

Phenomenology of continuum angular distributions. II. Griffin preequilibrium model

C. Kalbach

Triangle Universities Nuclear Laboratory, Duke University, Durham, North Carolina

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The Griffin or exciton model for preequilibrium nuclear reactions has been extended so that particle unbound states in the equilibrating system are distinguished from bound states. This permits the preequilibrium cross section to be divided into components due to multistep direct and multistep compound processes for calculation of emitted particle angular distributions. Preequilibrium models traditionally assume that all states of a given particle-hole number are equally likely whereas present work indicates that unbound states are preferentially populated. Nevertheless, the emission energy spectra calculated are not significantly changed from earlier versions of the model.

NUCLEAR REACTIONS Extended Griffin preequilibrium statistical model to calculate multistep direct and multistep compound cross sections separately. Tested equal *a priori* probabilities assumption.

I. INTRODUCTION

The aim of this and the preceding paper is to provide a simple general means of calculating unmeasured angular distributions for a wide variety of continuum reactions in applied areas such as reactor shielding design or materials radiation damage studies. A phenomenological approach has been taken, since proposed theoretical approaches disagree as to the important reaction parameters and involve serious approximations and/or computational complexities.

In a detailed quantum mechanical treatment of preequilibrium statistical processes, Feshbach, Kerman, and Koonin¹ have divided the reaction cross section into two parts, with the first exhibiting forward peaked angular distributions and the second symmetric angular distributions. They are termed statistical multistep direct (MSD) and statistical multistep compound (MSC), respectively. These basic ideas have been utilized in a study of the phenomenology of continuum angular distributions as discussed in Ref. 2, hereafter referred to as I. The present paper describes the modification of these ideas, for use in the more classical Griffin or exciton model,³⁻¹⁷ and their implementation. In all of this work the Griffin model is not used to actually calculate angular distributions, but only to estimate that part of the cross section which might be expected to show forward peaking. The parametrized systematics observed in the data in I is to be used to calculate the angular distributions.

In implementing the ideas of MSD and MSC processes in the Griffin model, it is necessary to divide the classes of states into bound and unbound subsets. This also permits the partial testing of the hypothesis of equal *a priori* occupation probabilities, one of the model's basic assumptions.

This hypothesis, which has been carried over into the hybrid¹⁸ and geometry dependent hybrid¹⁹ models, states that during equilibration all of the states in a given class (i.e., with a given number of particle and hole degrees of freedom) are equally likely to be populated. In the MSD/MSC formalism it is possible to compare the occupation probabilities of the bound and unbound subsets of each class.

In the next section the ideas of MSD and MSC are discussed in more detail, particularly with regard to their application to the Griffin model. Section III describes the actual extensions to the Griffin model. Section IV gives the results of numerical calculations, and Sec. V contains the summary and conclusions.

II. MSD AND MSC PROCESSES

Feshbach, Kerman, and Koonin¹ define statistical multistep direct processes as those in which the system passes through a series of configurations or stages, each of which contains a continuum particle. Statistical multistep compound processes are those in which all of the particles are bound at each stage of the reaction. The MSD processes are expected to show interference effects, while the MSC processes are not.

Some basic differences between their quantum mechanical theory and the Griffin model make it necessary to modify these ideas somewhat. In the Griffin model the bound and unbound states are treated on the same basis, and we have assumed that both MSD and MSC cross sections are calculated. In a quantum mechanical sense it may seem troublesome to treat the configurations containing an energetic continuum particle as well defined states of the composite system, but the Griffin model is basically classical in its approach and

seems to do quite well at reproducing angle integrated spectra with only one universal parameter.

In the Griffin model it is assumed that only states with at least one particle in the continuum are directly coupled to the exit channels, and that each such state couples only to exit channels corresponding to emission of (one of) its continuum particle(s). This means that for emission to occur from a bound state, it must first undergo a two-body interaction to an unbound state. As a result, it has been necessary to modify and extend the definition of MSC. A system cannot, strictly speaking, pass only through bound configurations, since emission must occur from an unbound one. Further, there is the additional class of reactions in which the system may have a continuum particle for several stages and then become totally bound for additional stages until a two-body interaction again makes the system unbound so that emission can occur. It is assumed here that any time the system passes through a totally bound configuration, all interference effects are lost and the process is MSC. More physically speaking, one might say that when a state is totally bound, the "leading particle" approximation of Mantzouranis *et al.*²⁰ is no longer valid and the system has lost its "memory" of the initial projectile direction. The original definition of MSD is retained.

III. THEORY

In the Griffin model, the composite nucleus is assumed to be formed in a simple class of states specified by its numbers of particle and hole degrees of freedom, $(p, h) = (p_0, h_0)$. Equilibration occurs through a series of residual two-body interactions which take the system to more complex configurations. Particle emission is allowed from all of the classes of states specified by different numbers of degrees of freedom. The quantities needed in describing a nuclear reaction are the state densities in the composite and residual nuclei, the two-body interaction rates, and the particle emission rates for each class of states.

The starting point for the present work is the formalism described in Refs. 17 and 21 and included in the computer code PRECO-B.²² In extending this formalism for the separate calculation of MSD and MSC cross sections, it is always going to be assumed that the available excitation energy E in the intermediate nucleus is large enough for there to be unbound states.

