion in the context of abnormal matter, see Nyman and Rho.[10]

One can write the energy density as

$$
\begin{aligned}
F(S,D,\Delta) = {} & U(\bar{\phi},\bar{\omega}^\mu) + 2\kappa \int \frac{d\vec{p}}{(2\pi)^3} (\vec{p}^{\,2} + m_N^{02})^{1/2}\, \theta(k_F - |\vec{p}|) \\
& + \int \frac{d^4p}{(2\pi)^4} \operatorname{tr}\{\ln[S(p)/S^0(p)] - S(p)/\bar{S}^0(p) + 1\} - \tfrac{1}{2} \int \frac{d^4p}{(2\pi)^4} \operatorname{tr}\{\ln[D(p)/D^0(p)] - D(p)/\bar{D}^0(p) + 1\} \\
& - \tfrac{1}{2} \int \frac{d^4p}{(2\pi)^4} \operatorname{tr}\{\ln[\Delta(p)/\Delta^0(p)] - \Delta(p)/\bar{\Delta}^0(p) + 1\} + \sum_{l=2}^{\infty} f_l(S,D,\Delta) - \text{subtractions}\,. \qquad (4)
\end{aligned}
$$

Here S^0, D^0, and Δ^0 are the bare perturbative propagators for nucleons, vector mesons, and scalar mesons. $\bar{S}^0$, $\bar{D}^0$, and $\bar{\Delta}^0$ are the propagators in the presence of the mean meson fields $\bar{\phi}$ and $\bar{\omega}^\mu$, and S, D, and Δ are the full renormalized propagators. The mean field potential energy is $U(\bar{\phi},\bar{\omega}^\mu)$ and κ, the isospin degeneracy, is equal to 2 for nuclear matter. The f_l is the sum of all l loop, two line irreducible energy diagrams with the bare perturbative propagators replaced with the full propagators.

The above energy density functional possesses some useful variational properties. There are the usual minimization conditions on the mean meson fields

$$\frac{\delta}{\delta\bar{\phi}} F = 0, \quad \frac{\delta}{\delta\bar{\omega}^\mu} F = 0\,. \qquad (5)$$

There are also the following minimization conditions:

$$\frac{\delta}{\delta S(p)}F=0,\quad \frac{\delta}{\delta D(p)}F=0,\quad \frac{\delta}{\delta\Delta(p)}F=0. \tag{6}$$

These last three conditions can easily be shown to lead to the following Schwinger-Dyson equations:

$$\begin{aligned} S^{-1}(p)-\overline{S}_0^{-1}(p) &= -\frac{\delta}{\delta S(p)}\sum_{l=2}^{\infty} f_l(S,D,\Delta),\\ D^{-1}(p)-\overline{D}_0^{-1}(p) &= 2\frac{\delta}{\delta D(p)}\sum_{l=2}^{\infty} f_l(S,D,\Delta),\\ \Delta^{-1}(p)-\overline{\Delta}_0^{-1}(p) &= 2\frac{\delta}{\delta \Delta(p)}\sum_{l=2}^{\infty} f_l(S,D,\Delta). \end{aligned} \tag{7}$$

As an example, we show in Fig. 2 the quantity $f_2(S,D,\Delta)$ for the basic Walecka Lagrangian $\mathcal{L}_W$, and the resulting Schwinger-Dyson equations which also serve to define the nucleon and meson self-energy parts.

IV. APPROXIMATE SOLUTIONS

Unfortunately there is no known closed formula for the energy density functional F. Each f_l must be constructed on a case by case basis using perturbation theory. This is one of the difficulties in finding a truly practical, truly nonperturbative variational solution.

The approximations to be used subsequently may be described in several steps. First we keep only the two-loop exchange diagrams, the f_2 of Fig. 2. If we were to include pions at this stage then of cours there would be additional diagrams. The primary motivation here is one of practicality, with the additional thought that this particular truncation should be adequate to eliminate tachyons in those theories with cubic meson couplings.

