Calculation of the parity mixing between the lowest 0^-,0 and 0^+,1 levels in ^{18}F

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The parity mixing in the 1040, 1080 keV doublet ($J,T = 0^+,1$ and $0^-,0$, respectively) of ^{18}F has been calculated in the frame of the spherical shell model with different residual interactions. In this case only the isovector part of the parity violating potential is effective, and only the dominating one-pion-exchange term has been considered. The shell model space includes the $0p_{1/2}$, $0d_{5/2}$, and $1s_{1/2}$ orbits. The influence of a further truncation of the model space has also been investigated. The results appear not to depend drastically on the choice of the residual interaction among those reported in the literature. The same conclusion appears to hold for the matrix element of the one-pion-exchange potential between the $1/2^-$ and $1/2^+$ levels at 2788 and 2796 keV in ^{21}Ne.

[NUCLEAR STRUCTURE Parity mixing calculation for the 0^+1, 0^-0 doublet in ^{18}F.]

The measurement of the circular polarization of the 1080 keV gamma ray deexciting the lowest $0^-,0$ level of ^{18}F was proposed years ago by Henley[1] as one of the best opportunities to investigate the isovector part of the parity violating nucleon-nucleon interaction. The interest in this case increased after the advent of the Weinberg-Salam theory.[2] In fact, due to the neutral weak current contribution, the isovector parity violating potential induced by the exchange of one pion (OPEP) is expected to be considerably enhanced over the predictions obtained with charged currents only. According to a recent calculation[3] performed in the frame of QCD, the enhancement factor falls in the range of 16–29. The expected values of the circular polarization P_γ has been calculated by Gari *et al.*[4] From their calculations, taking into account the neutral current enhancement of OPEP, and assuming for the Weinberg angle $\sin^2\theta_W = 0.25$, P_γ comes out to be in the range of $5-9\times10^{-3}$.

Recent experimental investigations[5,6], however, have not detected any significant parity-mixing effect. Moreover, some doubts have been raised[7] on the reliability of the nuclear structure calculation in deriving the parity mixing from the parity violating nucleon-nucleon potential. Small differences in the wave functions, in fact, could drastically change the calculated matrix elements if large cancellations occur. Actually, the results have been reported[7] not to be stable against reasonable changes of shell model parameters in a calculation concerning ^{21}Ne, and doubts[7] have been extended to the ^{18}F case. It appears therefore worthwhile to repeat the calculation with different sets of shell model parameters.

The circular polarization of the 1080 keV gamma ray is given by[8]

$$P_\gamma = -2\frac{(1^+,0\|M1\|0^+,1)}{(1^+,0\|E1\|0^-,0)}\frac{\langle 0^+,1\,|V_{\mathrm{PNC}}|\,0^-,0\rangle}{E(0^-,0)-E(0^+,1)},$$

where $(1^+,0\|M1\|0^+,1)$ and $(1^+,0\|E1\|0^-,0)$ are the reduced matrix elements for the electromagnetic decay of the $0^+,1$ (1040 keV) and of the $0^-,0$ (1080 keV) level to the $1^+,0$ ground state, and $\langle 0^+,1\,|V_{\mathrm{PNC}}|\,0^-,0\rangle$ is the matrix element of the parity violating interaction. Due to the different isospin of the levels, only the isovector part of V_{PNC} (dominated by OPEP) contributes.

The parity violating OPEP between two nucleons can be written in standard notation[9]

$$V^\pi_{\mathrm{PNC}} = \frac{G_{\pi NN}\,n_0^-}{8\pi\sqrt{2m_\pi}\,M}(\vec\sigma_1+\vec\sigma_2)\cdot[(\vec p_1-\vec p_2),\ e^{-m_\pi r}/r]_-$$

$$\times(\tau_1^-\tau_2^+-\tau_1^+\tau_2^-).$$

In the Cabibbo model the value of n_0^- deduced from Eq. VIII-12 of Ref. 9 with the decay amplitudes given in Ref. 10 comes out to be $0.177\times10^5\ \mathrm{s}^{-1/2}$. This value is enhanced[3] by a factor of 16–29 when neutral currents are included.

The absolute value of the ratio $R = (1^+,0\|M1\|0^+,1)/(1^+,0\|E1\|0^-,0)$ can be deduced from the experimental values for the mean lives of the parent levels. From the values given in Ref. 11 [i.e., $\tau_m(0^+,1) = 4^{+3}_{-2}$ fs and $\tau_m(0^-,0) = 27.5\pm1.9$ ps] one obtains $|R| = 88$. However, the matrix element $\langle 0^+,1\,|V^\pi_{\mathrm{PNC}}|\,0^-,0\rangle$ must be calculated starting from some model wave functions for the $0^+,1$ and $0^-,0$ states.

