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Influence of the Coulomb-distortion effect on proton-proton observables

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The effect of the Coulomb distortion of the strong interaction is studied on the basis of nucleon-nucleon observables. In particular, cross sections, polarizations, spin-correlation parameters, and spin-transfer coefficients are considered for proton-proton as well as neutron-neutron scattering at laboratory kinetic energies $E_{\mathrm{lab}} = 10$, 20, and 50 MeV. The calculations are performed for the meson-theoretical Paris potential, the nonlocal separable Graz potential, and also using the Arndt-Hackman-Roper parametrization of proton-proton scattering phase shifts.

[NUCLEAR REACTIONS N-N interaction; Coulomb corrections in polarization observables.]

I. INTRODUCTION

In recent years much effort has been invested in order to obtain reliable phase-shift analyses of nucleon-nucleon (N-N) experimental data, cf., e.g., Refs. 1–6. The situation looks quite satisfactory for the proton-proton (p-p) system (because of the large number of rather accurate data, which provide a solid basis for the analysis), but the neutron-proton (n-p) system still exhibits various insufficiencies, while the neutron-neutron (n-n) system remains totally unexplored in this respect. Because up till now not enough observables were measured for n-p scattering so that an unambiguous n-p phase-shift analysis was feasible, one was forced to supply the isospin triplet part with p-p data; this method is followed, e.g., in combined p-p and n-p phase-shift analyses. In such attempts it is usually assumed that the p-p "nuclear" scattering amplitude, i.e., the one which is obtained from the total amplitude by subtracting the pure-Coulomb amplitude (and possibly other electromagnetic amplitudes, such as vacuum polarization, etc.), can also be considered as the n-p amplitude and therefore can serve directly as isospin $T=1$ input for the n-p system. However, the "nuclear" p-p amplitude thus obtained is not purely nuclear, rather it contains further electromagnetic effects (for their discussion see, e.g., Refs. 7 and 8), for which it still has to be corrected. Of these, the Coulomb-distortion (CD) effect (or simply Coulomb correction as it is often called) of the nuclear interaction is commonly considered to be the most important one. For two-hadron systems this effect has been intensively studied in the context of scattering phase shifts, cf. the review by Hamilton.[9] The main purpose of the present paper is to investigate the influence of the CD effect on p-p observables. Such a study can be expected to shed light on the question of the reliability of the assumption outlined above, namely, whether or not it is justified to substitute the $T=1$ part of the n-p scattering system by the Coulomb-subtracted p-p amplitude.[10] In particular we calculate the influence on observables of the phase-shift difference

$$\Delta = \delta^{SC}(p\text{-}p) - \delta^{S}(p\text{-}p \text{ purely nuclear}),$$

i.e., of the CD effect as reflected by the phase parameters. Here $\delta^{SC}(p\text{-}p)$ is defined by the difference of the total and the Coulomb phase shifts

$$\delta^{SC}(p\text{-}p) = \delta^{\mathrm{tot}}(p\text{-}p) - \sigma^{C}(p\text{-}p \text{ purely Coulomb}).$$

Since in the framework of our approach we can employ a unique prescription for Δ, we are in the position to make predictions about n-n observables under the assumption that other (less important) electromagnetic effects may be neglected.

We perform our investigation by means of two potential models for the strong N-N interaction, namely, the Paris[11] and the Graz[12] potentials, as well as with a phenomenological phase-shift parametrization, namely, the Arndt-Hackman-Roper (AHR) p-p phase-shift analysis.[2] Thus the second aim of our work is to study p-p, likewise n-n, observables with respect to model differences. By comparing the results yielded by each one of the three models considered we can learn about their properties—most interestingly of the potential models—as reflected on the basis of experimental scattering observables.

In the following section we provide a short summary of the formalism that we used. Section III describes the various methods, how we obtained our results, and discusses in detail those observables that we present in this paper. Section IV contains our conclusion.

II. PROTON-PROTON AND NEUTRON-NEUTRON OBSERVABLES

Though a full account of the formalism for the evaluation of observables for both the p-p and n-n systems can be found in the literature (see, e.g., Refs. 13–17), we summarize the most important formulas that we used; also because in some places in the cited literature, we found a few of them to contain errors.

A. The proton-proton case

Denote by $\vec{r}$ and $\vec{k}$ the coordinate, respectively, the wave number of the relative motion of the protons and let $\{SM_S\}$ be their total spin S with z component M_S. Their scattering wave function at large distances can be expressed in the form[17–19]

$$\psi^{(+)}_{SM_S}(\vec{k},\vec{r}) = (2\pi)^{-3/2}\exp\{i[\vec{k}\cdot\vec{r}+\gamma\ln(kr-\vec{k}\cdot\vec{r})]\}\chi_S^{M_S} + (2\pi)^{-3/2}\frac{1}{r}\exp[i(kr-\gamma\ln 2kr+2\sigma_0)]M\chi_S^{M_S}, \tag{2.1}$$

with $\chi_S^{M_S}$ representing the spin state and

$$\gamma=\frac{\mu e^2}{\hbar^2 k}, \quad \mu, \text{ the reduced mass}$$

$$\sigma_0=\arg[\Gamma(1+i\gamma)].$$

Equation (2.1) defines the spin-scattering operator M, which was first discussed by Wolfenstein and Ashkin.[13,20] Once we know it, we can calculate all relevant observables such as the differential cross section, polarization, depolarization, spin-transfer coefficients, and spin-correlation parameters, etc.

Requiring invariance under time reversal, space reflections, and rotations, the operator M is related to the usual transition operator T by[21,22]

$$M(k,\theta,\varphi) = -\frac{(2\pi)^2\mu}{\hbar^2}T(\vec{k}_i,\vec{k}_f;k), \quad |\vec{k}_i| = |\vec{k}_f| = k. \tag{2.2}$$

In terms of the scattering amplitude this relation reads

$$M(k,\theta,\varphi)=f(k,\theta,\varphi)=f^C(k,\theta,\varphi)+f^{SC}(k,\theta,\varphi), \tag{2.3}$$

with f^C and f^{SC} representing the pure Coulomb as well as Coulomb-distorted strong amplitudes.

The spin-scattering amplitude $M(k,\theta,\varphi)$ of Eqs. (2.2) and (2.3) is still to be considered as an operator in spin space acting on the initial spin state $\chi_S^{M_S}$. After performing a partial wave decomposition of the strong-interaction part f^{SC} of the total scattering amplitude f we can express the matrix elements $\langle S'M_{S'}|M(k,\theta,\varphi)|SM_S\rangle$ via the following formulas[15,16,23]:

(i) In the spin singlet denoting $M_{ss}=\langle 00|M|00\rangle$,

$$M_{ss}=f_s^C+\frac{2}{ik}\sum_{\text{even } L}P_L(\cos\theta)(2L+1)\frac{\alpha_L}{2}. \tag{2.4}$$

(ii) In the spin triplet denoting $M_{M_{S'}M_S} = \langle 1M_{S'}|M|1M_S\rangle$,

$$M_{11}=f_t^C+\frac{2}{ik}\sum_{\text{odd } L}P_L(\cos\theta)\tfrac{1}{4}\{(L+2)\alpha_{L,L+1}+(2L+1)\alpha_L+(L-1)\alpha_{L,L-1} -[(L+1)(L+2)]^{1/2}\alpha^{L+1}-[L(L-1)]^{1/2}\alpha^{L-1}\}, \tag{2.5a}$$

$$M_{00}=f_t^C+\frac{2}{ik}\sum_{\text{odd } L}P_L(\cos\theta)\tfrac{1}{2}\{(L+1)\alpha_{L,L+1}+L\alpha_{L,L-1}+[(L+1)(L+2)]^{1/2}\alpha^{L+1} +[L(L-1)]^{1/2}\alpha^{L-1}\}, \tag{2.5b}$$

$$M_{01}=\frac{2}{ik}e^{i\varphi}\sum_{\text{odd } L}P_L{}^1(\cos\theta)\frac{\sqrt{2}}{4}\left[-\frac{(L+2)}{(L+1)}\alpha_{L,L+1}+\frac{2L+1}{L(L+1)}\alpha_L+\frac{L-1}{L}\alpha_{L,L-1}+\left(\frac{L+2}{L+1}\right)^{1/2}\alpha^{L+1} -\left(\frac{L-1}{L}\right)^{1/2}\alpha^{L-1}\right], \tag{2.5c}$$

$$M_{10}=\frac{2}{ik}e^{-i\varphi}\sum_{\text{odd } L}P_L{}^1(\cos\theta)\frac{\sqrt{2}}{4}\left[\alpha_{L,L+1}-\alpha_{L,L-1}+\left(\frac{L+2}{L+1}\right)^{1/2}\alpha^{L+1}-\frac{\sqrt{2}}{4}\left(\frac{L-1}{L}\right)^{1/2}\alpha^{L-1}\right], \tag{2.5d}$$

$$M_{1-1}=\frac{2}{ik}e^{-2i\varphi}\sum_{\text{odd } L}P_L{}^2(\cos\theta)\frac{1}{4}\left\{\frac{1}{L+1}\alpha_{L,L+1}-\frac{2L+1}{L(L+1)}\alpha_L+\frac{1}{L}\alpha_{L,L-1}-[(L+1)(L+2)]^{1/2}\alpha^{L+1} -[L(L-1)]^{1/2}\alpha^{L-1}\right\}. \tag{2.5e}$$