A. Particle-hole state densities

The general expression for the density of states specified by p , h , and E is

$$\omega(p, h, E) = \frac{g_0^n [E - A_{p,h}]^{n-1}}{p! h! (n-1)!}, \quad (1)$$

$$A_{p,h} = \frac{p_m^2}{g_0} - \frac{p^2 + h^2 + n}{4g_0}, \quad (2)$$

where the exciton number is given by $n = p + h$, g_0 is the density of the equally spaced single particle states, and $p_m = \text{maximum}(p, h)$. The quantity p_m^2/g_0 is the minimum energy for that class of states required by the Pauli exclusion principle.

The quantity to be derived is the density of states specified by p , h , and E which have at least one unbound or continuum particle. Let S denote the minimum excitation energy which a particle must have in order to be unbound. The density of unbound states, $\omega^{(u)}(p, h, E)$, is therefore given by the full state density of (1) multiplied by the probability that the highest energy particle degree of freedom will have an excitation energy at least equal to S . Thus we get

$$\begin{aligned} \omega^{(u)}(p, h, E) &= \frac{\int_S^{E-A_{p-1,h}} g_0 de \omega(p-1, h, E-e)}{\int_{A_{p,h}-A_{p-1,h}}^{E-A_{p-1,h}} g_0 de \omega(p-1, h, E-e)} \\ &\quad \times \omega(p, h, E) \\ &= \frac{g_0^n [E - A_{p-1,h} - S]^{n-1}}{p! h! (n-1)!}. \end{aligned} \quad (3)$$

This assumes that the remaining degrees of freedom can carry any remaining excitation energy of the system. In fact, however, the hole degrees of freedom are limited by the depth of the potential well, and the remaining particle degrees of freedom must have less energy than the particle of interest. Thus for the denominator of (3), the lower limit of integration should be the minimum energy e_m of the maximum energy particle,

$$e_m(p) = \text{maximum}[(E - hV)/p, A_{p,h} - A_{p-1,h}], \quad (4)$$

where V is the depth of the potential well. Similarly, for the numerator of (3), the lower limit of integration must be changed from S to

$$e_{mu}(p) = \text{maximum}[e_m(p), S]. \quad (5)$$

Thus we get

$$\begin{aligned} \omega^{(u)}(p, h, E) &= \frac{\int_{e_{mu}(p)}^{E-A_{p-1,h}} g_0 de \omega(p-1, h, E-e)}{\int_{e_m(p)}^{E-A_{p-1,h}} g_0 de \omega(p-1, h, E-e)} \omega(p, h, E) \\ &= \frac{g_0^n [E - A_{p-1,h} - e_{mu}(p)]^{n-1}}{p! h! (n-1)!} \\ &\quad \times \left[\frac{E - A_{p,h}}{E - A_{p-1,h} - e_m(p)} \right]^{n-1}. \end{aligned} \quad (6)$$

Clearly there are three distinct cases here. In the first, $e_{mu}(p) = S$ and $e_m(p) = A_{p,h} - A_{p-1,h}$ so that (6) reduces to our original formula (3). In the

other extreme, $e_{mu}(p) = e_m(p) \geq S$. In this case all of the states specified by (p, h) are unbound and (6) reduces to (1). Finally there is the intermediate case where $e_{mu}(p) = S$ but $S > e_m(p) > A_{p,h} - A_{p-1,h}$.

The density of bound states specified by $p, h,$ and E is now found by a simple difference:

$$\omega^{(b)}(p, h, E) = \omega(p, h, E) - \omega^{(u)}(p, h, E). \quad (7)$$

B. Corrections and approximations to the particle-hole state densities

In previous work, the various additional corrections to the composite nucleus state densities were made only for the initial states specified by $(p, h) = (p_0, h_0)$. In the present work, it was decided to make the corrections for states with $h = 0, 1,$ and 2 . Typically $h_0 = 0$ or 1 .

The finite well depth correction to the overall state densities of (1) is contained in an extra factor f given by

$$f_{mu}(p) = 1 - h \left[\frac{E - e_{mu}(p) - V}{E - e_{mu}(p)} \right]^{n-1}, \quad \text{for } E - e_{mu}(p) > V \text{ and } h \leq 2$$

$$= 1, \quad \text{for } E - e_{mu}(p) \leq V \text{ or } h > 2 \quad (11)$$

$$f_m(p) = 1 - h \left[\frac{E - e_m(p) - V}{E - e_m(p)} \right]^{n-1}, \quad \text{for } E - e_m(p) > V \text{ and } h \leq 2$$

$$= 1, \quad \text{for } E - e_m(p) \leq V \text{ or } h > 2 \quad (12)$$

$$g_u(p) = g_0 \left[\frac{p-1}{n-1} \left(\frac{V+(E-S)/n}{V} \right)^{1/2} + \frac{h}{n-1} \left(\frac{V-(E-S)/n}{V} \right)^{1/2} \right], \quad \text{for } h \leq 2$$

$$= g_0, \quad \text{for } h > 2 \quad (13)$$

$$g_p(p) = g_0 \left(\frac{V+S+(E-S)/n}{V} \right)^{1/2}, \quad \text{for } h \leq 2$$

$$= g_0 \left(\frac{V+S}{V} \right)^{1/2}, \quad \text{for } h > 2. \quad (14)$$

In addition the A 's in the correction factors of (6) have been neglected because they are quite small in the states for which the corrections are made. Thus (6) becomes, in practice,

