Coulomb correction for ^{40}Ca from a relativistic optical model

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(Received 7 July 1980)

An effective Coulomb correction term arising from the second order Dirac equation is calculated for ^{40}Ca. The optical potential used consists of a mixture of a Lorentz scalar potential and the timelike component of a Lorentz four-vector potential. These calculations show that the real Coulomb correction term resulting naturally from the Dirac equation is in agreement with empirics.

[NUCLEAR REACTIONS Relativistic optical potential, mean field theory of nuclear matter; Coulomb correction for ^{40}Ca.]

In a recent letter[1] we described a relativistic optical model potential based on meson exchange considerations in a relativistic mean field theory of nuclear matter.[2] One motivation for this work was the success we have had in analyzing intermediate energy proton-nucleus scattering experiments using the Dirac equation.[3] Another is the revival of interest in relativistic treatments of the nucleus and nuclear matter.[2–24]

The specification of the Lorentz character of an optical potential is a basic feature of models which employ a relativistic wave equation. In the references above, the potentials consist, in general, of a mixture of a Lorentz scalar potential U_s and the timelike component of a Lorentz four-vector potential U_0. The potentials U_s and U_0 and a tensor potential are the only ones which remain in the Dirac equation after applying conservation law constraints to static local interactions for scattering by a spin-zero target. The potential U_s is often associated with a neutral scalar field arising from two-pion exchange processes and is simulated by the exchange of a neutral scalar meson, the σ, while the potential U_0 may be associated with the field of the neutral vector ω meson. The tensor potential is usually neglected for isospin zero targets.

In this work we extend our previous calculations of general features of the optical model potential, such as volume integrals and rms radii, to the calculation of angular distributions and polarizations. Comparison is made with the p-^{40}Ca elastic scattering cross sections[25] and polarizations[26] at 26 MeV as well as with the recent data of Rapaport *et al.*[27] for n-^{40}Ca at 26.3 Mev. A comparison of the central optical potential volume integrals from the analyses of both neutron and proton data allows a determination of the empirical Coulomb correc tion volume integral.[27]

The Dirac equation used is given by

$$\{\alpha\cdot\vec{p}+\beta[m+U_s(r)]+U_0(r)+V_C(r)]\}\psi(\vec{r})=E\psi(\vec{r}), \quad (1)$$

where $V_C(r)$ is the Coulomb potential for protons, m the nucleon mass, and E its c.m. energy. In order to compare with nonrelativistic optical models, we write (1) in second order form. The equation for the upper two components is

$$(p^2+U_{eff}+U_{so}\vec{\sigma}\cdot\vec{L})\psi_u=[(E-V_C)^2-m^2]\psi_u\,, \quad (2)$$

where

$$U_{eff}=2EU_0+2mU_0-U_0^2+U_s^2-2V_CU_0+U_Di\vec{r}\cdot\vec{p}\,, \quad (3)$$

$$U_{so}=-\frac{1}{rA}\frac{\partial}{\partial r}A=-U_D\,, \quad (4)$$

and

$$A=E+m+U_s-U_0-V_C\,. \quad (5)$$

Of the effective potentials in Eq. (2) the Thomas spin orbit term U_{so} and the Darwin term U_D, are well known. Additionally, the effective central potential U_{eff} contains squares of the nuclear potentials U_0 and U_s, a nuclear-Coulomb cross term V_CU_0, and an explicit energy dependence form the EU_0 term. Their occurrence is a natural consequence of the use of a relativistic wave equation.[28] Their importance as distinguishable features of an optical model depends on the mixture of U_s and U_0.

The cross term V_CU_0 in U_{eff} is a Coulomb correction term of the type commonly associated with the Schrödinger equation for an energy dependent

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potential. If U_0 is complex then $V_C U_0$ is complex. A complex Coulomb correction term, long recognized as a possibility, has recently been observed in an empirical analysis.[29] Complex Coulomb correction terms also result from nonrelativistic microscopic calculations[30,31] of the optical potential.