The remaining elements can be obtained by the symmetry relations

$$
\begin{aligned}
&M_{-1-1}(\theta,\varphi)=M_{11}(\theta,-\varphi)\,,\\
&M_{0-1}(\theta,\varphi)=-M_{01}(\theta,-\varphi)\,,\\
&M_{-10}(\theta,\varphi)=-M_{10}(\theta,-\varphi)\,,\\
&M_{-11}(\theta,\varphi)=M_{1-1}(\theta,-\varphi)\,.
\end{aligned}
\tag{2.5f}
$$

In Eqs. (2.4) and (2.5) $P_L^M(\cos\theta)$ represents the associated Legendre function of the first kind[24]; f_s^C and f_t^C are the Coulomb scattering amplitudes in the singlet and triplet states, respectively,

$$f_s^C(\theta)=f^C(\theta)+f^C(\pi-\theta)\,, \tag{2.6a}$$

$$f_t^C(\theta)=f^C(\theta)-f^C(\pi-\theta)\,, \tag{2.6b}$$

$$f^C(\theta)=\frac{-\gamma}{2k\sin^2(\theta/2)}\exp[-2i\gamma\ln\sin^2(\theta/2)]\,. \tag{2.7}$$

The quantities α occurring in Eqs. (2.4) and (2.5) essentially consist of the on-shell elements of the Coulomb-distorted strong amplitude $T^{SC}=T-T^C$ in the various partial-wave states. In terms of phase shifts they are given by

$$\left.\begin{array}{l} S=0 \\ S=1,\ L=j \end{array}\right\}\alpha_L=\exp(2i\eta_L)[\exp(2i\delta_L^{SC})-1]\,, \tag{2.8}$$

$S=1,\ L=j\pm1$:

$$\alpha_{j\pm1,j}=\exp(2i\eta_{j\pm1})[\exp(2i\delta_{j\pm1}^{SC})\cos2\epsilon_j^{SC}-1]\,, \tag{2.9a}$$

$$\alpha^j=i\sin2\epsilon_j^{SC}\exp i(\delta_{j-1}^{SC}+\eta_{j-1}+\delta_{j+1}^{SC}+\eta_{j+1})\,. \tag{2.9b}$$

Here η_L stands for the difference of pure Coulomb phases

$$\eta_L=\sigma_L-\sigma_0=\arg[\Gamma(L+1+i\gamma)]-\arg[\Gamma(1+i\gamma)]$$

$$=\sum_{n=1}^{L}\arctan\frac{\gamma}{n}\,, \tag{2.10}$$

whereas δ^{SC} represents the Coulomb-distorted strong phase shift, which for some partial-wave state L is defined by

$$\delta_L^{SC}=\delta_L^{\text{tot}}-\sigma_L\,. \tag{2.11}$$

Note that in deriving Eqs. (2.8) and (2.9) we used the Stapp parametrization of the scattering matrix.[23] Therefore the phase parameters δ^{SC} and ϵ^{SC} represent the so-called "bar" phase parameters.

At this place a remark concerning the mixing parameter may be in order: While for the Coulomb-distorted nuclear phase shifts the relation (2.11) holds, we have for ϵ^{SC},

$$\epsilon_j^{\text{tot}}=\epsilon_j^{SC}\,, \tag{2.12}$$

i.e., in the total ϵ parameter there is no pure Coulomb contribution. The only influence on it by the Coulomb potential comes in via the CD effect (and possibly other less important Coulomb effects, which will not be considered here, as we mentioned in the Introduction). The role of the distortion effect with respect to ϵ is further discussed in the subsequent section.

B. The neutron-neutron case

For the n-n system the formalism simplifies considerably. Loosely speaking we get the appropriate formulas from the ones in the preceding subsection by switching off the Coulomb potential; then all major (direct) Coulomb effects on the nuclear interaction vanish. For the n-n system the well-known formulas for scattering from short-range interactions apply. Thus, e.g., the analog of Eq. (2.1) reads

$$\psi_{SM_S}^{(+)}(\vec{k},\vec{r})=(2\pi)^{-3/2}e^{i\vec{k}\cdot\vec{r}}\chi_S^{M_S}+(2\pi)^{-3/2}\frac{1}{r}e^{ikr}M\chi_S^{M_S}\,, \tag{2.13}$$

where M now only contains the strong scattering amplitude[22]

$$M(k,\theta,\varphi)=f(k,\theta,\varphi)=f^S(k,\theta,\varphi)\,. \tag{2.14}$$

For n-n scattering the matrix elements of the spin-scattering amplitude are obtained from Eqs. (2.4) and (2.5) by omitting the pure Coulomb amplitudes. Equivalently the quantities α_L now contain only the purely nuclear phase shifts

$$\left.\begin{array}{l} S=0 \\ S=1,\ L=j \end{array}\right\}\alpha_L=\exp(2i\delta_L^S)-1\,, \tag{2.15}$$

$$S=1,\ L=j\pm1:\quad \alpha_{j\pm1,j}=\exp(2i\delta_{j\pm1}^S)\cos2\epsilon_j-1\,, \tag{2.16a}$$

$$\alpha^j=i\sin2\epsilon_j\exp i(\delta_{j-1}^S+\delta_{j+1}^S)\,. \tag{2.16b}$$

In this section our aim was to find the matrix elements of the spin-scattering amplitude for both the p-p and n-n systems once the phase parameters are given. Then their notion allows us to calculate all the observables that we are interested in. The formulas expressing them in terms of M-matrix elements together with a complete list of all possible observables can, e.g., be found in Refs. 15 and 17 or in the book by Moravcsik.[25]

III. RESULTS

A. Basis and scope

Before describing the results of our study in detail let us first outline the methods and how we obtained them. As was already mentioned in the

Introduction, we calculated observables for three cases, namely two potential models and one phenomenological phase-shift analysis: the Paris potential,[11] the Graz potential,[12] and the p-p scattering analysis of Arndt, Hackman, and Roper (AHR).[2] We performed the calculations at three energies, namely at $E_{\rm lab}=10$, 20, and 50 MeV.

(i) Full p-p observables were obtained by inserting the Coulomb-distorted nuclear phase shifts δ^{SC} into Eqs. (2.8) and (2.9). The quantities α were used together with the pure Coulomb amplitudes (2.6) and (2.7) for evaluating the matrix elements of the spin-scattering amplitude via Eqs. (2.4) and (2.5). Once the M-matrix elements were known, the p-p observables could easily be deduced.

(ii) The influence of the Coulomb-distortion effect on full p-p observables was investigated in the next step. For this purpose we used the purely nuclear phase shifts δ^S instead of δ^{SC} and followed the same procedure as under (i). Note that for the two potential models the phase shifts δ^S are defined unambiguously by the solution of the nuclear problem alone; for the AHR parametrization we had to look for a prescription to subtract the CD effect in the phase shifts δ^{SC}. Such a prescription was already given by Plessas *et al.*[26] and later on as an improved version by Fröhlich *et al.*,[27]

$$\Delta_L=\delta_L^{SC}-\delta_L^S=-\frac{\mu e^2}{\hbar^2}a_L\left[\frac{d}{dk}\delta_L^S(k)+\frac{1}{2k}\sin 2\delta_L^S(k)\right], \tag{3.1}$$

with a_L being some numerical constant for each partial wave. Though this formula is only approximative to first order in e^2, it yields reasonable results at least at those energies where we performed our calculations, namely at $E_{\rm lab}=10$, 20, and 50 MeV. We remark that Eq. (3.1) applies only to uncoupled partial waves. An analogous formula holds also for the Coulomb-distortion effect in scattering parameters for coupled partial-wave states, in particular for the mixing parameters ϵ_j^{SC}. In the course of our study it turned out that with respect to the latter quantities the Coulomb-distortion effect is quite negligible[28]; thus only the phase shifts had to be treated according to Eq. (3.1) for coupled as well as for uncoupled partial-wave states.