$$\omega^{(u)}(p, h, E) = \frac{g_p(p) g_u^{n-1}(p) [E - A_{p-1,h} - e_{mu}(p)]^{n-1}}{p! h! (n-1)!} F(p), \quad (15)$$

$$F(p) = \frac{f(p) f_{mu}(p)}{f_m(p)} \left[\frac{E}{E - e_m(p)} \right]^{n-1}, \quad (16)$$

$$e_m(p) = \text{maximum}[0, (E - hV)/p], \quad \text{for } h \leq 2$$

$$= 0, \quad \text{for } h > 2, \quad (17)$$

$$e_{mu}(p) = \text{maximum}[e_m(p), S]. \quad (18)$$

$$f(p) = 1 - h \left(\frac{E - V}{E} \right)^{n-1}, \quad \text{for } E > V \text{ and } h \leq 2$$

$$= 1, \quad \text{for } E \leq V \text{ or } h > 2. \quad (8)$$

In addition, the single particle state density has been modified to account for long range deviations from the equispacing model,

$$g(p) = g_0 \left[\frac{p}{n} \left(\frac{V+E/n}{V} \right)^{1/2} + \frac{h}{n} \left(\frac{V-E/n}{V} \right)^{1/2} \right], \quad \text{for } h \leq 2$$

$$= g_0, \quad \text{for } h > 2 \quad (9)$$

so that (1) becomes

$$\omega(p, h, E) = \frac{g^n(p) [E - A_{p,h}]^{n-1}}{p! h! (n-1)!} f(p). \quad (10)$$

Similar corrections can be made to the density of unbound states. In this case we have one finite well depth factor for each of the three types of energy factors, and we use a separate single particle state density g_p , for the unbound particle. The pertinent values are

Thus the quantity $F(p)$ reduces to unity for $h > 2$. The quantities p and h are, of course, related by $p - h = p_0 - h_0$, since it is assumed that particle and hole degrees of freedom are always created together in pairs.

C. Particle emission rates

The emission rates used previously are averaged over all initial states, bound and unbound. For emission of a particle of type b and energy ϵ from a state specified by $p, h,$ and E they are

$$W_b(p, h, \epsilon) d\epsilon = \frac{(2S_b + 1)}{\pi^2 \hbar^3} A_b \epsilon \sigma_b(\epsilon) d\epsilon Q_b(p) \frac{\omega(p - A_b, h, U)}{\omega(p, h, E)}, \quad (19)$$

where $U = E - \epsilon - B_b$, and s_b , $\sigma_b(\epsilon)$, A_b , and B_b are, respectively, the spin, inverse reaction cross section, mass number, and binding energy for a particle of type b . The quantity $Q_b(p)$ is a weighting factor accounting for the distinguishability of protons and neutrons.

In the present case, the emission comes only from unbound states, and all of the inverse process of absorption initially populates unbound states. Accordingly the average rate for emitting a particle of type b and energy ϵ from an unbound state of a given p , h , and E is

$$\begin{aligned} W_b^{(u)}(p, h, \epsilon) d\epsilon &= \frac{(2s_b + 1)}{\pi^2 \hbar^3} A_b \sigma_b(\epsilon) d\epsilon Q_b(p) \frac{\omega(p - A_b, h, U)}{\omega^{(u)}(p, h, E)} \\ &= W_b(p, h, \epsilon) d\epsilon \frac{\omega(p, h, E)}{\omega^{(u)}(p, h, E)}. \end{aligned} \quad (20)$$

The average rate for bound states is, of course, zero.

In the present work, the additional correction for "auxilliary configurations" introduced in Ref. 17 has been dropped since it is important primarily at the equilibrium limit. Only closed form calculations are considered in the MSD/MSC formalism, and equilibrium components are taken from the Weisskopf-Ewing evaporation model.

D. Residual two-body interaction rates

In the Griffin model, three types of residual two-body interactions are possible: the creation and destruction of particle-hole pairs, and the exchange of energy between two degrees of freedom. The rate for these processes are denoted λ_+ , λ_- , and λ_0 , respectively. In most existing formulations of the model, only pair creation and pair annihilation are significant since exciton scattering does not change the class of states. It can,

however, take the system from a bound to an unbound state or vice versa and must be considered in an MSD/MSC formalism.

When the difference between bound and unbound states is considered explicitly, the transition rates depend on the bound or unbound character of the initial and final states. These are designated with the superscripts u and b . Thus $\lambda_+^{(ub)}(p, h, E)$ is the average rate for creating a particle-hole pair in an unbound state specified by p , h , and E and going to a bound final state specified by $p+1$, $h+1$, and E . The transition rates needed are

$$\begin{array}{cccc} \lambda_+^{(uu)} & \lambda_+^{(un)} & \lambda_+^{(bu)} & \lambda_+^{(bb)} \\ & \lambda_0^{(ub)} & \lambda_0^{(bu)} & \\ (\lambda_-^{(uu)}) & \lambda_-^{(ub)} & \lambda_-^{(bu)} & (\lambda_-^{(bb)}) \end{array}$$

where the two quantities in parentheses would be needed in a master equation approach but will not be needed here, since only the simpler closed form approach will be used.