Once we have decided upon which loop diagrams to keep, then the nucleon and meson self-energies should be determined self-consistently as functions of momentum p. Solving coupled nonlinear integral equations is hard enough, but this problem has the additional complication that infinite renormalizations need to be done. Possible conceptual as well as calculational difficulties have been discussed to some extent by Baym and Grinstein.[11] Therefore our second approximation is to pretend that the full propagators have the same form as the free propagators with the vacuum masses replaced by density dependent effective masses m_N^*, m_s^*, and m_v^*. The meson masses are then determined by satisfying the Schwinger-Dyson equations at zero momentum only. In fact the vector meson self-energy turns out to have the form

$$\Pi_v^{ex}(0)=-Ag^{\mu\nu}+Bg^{\mu 0}g^{\nu 0}, \tag{8}$$

where A and B are constants. Only the $g^{\mu\nu}$ term is kept self-consistently, since the $g^{\mu 0}g^{\nu 0}$ term changes the form of the propagator.

It also turns out that the vector exchange contribution to the nucleon self-energy, Σ_v^{ex}, depends upon the particular form of the vector propagator one uses, i.e.,

$$\frac{1}{p^2-m_v^{*2}}g^{\mu\nu}$$

or

$$\frac{1}{p^2-m_v^{*2}}\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{m_v^{*2}}\right).$$

One would not expect the $p^\mu p^\nu$ term to contribute to observable quantities. Most likely this is a difficulty to be faced by most treatments which are based on approximate solutions for the full propagators. An example of this difficulty in a field theory approach to NN scattering has been found by Bessis, Mery, and Turchetti.[12] Another example of this difficulty, in the context of a nonperturbative approach to the electron propagator in QED, has been pointed out by Atkinson and Fry.[13] In fact we have used the first form of the vector propagator without the $p^\mu p^\nu$ term.

In principle m_N^* may be determined either from the Schwinger-Dyson equation or by minimizing the total energy, because of the variational principle. However, in view of the fact that we have only an approximate solution, it was thought better to determine m_N^* by minimizing the total energy directly with respect to m_N^*.

The total energy density may be written as

$$\epsilon=\epsilon_{FG}+\epsilon_s^{dir}+\epsilon_v^{dir}+\epsilon_s^{ex}+\epsilon_v^{ex}. \tag{9}$$

Here ϵ_{FG} is the noninteracting Fermi gas result. The direct energies are

$$\begin{aligned} \epsilon_s^{dir} &= \frac{m_s^2}{2g_s^2}(m_N+\Sigma^{ex}-m_N^*)^2,\\ \epsilon_v^{dir} &= \frac{g_v^2}{2m_v^2}n^2, \end{aligned} \tag{10}$$

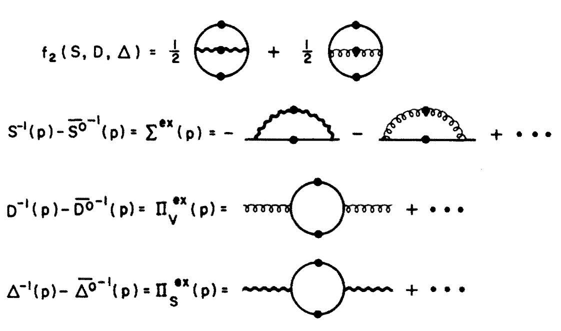

FIG. 2. The energy density functional f_2 for the Walecka model, and the associated contributions to the self-energies. The curly line is a massive vector meson.

since one has that

$$\bar{\omega}^{\mu} = \frac{g_v}{m_v^2} n g^{\mu 0} ,$$
$$m_N^* = m_N - g_s \bar{\phi} + \Sigma^{ex} \tag{11}$$

The baryon density and Fermi momentum are related by

$$n = \frac{2}{3\pi^2} k_F^3 . \tag{12}$$

The exchange energies ϵ_s^{ex} and ϵ_v^{ex} have been computed previously by Bolsterli[14] and by Chin,[2] and the results are reproduced in Appendix B. One assumes that the diagrams are renormalized perturbatively, and then the vacuum masses m_N, m_s, and m_v are replaced by the density dependent masses m_N^*, m_s^*, and m_v^*. This then modifies the vacuum fluctuation energy which arises because of the differences in the Dirac seas in the case of the nucleons and because of the differences in the zero point energies in the case of the mesons. This quantity $\Delta\epsilon_{vac}$ is difficult to compute. It has been done in two particular cases, by Lee and Margulies[7] and by Chin,[2] who found that it was not a particularly important contribution. Therefore we neglect it here.

The results of a numerical calculation, based on Eq. (9) and the equations in Appendix B are shown in Figs. 3 to 6. In all figures the solid lines refer to the results obtained with Eq. (9), including the (pseudo) self-consistent exchange energies, while the dashed lines refer to a purely mean field calculation without exchange energies.

