Beta decay of ^{24}Al

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(Received 3 October 1980)

The β^+ decay of ^{24}Al has been studied with a Ge(Li)-NaI(Tl) escape-suppression spectrometer. The number of ^{24}Mg γ transitions observed following ^{24}Al decay has been increased from 19 to 44. Definite new β^+ branches were observed to ^{24}Mg levels at 105 76, 108 21, and 113 14 keV. An enhanced $E2$ transition from the 4^+ 9301-keV level to the 2^+ 7349-keV level was sought for and found. A probable 8439→7349 $4^+ \rightarrow 2^+$ transition was also observed. The contribution of these new experimental results to an elucidation of the band structure of ^{24}Mg is discussed.

[RADIOACTIVITY ^{24}Al [from ^{24}Mg(p,n)]; measured E_γ, I_γ, β^+ and γ branchings; deduced $B(E2)$, $\log ft$.]

I. INTRODUCTION

With the possible exception of ^{20}Ne, ^{24}Mg has been the most important even-even sd-shell nucleus for testing nuclear models. In particular, early tests of various manifestations of collective motion were made in ^{24}Mg. These include the Nilsson model, SU(3) symmetries, and Hartree-Fock calculations.[1,2]

Recently shell-model calculations have been carried out for ^{24}Mg in a complete sd-shell basis by both the Glasgow group[3,4] and by Wildenthal and his co-workers.[5,6] These calculations which use the Glasgow shell-model code and the Preedom-Wildenthal[5] or Chung-Wildenthal[6] effective interaction appear to be very successful, although very little besides energy spectra have been published so far. In the model space used in these calculations, ^{24}Mg consists of eight nucleons in the sd shell outside an inert ^{16}O core. The number of active nucleons is large enough that a physical interpretation of the calculations is difficult. An obvious help in understanding the predictions is to express the shell-model wave functions in a collective framework such as the Nilsson model or in SU(3) symmetries. For example, expansion in terms of SU(3) symmetry has been done with several different interactions and model spaces for the low-lying levels of the $K=0$ ground-state band and the $K=2$ band built on the 2^+ level at 4238 keV. An example is the work of Wathne and Engeland.[7] In brief, the theoretical evidence for $K=0$ and $K=2$ bands in ^{24}Mg is overwhelming. At the same time the experimental information[2] on the states belonging to these bands is relatively complete and also strongly indicative of band structure.

Our motivation for the present experimental study was to address the question: Are there other even-parity bands based on bound levels of ^{24}Mg, and, if so, how well developed are they? In trying to answer this question one finds that both the spectroscopic information and theoretical predictions on these bands are quite limited. First, the spectroscopic information available on the levels in question is much more scarce than for the $K=0$ and $K=2$ bands.[2] Second, the theoretical predictions available as a guide to future experiments on the higher-lying states of these excited bands are quite meager.

The search for further even-parity bands in ^{24}Mg has a natural starting point with the first excited $J^\pi=0^+$ state which is at 6432 keV. If this is the band head of a $K=0^+$ band then possibilities for the 2^+ state of this band are at 7348 and 8653 keV. In a previous search for a band built on the 6432-keV level the possibility that the 8653-keV level was the 2^+ member of this band was explored.[8] A search for the 4^+ level was unsuccessful. We shall explore the evidence for the placement of the $J^\pi=2^+$ 7348-keV level in this band. There are two pieces of information which suggest a connection between the 2^+ 7348- and 0^+ 6432-keV levels. First, Feldmeier, Manakos, and Wolff[9] in a SU(3)-truncated shell-model calculation gave a prediction of 15 Weisskopf units (W.u.) (Ref. 10) for the 7348 → 6432 $E2$ transition. This is certainly large enough to suggest an intraband transition. Second, in their breakdown of shell-model wave functions into SU(3) symmetries, Wathne and Engeland[7] found the composition of these two states to be remarkably similar. Unfortunately, the branching ratio of the 7348 → 6432 transition is expected to be extremely small and an experiment to measure this $E2$ rate—even if highly enhanced—would be very difficult.

Recent experiments[11] have determined $J^\pi=4^+$ for both the 8437- and 9298-keV levels of ^{24}Mg. These are then the third and fourth $J^\pi=4^+$ levels of ^{24}Mg with the first two belonging to the $K=0$ and $K=2$ bands, respectively. As pointed out by Endt and van der Leun[2] and by Wright,[12] both of these levels are members of energy doublets/triplets and the difficulty of untangling these multiplets has kept the properties of these levels from being well determined.

The present study is of the β^+ decay of $J^\pi = 4^+$ ^{24}Al. Allowed β^+ decay will take place to $J^\pi = 3^+$, 4^+, and 5^+ states of ^{24}Mg. We use the specificity of this reaction to select the 4^+ levels at 8.44 and 9.30 MeV with negligible population of the other members of the doublet at 8.44 MeV and the triplet at 9.30 MeV. It has already been found that both levels are formed in ^{24}Al$(\beta^+)^{24}$Mg.[2,13,14] Our specific aim was to search for γ decays from these two levels to lower-lying 2^+ states.

II. EXPERIMENTAL PROCEDURE AND ANALYSIS

^{24}Al with $T_{1/2} = 2.060 \pm 0.010$ s (Ref. 15) was produced via the ^{24}Mg$(p,n)^{24}$Al reaction ($Q = -14.661$ MeV).[2] A 99.94% pure ^{24}Mg foil, 3.8×10^{-3} cm thick, was mounted on a Delrin rabbit. The ^{24}Mg was bombarded through a 2.5×10^{-3} cm thick Ta entrance window with a 75-nA, 18-MeV proton beam. The rabbit system, described previously,[16] allowed cycling of the ^{24}Mg between the bombard position and a count position which was 4 m away and on the other side of a concrete wall. A bombard-count cycle was as follows:

0–2.0 s, bombard
2.0–2.5 s, transfer (1)
2.5–4.5 s, count
4.5–5.0 s, transfer.

Gamma-ray spectra were collected using a Ge(Li)-NaI(Tl) escape-suppression (anti-Compton) spectrometer. The Ge(Li) detector had an efficiency of 19.5% and a resolution of 2.1 keV for 1332-keV γ rays. It was placed at the center of a 25.4×20.3 cm NaI(Tl) veto detector and was 9.0 cm from the ^{24}Al source. The configuration of the shield was similar to that reported by Konijn, Goudsmit, and Langeman.[17]

In order to stop positrons emanating from the target and to suppress low-energy γ rays and x rays, a plug consisting of 0.72 cm of brass followed by 0.66 cm of lead was inserted in the 5-cm long conical lead collimator between the ^{24}Al source and the Ge(Li) detector.

Two sets of 8192-channel spectra were recorded simultaneously by splitting the output from the Ge(Li) preamplifier. One had a dispersion of 0.51 keV/channel and extended to 4185 keV, the other had a dispersion of 1.41 keV/channel and extended to 11 588 keV. Each set consisted of the total counts (normal singles spectra) and a spectrum of those counts rejected by the NaI(Tl) shield. The anti-Compton spectra which were analyzed were obtained by subtracting the rejected spectra from the total spectra. The reduction of the Compton background was by factors of 5–8, with maximum reduction for γ rays of energy 5–6 MeV. Retaining the rejected spectra was an aid in identifying one- and two-escape peaks which were not entirely eliminated by the anticoincidence condition.