We remind the reader that in this step of our study we retained the pure Coulomb interaction; merely the CD effect was switched off in order to investigate its importance with respect to p-p observables.

(iii) Coulomb-distorted nuclear observables for p-p scattering were obtained by setting the Coulomb amplitude f^C in Eq. (2.7) equal to zero, but following the same procedure as under (i) in all other respects. In this way we calculated nuclear observables corresponding to the Coulomb-distorted nuclear phase shifts δ^{SC}.

(iv) Purely nuclear observables for p-p scattering were calculated in the last step. For this purpose the pure Coulomb interaction as well as the CD effect were removed. Neglecting all other less important Coulomb effects, the observables we found in this way can equally well be regarded as n-n scattering observables (only the purely nuclear phase shifts δ^S served as input).

For the calculations, according to each one of the foregoing cases, we took into account partial-wave states and phase parameters, as appear in Table I; whenever needed, the pure Coulomb phases as well as the pure Coulomb amplitude were incorporated as following from Eqs. (2.10) and (2.7), respectively.

All the results that were obtained by the procedures outlined above on the one hand provide the basis for comparing the properties exhibited by the meson-theoretical Paris potential, the phenomenological separable Graz potential, and the AHR experimental data within each one of the cases (i)–(iv); on the other hand they allow for a study of the influence of Coulomb effects, in particular and most interestingly, of the CD effect. For the latter purpose we only need to contrast cases (i) and (ii) as well as cases (iii) and (iv) to each other. By examining the situation for AHR we additionally get an idea of the relevance of the approximation formula (3.1).

B. Discussion of results

In the course of our study we computed the whole manifold of p-p observables. For many of them, especially for the more complicated ones such as some triple-scattering parameters, it turned out that the CD effect, whose study was our prior aim, played only a minor role; these observables are not considered in the present paper. In the following we are going to discuss in detail only those observables which are considerably sensitive to this effect; for their explicit definition see, e.g., Refs. 15 and 17. A schematic picture of them can be found in Fig. 1.

1. Differential cross section I_0 (Fig. 2)

Let us first outline the gross features of the p-p differential cross section. It can be read nicely from the lower group of curves marked "p-p" in Figs. 2(a)–2(c). They correspond to the cases (i) and (ii) of subsection A, namely where the pure Coulomb interaction was included. The solid line, the long-dashed line, and the long-dashed dotted line re-

TABLE I. Partial-wave states and phase parameters (in degrees) used for the calculation of observables according to the various procedures described in Sec. IIIA. The states $^3H_5$ and $^3H_6$ as well as higher partial waves were neglected. (a) The purely nuclear phase shifts δ^S for the AHR case were deduced as explained in Sec. IIIA, namely, by employing the approximation formula (3.1). (b) Phase parameters for partial-wave states not covered by the Graz potential were substituted by the corresponding AHR data.

Energy	Model	Phase	$\delta(^1S_0)$	$\delta(^1D_2)$	$\delta(^1G_4)$	$\delta(^3P_1)$	$\delta(^3F_3)$	$\delta(^3P_0)$	$\delta(^3P_2)$	ϵ_2	$\delta(^3F_2)$	$\delta(^3F_4)$	ϵ_4	$\delta(^3H_4)$
E_{lab}=10 MeV	AHR	δ^{SC}	0.5491E+2	0.1656E+0	0.3532E−2	−0.2056E+1	−0.3247E−1	0.3614E+1	0.6753E+0	−0.1984E+0	0.1275E−1	0.2039E−2	−0.4071E−2	0.1673E−3
		δ^S(a)	0.5651E+2	0.1803E+0	0.3818E−2	−0.2265E+1	−0.3520E−1	0.3984E+1	0.7605E+0	−0.1984E+0	0.1385E−1	0.2257E−2	−0.4071E−2	0.1809E−3
	Paris	δ^{SC}	0.5518E+2	0.1800E+0	0.0000E+0	−0.2120E+1	−0.3000E−1	0.3890E+1	0.6400E+0	−0.2100E+0	0.1000E−1	0.0000E+0	−0.0000E+0	0.0000E+0
		δ^S	0.5695E+2	0.1900E+0	0.0000E+0	−0.2310E+1	−0.4000E−1	0.4260E+1	0.7400E+0	−0.2300E+0	0.2000E−1	0.0000E+0	−0.0000E+0	0.0000E+0
	Graz	δ^{SC}	0.5471E+2	0.5994E−1	(b)	−0.1408E+1	(b)	0.4066E+1	0.5805E+0	(b)	(b)	(b)	(b)	(b)
		δ^S	0.5678E+2	0.6781E−1	(b)	−0.1579E+1	(b)	0.4482E+1	0.6652E+0	(b)	(b)	(b)	(b)	(b)
E_{lab}=20 MeV	AHR	δ^{SC}	0.5051E+2	0.5043E+0	0.2447E−1	−0.4079E+1	−0.1513E+0	0.7090E+1	0.1850E+1	−0.5914E+0	0.6358E−1	0.1663E−1	−0.2863E−1	0.2028E−2
		δ^S(a)	0.5107E+2	0.5312E+0	0.2564E−1	−0.4331E+1	−0.1583E+0	0.7496E+1	0.2003E+1	−0.5914E+0	0.6675E−1	0.1802E−1	−0.2863E−1	0.2170E−2
	Paris	δ^{SC}	0.5081E+2	0.5500E+0	0.3000E−1	−0.4180E+1	−0.1600E+0	0.7560E+1	0.1800E+1	−0.6300E+0	0.7000E−1	0.1000E−1	−0.3000E−1	0.0000E+0
		δ^S	0.5100E+2	0.5700E+0	0.3000E−1	−0.4390E+1	−0.1700E+0	0.7920E+1	0.1960E+1	−0.6600E+0	0.7000E−1	0.2000E−1	−0.3000E−1	0.0000E+0
	Graz	δ^{SC}	0.4966E+2	0.2975E+0	(b)	−0.3541E+1	(b)	0.8043E+1	0.1630E+1	(b)	(b)	(b)	(b)	(b)
		δ^S	0.5001E+2	0.3215E+0	(b)	−0.3796E+1	(b)	0.8438E+1	0.1782E+1	(b)	(b)	(b)	(b)	(b)
E_{lab}=50 MeV	AHR	δ^{SC}	0.3898E+2	0.1652E+1	0.1619E+0	−0.8323E+1	−0.6735E+0	0.1188E+2	0.5720E+1	−0.1662E+1	0.3136E+0	0.1624E+0	−0.1884E+0	0.2455E−1
		δ^S(a)	0.3883E+2	0.1700E+1	0.1654E+0	−0.8605E+1	−0.6888E+0	0.1213E+2	0.5975E+1	−0.1662E+1	0.3212E+0	0.1679E+0	−0.1884E+0	0.2508E−1
	Paris	δ^{SC}	0.3875E+2	0.1810E+1	0.1700E+0	−0.8420E+1	−0.7300E+0	0.1183E+2	0.5740E+1	−0.1780E+1	0.3500E+0	0.1400E+0	−0.2100E+0	0.3000E−1
		δ^S	0.3813E+2	0.1860E+1	0.1700E+0	−0.8650E+1	−0.7400E+0	0.1194E+2	0.5980E+1	−0.1800E+1	0.3600E+0	0.1400E+0	−0.2100E+0	0.3000E−1
	Graz	δ^{SC}	0.3642E+2	0.1845E+1	(b)	−0.9748E+1	(b)	0.1097E+2	0.5587E+1	(b)	(b)	(b)	(b)	(b)
		δ^S	0.3592E+2	0.1915E+1	(b)	−0.1007E+2	(b)	0.1105E+2	0.5855E+1	(b)	(b)	(b)	(b)	(b)

present the results for, respectively, the AHR, Graz, and Paris models according to case (i), while the dotted line, the short-dashed line, and the short-dashed dotted line correspond to the same models treated according to case (ii). Evidently all six curves exhibit the well-known Coulomb singularity at forward scattering angles. The influence of the strong interaction comes into play only at larger scattering angles. But it gains importance with increasing energy thus forcing the Coulomb dip back to smaller and smaller angles [cf. Figs. 2(a)–2(c)].