In Fig. 3 we show the energy per nucleon $E = \epsilon/n - m_N$ versus k_F and n/n_0. Normal nuclear matter is taken to be bound by 16 MeV at $k_F = 263$ MeV. The vacuum masses were taken to be $m_v = 783$ MeV and $m_s = 550$ MeV, as Chin[2] has used. Then, by fitting to the minimum of the energy curve, we find $g_s^2/4\pi = 9.33$ and $g_v^2/4\pi = 14.37$ for the mean field, and $g_s^2/4\pi = 14.70$ and $g_v^2/4\pi = 21.75$ with the exchange energies included. The first thing to notice about the figure is that the curves look qualitatively similar, although the mean field typically lies lower in energy. However, quantitatively the compressibility K is quite different, being 500 MeV for the mean field and 3 GeV for the exchange calculation. This is to be compared with a best experimental determination of $K = 210 \pm 30$ MeV.[15] The mean field results depend only on the ratios g_v/m_v and g_s/m_s. With the exchange energies the results depend on the four quantities g_v, g_s, m_s, and m_v. Thus K is determined uniquely in a mean field calculation with $\mathcal{L}_W$ once the energy and density of normal matter is specified.

With the exchange energies it is conceivable, though not likely, that by varying m_s and m_v within reasonable bounds one might be able to obtain a more suitable value for K. One might also obtain a better value for K by adding ϕ^3 and ϕ^4 terms to $\mathcal{L}_W$.[16] Both of these investigations lie outside the spirit of this paper.

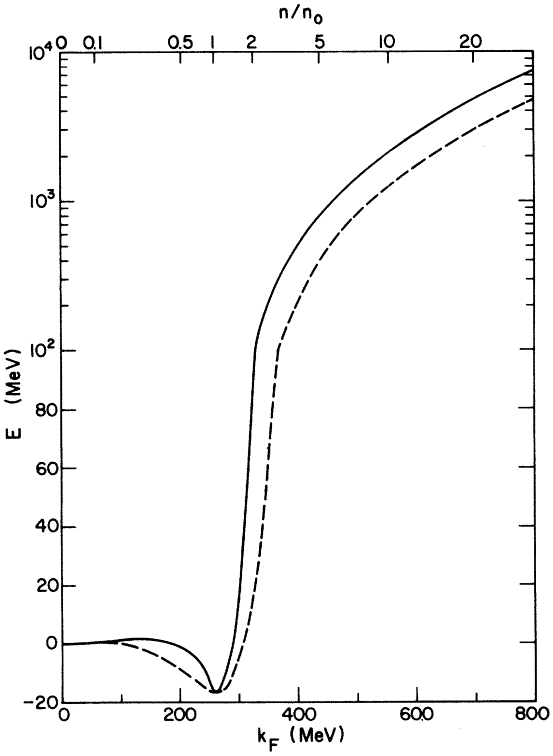

FIG. 3. The energy per nucleon versus Fermi momentum k_F and compression n/n_0. The dashed line is the mean field and the solid is the exchange modified result, Eq. (9). Note the change from a linear to a logarithmic scale at 100 MeV.

In Fig. 4 we show the contributions from the five component energies of Eq. (9). The first thing to notice is that the direct vector energy E_v^{dir} indeed dominates at high density.[2] The exchange energies are generally quite small, especially at high density. However, at lower densities there is a big cancellation among the various components. For instance, at normal density

$$\begin{aligned} E_{FG} - m_N &= -593 \text{ MeV} , \\ E_v^{dir} &= 270 , \\ E_s^{dir} &= 320 , \\ E_v^{ex} &= -52 , \\ E_s^{ex} &= 39 . \end{aligned} \tag{13}$$

Because of this the results are very sensitive to the input parameters and to the inclusion or exclusion of the exchange energies.