Data were accumulated for 17.5 h. The final low-gain anti-Compton spectrum contained a total of 56×10^6 counts and the corresponding rejected spectrum contained 78×10^6 counts. The energy resolution for all the accumulated spectra varied from 2.28-keV full width at half maximum (FWHM) at 1369 keV to 5.75-keV FWHM at 7069 keV. Gamma-ray intensities and peak positions were extracted using the program SAMPO.[18] Portions of the low-gain spectra are illustrated in Figs. 1–3. Figure 1 illustrates the improvement in peak/background offered by the escape suppression, while Figs. 2 and 3 illustrate the evidence for some previously unobserved cascades.

The energy calibration for the ^{24}Al spectra was arrived at in several successive steps. First, the nonlinearity of the high-gain spectrum ($E_\gamma < 4185$ keV) was determined from a separate ^{56}Co calibration. The energies for $E_\gamma < 2900$ keV were obtained using as energy standards the ^{21}Ne 350.725(6) $1 \to 0$ transition[19,20] [from ^{24}Mg-$(p,\alpha)^{21}$Na$(\beta^+)^{21}$Ne] and the ^{24}Mg $1 \to 0$ and $2 \to 1$ transitions of 1368.633(6) and 2754.030(14) keV as given by Greenwood, Helmer, and Gehrke.[21] Then utilizing the relations between ^{24}Al cascade and crossover γ rays, the calibration was extended in several successive steps to $E_\gamma < 3900$ keV, < 5000 keV, and finally to the whole energy region of interest. For the higher energies, one- and two-escape peaks from the rejected spectrum were also used in the energy determinations.

The efficiency versus energy curve for the anti-Compton spectrum was obtained from a ^{56}Co spectrum taken after the ^{24}Al data were accumulated. This gives an accurate efficiency calibration for the energy interval ~ 700–3700 keV.[21,22] One further calibration point at 7069 keV was provided by the low-gain ^{24}Al spectrum as follows. The $J^\pi = 4^+$ ^{24}Al ground state must have a negligible direct branch to the 2^+ ^{24}Mg 1369-keV level (i.e., $\Delta J = 2$, no β decay is twice forbidden), so that the γ-ray flux into the 1369-keV level must be equal to the γ-ray flux out. Then, anticipating our results, some 33% of the ^{24}Al γ flux is in the ^{24}Mg 7069-keV $8439 \to 1369$ transition and almost all of the remaining feeding of the 1369-keV level is via the 2754-keV $4123 \to 1369$ transition for which the efficiency is accurately obtained from the ^{56}Co efficiency calibration. Thus, the relative efficiency at 7069 keV is accurately provided ($\lesssim \pm 2\%$) from this in-

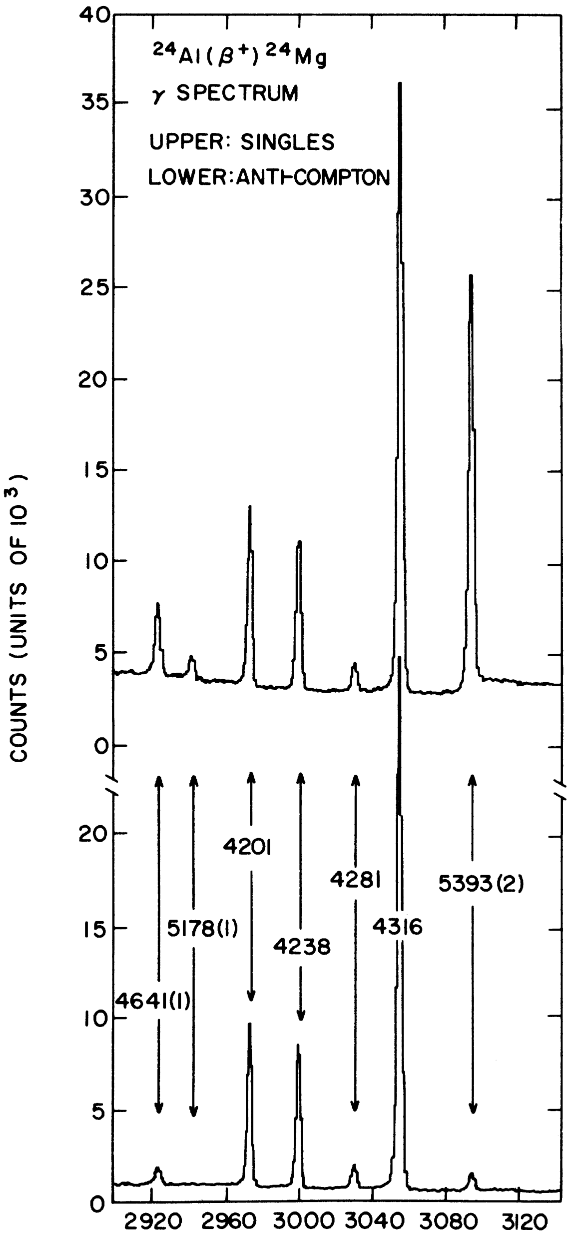

FIG. 1. Partial γ-ray spectra from $^{24}Al(\beta^+)^{24}Mg$. Singles data (upper) are shown as well as singles minus the rejects (lower). The peaks are labeled by their energies in keV and are identified in Table I. Note the reduced background in the anti-Compton spectrum, as well as the reduction of one- and two-escape peaks, which are labeled as (1) and (2), respectively.

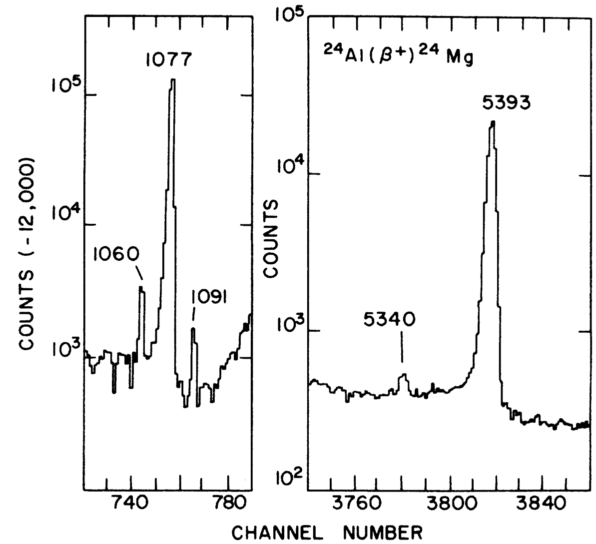

FIG. 2. Portions of the anti-Compton γ spectrum from $^{24}Al(\beta^+)^{24}Mg$ illustrating the evidence for weak, previously unobserved transitions. The γ peaks are labeled by their energies in keV and are identified in Table I. For purposes of display, 12 000 counts have been subtracted from the spectrum on the left. The 1060-, 1091-, and 5340-keV transitions were not previously observed.