The differences between curves corresponding to case (i) and (ii) stem from the different input of "strong" phases (δ^{SC} or δ^S). They exhibit the role of the CD effect on p-p observables. When considered as a function of the scattering angle, it can be seen that the effect shows up whenever the strong interaction gathers relative importance; of course the effect is primarily correlated to the strong interaction. It results in shifting the curves without changing their shapes in exactly those domains where they are governed by the strong potential (this shift is even much better reflected

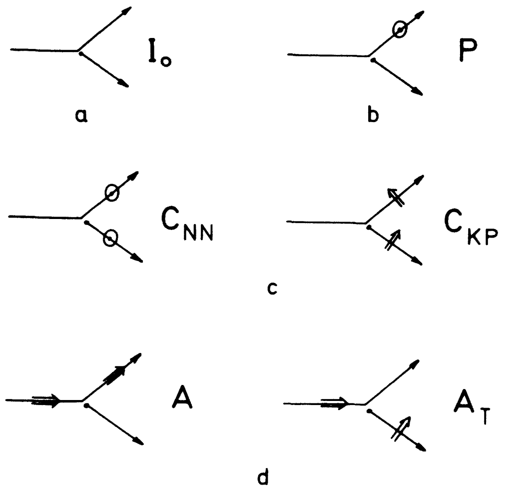

FIG. 1. Schematic representation of N-N observables, which are considered in detail in the present paper. Circles mark spin orientations normal to the scattering plane, whereas arrows correspond to spin directions lying in the scattering plane: (a) differential cross section I_0, (b) polarization P, (c) spin-correlation parameters C_{NN} and C_{KP}, (d) spin-transfer coefficients A and A_T.

by the "n-n" curves, which we are going to discuss in a moment). In these very domains the effect is quite independent of the scattering angle. But the CD effect depends on the energy; in particular, it is most important at lower energies [Figs. 2(a) and 2(b)]. At $E_{\text{lab}} = 10$ MeV the differences caused by it even exceed the differences between the various models (AHR, Graz, or Paris); at $E_{\text{lab}} = 20$ MeV they are still of the same magnitude. Only at $E_{\text{lab}} = 50$ MeV the Coulomb effect seems to become negligible.

Next we examine in detail the upper group of curves marked "n-n." They correspond to the cases (iii) and (iv) of subsection A, i.e., where the pure Coulomb interaction was left out.[29] The relation of the curves to each other and the various models is the same as before (cf. also the figure caption). Again the differences of curves corresponding to cases (iii) and (iv) exhibit the CD effect, this time on Coulomb-subtracted nuclear observables. What was already said before about its characteristics remains true; only the singular structure of the differential cross section at forward angles is wiped out. Thus the CD effect results in shifting the curves over the whole range of scattering angles; switching it off causes a uniform rise in I_0, which is greater the lower the energy.

The CD effect is seen to be of the same order of magnitude for all three models, the two potentials as well as for AHR; though for obtaining the scattering phase shifts δ^{SC} and δ^{S}, respectively, in each one of these cases a different method was followed. For the Paris potential the Schrödinger equation was solved, whereas for the Graz potential the Lippmann-Schwinger formalism was employed. For the AHR case the Coulomb-distortion effect Δ in phase shifts was treated (approximatively) according to formula (3.1); since the corresponding results are in keeping with the (exact) ones of the potential models, we can conclude that the approximation (3.1) is practical.

Now we compare p-p with n-n cross sections. By doing so we get an idea of the alternate play of the Coulomb and the strong interaction. The Coulomb interaction dominates at the lower energies also in the nonforward scattering range. Only at higher energies does the nuclear potential succeed in balancing the Coulomb interaction; then the n-n curves approach the p-p curves at least for scattering angles around $\pi/2$ [cf. Fig. 2(c)]. If we include the CD effect in the present consideration, we see that it results in weakening the action of the strong potential—a fact which is reflected by the common rise of all curves, when the effect is switched off.

Let us now say a few more words about model differences, in particular about differences in the various cross sections themselves as yielded by the AHR, Graz, and Paris models. In general these differences are relatively small, though by no means unimportant. The deviations of the Graz potential from AHR and the Paris potential at 50 MeV are remarkable. With respect to the nuclear contribution to I_0 the Graz potential yields too low a result and does not agree quite well with the data. The reason for this behavior obviously lies in the $^1S_0$ partial wave, where the Graz potential is not attractive enough at the energies in question, above all at 50 MeV; we will come back to this point later on. The Paris potential in turn seems to fit the experimental data equally well or even better than the AHR phase parametrization.

At this place we would also like to make a remark with respect to phase-shift analyses: In combined p-p and n-p analyses various Coulomb effects are often neglected. In particular, it is sometimes assumed that the isospin $T=1$ part of the n-p system can be substituted directly by Coulomb-subtracted p-p data [corresponding to case (iii)]. By this assumption one disregards the fact that these data still contain (indirect) Coulomb contributions. As explained above, our results allow us to estimate the magnitude of the CD effect in p-p observables just by comparing cases (i) and (ii). In view of the relative importance of this effect—notice that for the differential cross section it exceeds the experimental errors at all energies considered here—our results can be re-