In Fig. 5 we plot the effective masses as func-

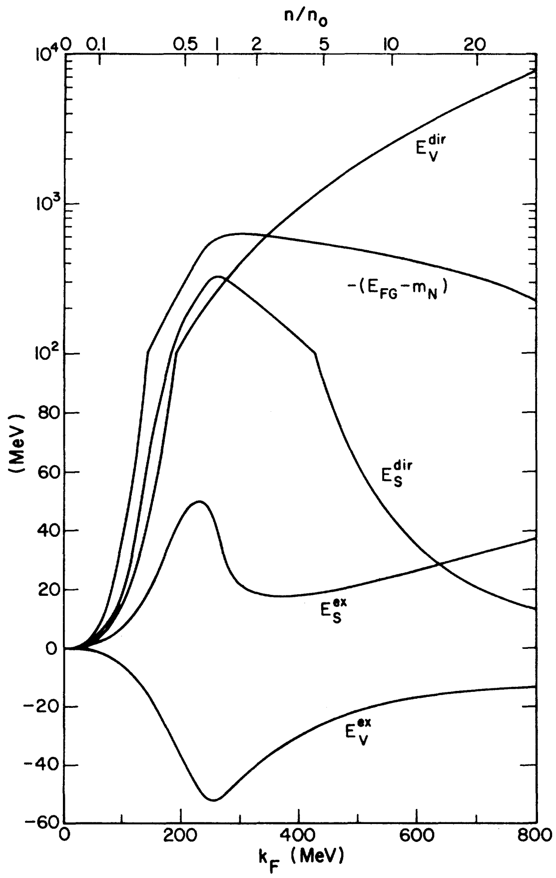

FIG. 4. The Fermi gas, direct, and exchange energy contributions to Fig. 3.

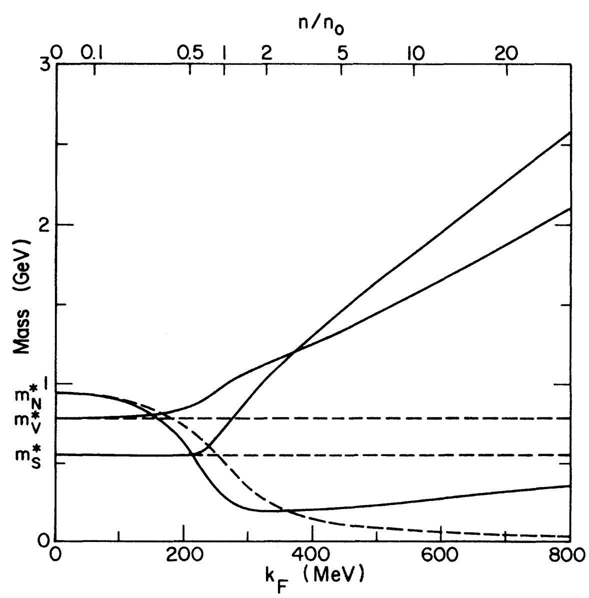

FIG. 5. Values of the nucleon, scalar, and vector meson effective masses in the mean field (dashed lines) and exchange modified (solid lines) calculations.

tions of k_F and n/n_0. The important point to notice about this graph is that the meson masses begin to increase dramatically around normal nuclear density. In fact at that point $m_v^* = 1$ GeV and $m_s^* = 720$ MeV. Above several times normal density the meson masses are roughly proportional to k_F. This is the primary reason that the exchange energies are suppressed at high density, i.e., the meson propagators go like $1/(k^2 - m_{s,v}^{*2})$.

Although the pion is not included in $\mathcal{L}_W$, we can still make an estimate of what m_π^* would be if the pion were included in a consistent treatment via Eq. (2). Referring to Fig. 1(b) we have

$$m_\pi^{*2} = m_\pi^2 - g_{\phi\pi} m_s \bar{\phi} + \Pi_{ps}. \qquad (14)$$

For the parameters used in the exchange calculation one needs $g_{\phi\pi} = 7.7$ in order to have a small $a_0^{(+)}$. The result of evaluating Eq. (14) is shown in Fig. 6. The tachyon problem arising in a strict mean field approach is gone. As with the other mesons m_π^* increases nearly linearly with k_F at high density. Notice however that at normal density $m_\pi^*/m_\pi = 4.8$. It is ironic that instead of the tachyon problem one has gone to the opposite extreme of having an unacceptably massive pion propagating in nuclear matter. Undoubtedly the presence of nuclear matter plus the large coupling constants have destroyed the delicate πN s-wave cancellations discussed in Sec. II.

