Entropy generation and hydrodynamic flow in relativistic heavy ion collisions

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The spectra of pions, protons, and deuterons from collisions of Ar on KCl at a beam energy of 800 MeV per nucleon have been computed in a conventional hydrodynamic approach. Extreme softening of the finite temperature form of the nuclear equation of state leads to significant differences in the number of emitted particles of each species, although there is still too little entropy and too few pions produced as compared with experiment.

[NUCLEAR REACTIONS Hydrodynamic calculations, spectra of π, p, d; Ar + KCl, 800 MeV/A; entropy, equation of state.]

One of the primary goals of relativistic heavy ion physics is the study of the properties of high temperature, high density hadronic matter. Although the question of how much of the kinetic energy of the colliding nuclei's relative motion if actually thermalized is an interesting and as yet unsettled problem,[1] it was only recently that arguments were given[2] which have allowed one to infer some properties of hot, dense matter from experiment once the assumption of thermalization was made. The essential idea is that the observed ratio of deuterons to protons at large transverse momenta is a direct measure of the entropy of the fireball. We present here the results of a detailed numerical calculation, based on a hydrodynamical model, which confirm those earlier arguments to a large extent.

The hydrodynamic computer code we used has been discussed and utilized in calculating the charged baryon spectra for various reactions in several other papers.[3-5] It is three dimensional and relativistically correct, and the method of solution is a particle-in-cell finite difference approximation. We have extended this hydrodynamic model to incorporate the emission of pions and deuterons, as well as nucleons, in the following way. During the initial stage of the collision the system is compressed. When the system subsequently expands it will eventually reach such a low density that a hydrodynamic description is no longer appropriate. When each element of fluid reaches this freeze-out density, we associate with it a temperature and baryon chemical potential determined by equating the internal energy and entropy with those of a noninteracting relativistic gas of particles. The particles we currently include are π^+, π^0, π^-, p, n, d, d^*, t, and ^{3}He, where d^* is the excited state of the deuteron. As discussed in Ref. 6 the inclusion of heavier fragments such as ^{4}H, ^{4}He, ^{4}Li, ^{5}He, ^{5}Li, and their excited states would primarily influence the calculated spectra of t and ^{3}He. The momentum distribution of a particle of type i in the rest frame of the fluid element is then given by

$$\frac{d^3N_i}{dp^3}=\frac{(2S_i+1)}{(2\pi)^3}\,V\,\frac{1}{\exp[(E-\mu_i)/T]\pm 1}+\text{resonance}, \quad (1)$$

where V is the elemental volume, T the temperature, and μ_i and S_i the chemical potential and spin of the particle. The resonance contribution only occurs for p and n, and is due to a thermal folding of the decay process $d^*\to p+n$. This part of the model is reminiscent of the calculations done with the nuclear firestreak.[6]

Simple estimates suggest that the freeze-out density should be somewhat less than normal nuclear density. We have used a value of 80% of normal density, and have checked that lowering it to 70% or 60% has a negligible effect. Since there are only a finite number of computational particles in a cell (27 at normal density) and since the code increments in finite time steps, the fluid elements freeze-out at densities less than 80%, typically in the range 40% to 80%. We have also placed a cutoff on the internal excitation energy of 5 MeV per nucleon, measured relative to zero energy, since those fluid elements will contribute predominantly to the production of heavier fragments.[6]

To test the response of the computed spectra of π, p, and d we have employed four parametrizations of the equation of state used in the hydrodynamic code. The energy per nucleon is

$$E(n,\mathcal{E})=E_0(n)+\mathcal{E}\,, \quad (2)$$

where $\mathcal{E}$ is the thermal excitation energy, n is the baryon density, and $E_0(n)$ is the ground state energy. We use two forms for the latter,

$$E_0(n)=-8\ \text{MeV}+\tfrac{2}{9}K[(n/n_0)^{1/2}-1]^2 \quad (3)$$

and

$$E_0(n) = -8\text{ MeV} + \tfrac{1}{9}K[n_0/n - 1 + \ln(n/n_0)]\,, \quad (4)$$

referred to as NS[5] and KS,[2] respectively. A common value of the compressibility coefficient of $K = 200$ MeV was used.

We have also used two forms for the entropy per nucleon. In one case it is just the nonrelativistic Fermi-gas formula $S = S_{FG}(x)$, which depends only on the quantity $x = \mathcal{E}n^{-2/3}$. In the other case we use

$$S = S_{FG} + \tfrac{3}{2}\left[\left(1 + \tfrac{8}{9}\frac{\mathcal{E}}{T_0}\right)^{1/2} - 1\right]\,, \quad (5)$$

with $T_0 = 20$ MeV. The meaning of the parameter T_0 can be found by examining the limit of low degeneracy and moderate temperature, $T/T_0 \lesssim 3$,

$$\mathcal{E} = \tfrac{3}{2}T\left(1 + \tfrac{1}{3}\frac{T}{T_0}\right)\,, \quad (6)$$

$$S = 2.5 + \ln\left[\frac{4}{n}\left(\frac{m\mathcal{E}}{3\pi}\right)^{3/2}\right] + \frac{T}{T_0}\,. \quad (7)$$

This suggests that the effective number of degrees of freedon, which is 4 in the Fermi-gas model (2 spin $\times$ 2 isospin), grows exponentially as 4 exp (T/T_0). Although this is reminiscent of a Hagedorn gas with an exponentially rising mass spectrum,[7] here there is no limiting temperature. A value for T_0 of 20 MeV is small but physically not unreasonable.