All of the two-body interaction rates used in the Griffin model are assumed to be given by time-dependent perturbation theory and consist of the density of accessible final states multiplied by the quantity $2\pi M^2/\hbar$. Assuming that the same effective mean square matrix element M^2 applies for both bound and unbound states, the derivation of all of the transition rates listed above is equivalent to the derivation of the density of accessible final states. These are denoted in the same way as the transition rates so that, for example, $\lambda_+^{(ub)}(p, h, E) = (2\pi M^2/\hbar) \omega_+^{(ub)}(p, h, E)$.

The general method of derivation is demonstrated for the case of $\omega_+^{(uu)}(p, h, E)$. There are three contributions corresponding to pair excitation by the unbound particle, a bound particle or a hole. Thus we get

$$\begin{aligned} \omega_+^{(uu)}(p, h, E) &= \frac{1}{p} \int_{e_{m_u(p+1)+A_p, h+1-A_{p-1}, h}}^{E-A_{p-1}, h} de \left[\frac{\omega_p^{(u)}(1, 0, e) \omega(p-1, h, E-e)}{\omega^{(u)}(p, h, E)} \right] \omega_p^{(u)}(2, 1, e) \\ &+ \frac{p-1}{p} \int_{e_{m_u(p+1)-A_{p-2}, h}}^{E-A_{p-2}, h-e_{m_u(p)}} de \left[\frac{\omega_p(1, 0, e) \omega^{(u)}(p-1, h, E-e)}{\omega^{(u)}(p, h, E)} \right] \omega_p(2, 1, e) \\ &+ \int_{e_{m_u(p+1)-A_{p-1}, h-1}}^{E-A_{p-1}, h-1-e_{m_u(p)}} de \left[\frac{\omega_p(0, 1, e) \omega^{(u)}(p, h-1, E-e)}{\omega^{(u)}(p, h, E)} \right] \omega_p(1, 2, e). \end{aligned} \quad (21)$$

In each integral the term in brackets represents the probability of having the appropriate active degree of freedom with energy e . The remaining state density is the final state density for the active degree of freedom and the pair it excites. All

of the state densities in (21) have the general form of (10) or (15), but those in the numerator differ somewhat in the application of the various correction factors because we are there dealing with state densities for *parts* of the system. For each

product of state densities in the numerator of a bracketed factor, only one finite well depth correction is applied: the one appropriate to the system as a whole in its initial state, or in this case $F(p)$. Another set of finite well depth corrections is applied to each of the final state densities for the active degree(s) of freedom. Because of the reduced number of energy factors, two quantities analogous to $F(p)$ are defined for use in pair creation and exciton scattering final state densities. They are

$$F_+(p) = \frac{f(p)f_{mu}(p)}{f_m(p)} \left[\frac{E}{E - e_m(p)} \right]^2, \quad (22a)$$

$$F_0(p) = \frac{f(p)f_{mu}(p)}{f_m(p)} \frac{E}{E - e_m(p)}. \quad (22b)$$

(For pair annihilation these corrections are ignored.) Further, in evaluating the A 's in the state

densities for the active degree(s) of freedom, the presence of the degrees of freedom in the inactive core must be considered. This is the meaning of the subscript p on these state densities. Thus the final state density for the active particles and holes in the second integral of (21) is

$$\omega_p(2, 1, e) = \frac{g^3(p+1)[e - A_{p,h+1} + A_{p-2,h}]^2}{2!1!2!} F_+(p+1). \quad (23)$$

The presence of the passive degrees of freedom and their energy requirements according to the Pauli principle are also reflected in the limits of integration in (21). When a change of variable is used in each of the integrals in order to get a factor of the type e^2 for the final state density of the active trio, then they become virtually identical, and one obtains

$$\begin{aligned} \omega_p^{(uu)}(p, h, E) &= \frac{[g_p(p+1) + (p-1+h)g_u(p+1)]g_u^2(p+1)(n-1)F_+(p+1)}{4[E - A_{p-1,h} - e_{mu}(p)]^{n-1}} \\ &\quad \times \int_0^{E - A_{p,h+1} - e_{mu}(p+1)} de' (e')^2 [E - A_{p,h+1} - e_{mu}(p+1) - e']^{n-2} \\ &= \frac{g_a(p+1)g_u^2(p+1)F_+(p+1)}{2(n+1)} \frac{[E - A_{p,h+1} - e_{mu}(p+1)]^{n-1}}{[E - A_{p-1,h} - e_{mu}(p)]^{n-1}}, \end{aligned} \quad (24)$$

where

$$\begin{aligned} g_a(p+1) &= \frac{n-1}{n} g_u(p+1) + \frac{1}{n} g_p(p+1), \quad \text{for } h \leq 1 \\ &= g_0, \quad \text{for } h > 1. \end{aligned} \quad (25)$$

An additional correction needs to be made to (24). In obtaining the contribution from pair excitation by the unbound particle, it was assumed that only one particle would be unbound. If, however, $E > 2S$, then it is possible for there to be states with more than one unbound particle, and some of the states which would have been calculated as belonging to $\omega_p^{(ub)}(p, h, E)$ actually belong in $\omega_p^{(uu)}(p, h, E)$. To correct this, we use the fraction $m(p)$ of the initial states which are multiply unbound. This fraction of the pair creation interactions which would otherwise have led to bound states is now assumed to lead to unbound states. The density of multiply unbound states can be derived in the same way as the density of all unbound states and represents [neglecting the $A_{p,h}$ which are small when $m(p)$ is large] a fraction given by

$$\begin{aligned} m(p) &= \left[\frac{E - 2e_{mu}(p)}{E - e_{mu}(p)} \right]^{n-1} \left[\frac{E - e_m(p)}{E - 2e_m(p)} \right]^{n-1}, \quad \text{for } E > 2e_{mu}(p) \\ &= 0, \quad \text{for } E \leq 2e_{mu}(p). \end{aligned} \quad (26)$$