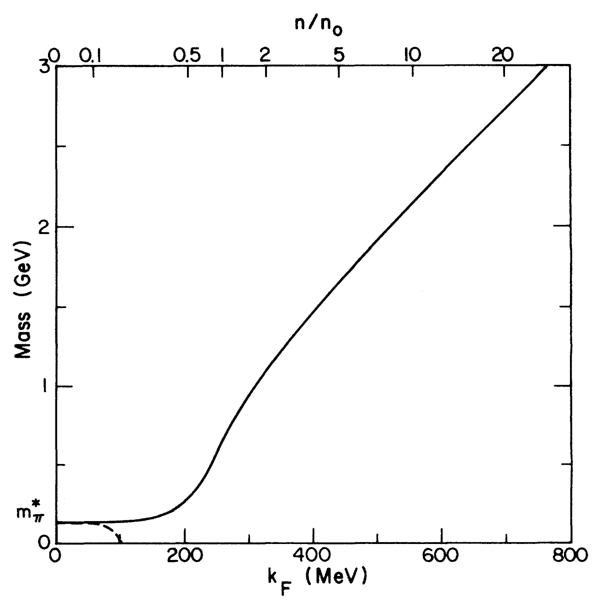

FIG. 6. Estimate of the pion effective mass from Eq. (14). Notice that the mean field calculation (dashed line) predicts an unphysical tachyon at below normal density, whereas the calculation including exchange effects (solid line) gives a very massive pion at normal density.

V. CONCLUSION

In this paper we have pointed out that the model nuclear Lagrangian of Walecka and Serot, when solved in mean field approximation, gives rise to a tachyonic pion at significantly less than normal nuclear density. The cause of this behavior is not hard to locate. For s-wave pion-nucleon scattering there is a delicate cancellation between two diagrams. The mean field approximation includes the equivalent of only one of these diagrams, the attractive tadpole pion self-energy. Keeping both diagrams is equivalent to including the exchange energy terms in a self-consistent expansion of the energy density functional about the full propogators as well as the mean meson fields. Unfortunately the model then becomes extremely difficult to solve. Our principal approximations were to keep only the lowest order diagrams in a loop expansion of two-line irreducible diagrams, and to satisfy the Schwinger-Dyson equations only at $p=0$. Although this approximation scheme cures the problem of a tachyonic pion at subnuclear densities, it leads to a far too massive pion at normal nuclear density.

Systematic improvements to this scheme are possible. One could attempt to solve the truncated Schwinger-Dyson equations not just at $p=0$ but at all momenta. This is necessary for a proper treatment of pion condensation in this model. The difficult problem of renormalizing such a self-consistent scheme and the associated vacuum fluctuation energy remains an open question.

If one wants to use a relativistic quantum field theory approach to nuclear matter then the options available seem to be threefold. First, we could choose to ignore the pion altogether. Second, we could insist on a renormalizable Lagrangian, in which case a coupling of the form $\phi\vec{\pi}^2$ seems to be inevitable in order to obtain the correct s-wave scattering lengths. Third, we could drop the requirement of renormalizability so that a pseudovector pion-nucleon coupling is allowed. The first option can be thrown out immediately in view of the importance of the pion in nuclear physics. The second option is possible but, as discussed in this paper, one must go beyond the mean field approximation in order to avoid a tachyonic pion at subnuclear densities.

The third option we consider to be the most viable. The mean field approximation may be applied without the appearance of unphysical tachyons. Pion condensation may also be more easily considered. Although the Lagrangian is then nonrenormalizable, which is mathematically unpleasant, this really should be of secondary concern. We know that hadrons are actually composites of quarks and gluons and not point particles. Hence a better description of a system of hadrons may follow from a nonrenormalizable or nonlocal field theory rather than from a local renormalizable one. One may then take the point of view that an economical description of many properties of finite nuclei and infinite nuclear matter is afforded by such an effective nonrenormalizable nuclear Lagrangian when used in conjunction with a definite physical approximation scheme (such as mean field). Such a description will only be valid up to densities of two to ten times normal when a phase transition to quark matter takes place.

ACKNOWLEDGMENTS

I am gratefully indebted to M. Bolsterli and especially to D. K. Campbell for a number of enlightening discussions.

APPENDIX A

In this appendix we calculate the effective nucleon and meson masses in the mean field approximation to the Walecka-Serot Lagrangian.