The pressure can now be determined in the usual manner as

$$P = n^2\frac{d}{dn}E_0(n) + n^2\left(\frac{\partial \mathcal{E}}{\partial n}\right)_s\,, \quad (8)$$

where the compressional and thermal contributions are evident. One equation of state is said to be softer than another if, for the same n and $\mathcal{E}$, its pressure is less. We find that $P_{comp}(\text{NS}) > P_{comp}(\text{KS})$ and $P_{therm}(T_0 = \infty) > P_{therm}(T_0 = 20\text{ MeV})$.

We have performed calculations for collisions at 800 MeV per nucleon between charge-symmetric, equal size nuclei of mass number 40. In Fig. 1 the entropy is plotted as a function of time for central collisions. In this hydrodynamic model all of the entropy is created by shock heating. Entropy is created up to cycles 20–25, at which time maximum compression (5.3-6.7 times normal density) is acheived. After this time the entropy remains constant to within numerical fluctuations in the code. Notice that the results are much more sensitive to the finite temperature form of the equation of state than to the ground state form, and that the softer the equation of state the more entropy is produced. The average temperature at the time of freeze-out ranges from 33 to 39 MeV.

In Figs. 2-4 we plot the spectra of p, d, and

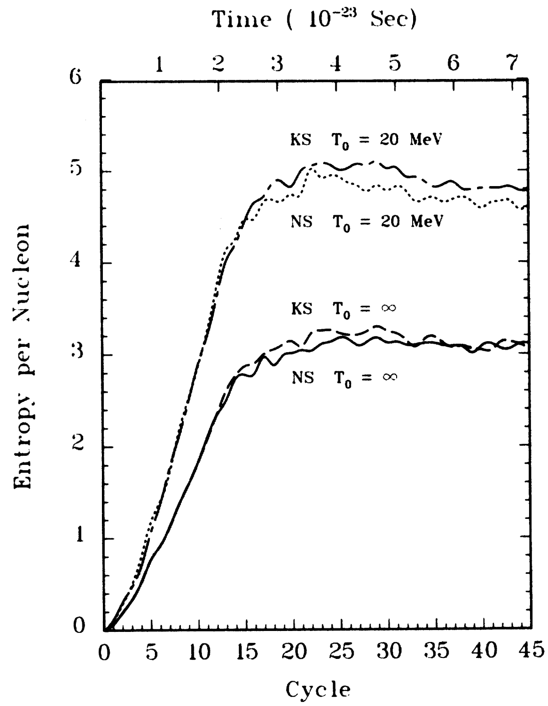

FIG. 1. The entropy per nucleon as a function of time and computational cycle for central collisions of mass 40 nuclei at 800 MeV/nucleon. The four equations of state are described in the text.

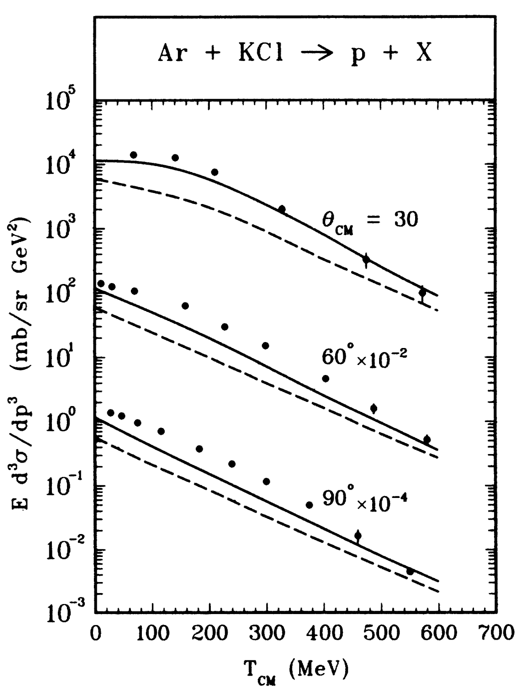

FIG. 2. The proton invariant cross section, plotted in the center of mass frame, at a beam energy of 800 MeV per nucleon. The solid line refers to KS $T_0 = 20$ MeV, the dashed line to NS $T_0 = \infty$, and the data are from Ref. 8.