The final result for $\omega_p^{(uu)}(p, h, E)$ then becomes

$$\omega_p^{(uu)}(p, h, E) = \frac{g_a(p+1)g_u^2(p+1)F_+(p+1)}{2(n+1)} \frac{x_2^{n+1}(p+1)}{x_2^{n-1}(p)} + \frac{m(p)}{1 - m(p)} \omega_p^{(ub)}(p, h, E), \quad (27)$$

where for simplicity in this and the following equations we have defined the quantities

$$x_1(p) = E - A_{p,h}, \quad (28a)$$

$$x_2(p) = E - A_{p-1,h} - e_{mu}(p), \quad (28b)$$

$$x_3(p) = \text{maximum}[E - A_{p-2,h} - 2e_{mu}(p), 0]. \quad (28c)$$

In evaluating these quantities it must be remembered that a one unit change in p implies a corresponding one unit change in h so that, for instance, $x_1(p+1) = E - A_{p+1, h+1}$.

The other final state densities for the two-body interactions are derived in an analogous way. A few simplifications have been made in the g values and in the A subscripts, and the f and F values have been neglected in deriving pair annihilation state densities.

The remaining final state densities for pair creation are

$$\omega_+^{(ub)}(p, h, E) = [1 - m(p)] \frac{1}{2n(n+1)} \left(g^3(p+1)f(p+1) \left\{ x_1(p+1)x_2(p) + \frac{n^2}{2} [x_1(p+1) - x_2(p)]^2 + \frac{n}{2} [x_1^2(p+1) - x_2^2(p)] \right\} \right. \\ \left. - g_p(p+1)g_u^2(p+1) \frac{x_2^{n+1}(p+1)}{x_2^{n-1}(p)} F_+(p+1) \right), \quad (29)$$

$$\omega_+^{(bu)}(p, h, E) = \frac{g_p(p+1)g_u^2(p+1)h}{2n(n+1)} F_+(p+1) \frac{x_2^{n+1}(p+1) - x_3^{n+1}(p+1)F'(p) - (n^2/2)(V-S)^2[G_1^{n-1} - G_2^{n-1}F'(p)]}{x_1^{n-1}(p)f(p) - x_2^{n-1}(p)F(p)}, \quad (30)$$

where

$$F'(p) = \{E/[E - e_m(p)]\}^{n-1}, \quad (31)$$

$$G_1 = \text{maximum}(E - V, 0), \quad (32)$$

$$G_2 = \text{maximum}(E - V - S, 0), \quad (33)$$

and finally

$$\omega_+^{(bb)}(p, h, E) = \frac{1}{2n(n+1)} \frac{1}{x_1^{n-1}(p)f(p) - x_2^{n-1}(p)F(p)} \\ \times (ng^3(p+1)x_1^{n+1}(p+1)f(p)f(p+1) - (n-1)g_u^3(p+1)x_2^{n+1}(p+1)F(p)F_+(p+1) \\ - g^3(p+1)x_2^{n-1}(p)F(p)f(p+1)\{(n^2/2)[x_1(p+1) - x_2(p)]^2 + (n/2)[x_1^2(p+1) - x_2^2(p)] + x_1(p+1)x_2(p)\}) \\ - \omega_+^{(bu)}(p, h, E). \quad (34)$$

The final state densities for exciton scattering are

$$\omega_0^{(ub)}(p, h, E) = [1 - m(p)] \frac{g_0^2(p+2h-1)}{2} \times [x_1(p)f(p) - x_2(p)F_0(p)] \quad (35)$$

and

$$\omega_0^{(bu)}(p, h, E) = \frac{p(p-1) + 4ph}{4} g_u(p)g_p(p) [x_1(p)f(p) - x_2(p)F_0(p)] \frac{x_2^{n-1}(p)f(p) - x_3^{n-1}(p)F(p)}{x_1^{n-1}(p)f(p) - x_2^{n-1}(p)F(p)}. \quad (36)$$

The final state densities for pair annihilation are

$$\omega_-^{(ub)}(p, h, E) = [1 - m(p)] g_h \frac{h(h-1)}{2}, \quad (37)$$

with (assuming $V > S$)

$$g_h = g_0 [(V - S)/V]^{1/2} \quad (38)$$

and

$$\omega_-^{(bu)}(p, h, E) = \frac{p(p-1)h}{2} g_p(p-1), \quad \text{for } n \leq 3 \\ = \frac{p(p-1)h(n-1)(n-2)(n-3)g_p(p-1)}{2[x_1^{n-1}(p) - x_2^{n-1}(p)]} x_2^{n-3}(p-1) \left[\frac{x_2^2(p-1)}{n-1} - \frac{2x_2(p-1)x_1(p)}{n-2} + \frac{x_1^2(p)}{n-3} \right] \\ - x_3^{n-3}(p-1) \left[\frac{x_3^2(p-1)}{n-1} - \frac{2x_3(p-1)x_2(p)}{n-2} + \frac{x_2^2(p)}{n-3} \right] - \frac{x_2^{n-1}(p) - x_3^{n-1}(p)}{(n-1)(n-2)(n-3)}, \quad \text{for } n > 3. \quad (39)$$