The finite density Lagrangian is

$$\begin{aligned}\mathfrak{L}_W+\mathfrak{L}_\pi=\bar{\psi}(i\partial\!\!\!/-m_N+g_s\phi-g_v\omega\!\!\!/-ig_\pi\gamma_5\vec{\tau}\cdot\vec{\pi}+\mu\gamma^0)\psi\\ +\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi-\tfrac{1}{2}m_s{}^2\phi^2+\tfrac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}-\tfrac{1}{2}m_\pi{}^2\vec{\pi}^2\\ +\tfrac{1}{2}g_{\phi\pi}m_s\phi\vec{\pi}^2-\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}+\tfrac{1}{2}m_v{}^2\omega^\mu\omega_\mu.\end{aligned}\tag{A1}$$

We anticipate that the scalar and vector meson fields will acquire finite constant components so we write

$$\phi=\bar{\phi}+\phi',\quad \omega^\mu=\bar{\omega}^\mu+\omega'^\mu,\quad \vec{\pi}=\vec{\pi};\tag{A2}$$

where

$$\langle\phi\rangle=\bar{\phi},\quad \langle\omega^\mu\rangle=\bar{\omega}^\mu,\quad \langle\vec{\pi}\rangle=0.\tag{A3}$$

Notice that we do not allow for a pion condensate. The $\bar{\phi}$ and $\bar{\omega}^\mu$ are c numbers and the ϕ', ω'^μ, and $\vec{\pi}$ are quantum fields. The resulting *linearized* field equations, appropriated averaged over the ground state of the system, are

$$\begin{aligned}&\Box\vec{\pi}=-ig_\pi\langle\bar{\psi}\gamma_5\vec{\tau}\psi\rangle-\vec{\pi}(m_\pi{}^2-g_{\phi\pi}m_s\bar{\phi}),\\ &\Box\phi'=(-m_s{}^2\bar{\phi}+g_s\langle\bar{\psi}\psi\rangle)-m_s{}^2\phi',\\ &\Box\omega'^\mu=(g_v\langle\bar{\psi}\gamma^\mu\psi\rangle-m_v{}^2\bar{\omega}^\mu)-m_v{}^2\omega'^\mu,\\ &\bar{\psi}(i\partial\!\!\!/-m_N+g_s\bar{\phi}+\gamma^0\mu-\bar{\omega}\!\!\!/)\psi=0.\end{aligned}\tag{A4}$$

Thus we see that

$$\begin{aligned}&\bar{\phi}=\frac{g_s}{m_s{}^2}\langle\bar{\psi}\psi\rangle=\frac{g_s}{m_s{}^2}n_s,\\ &\bar{\omega}^\mu=\frac{g_v}{m_v{}^2}\langle\bar{\psi}\gamma^\mu\psi\rangle=\frac{g_v}{m_v{}^2}n\delta^{\mu 0},\\ &\langle\bar{\psi}\gamma_5\vec{\tau}\psi\rangle=0.\end{aligned}\tag{A5}$$

The effective masses and chemical potential are

$$m_v^* = m_v ,$$
$$m_s^* = m_s ,$$
$$m_\pi^{*2} = m_\pi^2 - g_{\phi\pi} m_s \bar{\phi} = m_\pi^2 - \frac{g_{\phi\pi} g_s}{m_s} n_s , \qquad \text{(A6)}$$
$$m_N^* = m_N - g_s \bar{\phi} = m_N - \frac{g_s^2}{m_s^2} n_s ,$$
$$\mu^* = \mu - g_v \bar{\omega}^0 = \mu - \frac{g_v^2}{m_v^2} n .$$

The relationship between the Fermi momentum k_F and μ^* is

$$\mu^{*2} = k_F^2 + m_N^{*2} . \qquad \text{(A7)}$$

The scalar and baryon densities are

$$n_s = \frac{2\kappa}{(2\pi)^3} \int d\vec{p} \frac{m_N^*}{(m_N^{*2} + \vec{p}^2)^{1/2}} \theta(k_F - |\vec{p}|) ,$$
$$n = \frac{2\kappa}{(2\pi)^3} \int d\vec{p}\, \theta(k_F - |\vec{p}|) . \qquad \text{(A8)}$$

Here κ is the isospin degeneracy, equal to 2 for nuclear matter.

APPENDIX B

In this appendix we quote the results for the energy densities and self-energies as used in Sec. IV of the text. In all cases we set the isospin degeneracy factor $\kappa = 2$.

The energy density of a Fermi gas is

$$\epsilon_{FG} = \frac{4}{(2\pi)^3} \int d\vec{p} (\vec{p}^2 + m_N^{*2})^{1/2} \theta(k_F - |\vec{p}|)$$
$$= \frac{1}{(2\pi)^2} \left[2E_F k_F^3 + m_N^{*2} E_F k_F - m_N^{*4} \ln\left(\frac{k_F + E_F}{m_N^*}\right) \right] . \qquad \text{(B1)}$$

Here $E_F = \mu^* = (k_F^2 + m_N^{*2})^{1/2}$.