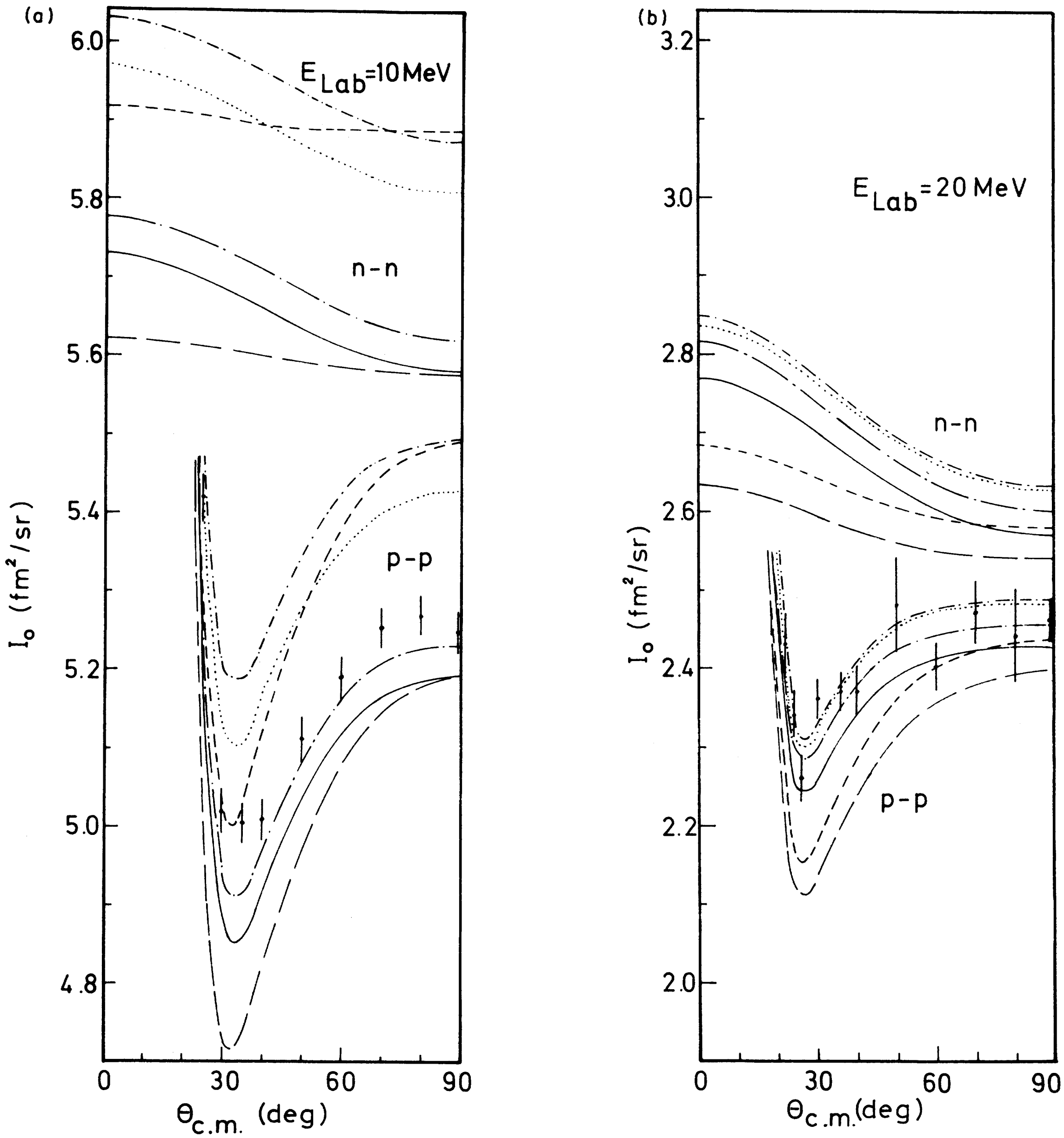

FIG. 2. Differential cross sections I_0 for p-p (with) and n-n scattering (without inclusion of the pure Coulomb interaction). The description of the curves in relation to the case studies of Sec. III A is as follows: ———— AHR, —— —— Graz, and —— • —— Paris, with case (i) for p-p and case (iii) for n-n, i.e., with the CD effect included; •••••••• AHR, --------- Graz, - • - • - • - Paris, with case (ii) for p-p and case (iv) for n-n, i.e., with the CD effect excluded. For the differential cross section the symmetry relation $I_0(\pi-\theta)=I_0(\theta)$ holds. Experimental data at 10, 20, and 50 MeV were taken from Refs. 30–32; they refer particularly to energies $E_{1ab}=9.918 \pm 0.020$ MeV, $E_{1ab}=19.8$ MeV, and $E_{1ab}=49.41 \pm 0.06$ MeV, respectively. In Fig. 2(c) the curves for Paris practically coincide with those of AHR.

garded as an indication that it should not be neglected. In fact this is confirmed by the work of Gersten,[10] who investigated various Coulomb effects on the basis of n-p observables. He found the CD effect to be of about the same magnitude as other isospin-breaking contributions (in particular those from mass differences of charged and neutral pions, which can be exchanged in the n-p system). All these findings suggest that the CD effect should be taken into account in combined analyses whenever the $T=1$ part is taken from the p-p system. A practical way for doing so is provided,

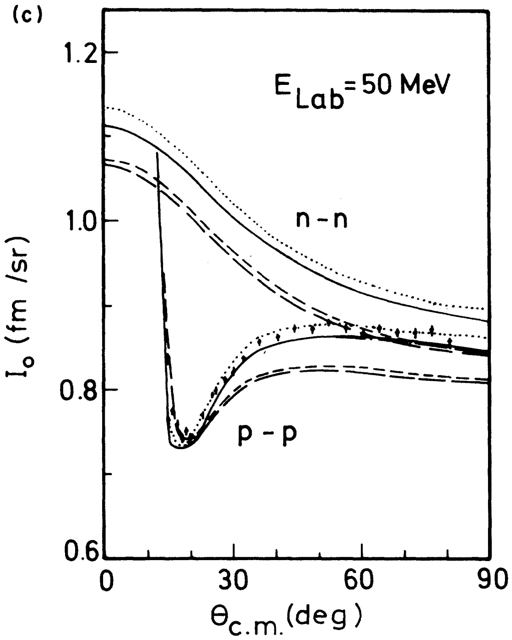

FIG. 2. *(Continued)*.

e.g., by the approximation formula (3.1); as we indicated above when discussing the AHR case it yields reasonable results. Such a procedure was in fact followed in a recent phase-shift analysis by Bugg *et al.*[3] With respect to experimental explorations of the *n-n* system the magnitude of the CD effect [above all as revealed by a comparison of cases (iii) and (iv)] gives an idea of the desirable order of magnitude of the error in possible *n-n* cross section measurements that would still allow us to decide on the question of charge symmetry (and/or independence).

Finally we note that most of the features of the CD effect described above can be traced back to the situation in the phase parameters mainly in the $^1S_0$ state, which of course plays the most important role for I_0: at $E_{\text{lab}} = 10$ MeV the differences $\Delta = \delta^{SC} - \delta^S$ coming from the Coulomb-distortion effect are of considerable magnitude, they are larger than model differences (cf. the first column of Table I). In the energy range considered the strong interaction is predominantly attractive; again it is seen that the CD effect results in weakening this attractive property of the nuclear potential: when it is switched off, i.e., for δ^S, the phase shift is much larger (positive). With increasing energy the difference diminishes and the Coulomb effect becomes negligible. Also reflected by the $^1S_0$ phase shift is the deviation of the Graz potential from AHR and the Paris potential, in particular at 50 MeV. The phase shift appears to be too small, the potential much less attractive, and as a consequence the differential cross section curves happen to lie at very small values.

2. *Polarization P (Fig. 3)*

The next observable where we found Coulomb effects to be important is the polarization. Corresponding results are shown in Fig. 3 in the same manner as before for I_0. Let us begin the discussion again with the *p-p* curves. At $E_{\text{lab}} = 10$ MeV the polarization is governed by the Coulomb interaction, the influence of the strong potential is still rather weak; therefore we see the pronounced dip at small scattering angles. With increasing energy the strong interaction comes into play more and more, the polarization goes to positive values, and the depth of the Coulomb dip is reduced.

Relative to the magnitude of P the CD effect for the *p-p* case is most important at 20 MeV, there coming up to about 30% of the polarization itself; at 50 MeV it still amounts to about 10%, while at the lower energy of $E_{\text{lab}} = 10$ MeV it shows up rather insignificantly. With respect to the latter energy point, however, a comparison with the *n-n* case tells us that the CD effect on the strong interaction is by no means unimportant also at this energy. For the *n-n* curves it again amounts to about 30% of P, only for the *p-p* case it is veiled by the predominant Coulomb interaction. A further examination of the *n-n* curves confirms a finding that we had already obtained with I_0, namely that the CD effect weakens the action of the strong potential. For the polarization this fact is reflected by a reduction of its magnitude; when the CD effect is switched off, the polarization is raised by some multiplicative factor which is again quite independent of the scattering angle. This factor grows smaller with increasing energy (being about $\frac{1}{3}$, $\frac{1}{5}$, and $\frac{1}{10}$ at $E_{\text{lab}} = 10$, 20, and 50 MeV, respectively).

Let us have a further look at model differences. As is seen especially from the *n-n* curves all models yield similar results except for the Graz potential, which slightly deviates from the Paris potential and AHR again at 50 MeV. For the *p-p* curves the agreement with experimental data may be regarded as satisfactory for all models. We remark that most of the properties of the polarization, which we just discussed, may again be traced back to phase shifts, but this time of the P-wave states. They are known to play quite an important role for the polarization, cf., e.g., Ref. 33. In particular, the behavior of the CD effect can be correlated to the Δ's of P-wave phase shifts; likewise the deviation of the Graz model is redetected there.