ternal calibration. The form of the efficiency versus energy curve was assumed to be exponential for $E_\gamma > 3.5$ MeV. Justification for this is provided by previously published efficiency curves such as the data of Young *et al.*[23] The relative efficiency was assumed to have a ±2% accuracy for $800 < E_\gamma < 3500$ keV and at 7069 keV, a ±10% accuracy at 5300 keV, and a ±20% accuracy at 10 000 keV. The accuracy was assumed to be given by a

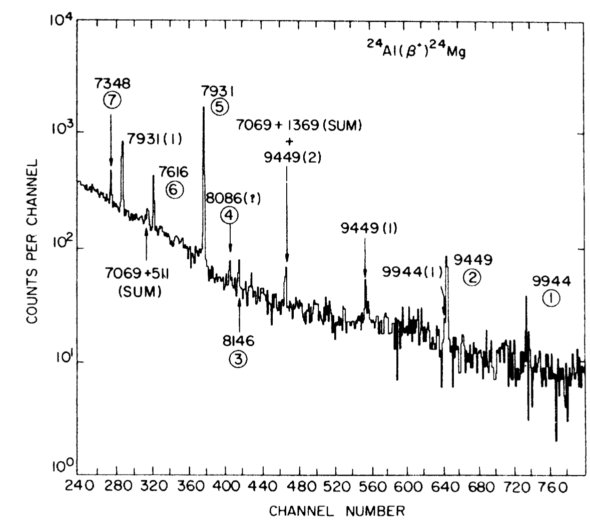

FIG. 3. The high-energy portion of the $^{24}Al(\beta^+)^{24}Mg$ anti-Compton spectrum. The γ peaks are labeled by their energies in keV. One- and two-escape peaks are labeled as (1) and (2), respectively. The peaks labeled SUM result from simultaneous detection of two coincident γ rays. One questionable peak is labeled by (?). Seven peaks are numbered sequentially starting with the highest energy. These correspond to the seven highest-energy transitions of Table I.

TABLE I. Gamma rays observed from $^{24}Al(\beta^+)^{24}Mg$.

Energy[a] (keV)	Assignment[b]	Relative intensity[c] (%)	Energy[a] (keV)	Assignment[b]	Relative intensity[c] (%)
426.00(10)	^{24}Al(426 → 0)[d]	0.29(4)	2869.50(6)	4238 → 1369	1.097(28)
587.95(6)	*U*	0.149(8)	3203.88(8)	8439 → 5235	3.085(66)
775.40(20)	6010 → 5235	0.053(8)	3378.27(80)	7616 → 4238	0.043(7)
822.00(60)	8439 → 7616	0.021(8)	3493.32(*N*)	7616 → 4123	0.04(1)
860.20(60)[e]	9301 → 8439(*Q*)	0.022(11)	3505.61(9)	9516 → 6010	1.98(6)
909.09(6)	*U*	0.122(6)	3866.14(10)	5235 → 1369	5.26(22)
996.83(10)	5235 → 4238	0.137(7)	4200.54(13)	8439 → 4238	4.02(22)
1059.78(8)	10576 → 9516	0.285(17)	4237.96(6)	4238 → 0	3.61(21)
1076.86(4)	9516 → 8439	14.84(31)	4280.62(13)	9516 → 5235	0.66(4)
1090.67(10)	8439 → 7349	0.140(7)	4316.00(12)	8439 → 4123	14.20(86)
1274.71(10)[f]	10576 → 9301	0.106(6)	4641.19(9)	6010 → 1369	3.42(25)
1368.633(6)	1369 → 0(*C*)	96.0(2.5)	5060.68(80)	9301 → 4238	0.036(13)
1434.11(20)	*U*	0.063(6)	5177.51(20)	9301 → 4123	0.98(10)
1704.77(80)	9516 → 7812	0.016(4)	5340.30(40)	10576 → 5235	0.115(13)
1771.92(7)	6010 → 4238	0.40(1)	5392.68(9)	9516 → 4123	18.3(18)
1887.52(20)[g]	6010 → 4123	0.056(6)	5979.52(80)	7349 → 1369	0.093(9)
1899.70(6)	9516 → 7616	0.82(2)	6246.89(11)	7616 → 1369	0.54(4)
1952.38(20)	9301 → 7349	0.094(6)	7069.50(12)	8439 → 1369	43.0(1.3)
2127.51(20)	*U*	0.054(7)	7348.17(90)	7349 → 0	0.153(16)
2136.58(15)	10576 → 8439	0.168(9)	7615.17(90)	7616 → 0	0.224(15)
2381.03(30)	7616 → 5235	0.037(10)	7930.87(15)	9301 → 1369	1.34(10)
2428.97(15)	8439 → 6010	0.774(18)	8085.66(*N*)	9456 → 1369(*Q*)	0.020(10)
2566.96(20)	*U*	0.065(7)	8146.04(*N*)	9516 → 1369	0.028(7)
2577.44(80)	7812 → 5235(*Q*)	0.030(12)	9450.09(40)	10821 → 1369	0.110(20)
2754.030(14)	4123 → 1369(*C*)	41.19(90)	9943.46(150)	11314 → 1369	0.027(6)

[a] Uncorrected for nuclear recoil. The number in parentheses is the uncertainty in the last figure. An *N* means the γ-ray energy is calculated from the transition energy.
[b] ^{24}Mg $E_i \to E_f$ in keV. *U* denotes an unknown, *Q* a questionable assignment, *C* a γ ray used for energy calibrations.
[c] Normalized such that the flux into the ^{24}Mg ground state is 100.00.
[d] The dominant decay mode of $^{24}Al^m$ (see text).
[e] Possibly a doublet with an unknown contaminant.
[f] Contains an unknown contribution from the $^{22}Na(\beta^-)^{22}Ne$ 1274-keV γ ray.
[g] Possibly contains a contribution from the ^{24}Mg 10328 → 8439 transition.

linear interpolation between these points for E_γ >3500 keV. The energies, relative intensities, and decay scheme assignments of the observed γ rays are listed in Table I.

In all, the energies and intensities of 57 definite γ rays were extracted from the anti-Compton spectrum. Forty-four of these were assigned to ^{24}Al β^+ decay. Eight others were assigned to various obvious activities such as $^{23}Mg(\beta^+)^{23}Na$ from $^{24}Mg(p,d)^{23}Mg$ or $^{40}K(EC)^{40}Ar$. Five unidentified lines are listed in Table I.

The bombard-count cycle discriminated strongly against γ rays from the decay of $^{24}Al^m$ which has a half-life of 129 ± 5 ms.[2] The dominant decay mode of this $J^\pi = 1^+$ metastable state is via $M3$ decay to the 4^+ ground state. The energy we observe for this transition, 426.00 ± 0.10 keV, is in only fair agreement with the value of 425.8 ± 0.1 keV reported by Houkanen *et al.*[14] From the observed intensity of this branch (see Table I) and previous relative intensities[14] all other branches from $^{24}Al^m$ are inferred to be negligible in the present study.

The level energies and γ-ray branching ratios derived from the results of Table I are collected in Table II. The various γ-ray combinations contributing to a determination of the level energies were in good agreement and the uncertainties assigned to the level energies are considered to be conservative. The assignments of γ-ray peaks to ^{24}Mg transitions were first made on the basis of energy and from consideration of previous spectroscopic information. Since the energy determinations were quite accurate, the target impurities were minimal, and the number of ^{24}Mg levels directly fed via ^{24}Al is small, it is unlikely that any transition is misassigned or is a doublet. All possible transitions between the known ^{24}Mg levels below 9-MeV excitation and states for $9 < E_x < 11$ MeV with known or possible 3^+, 4^+, or 5^+ assign-

TABLE II. ^{24}Mg γ-ray transitions observed in the β^+ decay of ^{24}Al.