The calculations reported in this work have been performed in the frame of the spherical shell model and are limited to a configuration space including the $0p_{1/2}$, $0d_{5/2}$, and $1s_{1/2}$ valence orbits as in Refs. 12–14. The calculations have been repeated for three different residual inter-

actions, namely interaction I of Ref. 12 and interactions F and Z of Ref. 14. The last one has been used in the calculation by Gari *et al.*,[4] while interactions F and Z have been used by Brandenburg *et al.*[7] for a similar calculation in ^{21}Ne and have been reported to give, for this nucleus, quite different values of P_γ. The interaction I is derived almost entirely from the Hamada-Johnston potential through a reaction matrix calculation of the Kuo-Brown type. The interaction F is a purely phenomenological one, while the interaction Z (a modification of interaction II of Ref. 12) is somewhat intermediate between the other two.

Two independent computing procedures have been used for the calculation of the level wave functions and of the matrix element $\langle 0^+,1 \,|V^\pi_{\rm PNC}|\, 0^-,0\rangle$. One of these, running on the CDC 6600 computer of CINECA (Bologna), includes as a first step the Oak Ridge-Rochester shell model code[15] and is completely developed in the j-j coupling scheme. The other one, running on the IBM 370/158 computer of CNUCE (Pisa) has been written in the m scheme and uses a version of the Glasgow-Manchester shell model code.[16] The two procedures have as a common input, in addition to the single particle energies and residual interactions, the two-body matrix elements of $V^\pi_{\rm PNC}$. The latter have been calculated using the radial wave functions of an harmonic oscillator with $\hbar\omega = 15$ MeV.

Corrections due to short range correlations have not been considered. In fact, Gari has shown[4,17] that their effect on the parity mixing is small ($\lesssim 10\%$) as long as the mixing is dominated by the long-range OPEP. The contamination of the wave functions due to the center-of-mass motion has been evaluated with the procedure described in Ref. 18 and has been found to be less than 4% (in strength) in the worst case, i.e., for the $0^-,0$ state.

In Table I are shown, for each of the three residual interactions, the calculated excitation energies of the $0^+,1$ and $0^-,0$ states relative to the $1^+,0$ ground state, the reduced amplitude for the $M1$ transition $0^+,1 \to 1^+,0$, and the value of $\langle 0^+,1 \,|V^\pi_{\rm PNC}|\, 0^-,0\rangle$. The sign of the matrix element of $V^\pi_{\rm PNC}$ depends on the (arbitrary) choice of the phases of the relevant wave functions.[19] The relative sign of the values given in the 4th row of Table I is, however, significant. In fact, the relative sign of the wave functions calculated for a given state with two different interactions can be fixed by the requirement that their scalar product be positive. This criterion is meaningful in the present case since the overlap (scalar product) of the wave functions turns out to be always larger than 0.95. From a comparison of the different columns of Table I, one sees that the predictions obtained with the F and Z interactions are quite similar (apart from the position of the 0^+, 1 level), whereas interaction I does not even reproduce the relative position of the three levels and leads to a value of P_γ significantly larger than the other two. It must be stressed here that the Z and F interactions have been used with good results in several instances,[13,14,20] while the I interaction introduced in Ref. 12 has since never been applied, to our knowledge, for actual spectroscopic calculations.

The effect of a different choice of the harmonic oscillator constant $\hbar\omega$ has also been investigated. We found that the value of $\langle 0^+,1 \,|V^\pi_{\rm PNC}|\, 0^-,0\rangle$, ob-

TABLE I. Results of shell model calculations in ^{18}F with different residual interactions compared with experimental values given in the last column. The reduced matrix element of the $M1$ operator is calculated with the free particle values of the gyromagnetic ratio; the experimental value is deduced from the mean life of the $0^+,1$ level $\tau_m = 4^{+3}_{-2}$ fs. The expected polarization of the 1080 keV γ ray is proportional to the matrix element of $V^\pi_{\rm PNC}$. For the Z interaction P_γ turns out to be 0.41×10^{-3} in the Cabibbo model and $(6.5-11.3)\times10^{-3}$ with the enhancement obtained by Buccella, Maiani, Lusignoli, and Pugliese from Weinberg-Salam theory and QCD. According to present experimental results, an upper limit for P_γ is about 5×10^{-3}.