For further detailed studies, especially of the

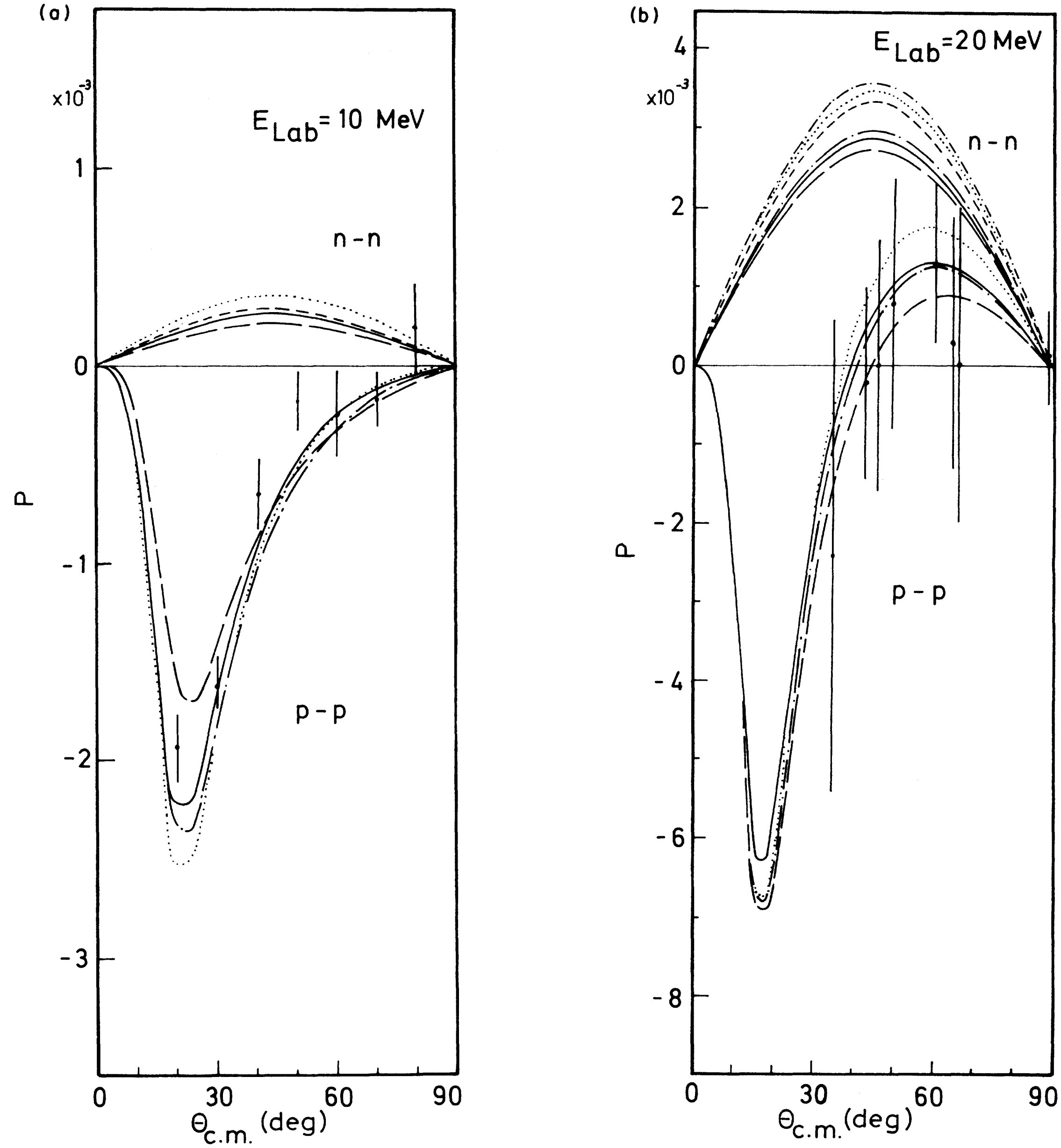

FIG. 3. Polarization P for p-p (with) and n-n scattering (without inclusion of the pure Coulomb interaction). The description of the curves in relation to the case studies of Sec. IIIA is as in Fig. 2. For the polarization the symmetry relation $P(\pi-\theta)=-P(\theta)$ holds. Experimental data at $E_{\text{lab}}=10$, 20, and 50 MeV were taken from Refs. 36–39; they refer particularly to energies $E_{\text{lab}}=10.00\pm0.05$ MeV, $E_{\text{lab}}=20.2\pm0.02$ MeV, $E_{\text{lab}}=49.9\pm0.1$ MeV, and $E_{\text{lab}}=52.34$ MeV, respectively. In Figs. 3(a) and 3(b) case (ii) curves are given only for AHR, since the influence of the Coulomb-distortion on the p-p polarization is analogous and very similar in shape for the other models. In Fig. 3(a) the n-n curves for Paris practically coincide with those of AHR.

influences of potential properties on the polarization, we refer the reader to the recent papers by Bittner and Kretschmer[34] and Coté *et al.*,[35] where the properties of the Paris potential with respect to the polarization are discussed also at $E_{\text{lab}}=6.141$ and $E_{\text{lab}}=16.0$ MeV. Let us now go on to the description of triple-scattering parameters.

3. Spin-correlation parameters C_{NN} and C_{KP} (Figs. 4 and 5)

We decided to present the observables C_{NN} and C_{KP} in figures only for the most interesting cases. In particular, Fig. 4 shows the parameter C_{NN} at $E_{\text{lab}}=50$ MeV. Only at this energy the strong-in-

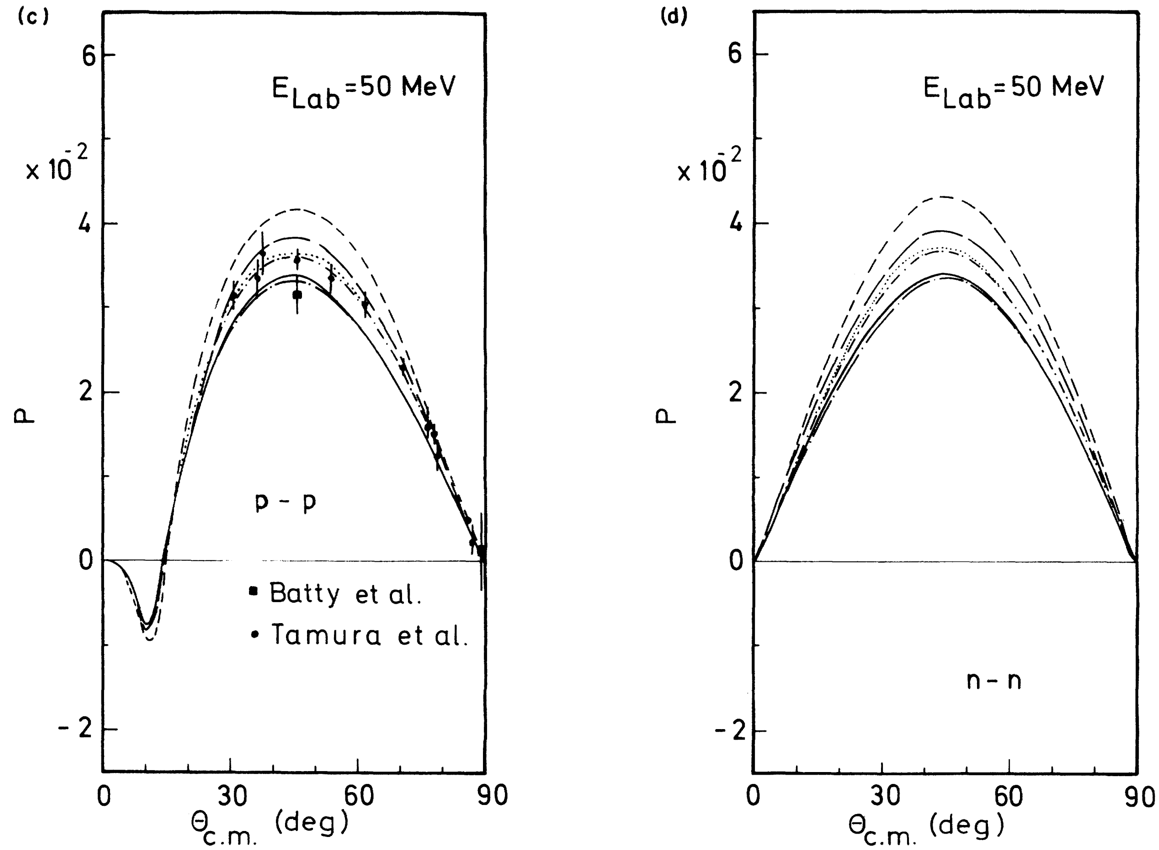

FIG. 2. (*Continued*).

teraction influence on C_{NN} shows up remarkably[40]; at the lower energies of 10 and 20 MeV the shape of C_{NN} for p-p scattering is determined mainly by the pure Coulomb interaction (i.e., a pronounced peak at forward angles followed by a rather deep minimum at larger scattering angles), while for the n-n case it does not exhibit much structure. The reason for this latter behavior lies in the fact that the spin-correlation parameter C_{NN} is predominantly governed by the tensor and spin-orbit ingredients of the strong interaction—a fact which is quite well manifested for the n-p system.[41,42] For the p-p system, however, the tensor and spin-orbit parts come into play only at higher energies, a property which, e.g., becomes evident by noticing that coupled partial waves occur only for higher angular momenta ($^3P_2$-$^3F_2$, $^3F_4$-$^3H_4$, ...) and that their mixing is weak and grows only slowly with increasing energy (cf. the columns for ϵ_2 and ϵ_4 in Table I).