J_i^π	E_i[a] (keV)	E_f (keV)	E_γ (keV)	Branching ratio (%) Present	Branching ratio (%) Previous[b]
2^+	1368.675(6)	0	1369	100	100
4^+	4122.875(15)	0	4123		$<10^{-3}$
		1369	2754	100	100
2^+	4238.360(60)	0	4238	76.7 ±0.7	78.9±0.5
		1369	2870	23.3 ±0.7	21.2±0.5
3^+	5235.200(80)	0	5235	<0.4	<0.04
		1369	3866	97.3 ±0.3	98.2±0.2
		4123	1112	<0.7	<0.5
		4238	997	2.7 ±0.3	1.8±0.2
4^+	6010.315(90)	0	6010	<0.8	<2
		1369	4641	86.6 ±1.2	87±1
		4123	1887	≤2.0 ±0.3[c]	<1
		4238	1772	10.1 ±1.0	13±1
		5235	775	1.3 ±0.2	<1
2^+	7349.05(40)	0	7348	62.0 ±2.4	62±2
		1369	5980	38.0 ±2.4	38±2
3^-	7616.470(90)	0	7615	25.3 ±2.0	23±2
		1369	6247	61.4 ±2.0	72±2
		4123	3493	4.3 ±0.8	5±1
		4238	3378	4.9 ±0.8	<5
		5235	2381	4.1 ±0.7	
		6010	1606	<3	<2
4^+	8439.295(90)	0	8438	<0.08	<2
		1369	7069	65.9 ±1.5	63±3
		4123	4316	21.7 ±0.9	23±3
		4238	4201	6.2 ±0.3	7±1
		5235	3204	4.8 ±0.2	6±1
		6010	2429	1.2 ±0.1	
		7349	1090	0.22±0.02	
		7616	822	0.03±0.02	1.0±0.5
4^+	9300.95(15)	0	9299	<0.2	
		1369	7931	54.0 ±2.0	57±5[d]
		4123	5177	40.0 ±1.8	43±5[d]
		4238	5061	1.5 ±0.3	<5[e]
		5235	4065	<2.0	
		6010	3290	<2.4	
		7349	1952	4.1 ±0.4	
		7616	1684	<0.8	
		8439	860	<0.9 ±0.2	
4^+	9516.210(80)	0	9514	<0.05	<5
		1369	8146	0.08±0.04	<0.2
		4123	5393	49.5 ±1.2	50±3
		4238	5277	<0.7	1.0±0.5
		5235	4281	1.8 ±0.2	3.8±0.4
		6010	3506	5.3 ±0.3	6±1
		7349	2167	<0.07	
		7616	1900	2.3 ±0.2	1.5±0.5
		7812	1704	0.04±0.02	
		8439	1077	41.0 ±1.0	38±3
$(5)^+$	10575.983(80)[f]	0	10573	<1.5	<2
		1369	9205	<2	
		4123	6452	<4	<10
		4238	6337	<4	<15
		5235	5340	17.1 ±1.9	28±10
		6010	4565	<3	13±4
		7349	3227	<2	<10
		7616	2959	<4	(11)±4[g]
		8439	2137	24.9 ±1.3	<25[h]
		9301	1275	≤15.7 ±0.9[i]	<7
		9516	1060	42.3 ±2.5	34±12
	10820.76(40)	1369	9449	(100)	
	11314.23(150)	1369	9944	(100)	

[a] The number in parentheses is the uncertainty in the least significant figure. The energies of the first two states are from Ref. 21. The other energies were calculated by combining the measured γ-ray energies as explained in the text.

[b] From Ref. 2.

[c] Some or all of this γ-ray intensity could arise from a 10328 → 8439 transition.

[d] From Ref. 13. There appears to be a transcription error in Table 24.11 of Endt and van der Leun (Ref. 2) for the decay of this level.

[e] From Ref. 34.

[f] The previous branching ratios are from Ref. 24.

[g] Uncertain.

[h] Exists but of unknown intensity.

[i] A possible ^{22}Na(β^-)^{22}Ne(1274.55-keV level) contribution cannot be resolved from this transition.

ments were considered in making the transition assignments. In almost all cases where other transitions were possibly degenerate with those of Tables I and II, the assignment could be excluded because more intense transitions from the initial level had been reported but were not observed in our spectra. Nevertheless it was felt desirable to substantiate certain aspects of the decay scheme by a $\gamma\gamma$-coincidence experiment. The coincidence measurement was made with the escape-suppression spectrometer as described earlier and a 12.7-cm diameter by 12.7-cm long NaI(Tl) detector at 180° to it. A 1.2-cm Pb absorber was interposed between the rabbit and the NaI(Tl). Thirty hours of coincidence data were event-mode recorded in a 8192 × 4096 matrix. From subsequent analyses of these data we were able to establish almost all of the coincidence relationships necessary to support the assignments of Tables I and II.

The observed yields of the γ peaks listed in Table I vary from 1.58×10^6 for the 1369-keV transition to 25 for the 9944-keV γ ray. For most of the transitions the uncertainties in the yields were small compared to that in the relative efficiency which, therefore, dominates most of the uncertainties assigned to the branching ratios in Table II.

As can be seen from the comparison to the previously adopted branching ratios of Endt and van der Leun,[2] our branching ratios are in good general agreement with previous results. Somewhat minor discrepancies are noted for the 4238-keV level, the 5235 → 4238 transition, the 6010 → 4123 transition, and the 9516 → 4238 and 9516 → 5235 transitions. There is only one major discrepancy which is the nonobservation of the 822-keV 8439 → 7616 transition at the level reported by Détraz[13] in the most extensive previous ^{24}Al(β^+)^{24}Mg study. We can only conclude that the 822-keV γ ray observed by Détraz arose from some other source.

The present γ-ray and level energies are, in general, considerably more precise than previous measurements. The agreement with previous results[2] is fair.

The two highest energy γ rays observed had energies of 9450.09(40) and 9943.46(150) keV and are shown in Fig. 3. [A 9440(10)-keV γ ray was also observed by Détraz[13] in his study of ^{24}Al β^+ decay.] These two γ rays were assigned to the decay of ^{24}Al because the energies and intensities preclude any other possibilities. (They were both too weak to show up in the coincidence spectra.) The intensities are only compatible with allowed β^+ decay, thus the ^{24}Mg states emitting these γ rays have $J^\pi = 3^+$, 4^+, or 5^+. For these alternatives we reject the possibility that either of these γ rays is a ground-state transition as untenable. Since the β^+ Q value is not large enough, neither can arise from a cascade to a ^{24}Mg level with $E_x > 4$ MeV. Thus we conclude both γ rays correspond to transitions to the 2^+ 1369-keV level as shown in Table II. It is possible that the 10 821- and 11 314-keV levels of this work can be associated with states observed at 10 824(3) and 11 318(3) keV in the ^{24}Mg(p,p')^{24}Mg reaction.[2]

The observed γ-ray intensities and assignments of Table II result in the β^+ branching ratios of Table III. The normalization of the relative intensities in Table I is such that the % β^+ branch into a given level is

$$\%\ \beta^+ \mathrm{BR} = I_\gamma\ (\mathrm{decay}) - I_\gamma\ (\mathrm{feeding})\,. \tag{2}$$

The β^+ branching ratio results are in excellent agreement with those of Détraz[13] with which they are compared, and also with the recent less complete results of Shibata, Imazato, Yamazaki, and Brown[14] (not shown). The logft values were calculated using $T_{1/2} = 2.060 \pm 0.010$ s,[15] $E_{\beta\max} = 12\,856 \pm 4$ keV,[2] and the currently determined branching ratios.