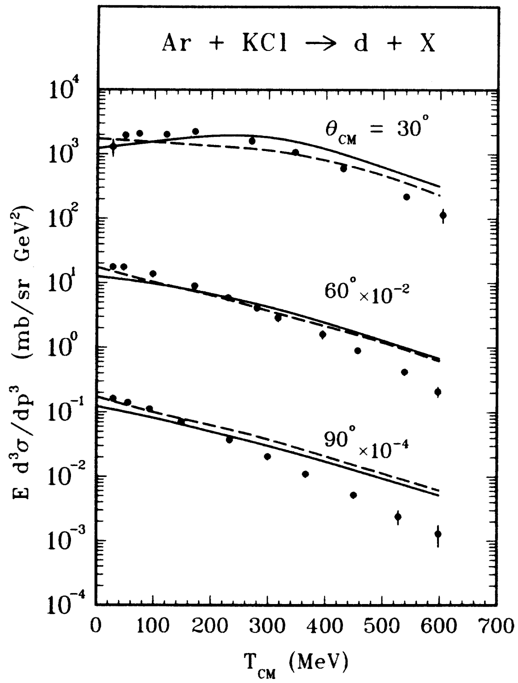

FIG. 3. The deuteron invariant cross section plotted in the center of mass frame. The labeling is the same as in Fig. 2.

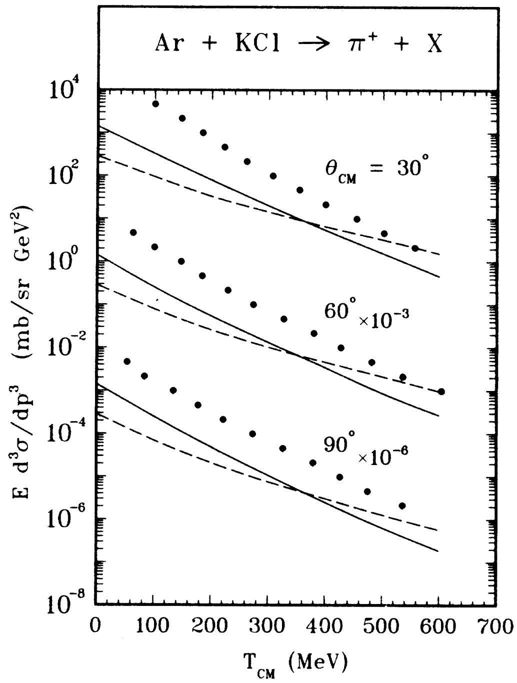

FIG. 4. The positive pion invariant cross section plotted in the center of mass frame. The labeling is the same as in Fig. 2.

π^+ along with the data of Nagamiya *et al.*[8] The dashed lines refer to the moderate equation of state NS $T_0=\infty$, and the solid lines refer to the extremely soft equation of state KS $T_0=20$ MeV. The calculated deuteron spectra are quite close to each other. There is roughly a factor of 2 difference between the calculated proton spectra. This is to be expected since a stiffer equation of state will produce less entropy and hence more protons will be bound up in heavier mass fragments, in our case, mass 3 nuclei. However, even with the extremely soft equation of state there are not enough free protons emitted. Notice also that, in contrast to purely thermal models which predict too many pions,[6] there are far too few pions produced in these calculations even with KS $T_0=20$ MeV. These observations tend to suggest that perhaps heat conduction and viscosity play a role in reducing the amount of energy contained in collective hydrodynamic flow and keeping it in the form of internal excitation energy and pions. One might also expect heat conduction and viscosity to have some effect on a system as light as Ar+KCl since the nucleon mean free path at this energy is probably not negligible compared to the size of the system.

In conclusion we would like to stress that the computed spectra of π, p, and d in a hydrodynamical model are sensitive to the equation of state, particularly its thermal composition. Although there are experimental and theoretical indications that heat conduction and viscosity may be important for a system as light as Ar+KCl, we would expect that sensitivity to remain even with their inclusion.

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[1]S. Nagamiya, in Proceedings of the Symposium on Heavy-Ion Physics, Brookhaven, 1979, edited by J. Barrette and P. Bond, BNL Report No. 51115 (unpublished), p. 131; and LBL Report No. 9494.
[2]P. J. Siemens and J. I. Kapusta, Phys. Rev. Lett. 43, 1486 (1979).
[3]A. A. Amsden, F. H. Harlow, and J. R. Nix, Phys. Rev. C 15, 2059 (1977).
[4]J. R. Nix and D. Strottman, Phys. Rev. C (to be published).
[5]A. J. Sierk and J. R. Nix, Phys. Rev. C 22, 1920 (1980).

[6]J. Gosset, J. I. Kapusta, and G. D. Westfall, Phys. Rev. C 18, 844 (1978).
[7]R. Hagedorn, in *Cargèse Lectures in Physics*, edited by E. Schatzmann (Gordon and Breach, New York, 1973), Vol. VI.
[8]S. Nagamiya, in Proceedings of the Fourth High Energy Heavy Ion Summer Study, Berkeley, 1978, edited by C. Webb, J. Kingston, and J. Mahoney, LBL Report No. 7766 (unpublished), p. 71; S. Nagamiya *et al.*, Phys. Lett. 81B, 147 (1979); M.-C. Lemaire *et al.*, *ibid.* 85B, 38 (1979).