The optical potentials used are written

$$U_0(r) = V_0(r) + iW_0(r)\,, \tag{6}$$

$$U_s(r) = V_s(r) + iW_s(r)\,. \tag{7}$$

The real parts of the optical potentials are constructed using a standard folding formula,[1,32]

$$\begin{aligned} V_s(r) &= \int \tilde{\rho}_s(\vec{r}\,')v_s(|\vec{r}-\vec{r}\,'|)d\vec{r}\,' \\ V_0(r) &= \int \tilde{\rho}_0(r')v_0(|\vec{r}-\vec{r}\,'|)d\vec{r}\,'\,. \end{aligned} \tag{8}$$

The effective baryon density $\tilde{\rho}_0$ is obtained by a double folding of projectile and target nucleons with the nuclear matter density ρ_0; the density ρ_0 is taken from an empirical formula of Negele.[33] The effective scalar density is approximated by $\tilde{\rho}_s(r) = [\rho_s/\rho_0]_{nm}\tilde{\rho}_0(r)$, where $[\rho_s/\rho_0]_{nm}$ is the scalar to baryonic density ratio in nuclear matter. The effective interaction is written as $v(r) = tf(r)$, where $f(r)$ is a form factor with rms radius determined by the mass of the exchanged meson and t is the volume integral of the effective interaction in nuclear matter. The density ratio and the values of t_0 and t_s are taken from Walecka's[2] relativistic mean field theory of nuclear matter as described in Ref. 1. The potentials $V_0(r)$ and $V_s(r)$ are completely specified by this construction.

As a test of the model we consider p-^{40}Ca at 26 MeV. We find that the data may be adequately represented with real potentials calculated as described above. As noted in Ref. 1, the volume integral of Re(U_{eff}) is about 15% smaller than the phenomenological p-^{40}Ca optical potential. The calculated volume integral may be brought into agreement with experiment by a four percent decrease in the strength of V_0. Such a change is well within the uncertainties in the input to this calculation.

The imaginary parts of the optical potentials are determined phenomenologically. Both Lorentz scalar and Lorentz vector absorptions are assumed to be Woods-Saxon derivative shapes[34] with parameters given by Rapaport *et al.*[27] The two strength parameters of W_0 and W_s are varied to obtain reasonable agreement with the proton scattering data. Figure 1 shows the data Ref. 25, and the calculated p-^{40}Ca cross section at 26 MeV. Figure 2 shows the data of Ref. 26 and the calculated polarization. These fits are comparable to results from empirical nonrelativistic optical model cal-

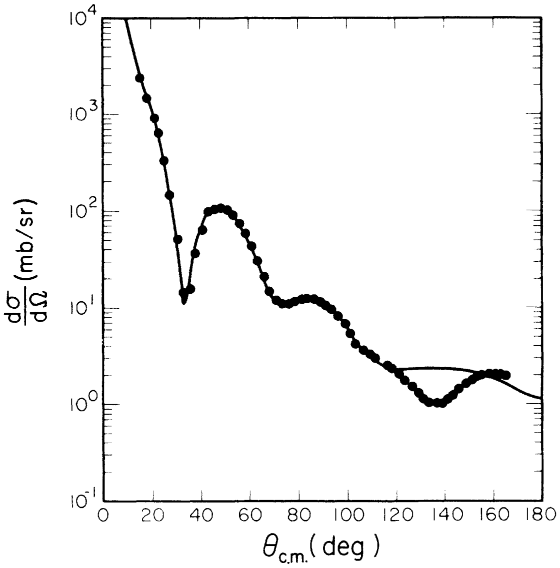

FIG. 1. Elastic scattering cross sections for p-^{40}Ca at 26.3 MeV. The smooth curve is the calculated cross section. Experimental data are from Ref. 25.

culations as well as the recent calculations by Brieva and Rook.[31]

The discrepancy between calculated and experimental cross sections at large angles is charac-

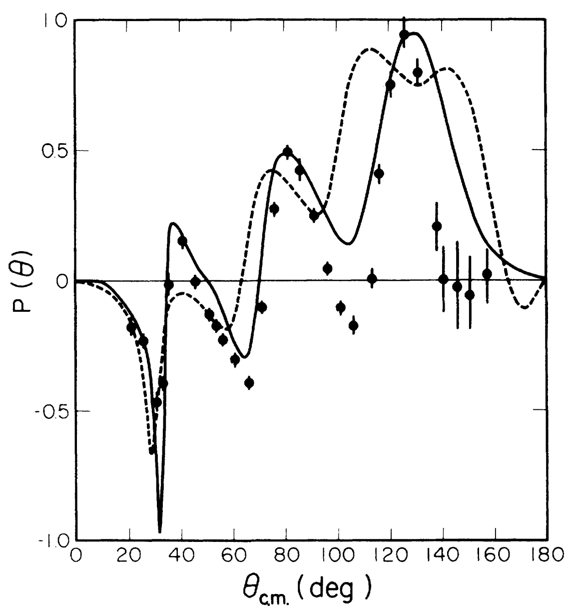

FIG. 2. Elastic scattering polarization for nucleons on ^{40}Ca. The smooth curve is the calculated p-^{40}Ca polarization at 26.3 MeV. The experimental proton data at this energy are from Ref. 26. The dashed curve is the predicted n-^{40}Ca polarization corresponding to the dashed curve in Fig. 3.