New ^{24}Al(β^+)^{24}Mg β^+ branches from the present work are those to the 10 576-, 10 821-, and 11 314-keV levels and possibly to the 7812- and 9456-keV levels.

The J^π assignments of Table III are those of Endt and van der Leun[2] for $E_x < 10$ MeV. The three $(3,4,5)^+$ assignments for $E_x > 10$ MeV follow from the allowed character of the β decay demanded by the observed logft values.

The probable branch to the 7812-keV level is based on the observation of a 2577.44(80)-keV γ ray which, if it were assigned to the 7812 → 5235

TABLE III. Positron decay of ^{24}Al.

^{24}Mg Level J^π	(keV)	Positron yield (%) Present	Previous[a]	logft Present	Previous
2^+	1 369	<4.0		>6.98	
4^+	4 123	7.7 ± 1.0	7.0 ± 4.5	6.13 ± 0.06	6.2 ± 0.3
2^+	4 238	<0.20		>7.68	
3^+	5 235	1.40 ± 0.13	1.7 ± 0.8	6.59 ± 0.04	6.5 ± 0.3
4^+	6 010	1.2 ± 0.1	1.3 ± 0.8	6.43 ± 0.04	6.4 ± 0.3
2^+	7 348	<0.03		>7.59	
3^-	7 616	<0.06		>7.19	
5^+	7 812	0.05 ± 0.02[b]		7.19 ± 0.17[b]	
4^+	8 439	50.0 ± 2.0	48 ± 6	3.93 ± 0.02	3.95 ± 0.06
4^+	9 301	2.5 ± 0.2	2.4 ± 0.5	4.80 ± 0.04	4.82 ± 0.09
3^+	9 456	0.033 ± 0.017[b]		6.63 ± 0.30[b]	
4^+	9 516	37.0 ± 1.5	39 ± 3	3.510 ± 0.018	3.489 ± 0.034
$(3,4,5)^+$	10 576	0.67 ± 0.06		4.50 ± 0.04	
(4^+)	10 580[c]	<0.2		>5.4	
$(3,4,5)^+$	10 821	0.11 ± 0.01[d]	⩾0.01 ± 0.05	5.08 ± 0.08	
$(3,4,5)^+$	11 314	[(2.6 ± 0.8) × 10^{-2}][d]		5.19 ± 0.14	

[a] Reference 7.

[b] Uncertain (see text).

[c] The other member of the 10.58-MeV doublet (Refs. 8 and 24) for which decay to the 1.37- and 4.12-MeV levels is assumed to amount to at least 40%.

[d] Assumes 100% γ branching to the ^{24}Mg 1369-keV level.

transition (a $60\pm3\%$ branch[2]), would give 7812.79(80) keV for the excitation energy of this level. [Another possible placement of the 2577-keV γ ray is to an $11.01\rightarrow8.44$ transition (see Ref. 11).] The probable branch to the 9456-keV level is based on the observation of a γ-ray peak (see Fig. 3) for which the energy is in agreement with that expected for the $9456\rightarrow1369$ transition (a $61\pm3\%$ branch[2]). Both of these β^+ branches are interpreted as limits because the γ-ray peaks were very weak and because such weak intensities could very well arise from β feeding via higher-lying ^{24}Mg levels.

Our results for the 10 576-keV level are a confirmation of the ^{23}Na$(p,\gamma)^{24}$Mg studies of Boydell and Sargood.[24] These authors noted a major difference in the γ-decay modes of a "10.58-MeV level" when fed by two different (p,γ) resonances. They therefore proposed a doublet with one state having the decay modes with which our results are compared in Table II. Except for the branch to the 7616-keV level, the results are in fair agreement. Boydell and Sargood proposed $J=3$, 4, or 5 for their 10.58-MeV level. This is consistent with the assignment $J^\pi=3^+$, 4^+, or 5^+ resulting from the logft value of Table III. We cannot state definitively that ^{24}Al(β^+) does not feed the other member of the 10.58-MeV doublet.[24] However, the various cascade sums give good agreement in the calculation of the excitation energy, 10 525.983(80) keV, for the level formed. Thus, the two 10.58-MeV levels would have to be separated by ≤ 0.2 keV if indeed both are formed significantly in the β^+ decay.[25]

The main motivation for the $\gamma\gamma$-coincidence measurement was to obtain additional evidence confirming the placement of the 1952- and 1091-keV transitions. The data allow a definite assignment of the 1952-keV γ ray to the $9301\rightarrow7349$ transition. Coincidence spectra are shown in Fig. 4. This is but one of several scans of the coincidence data which, taken together, establish the assignment. The 1091-keV peak (see Fig. 2) was considerably weaker, relative to the background, than the 1952-keV peak. The coincidence data gave evidence supporting the assignment of Table I but fell short of being completely definite. We regard this assignment as slightly uncertain but it will be assumed definite for purposes of discussion.

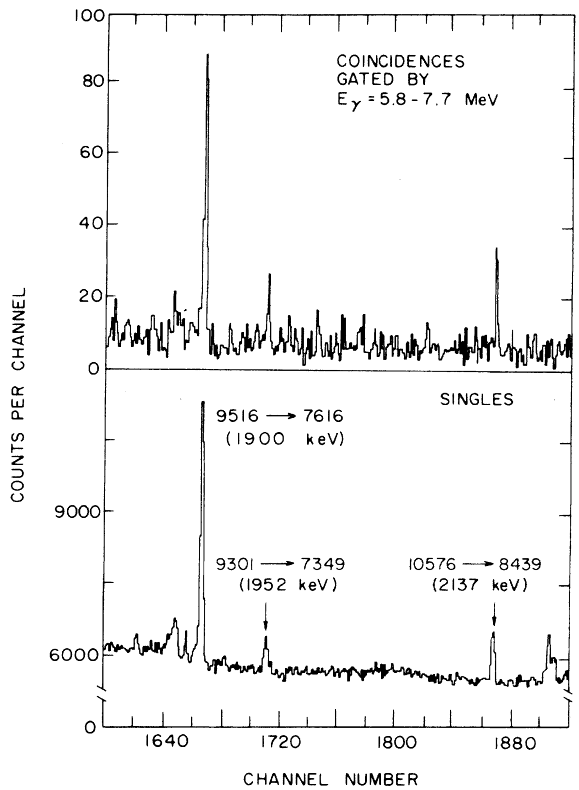

FIG. 4. Partial spectra illustrating the $\gamma\gamma$-coincidence data from ^{24}Al$(\beta^+)^{24}$Mg. Peaks are labeled in the singles spectrum (lower) by the levels (in keV) between which the corresponding transitions occur and by their energies. The three peaks which are labeled occur in both the coincidence and singles spectra as is expected from their assignments.