A comparison of the corresponding curves of Fig. 4 clearly shows that the CD effect is rather unimportant for C_{NN} in both the p-p and n-n systems. In particular it is smaller than model differences and also smaller than experimental errors at about that energy (cf. the compilation of N-N data by Bystricky and Lehar).[43] Together with the arguments given previously, this latter finding, when traced back to phase parameters, also suggests that the Coulomb distortion may be considered as weak in coupled p-p partial waves and especially in the concomitant mixing parameters.

The description of the CD effect just given for C_{NN} in more or less the same way also applies to the observable C_{KP} represented in Fig. 5. But there we detect a remarkable model difference, namely, the Graz potential deviates considerably from AHR and the Paris potential especially at E_{lab} = 10 and 20 MeV. In order to explain this property we have drawn the n-n curves for C_{KP} at all three energies [Figs. 5(b)–5(d)]. It is seen that the results for the Graz model begin with negative values (10 MeV), go to positive ones, which are still too small at 20 MeV, and finally (at 50 MeV) exceed the AHR and Paris results. This behavior can be traced back to the phase shifts of particularly the P waves. While the Graz potential is too weakly repulsive (too attractive) in the $^3P_1$ ($^3P_0$) state at E_{lab} = 10 and 20 MeV it

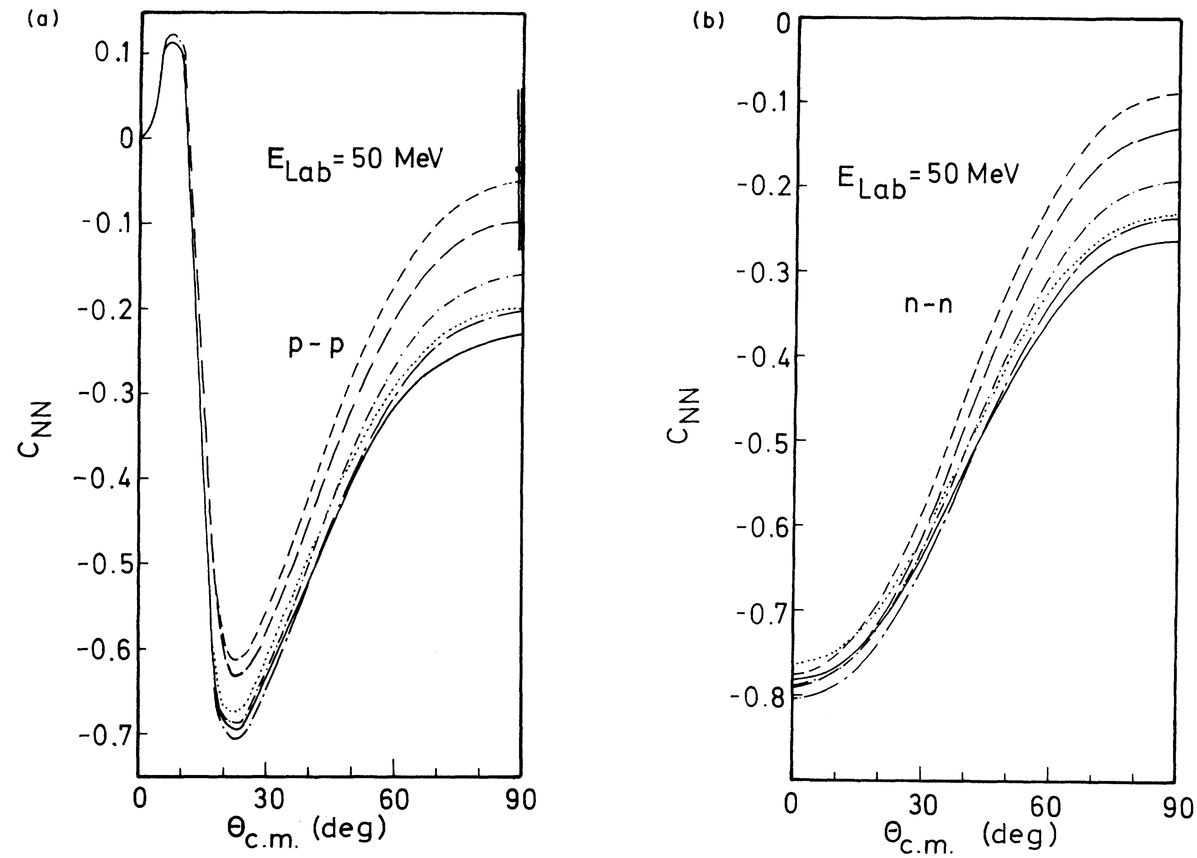

FIG. 4. Spin-correlation parameter C_{NN} for p-p (with) and n-n scattering (without inclusion of the pure Coulomb interaction). The description of the curves in relation to the case studies of Sec. IIIA is as in Fig. 2. For C_{NN} the symmetry relation $C_{NN}(\pi-\theta)=C_{NN}(\theta)$ holds. The experimental datum was taken from Ref. 44; it refers particularly to the energy E_{lab}= 52.3 MeV.

turns out to be too repulsive (too weakly attractive) in this state at E_{lab} = 50 MeV. Thus we can argue that C_{KP} is rather sensitive to P waves.

The action of the pure Coulomb interaction on C_{KP} is seen from Fig. 5(a). Its influence simply consists in preventing the curves from rising at forward scattering angles, this effect being less pronounced at the higher energies thus indicating the strengthening of the nuclear interaction with increasing energy.

4. Spin-transfer coefficient A and A_T (Fig. 6)

Though we have calculated three observables of that type, namely A, R, and D (A_T, R_T, and D_T, respectively), at all of the three energies, we decided to present only the parameter A at E_{lab} = 50 MeV; the reason being that all observables of that type revealed practically the same behavior. Above all the CD effect turned out to be unimportant for both the p-p and n-n systems at all energy points considered. In Figs. 6(a) and 6(b) the corresponding curves are given for E_{lab} = 50 MeV. Only at this energy the nuclear potential gets strong enough to force back the influence of the pure Coulomb potential for the p-p case; only at the same energy the observable A exhibits some structure for the n-n case. At the lower energies the pure Coulomb influence becomes predominant, thus leading to broader minima of A at forward and backward scattering angles.

Note that because of the symmetry relation between A and A_T the latter quantity can be deduced from the former one by reversing the angular dependence (cf. Fig. 6). The same is true for D and R as well.

All model predictions for A agree excellently with the experimental data. Differences between the models themselves also appear to be insignificant, thus suggesting that spin-transfer coefficients are rather insensitive to details of the nuclear potential, at least in the energy range considered here.