E. Closed form reaction equations

The closed form approach to preequilibrium calculations relies on the fact that for the simple states, populated early in the reaction, particle-hole pair creation dominates the two-body interactions. Thus the strength of the system is imagined to pass sequentially through configurations of increasing complexity until the most probable class of states at equilibrium (specified by \bar{p} and \bar{h}) is reached. Thus we have

$$(p_0, h_0) - (p_0 + 1, h_0 + 1) - (p_0 + 2, h_0 + 2) - \dots \\ - (\bar{p} - 1, \bar{h} - 1) - (\bar{p}, \bar{h}).$$

At each stage along the way, strength is also allowed to go into particle emission.

In the MSD/MSC formalism, the strength for each class of states specified by (p, h) is divided into the part which is unbound and the part which is bound. These quantities are denoted $S_u(p, h)$ and $S_b(p, h)$, respectively. All particle emission comes from the strength $S_u(p, h)$. In addition, the quantity $S_d(p, h)$ denotes the strength which passes through unbound states of the class (p, h) and which was unbound at all previous stages as well. This is the strength which is responsible for the MSD particle emission.

Initially it is assumed that the unbound states are populated in proportion to their relative state densities so that

$$S_u(p_0, h_0) = S_d(p_0, h_0) = \omega^{(u)}(p_0, h_0, E) / \omega(p_0, h_0, E), \quad (40a)$$

$$S_d(p+1, h+1) = S_d(p, h) \Gamma^{(uu)}(p, h), \quad (43a)$$

$$S_u(p+1, h+1) = S_u(p, h) [\Gamma^{(uu)}(p, h) + \Gamma_0^{(ub)}(p, h) \Gamma^{(bu)}(p, h)] + S_b(p, h) [\Gamma^{(bu)}(p, h) - \Gamma_0^{(bu)}(p, h) \Gamma^{(ub)}(p, h)], \quad (43b)$$

$$S_b(p+1, h+1) = S_b(p, h) [\Gamma^{(bb)}(p, h) + \Gamma_0^{(bu)}(p, h) \Gamma^{(ub)}(p, h)] + S_u(p, h) [\Gamma^{(ub)}(p, h) - \Gamma_0^{(ub)}(p, h) \Gamma^{(bu)}(p, h)]. \quad (43c)$$

The transitions contributing to $S_u(p+1, h+1)$ are shown schematically in Fig. 1. Some of the transition arrows terminate at unbound states specified by (p, h) or $(p-1, h-1)$. These all represent small amounts of strength, and it is assumed that all of this strength will, by pair creation, pass through unbound states specified by $(p+1, h+1)$. This assumption is based on the observation, discussed later, that pair creation tends to populate the same kind of state (bound or unbound) it started from.

In the closed form MSD/MSC formalism, the preequilibrium cross sections are determined from the strength variables. For the reaction $A(a, b)$, the overall energy differential cross sec-

$$S_b(p_0, h_0) = 1 - S_u(p_0, h_0). \quad (40b)$$

The strengths for the more complex states are found from a set of recursion relations involving branching ratios or reduced widths Γ , for different kinds of transitions. The branching ratios are labeled by superscripts in the same way as the transition rates themselves to indicate the bound or unbound character of the initial and final states. Dropping the energy label E , they are given by

$$\Gamma^{(uu)}(p, h) = \lambda_+^{(uu)}(p, h) T_u(p, h), \quad (41a)$$

$$\Gamma^{(ub)}(p, h) = [\lambda_+^{(ub)}(p, h) + \lambda_0^{(ub)}(p, h) \\ + \lambda_-^{(ub)}(p, h)] T_u(p, h), \quad (41b)$$

$$\Gamma_0^{(ub)}(p, h) = \lambda_0^{(ub)}(p, h) T_u(p, h), \quad (41c)$$

with the analogous quantities $\Gamma^{(bb)}$, $\Gamma^{(bu)}$, and $\Gamma_0^{(bu)}$ obtained by interchanging the u and b labels. The quantities T_u and T_b are the total lifetimes of the states in question and are given by

$$T_u(p, h) = \left[\lambda_+^{(uu)}(p, h) + \lambda_+^{(ub)}(p, h) \\ + \lambda_0^{(ub)}(p, h) + \lambda_-^{(ub)}(p, h) \\ + \sum_b \int W_b^{(u)}(p, h, \epsilon) d\epsilon \right]^{-1}, \quad (42a)$$

$$T_b(p, h) = [\lambda_+^{(bb)}(p, h) + \lambda_+^{(bu)}(p, h) \\ + \lambda_0^{(bu)}(p, h) + \lambda_-^{(bu)}(p, h)]^{-1}. \quad (42b)$$