III. DISCUSSION

A. β^+ decay

The experimental logft values of Table III are compared to theoretical predictions of Kelvin *et al.*[4] in Table IV, which also includes β^+-delayed α emission results of Torgerson, Oakey, and Macfarlane.[26] A comparison between these predictions and experiment was made previously by Kelvin *et al.*[4] The small changes between our results and previous results do not affect this comparison for the states with $E_x<9.5$ MeV. Our highly tentative association of predicted 4^+ states with experimental states for $E_x>10$ MeV is different from that of Kelvin *et al.*

Kelvin *et al.*[4] did not publish predictions for ^{24}Al(β^+) decay to the $J^\pi=5^+$ states of ^{24}Mg. This is unfortunate since a comparison of the predictions to experiment for the ^{24}Mg 7812- and 10 576-keV levels would be especially interesting in view of the great difference in their logft values.

As noted by Kelvin *et al.*, the general agreement is not very good and neither is the agreement between the predictions for the two different interactions. The agreement is expected to be best for the large matrix elements (small logft) and this tends to be true. However, there are only two branches which we regard as strong (log$ft<4$) and only two of medium strengh ($4\leq\log ft\leq5$). The

TABLE IV. Comparison of the logft values of Table III with the predictions of Kelvin *et al*. (Ref. 4).

E_f (keV)	J_{fn}[a]	logft Expt.	CWC[b]	PW[c]
4123	4_1	6.13	5.86	5.92
5235	3_1	6.59	6.09	5.88
6010	4_2	6.43	5.94	5.63
7812	5_1	≥7.19		
8439	4_3	3.93	3.92	3.95
9301	4_4	4.82	4.27	4.73
9456	3_2	>6.63		
9516	4_5 ($T=1$)	3.510	3.512	
10328	4_6		5.16	7.72
10576	5_2	4.50		
(10580)[d]	4_7	>5.4	4.10	4.82
10821	4_8	5.08	5.15	
11217	4_9	6.6	6.54	
11694	4_{10}	5.2	7.56	

[a] For $E_x \leq 9516$ keV the association of shell-model states with experiment is that of Kelvin *et al*. (Ref. 4). The assumption that the 10576-keV level is the 5_2^+ state follows Fifield *et al*. (Ref. 11). The associations for the remaining states for $E_x > 10$ MeV are highly speculative. The β^+ decay to the 11217- and 11694-keV levels was observed in the β^+-delayed α emission studies of Torgerson *et al*. (Ref. 26).

[b] The Chung-Wildenthal interaction (Ref. 6) with added effective Coulomb terms.

[c] The Preedom-Wildenthal interaction (Ref. 5).

[d] Reference 8.

comparison for the superallowed $\Delta T = 0$ decay to the analog of the ^{24}Al ground state at 9516 keV in ^{24}Mg is interesting. The ft value was calculated from[4]

$$ft = \frac{6250}{\langle 1\rangle^2 + 1.51\langle\sigma\rangle^2}, \quad (3)$$

where $\langle 1\rangle$ and $\langle\sigma\rangle$ are shorthand for the Fermi and Gamow-Teller matrix elements, respectively. For $T, T_z = 1, -1 \rightarrow T, T_z = 1, 0$, $\langle 1\rangle^2 = 2$ and for $\langle\sigma\rangle^2 = 0$, Eq. (3) gives log$ft = 3.495$. The fact that the predicted logft value of Kelvin *et al*. is larger than this value implies that their calculation gives a departure from the isospin-symmetry prediction for $\langle 1\rangle^2$, even if $\langle\sigma\rangle^2 = 0$, which amounts to at least 4% (larger if $\langle\sigma\rangle^2$ is not zero). Our experimental result supports this prediction indicating ≥ 4% isospin impurity in the ^{24}Mg 4^+, $T=1$ state at 9516 keV. The very small value of $\langle\sigma\rangle^2$ implied for this decay is very interesting; a possible explanation for it is offered in the next subsection.

Aside from the superallowed decay, that to the third 4^+ state at 8439 keV is dominant. The strength of this transition and the strong 9516 → 8439 $M1$ transition (Table II) led Détraz[13] to propose the 8439-keV level as the antianalog of the 9516-keV state. In the next subsection we

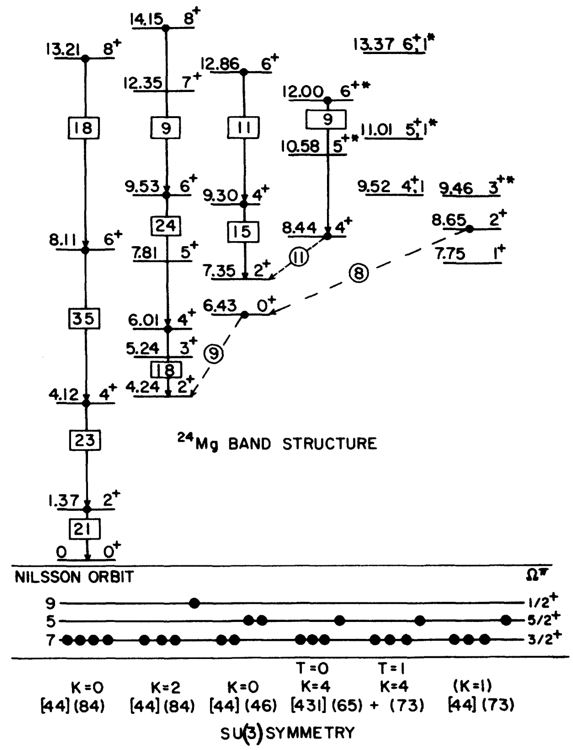

FIG. 5. Band structure of ^{24}Mg. All the levels of ^{24}Mg for $E_x < 9$ MeV and associated levels for $E_x > 9$ MeV are grouped into bands. The bands are labeled by what is proposed to be the simplest (or one of the simplest) occupations in a Nilsson scheme and by the dominant term in a SU(3) basis as discussed in the text. Uncertain spin-parity assignments are denoted with an asterisk. The numbers in squares or circles are $E2$ transition strengths in Weisskopf units.

shall propose an ordering of the even-parity levels of ^{24}Mg with $E_x < 9$ MeV into bands and it will be seen that the concept of an antianalog to the $J^\pi, T = 4^+, 1$ 9516-keV level of ^{24}Mg has a sound basis.

B. The band structure of ^{24}Mg

1. Classification in the Nilsson scheme

The unified model[27] based largely on the work of Nilsson[28] provides a convenient framework for the classification of the levels of ^{24}Mg into rotational bands. Such a classification is attempted in Fig. 5 which includes all even-parity levels of ^{24}Mg for $E_x < 9$ MeV. The levels are grouped into bands which are labeled by what is expected to be the dominant (or at least a significant) Nilsson level occupancy. We consider eight nucleons distributed among the Nilsson orbits, (#) 6, 7, 5, 9, 11, and 8. For ^{24}Mg we expect a deformation of $\beta \simeq 0.4$ and a spin-orbit parameter κ of ~0.08.[29] For these parameters the ordering of the orbits in energy is as given above with #5 and #9 essentially degenerate (in ^{25}Mg-^{25}Al the $\frac{5}{2}^+$-$\frac{1}{2}^+$ splitting is ~0.5 MeV indicating that the #5 orbit lies below the #9 orbit by this amount).