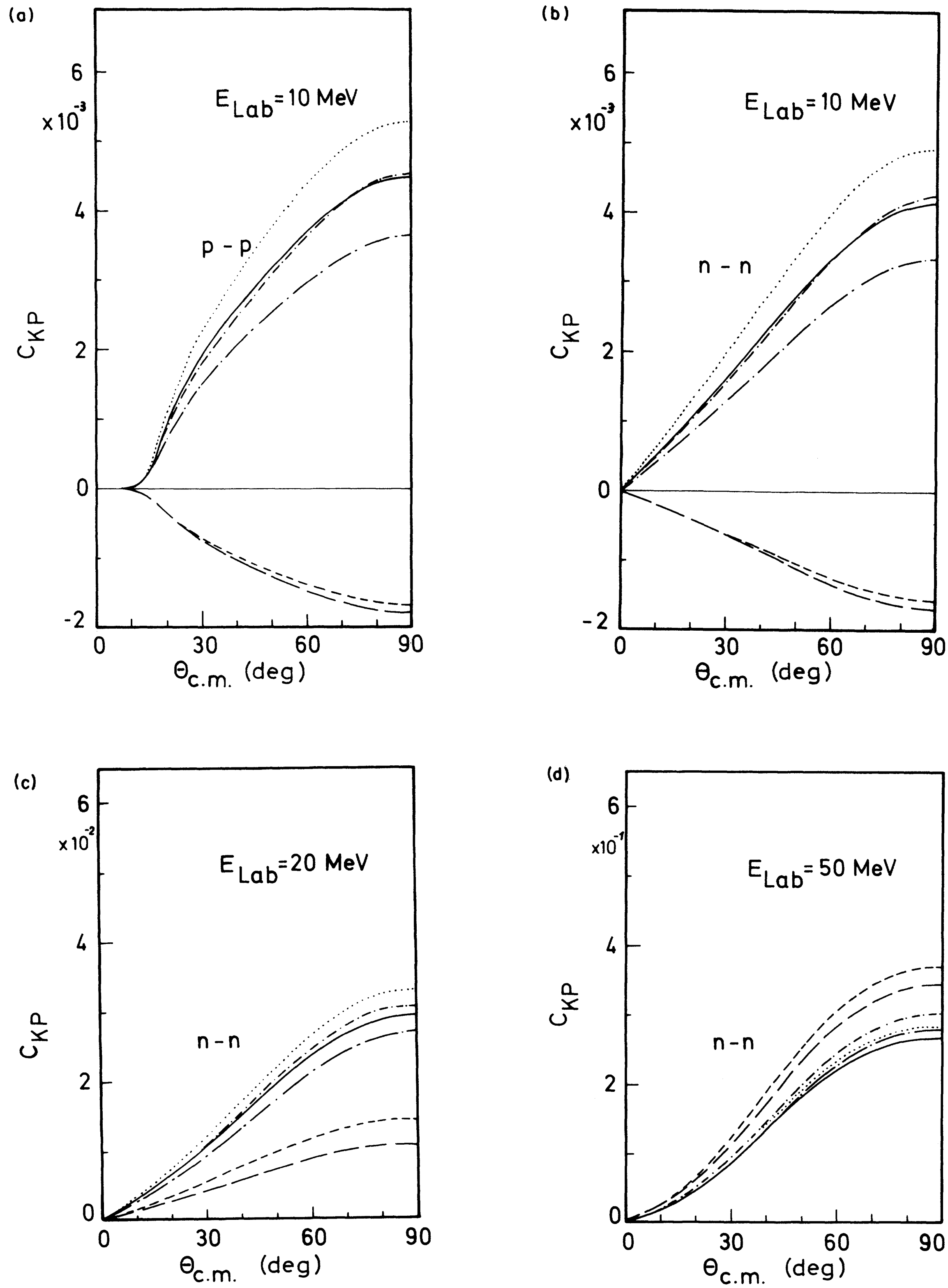

FIG. 5. Spin-correlation parameter C_{KP} for p-p (with) and n-n scattering (without inclusion of the pure Coulomb interaction). The description of the curves in relation to the case studies of Sec. IIIA is as in Fig. 2. For C_{KP} the symmetry relation $C_{KP}(\pi-\theta)=C_{KP}(\theta)$ holds.

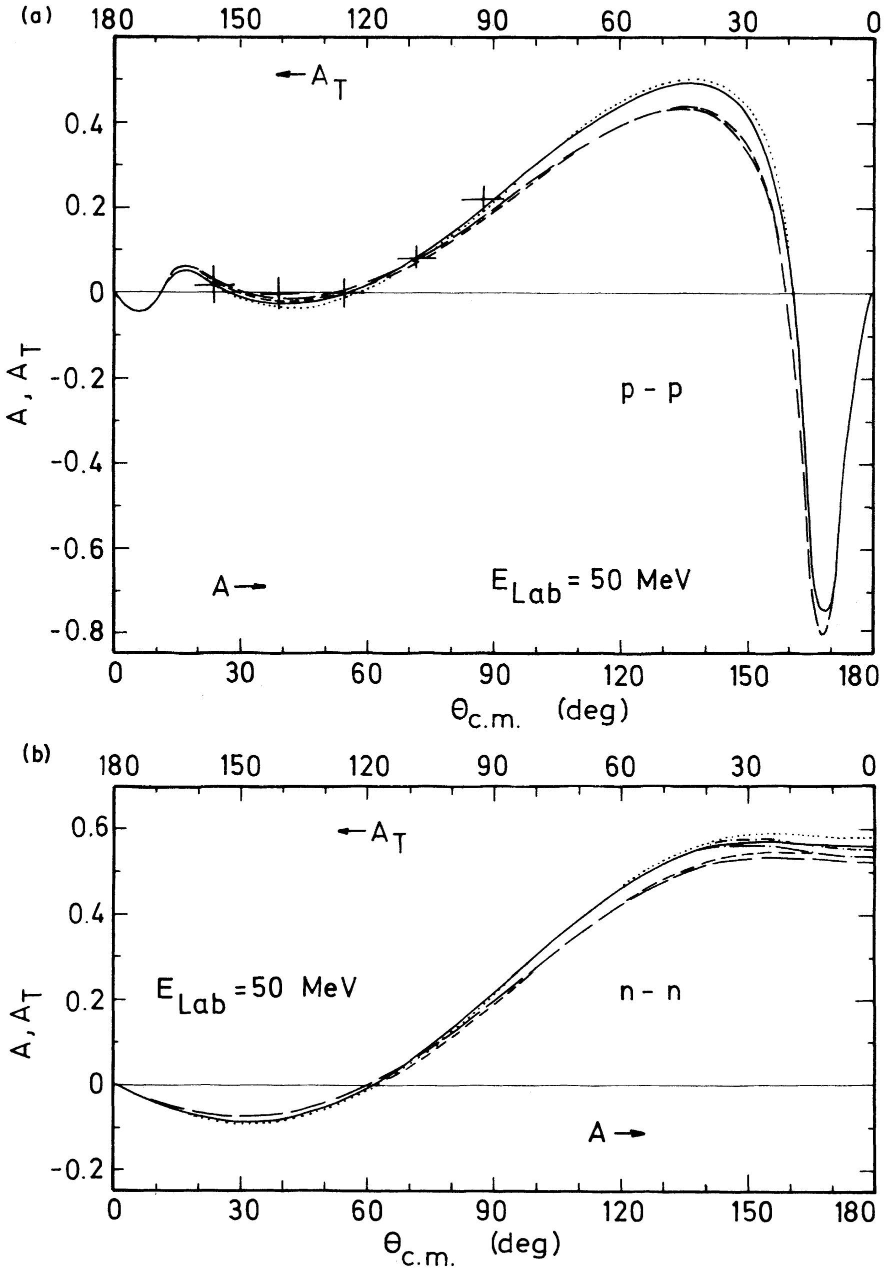

FIG. 6. Spin-transfer coefficients A and A_T for p-p (with) and n-n scattering (without inclusion of the pure Coulomb interaction). The description of the curves in relation to the case studies of Sec. IIIA is as in Fig. 2. For A and A_T the symmetry relation $A_T(\pi-\theta)=A(\theta)$ holds. Therefore the angular dependence of A_T has to be read from the upper scale in the figures. Experimental data were taken from Ref. 45. They refer particularly to the energy 47.5 ± 2.5 MeV. In Fig. 6(a) the curves for Paris practically coincide with those of AHR.

IV. CONCLUSION

The discussion in the foregoing section clearly revealed the role of the CD effect both in full p-p observables as well as in Coulomb subtracted nuclear observables. We noticed its importance particularly with respect to the differential cross section I_0 and also the polarization P. Though the effect is predominant at the lower energies this statement holds for all three energy points considered, i.e., at $E_{\rm lab}=10$, 20, and 50 MeV. The magnitude of the influence of the Coulomb distortion considerably exceeded model differences and also experimental errors (of nowadays feasible p-p experiments). The CD effect turned out to be rather insignificant for more complicated observables such as C_{NN}, C_{KP}, and $A(A_T)$ and also further ones which were not presented. As it turned out in handling the AHR case, the approximation formula (3.1) for describing the CD effect in scattering phase shifts worked quite well. It can certainly be considered as a reasonable prescription of how to remove the CD effect from Coulomb-distorted p-p phase shifts in a model-independent way; thus it might be useful in combined n-p and p-p analyses when taking into account the CD effect explicitly.

Slight model differences appeared in almost all observables that we considered, especially with respect to the Graz potential. A comparison of the results of the two potential models with the experimental data favors the Paris potential; it predicts all observables with satisfactory accuracy.

The various results we quoted for the n-n system will probably not be tested by experiment in the near future. Hitherto only a few measurements of n-n differential cross sections were performed (at higher energies). Under these circumstances our n-n curves may seem somewhat academic. But they can serve as a measure for the required accuracy in n-n experiments, which will possibly be undertaken in order to decide the question of charge symmetry on the level of scattering observables; in particular the experimental errors would have to fall below the magnitude of the CD effect in Coulomb-subtracted observables.

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[29]For this reason we have chosen the label "n-n," though these curves represent Coulomb-subtracted nuclear observables. Only the case (iv) curves are to be considered as n-n observables (under the assumption that all Coulomb effects other than the CD effect can be neglected).

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