The recursion relations are then given by

tion and the cross sections for MSD and MSC processes are given by

$$\frac{d\sigma}{d\epsilon}(a, b)_{\text{pre}} = \sigma_{\text{CN}} \sum_{p=p_0}^{\bar{p}} S_u(p, h) T_u(p, h) W_b^{(u)}(p, h, \epsilon), \quad (44a)$$

$$\frac{d\sigma}{d\epsilon}(a, b)_{\text{pre-MSD}} = \sigma_{\text{CN}} \sum_{p=p_0}^{\bar{p}} S_d(p, h) T_u(p, h) W_b^{(u)}(p, h, \epsilon), \quad (44b)$$

$$\frac{d\sigma}{d\epsilon}(a, b)_{\text{pre-MSC}} = \frac{d\sigma}{d\epsilon}(a, b)_{\text{pre}} - \frac{d\sigma}{d\epsilon}(a, b)_{\text{pre-MSD}}, \quad (44c)$$

where σ_{CN} is the cross section for forming the

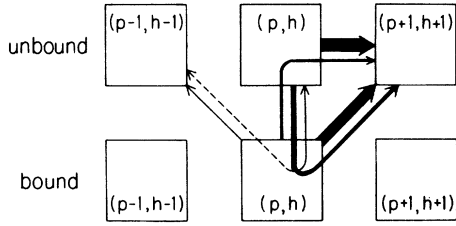


FIG. 1. Schematic diagram of two-body interactions producing strength in unbound states specified by $p + 1$, $h + 1$, and E . The boxes represent different classes of states and the heaviness of the lines indicates roughly the amount of strength involved. All strength shown ending in unbound states denoted (p, h) and $(p - 1, h - 1)$ is assumed to pass by pair creation to the final $(p + 1, h + 1)$ states.

composite nucleus. Any cross section not used in the preequilibrium phase of the reaction is assumed to go into equilibrium emission and to be MSC.

IV. CALCULATIONS AND RESULTS

Calculations were performed with the computer code PRECO. An earlier version, PRECO-B,²² was modified to calculate the MSD and MSC contributions to the preequilibrium cross section separately as described in Sec. III.

A. Input quantities

All input quantities were the same as those employed in using PRECO-B. These include nucleon²³ and alpha particle²⁴ reaction cross sections, and particle binding energies.²⁵ The effective mean square two-body matrix element is given²¹ by

$$\begin{aligned}
 M^2(p) &= \frac{k}{A^3} \frac{n}{E} \left(\frac{E/n}{7 \text{ MeV}} \frac{E/n}{2 \text{ MeV}} \right)^{1/2}, \quad \text{for } E/n < 2 \text{ MeV} \\
 &= \frac{k}{A^3} \frac{n}{E} \left(\frac{E/n}{7 \text{ MeV}} \right)^{1/2}, \quad \text{for } 2 \text{ MeV} \leq E/n < 7 \text{ MeV} \\
 &= \frac{k}{A^3} \frac{n}{E}, \quad \text{for } 7 \text{ MeV} \leq E/n \leq 15 \text{ MeV} \\
 &= \frac{k}{A^3} \frac{n}{E} \left(\frac{15 \text{ MeV}}{E/n} \right)^{1/2}, \quad \text{for } 15 \text{ MeV} < E/n
 \end{aligned}
 \tag{45}$$

where A is the mass number of the system and k is an empirical constant which has been found to be 135 MeV^3 . The depth of the nuclear potential well, V , has, as in previous work, been taken to be 38 MeV , and the initial configuration is $(p_0, h_0) = (A_a + 1, 1)$, where A_a is the mass number of the projectile.

The only new quantity needed is the energy S required for a particle degree of freedom to be

unbound. This effective separation energy is clearly different for each type of emitted particle. Here neutrons, protons, and alpha particles have been considered, and the added requirements of the Coulomb barriers have been taken into account. The quantity S has thus been defined as

$$S = \text{minimum}(B_n, B_p + C_p, B_\alpha + C_\alpha), \tag{46}$$

where B_n , B_p , and B_α are the binding energies of the different particle types while C_p and C_α are the Coulomb barriers taken to be

$$C_b = 0.75 \frac{Z_b Z_B}{A^{1/3}} \text{ MeV}. \tag{47}$$

Here Z_b and Z_B denote the atomic numbers of the emitted particle and residual nucleus, respectively. Other calculations have been run using the average (rather than the minimum) effective separation energy for the three channels. Still others have been made neglecting the Coulomb barriers. In all cases the resulting preequilibrium emission spectra are quite similar, indicating that they are fairly insensitive to the choice of S within physically reasonable limits.

B. State densities and transition rates

Figure 2 shows a comparison of unbound and total state densities for different particle-hole configurations. The calculations are for a ^{55}Co intermediate nucleus at 43.3 MeV of excitation. The two quantities are equal for the $(p, h) = (1, 0)$ states (not plotted), but at higher exciton num-

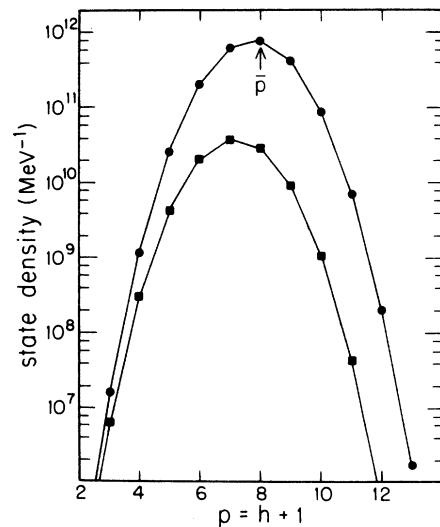


FIG. 2. State densities for a sample system as a function of particle and hole number. The circles denote the total state densities, while the squares show the densities of unbound states only. The calculations are for a ^{55}Co nucleus at 43.3 MeV of excitation.

bers the proportion of unbound states decreases rapidly as the available energy gets divided up among more and more degrees of freedom. The two distributions of states have the same general form, with the unbound state density looking very much like $\omega(p, h, E - S)$. The value of \bar{p} , the most probable particle number at equilibrium, is also indicated.