The Nilsson occupancies for the lowest two bands are the obvious dominant ones. For the excited $K=0$ band there are many possibilities all involving nucleons in the #5, #8, or #11 orbits. The simplest is shown—it is not necessarily dominant.

Daum[29] has considered the conjugate nuclei ^{24}Na-^{24}Al in the Nilsson model. He assigned the $J^\pi = 4^+$, $T=1$ ^{24}Al ground state to the $K=4$ band based on $(\#7)^3\#5$ as shown in Fig. 5. For this Nilsson occupancy in ^{24}Mg the nucleon in orbit #5 has an equal probability of being a neutron or a proton. Thus the $T=1$ band has an antianalog $T=0$ band and this we assign to the band built on the 8.44-MeV level.

Again for the $K=1$ band there are many possibilities. One of the simplest is obtained from the $(\#7)^3\#5$ configuration which can generate two $K=4$ bands ($T=0$ and $T=1$) from $\Omega_1+\Omega_2=\frac{5}{2}+\frac{3}{2}=4$ and also two $K=1$ bands from $\Omega_1-\Omega_2=\frac{5}{2}-\frac{3}{2}=1$.

2. Classification into SU(3) symmetries

An alternate description of the band structure of ^{24}Mg is in terms of SU(3) symmetries.[30-32] Neither the Nilsson nor the SU(3) description is expected to be rigorous and it is enlightening to consider both. The bands of Fig. 5 are labeled by the expected dominant SU(3) symmetry. The theoretical evidence for the assignments to the lowest $K=0$ and $K=2$ bands was discussed fully previously.[1,2] The evidence for the assignment of [44](46) to the excited $K=0$ band is the calculation of Wathne and Engeland.[7] The remaining assignments are based on energetics and the connection between the Nilsson and SU(3) models.[30-32]

The connection between the SU(3) classification and the Nilsson occupancies is given in Table V for all the symmetries assigned in Fig. 5 except [431](73) which is quite complicated. The connection is for the $\beta\to\infty$ asymptotic limit or equivalently for no spin-orbit force and no $1d$-$2s$ splitting. The [44] bands of Fig. 5 all have $T=0$; $T=1$ is not allowed. For [431], $T=0$ and $T=1$ are both allowed. Actually there is a multitude of $J^\pi=4^+$ states (each with its band) in [431](65) and [431](73). These can be specified by the quantum numbers K_s, K_L. The $K_s=1$, $K_L=3$ states, both $T=0$ and $T=1$, are expected to lie lowest[31] and this is the state we consider in Table V.

3. Evidence for the band assignments

The lowest $K=0$ and $K=2$ bands have been fully discussed in the literature.[1] The main evidence for the band structure is found in the strong intraband $B(E2)$ values and the weak interband $B(E2)$ values. The intraband $B(E2)$ values [in W.u. (Ref. 10)] are shown in the square boxes in Fig. 5. They are taken from the compilations of Endt[2,33] or from the ^{20}Ne$(\alpha,\gamma)^{24}$Mg work of Fifield *et al.*[11]

The nuclear spectroscopy of the $T=1$ levels of mass 24 is very poorly known. For instance, the analogs of the ^{24}Mg 11.01- and 13.37-MeV levels have not been established in ^{24}Ne or ^{24}Al and the spin-parity assignments of these levels have not been determined. However, the values suggested by Fifield *et al.*[11] and shown in Fig. 5 are probably

TABLE V. Nilsson orbit occupancies for some SU(3) symmetries.

[44] (84)

	$\#7^4$	$\#7^3$ #9	$\#7^2$ $\#9^2$	#7 $\#9^3$	$\#9^4$
$K=0$	$\frac{1}{6}$		$\frac{2}{3}$		$\frac{1}{6}$
$K=2$		$\frac{1}{2}$		$\frac{1}{2}$	

[44] (46)

	$(\#6)^4$ $(\#7, \#9)^2$ $(\#5, \#8, \#11)^2$	$(\#6)^3$ $(\#7, \#9)^4$ (#5, #8, #11)
$K=0$	$\frac{1}{7}$	$\frac{6}{7}$

[431] (65)

	$\#7^2$ #9 #11	$\#9^2$ #7 #5	$\#7^3$ #5
$K=4$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{1}{5}$

[44] (73)

	$(\#6)^4$ $(\#7, \#9)^3$ (#5, #8, #11)	$(\#6)^3$ $(\#7, \#9)^5$
$K=1$	$\frac{10}{13}$	$\frac{3}{13}$

correct. No intraband $B(E2)$ values are known for the $K=4$, $T=1$ band since these levels decay predominantly by $\Delta T=1$ dipole transitions.

Granting that the 9.52-MeV level is the bandhead of a $K=4$, $T=1$ band, the main arguments for the $K=4$, $T=0$ band come from ^{24}Al(β^+)^{24}Mg. The two strongest Gamow-Teller ^{24}Al β^+ decays are to the 8.44- and 10.58-MeV levels and since Gamow-Teller β decay has a $\Delta K \leq 1$ selection rule it is natural to assign $K=4$ to these two levels. The suggested spin-parity assignments shown in Fig. 5 for the 10.58- and 12.00-MeV levels are also from Fifield *et al.*[11]

It is instructive to consider the SU(3) predictions for the β^+ decay even though it is not expected that they will be obeyed in detail. For the SU(3) band structure of Fig. 5 the only allowed ^{24}Al β^+ transitions are to the 4^+ and 5^+ levels of the $K=4$ bands. It is pleasing, then, to note that the transitions to three such levels are the strongest observed (see Table IV).

In more detail the Gamow-Teller matrix element vanishes identically between states of the supermultiplet [431], with $K_s=1$, $K_L=3$, thus the very small value for this quantity which is discussed in the previous subsection has a natural explanation in the framework of the SU(3) model.

In the same SU(3) model the predicted logft values for decay to the 4^+ and 5^+ $T=0$ levels of the $K=4$ band are 3.24 and 4.16, respectively. The calculation was carried out for [431](73) SU(3) wave functions. The result for [431](65) is very closely the same and since the amplitudes for these two symmetries add incoherently the result is very insensitive to the weighting of these two symmetries in the $K=4$ bands. More important are the possible deviations from the assumption $K_s=1$, $K_L=3$. Thus, we expect that the logft values quoted are approximate lower limits and the fact that the ratio of the two is given approximately correctly is perhaps all that can be expected.