	Interaction			
	F	Z	I	Experiment
$E(0^+,1)-E(1^+,0)$ (MeV)	0.98	0.57	−1.62	1.040
$E(0^-,0)-E(1^+,0)$ (MeV)	1.34	1.32	2.25	1.080
$(1^+,0 \,\|\| M1 \,\|\| 0^+,1)$ (μ_N)	3.15	3.00	2.67	$\pm(3.6^{+1.4}_{-0.9})$
$\langle 0^+,1 \,\|\, V^\pi_{\rm PNC} \,\|\, 0^-,0\rangle$ (eV)	0.074	0.093	0.136	

TABLE II. Effect of the truncation of the model space on the results of the shell model calculations of Table I, for the Z interaction. The space dimensions increase (from left to right) from not more than 1 hole in the $0p_{1/2}$ to the full model space of Table I. The values of the ground state energy given in the first line are reported only to show the degree of convergence when the space is enlarged.

	Holes in $0p_{1/2}$			
	0–1	0–2	0–3	0–4
$E(1^+,0)$ (MeV)	−16.84	−20.06	−20.06	−20.43
$E(0^+,1)-E(1^+,0)$ (MeV)	2.15	1.17	1.17	0.57
$E(0^-,0)-E(1^+,0)$ (MeV)	−0.33	2.89	0.95	1.32
$(1^+,0\|\|M1\|\|0^+,1)$ (μ_N)	3.28	3.20	3.20	3.00
$\langle 0^+,1\|V^\pi_{\rm PNC}\|0^-,0\rangle$ (eV)	0.23	0.10	0.14	0.093

tained with the Z interaction, varies by ± 13% when $\hbar\omega$ is varied by ± 10%. Then we gained a rough idea of the influence of the assumed configuration space on the predicted value for the parity mixing by repeating the calculations, with the Z interaction , in configuration spaces consecutively restricted from the initial one (up to four holes in the $0_{p1/2}$ orbit) to a maximum of 3, 2, and 1 hole in the $0p_{1/2}$ orbit. This procedure is, of course, somewhat inconsistent since the proper interaction to be used depends also on the assumed configuration space, but it probably gives an indication of which configurations are "important" for the parity mixing. The results, shown in Table II, are seen to stabilize as soon as at least two $0p_{1/2}$ holes are allowed in the configuration space. We may add that the overlap of the relevant wave functions in the full space with the corresponding ones in the two-hole space is better than 0.90, while this figure decreases to 0.69 when only one $0p_{1/2}$ hole is allowed. The extension of the model space to include the $0p_{3/2}$ and $0d_{3/2}$ orbits would be desirable. However, to the best of our knowledge, no effective two-body interaction matched to the enlarged space is available in the literature.

Finally, the same procedure has been used to calculate the matrix element of $V^\pi_{\rm PNC}$ between the lowest $\frac{1}{2}^+,\frac{1}{2}$, and $\frac{1}{2}^-,\frac{1}{2}$ levels in ^{21}Ne. The calculation gives for $\langle\frac{1}{2}^+,\frac{1}{2}|V^\pi_{\rm PNC}|\frac{1}{2}^-,\frac{1}{2}\rangle$ the values 0.043 eV and 0.067 eV with the Z and F interactions, respectively. The change of sign reported by Brandenburg *et al.*[7] going from the F to the Z interaction (which brought to our attention the need of a more detailed investigation) is not confirmed. However, the absolute values of $\langle\frac{1}{2}^+,\frac{1}{2}|V^\pi_{\rm PNC}|\frac{1}{2}^-,\frac{1}{2}\rangle$ (coefficient ϵ in Table II of Ref. 7) are consistent with the present ones, as the small difference could be ascribed to the inclusion of short-range correlations. We could not find any reasonable explanation for this sign discrepancy. We are rather confident, however, that it is not due to a phase inconsistency in the present work, since the wave functions for each level with the F and Z interactions come out to be rather similar and their relative sign has been fixed with the same procedure as for ^{18}F. The scalar product is + 0.97 in both cases.

In conclusion, the calculation of parity mixing in ^{18}F in the frame of a currently used model is not exceedingly sensitive to the choice of the residual interaction (at least not at the level implied in Ref. 7). For what concerns the matrix element of $V^\pi_{\rm PNC}$, the same conclusion appears to hold also for the case of ^{21}Ne. The conclusion could be different in this case for the isoscalar parts of the parity violating potential, particularly due to the nonlocal nature of some of them. A calculation of these terms, which do not contribute to the parity mixing in ^{18}F, is, however, outside the limits of the present work. A possible contribution of other isovector terms, different from $V^\pi_{\rm PNC}$ considered here, cannot be excluded but can be expected to be small if the neutral current enhancement for the pion exchange term is large as predicted[3] by current theoretical calculations.

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