The classical, or direct, scalar and vector meson contributions to the energy density are

$$\epsilon_s^{\mathrm{dir}} = \tfrac{1}{2} m_s^2 \bar{\phi}^2 ,$$
$$\epsilon_v^{\mathrm{dir}} = \tfrac{1}{2} m_v^2 (\bar{\omega}^0)^2 . \qquad \text{(B2)}$$

The classical fields $\bar{\phi}$ and $\bar{\omega}^0$ are determined below.

The scalar meson exchange energy density has been calculated previously by Bolsterli[14] and Chin.[2] The corresponding diagram is given in Fig. 2.

$$\epsilon_s^{\mathrm{ex}} = g_s^2 \int \frac{d\vec{p}\, d\vec{q}}{(2\pi)^6} \frac{\theta(k_F - |\vec{p}|)\theta(k_F - |\vec{q}|)}{E_p E_q} \times \left[\frac{m_N^{*2} + p \cdot q}{m_s^{*2} - (p-q)^2} \right] , \qquad \text{(B3)}$$

where $p = (E_p, \vec{p})$ and $E_p = (\vec{p}^2 + m_N^{*2})^{1/2}$, etc. This can be written in the computationally more convenient form

$$\epsilon_s^{\mathrm{ex}} = \frac{g_s^2}{32\pi^4} \left[E_F k_F - m_N^{*2} \ln\left(\frac{k_F + E_F}{m_N^*}\right) \right]^2 + 2 g_s^2 (1 - \tfrac{1}{4} w_s)(m_N^*/2\pi)^4 I(w_s, \zeta) , \qquad \text{(B4)}$$

where

$$w_s = (m_s^*/m_N^*)^2 ,$$
$$\zeta = (E_F + k_F)/m_N^* , \qquad \text{(B5)}$$

and

$$I(w, \zeta) = \tfrac{1}{4} \int_1^\zeta du (1 - u^{-2}) \times \int_1^\zeta dv (1 - v^{-2}) \ln\left[\frac{(uv-1)^2 + wuv}{(u-v)^2 + wuv} \right] . \qquad \text{(B6)}$$

The vector meson exchange energy, Fig. 2, has also been calculated by the above authors. It is

$$\epsilon_v^{\mathrm{ex}} = -2 g_v^2 \int \frac{d\vec{p}\, d\vec{q}}{(2\pi)^6} \frac{\theta(k_F - |\vec{p}|)\theta(k_F - |\vec{q}|)}{E_p E_q} \times \left[\frac{2m_N^{*2} - p \cdot q}{m_v^{*2} - (p-q)^2} \right] , \qquad \text{(B7)}$$

or

$$\epsilon_v^{\mathrm{ex}} = \frac{g_v^2}{16\pi^4} \left[E_F k_F - m_N^{*2} \ln\left(\frac{k_F + E_F}{m_N^*}\right) \right]^2 - 2 g_v^2 (1 + \tfrac{1}{2} w_v)(m_N^*/2\pi)^4 I(W_v, \zeta) \qquad \text{(B8)}$$

with

$$W_v = (m_v^*/m_N^*)^2 . \qquad \text{(B9)}$$

The nucleon and meson self-energies, Fig. 2, have been calculated using the finite temperature formalism, analytically continuing to $p = 0$, and then letting the temperature go to zero.[17] The analytic continuation would be especially important at finite temperature since, for fermions, $p^0 = (2n+1)\pi T i + \mu^*$ is never zero for any integer n. We find that the scalar meson self-energy is

$$\Pi_s \equiv \Pi_s(p=0) = 4 g_s^2 \int \frac{d\vec{q}}{(2\pi)^3} \frac{\vec{q}^2}{E_q^3} \theta(k_F - |\vec{q}|)$$
$$= \frac{g_s^2}{\pi^2} \left[E_F k_F + \frac{2m_N^{*2} k_F}{E_F} - 3 m_N^{*2} \ln\left(\frac{E_F + k_F}{m_n^*}\right) \right] . \qquad \text{(B10)}$$

which is in agreement with a calculation by Lee and Margulies[7] [their Eq. (4.33)]. The vector meson self-energy tensor is

$$\Pi_v^{\mu\nu} \equiv \Pi_v^{\mu\nu}(p=0) = -A g^{\mu\nu} + B g^{\mu 0} g^{\nu 0}, \qquad \text{(B11)}$$