The evidence for the grouping of the levels at 7.75, 8.65, and 9.46 MeV into a $K=1$, $T=0$ band is highly speculative. However, these are the only levels remaining after the other band assignments of Fig. 5 are made, thus the spin sequence is suggestive of this band structure. The spin-parity of the 9.46-MeV level has not been determined[2]; the 3^+ assignment is suggested by comparison to the predictions of Kelvin *et al.*[4]

The suggested grouping of the ^{24}Mg levels at 6.43, 7.35, 9.30, and 12.86 MeV into a $K=0$, $T=0$ band is based mainly on the large $B(E2)$ values shown in Fig. 5 for the $6^+\to4^+$ and $4^+\to2^+$ transitions and the theoretical evidence discussed in Sec. I. The $6^+\to4^+$ strength is from Fifield *et al.*[11] while the $4^+\to2^+$ strength is obtained from the present branching ratios (Table II) and previous lifetime measurements. As an aid in this discussion, the known $B(E2)$ values for the decay of the ^{24}Mg 8.44-, 9.30-, and 9.52-MeV levels are collected in Table VI. These $B(E2)$ values result from the branching ratios of Table II and the listed mean lives. The mean life listed for the 9.52-MeV level is the adopted value of Endt and van der Leun.[2] For the 8.44- and 9.30-MeV levels the only published measurement is that of Meyer *et al.*[34] For the 2nd through 6th excited states of ^{24}Mg, the mean life results of Meyer *et al.* average $\sim\frac{1}{2}$ of the adopted value of Endt and van der Leun.[2] Thus we arbitraily increase by a factor of 2 the mean lives quoted by Meyer *et al.* for the 8.44- and 9.30-MeV levels. This we do for purposes of present discussion but we note the importance of additional lifetime measurements for these levels.

A perusal of Table VI yields the interesting fact that only two $B(E2)$ values are enhanced and both are $4^+\to2^+$ transitions to the 7.35-MeV level. That from the 9.30-MeV level gives us the evidence for a $K=0$ band, while that from the 8.44-MeV level clearly indicates mixing between the $K=0$ and $K=4$ 4^+ levels at 8.44 and 9.30 MeV, i.e., $E2$ transitions have a $\Delta K \leq 2$ selection rule. The relatively strong

TABLE VI. Some $E2$ decays for $J^\pi=4^+$ levels of ^{24}Mg.

E_i (keV)	E_f (keV)	Transition	$B(E2)^a$ (W.u.)	$\tau_i{}^b$ (fs)
8439	1369	$4_3^+\to2_1^+$	0.29	26±10
	4123	$4_3^+\to4_1^+$	≤1.1	
	4238	$4_3^+\to2_2^+$	0.36	
	5235	$4_3^+\to3_1^+$	≤1.1	
	6010	$4_3^+\to4_2^+$	≤1.1	
	7349	$4_3^+\to2_3^+$	11±5	
9301	1369	$4_4^+\to2_1^+$	0.17	20±8
	4123	$4_4^+\to4_1^+$	≤1.1	
	4238	$4_4^+\to2_2^+$	0.04	
	7349	$4_4^+\to2_3^+$	15±6	
9516	1369	$4_5^+\to2_1^+$	$<1.7\times10^{-4}$	25±10
	4238	$4_5^+\to2_2^+$	$<1.3\times10^{-2}$	
	7349	$4_5^+\to2_3^+$	$<1.1\times10^{-1}$	

[a] For the $\Delta J<2$ transitions, pure $E2$ was assumed, hence the limits. The uncertainties are derivable from those on the branching ratios of Table I and the mean lifetimes of column 5.

[b] For the 8439- and 9301-keV levels the mean lives are twice those of Ref. 34 as discussed in the text. The value for the 9516-keV level is from Ref. 2.

β^+ decay of ^{24}Al to the 9.30-MeV level is further evidence of this mixing.

Other interband transitions in ^{24}Mg are present, for instance, the relatively strong 8.65→6.43 $E2$ transition shown in Fig. 5 and the decay of the 6.43-MeV level.[2] These remind us not to take the band structure shown in Fig. 4 too literally. The $E2$ decay of the 9.52-MeV level also listed in Table VI indicates sizable inhibition which goes well beyond that expected from the $\Delta T=1$ selection rule.[35] The $\Delta K \leq 2$ selection rule provides an explanation for the weak decays to the $K=0$ levels but not for the decay to the $K=2$ 4238-keV level.

IV. SUMMARY

The present study of ^{24}Al(β^+)^{24}Mg yielded 44 ^{24}Mg γ transitions compared to the 19 previously reported.[13] Three β^+ branches were observed in addition to the six previously reported. One of these—to the 10 576-keV state—has the third lowest observed ^{24}Al logft value. In addition to this branch, the major new information obtained is the observation of 8.44→7.38 and 9.30→7.38 $E2$ transitions which are rather strongly collective. Because the 12.86→9.30 $E2$ transition is also strong we suggest placement of the ^{24}Mg levels at 7.38, 9.30, and 12.86 MeV in an excited $K=0$ band built on the 0^+ 6.43-MeV level. However, the rather strong 8.44→7.38 $E2$ transition indicates rather strong interband mixing.

Both $K=4$, $T=1$ and $K=4$, $T=0$ bands are proposed. The β^+ decay matrix elements connecting the 4^+ ^{24}Al ground state ($K=4$, $T=1$) with the $K=4$, $T=1$ 4^+ level at 9.52 MeV and the 4^+ and (5^+) $K=4$, $T=0$ levels at 8.44 and 10.58 MeV are calculated in the extreme assumption that all have the SU(3) symmetry [431](65) with $K_s=1$, $K_L=0$. The calculation provides an explanation for the small matrix element of $\langle\sigma\rangle^2$ in the superallowed decay and predicts strong transitions to the $K=4$, $T=0$ band with the ratio of decays to the 4^+ and 5^+ states in rough conformity to those for the 8.44- and 10.58-MeV levels.

The present discussion points up the need for more accurate lifetime measurements in ^{24}Mg and for additional spin-parity determinations—especially for those marked by an asterisk in Fig. 5. An expansion of the large basis shell-model wave functions[3-6] in terms of SU(3) wave functions would be of great interest.

Note added in proof. J. D. Garrett, H. T. Fortune, R. Middleton, and W. Scholz [Phys. Rev. C 18, 2032 (1978)] have proposed a band structure for ^{24}Mg similar to that given in Fig. 5. K. Kumar [International Conference on the Structure of Medium-Heavy Nuclei, Rhodes, Greece, 1979, Inst. Phys. Conf. Ser. No. 49, 169 (1980)] has calculated results for the even J, even parity, $T=0$ band structure of ^{24}Mg using the dynamic deformation theory. Kumar (unpublished) successfully predicts a $K=0$ band (β vibration, band head at 6.01 MeV) which can be associated with the $K=0$ band commencing at 6.43 MeV in Fig. 5. He calculates considerable mixing of this band with the lower two bands and with the lowest $K=4$ band also in agreement with results summarized in Fig. 5. However, the predicted β band has a 4^+-0^+ spacing of 5.5 MeV as opposed to 2.87 MeV in Fig. 5 and a close-lying second $K=0$ band ($\gamma\gamma$-vibration, band head 6.52 MeV) is predicted in disagreement with experiment (i.e., no 0^+ states other than those of Fig. 5 are known below 9 MeV in ^{24}Mg).

ACKNOWLEDGMENTS

We wish to thank I. F. Wright, who suggested this study, for discussions and communications concerning it. D. J. Millener guided us in the assignments of SU(3) symmetries. This research was supported by the U. S. Department of Energy, Division of Basic Energy Sciences, under Contract No. DE-AC02-76CH00016.

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