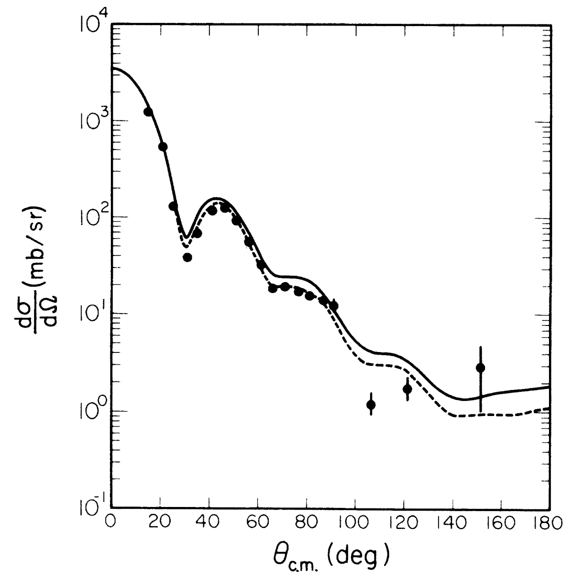

FIG. 3. Elastic scattering cross section for n-^{40}Ca at 26 MeV. The smooth curve is the calculated cross section using the optical potential determined from p-^{40}Ca. The dashed curve results when the absorption is increased as described in the text. The experimental data are from Ref. 27.

teristic of optical model calculations which do not include an explicit exchange interaction. In a recent paper Vosniakos *et al.*[35] have found that the inclusion of a complex l-dependent Majorana exchange potential in the optical model removes most of this large angle discrepancy. We have found that inclusion of a relativistic generalization of a Majorana exchange potential can account for the structure in the large angle cross sections for p-^{4}He at 800 MeV.[36] We are investigating the inclusion of such an exchange potential at low energies but do not discuss it in this paper as its effect on the Coulomb correction is small.

The volume integral of the Coulomb correction term defined by

$$J_c/A \equiv \frac{\int U_0(r)V_c(r)dv}{AE} \tag{9}$$

is calculated to be (18 - 2i) MeV fm^3. This result is in agreement with the nonrelativistic theoretical work of Jeukenne, Lejeune, and Mahaux[30] and Brieva and Rook.[31] The empirical value[29] determined from the difference in optical potentials for protons and neutrons at this energy is (22-15i) MeV fm^3. The uncertainty in these values is of the order of 10-15 %.[29] The smooth curve in Fig. 3 shows the calculated cross section for n-^{40}Ca at 26 MeV using the optical potential for p-^{40}Ca described above with the Coulomb potential set to zero. The neutron scattering data of Ref. 27 is also shown. As is indicated by the dashed curve in Fig. 3 the generally good agreement can be improved by increasing W_0 by 4%. In this case comparison of neutron and proton volume integrals yields an empirical Coulomb correction volume integral of (18-16i) MeV fm^3, which is in agreement with the nonrelativistic analysis of Ref. 29.

In the calculations discussed here the real potentials are energy independent. This means that the real Coulomb correction defined by Eq. (9) varies inversely with the energy, $E = T + M$. Thus, it is essentially constant at low energies ($T \lesssim 50$ MeV). One would not necessarily expect this situation to hold at intermediate energies due to explicit energy dependence in V_0(r). For example, we found a linear energy variation of the ratio $R_R = \int V_0(r)d\vec{r} / \int V_s(r)d\vec{r}$ in our fits to p-^{4}He data at intermediate energies.[3] We are currently investigating p-^{40}Ca at intermediate energies and our preliminary results indicate a similar linear variation in R_R.[37] Thus, we expect additional energy dependence in J_c when the entire energy range is considered.

We wish to acknowledge useful conversations and communications with C. Mahaux, B. Mulligan, and J. Rapaport. This work was supported in part by NSF Grant. No. PHY-7825532.