where

$$A = 2g_v^2 \int \frac{d\vec{q}}{(2\pi)^3} \frac{\theta(k_F - |\vec{q}|)}{E_q}\left(1 - \frac{2}{3}\frac{\vec{q}^{\,2}}{E_q^2}\right)$$
$$= \frac{g_v}{\pi^2}\left[\frac{1}{6}E_F k_F - \frac{2}{3} m_N^{*2}\frac{k_F}{E_F} + \frac{1}{2} m_N^{*2}\ln\left(\frac{E_F + k_F}{m_N^*}\right)\right],$$
$$B = 4g_v^2 \int \frac{d\vec{q}}{(2\pi)^3} \frac{\theta(k_F - |\vec{q}|)}{E_q}\left(1 - \frac{1}{3}\frac{\vec{q}^{\,2}}{E_q^2}\right). \qquad \text{(B12)}$$

The effective meson masses are then

$$m_s^{*2} = m_s^2 + \Pi_s, \quad m_v^{*2} = m_v^2 + A. \qquad \text{(B13)}$$

Although no self-consistent calculations have been done which include the pion, an estimate of m_π^{*2} has been made in Sec. IV. The pion self-energy, Fig. 1, is in our approximation

$$\Pi_{ps} \equiv \Pi_{ps}(p=0) = 4g_\pi^2 \int \frac{d\vec{q}}{(2\pi)^3} \frac{\theta(k_F - |\vec{q}|)}{E_q}$$
$$= \frac{g_\pi^2}{\pi^2}\left[E_F k_F - m_N^{*2}\ln\left(\frac{E_F + k_F}{m_N^*}\right)\right]. \qquad \text{(B14)}$$

The nucleon self-energy, Fig. 2, has nonvanishing contributions proportional to the unit matrix and to the matrix γ^0. The part proportional to γ^0, $\Sigma_v^0 + \Sigma_s^0$, is absorbed into the definition of μ^* and need not be calculated explicitly here.

$$\mu^* = \mu - g_v \bar{\omega}^0 - \Sigma_v^0 - \Sigma_s^0 = (k_F^2 + m_N^{*2})^{1/2}. \qquad \text{(B15)}$$

The parts proportional to the unit matrix are

$$\Sigma_v \equiv \Sigma_v(p=0) = g_v^2 m_N^* \int \frac{d\vec{q}}{(2\pi)^3} \frac{\theta(k_F - |\vec{q}|)}{E_q \omega_v (E_q + \omega_v)},$$
$$\omega_v = (m_v^{*2} + \vec{q}^{\,2})^{1/2}, \qquad \text{(B16)}$$

and

$$\Sigma_s \equiv \Sigma_s(p=0) = -\frac{g_s^2}{4} m_N^* \int \frac{d\vec{q}}{(2\pi)^3} \frac{\theta(k_F - |\vec{q}|)}{E_q \omega_s (E_q + \omega_s)},$$
$$\omega_s = (m_s^{*2} + \vec{q}^{\,2})^{1/2}. \qquad \text{(B17)}$$

The integrals appearing in Eqs. (B16) and (B17) can be evaluated in terms of elementary functions but the results are too lengthy to reproduce here. As noted in Sec. IV, Σ_v has been computed with the vector propagator $g_{\mu\nu}/(p^2 - m_v^{*2})$. The effective nucleon mass is

$$m_N^* = m_N - g_s \bar{\phi} + \Sigma_v + \Sigma_s. \qquad \text{(B18)}$$

The mean vector field $\bar{\omega}^0$ is determined by noticing that the pressure depends on $\bar{\omega}^0$ in the following way:

$$P = \frac{1}{2} m_v^2 \bar{\omega}^{02} + F(\mu - g_v \bar{\omega}^0). \qquad \text{(B19)}$$

The system is in thermodynamic equilibrium when P is at a maximum with respect to variations of the independent parameters, in particular $\bar{\omega}^0$. Hence

$$\frac{\partial P}{\partial \bar{\omega}^0} = m_v^2 \bar{\omega}^0 - g_v F'(\mu - g_v \bar{\omega}^0) = 0. \qquad \text{(B20)}$$

Noticing that the baryon density is given by

$$\frac{\partial P}{\partial \mu} = F'(\mu - g_v \bar{\omega}^0) = n, \qquad \text{(B21)}$$

we see that

$$\bar{\omega}^0 = \frac{g_v}{m_v^2} n. \qquad \text{(B22)}$$

The mean scalar field $\bar{\phi}$ is determined by Eq. (B18).

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