[1]L. G. Arnold and B. C. Clark, Phys. Lett. 84B, 46 (1979).
[2]J. D. Walecka, Ann. Phys. (N.Y.) 83, 491 (1974).
[3]L. G. Arnold, B. C. Clark, and R. L. Mercer, Phys. Rev. C 19, 917 (1979), and references therein.
[4]L. D. Miller, Phys. Rev. Lett. 28, 1281 (1972).
[5]L. D. Miller and A. E. S. Green, Phys. Rev. C 5, 241 (1972).
[6]L. D. Miller, Phys. Rev. C 9, 537 (1974); 14, 706 (1976).
[7]L. D. Miller, Ann. Phys. (N.Y.) 91, 40 (1975).
[8]J. V. Noble, Nucl. Phys. A329, 354 (1979).
[9]R. Brockmann and W. Weise, Phys. Rev. C 16, 1282 (1977).
[10]R. Brockmann, Phys. Rev. C 18, 1510 (1978).
[11]J. Boguta and J. Rafelski, Phys. Lett. 71B, 22 (1977).
[12]J. Boguta and A. R. Bodmer, Nucl. Phys. A292, 413 (1977).
[13]K. P. Lohs and J. Hüfner, Nucl. Phys. A296, 349 (1978).
[14]F. E. Serr and J. D. Walecka, Phys. Lett. 79B, 10 (1978).

[15]B. D. Serot, Phys. Lett. 86B, 146 (1979).
[16]B. D. Serot and J. D. Walecka, Phys. Lett. 87B, 172 (1979).
[17]S. A. Chin, Ann. Phys. (N.Y.) 108, 301 (1977).
[18]M. Brittan, Phys. Lett. 79B, 27 (1978).
[19]L. C. Liu and C. M. Shakin, Phys. Rev. C 20, 1195 (1979).
[20]M. R. Anastasio, L. S. Celenza, and C. M. Shakin, Report No. B.C.I.N.T. 80/051/97.
[21]M. R. Anastasio, L. S. Celenza, and C. M. Shakin, Report No. B.C.I.N.T. 80/052/99.
[22]M. Jaminon, C. Mahaux, and P. Rochus, Phys. Rev. Lett. 43, 1097 (1979).
[23]M. Jaminon and C. Mahaux, in *Proceedings of the Conference on the Meson Theory of Nuclear Forces and Nuclear Matter, Bad Honnef, 1979*, edited by K. Bleuler (to be published).
[24]M. Jaminon, C. Mahaux, and P. Rochus, Phys. Rev. C (to be published).
[25]K. H. Bray, K. S. Jayaraman, G. A. Moss, W. T. H. van Oers, D. O. Wells, and Y. I. Wu, Nucl. Phys. A167, 57 (1971). Tables of the data were provided by W. T. H. van Oers.
[26]D. L. Watson, J. Lowe, J. C. Dore, R. M. Craig, and D. J. Baugh, Nucl. Phys. A92, 193 (1967).
[27]J. Rapaport, V. Kulkarni, and R. W. Finlay, Nucl. Phys. A330, 15 (1979). Tables of the data were provided by J. Rapaport.
[28]For example, the latter three features discussed also appear in the effective potential for the Klein-Gordon equation.
[29]J. Rapaport, Phys. Lett. 92B, 233 (1980); and private communication.
[30]J. -P. Jeukenne, A. Lejeune, and C. Mahaux, Phys. Rev. C 15, 10 (1977); 16, 80 (1977).
[31]F. A. Brieva and J. R. Rook, Nucl. Phys. A307, 493 (1978).
[32]P. E. Hodgson, *Nuclear Reactions and Nuclear Structure* (Oxford U.P., London, 1971), Ch. 6; see also F. Petrovich in *Microscopic Optical Potentials*, edited by H. von Geramb (Springer, New York, 1979), Vol. 89, p. 155.
[33]J. W. Negele, Phys. Rev. C 1, 1260 (1970).
[34]We have done calculations with surface plus volume absorption. For simplicity we consider only the surface absorption in this work.
[35]F. K. Vosniakos, N. E. Davison, W. R. Falk, O. Abou-Zeid, and S. P. Kwan, Nucl. Phys. A332, 157 (1979).
[36]L. G. Arnold, B. C. Clark, and R. L. Mercer, Phys. Rev. C 21, 1899 (1980).
[37]B. C. Clark, L. G. Arnold, and R. L. Mercer, Bull. Am. Phys. Soc. 25, 520 (1980).