Figure 3 shows plots of the various two-body interaction rates as a function of particle number up through $p = \bar{p}$. Here two observations may be made:

(i) Pair creation dominates the two-body interactions for states near $p = p_0 = 2$, whereas other rates become comparable as p approaches \bar{p} . This is well known from earlier formulations of the Griffin model.

(ii) For the simpler states near $p = p_0$, it can be seen that $\lambda_+^{(uu)}(p, h) > \lambda_+^{(ub)}(p, h)$ and $\lambda_+^{(bb)}(p, h) > \lambda_+^{(bu)}(p, h)$.

This second observation indicates a preference for unbound states to populate other unbound states during pair creation and for bound states to populate bound states. This trend is seen more clearly in Fig. 4. Here the ratios $\omega_+^{(uu)}(p, h, E) / \omega_+^{(ub)}(p, h, E)$, or equivalently $\lambda_+^{(uu)}(p, h, E) / \lambda_+^{(ub)}(p, h, E)$, and $\omega_+^{(bu)}(p, h, E) / \omega_+^{(bb)}(p, h, E)$ are compared with the relative densities of unbound and bound configurations, $\omega^{(u)}(p+1, h+1, E) / \omega^{(b)}(p+1, h+1, E)$, in the class of states being populated. Pair creation from the $(p, h) = (1, 0)$ states (all of which are unbound) populates the unbound $(2, 1)$ states in proportion to their total

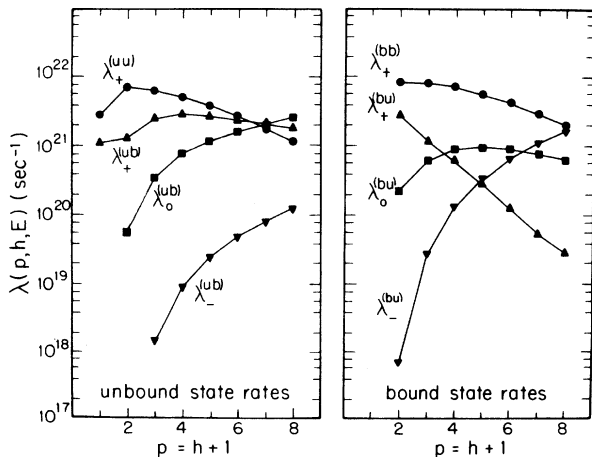


FIG. 3. Average two-body interaction rates in the MSD/MSC formalism as a function of the particle and hole numbers of the initial states in the interactions. (a) is for interactions starting from unbound states, (b) is for bound states. Calculations are for the same sample system as in Fig. 2.

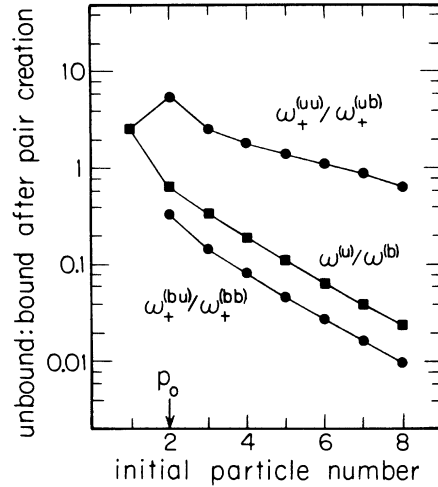


FIG. 4. Ratio of unbound to bound state densities for states reached in pair creation as a function of the particle and hole numbers of the starting configurations. The circles show the ratios for the states actually accessible from a single starting state in the MSD/MSC formalism while the squares show the ratio of the overall unbound and bound state densities for the final class of states. Calculations are for the same sample system as in Fig. 2.

abundance. For more complex configurations, however, unbound states preferentially populate unbound states and bound states preferentially populate bound states.

The explanation of this trend is readily understood. For pair creation to lead from an unbound to a bound state, there must be only one unbound particle and it must do the exciting. Thus only one of the $n = p + h$ excitons can accomplish this. Similarly, for pair creation to lead from a bound to an unbound state, the excitation must be accomplished by a deep hole degree of freedom. All other pair creations from a bound state will produce another bound state.

This preferential tendency for bound states to populate bound states and for unbound states to populate unbound states is important for the question of the equal probabilities assumption.

C. The equal *a priori* probabilities assumption

If the hypothesis of equal occupation probabilities for all states of a given particle-hole class is valid, then for each class of states the ratio $s_u(p, h) / [s_u(p, h) + s_b(p, h)] \equiv s_u(p, h) / s(p, h)$ should be equal to the fraction of those states which are unbound, $\omega^{(u)}(p, h, E) / \omega(p, h, E)$. That this is not the case is seen in Fig. 5. The calculations are begun in the $(2, 1)$ configuration with the equal probabilities condition satisfied. [The results of Fig. 4 imply that it would also be satisfied for

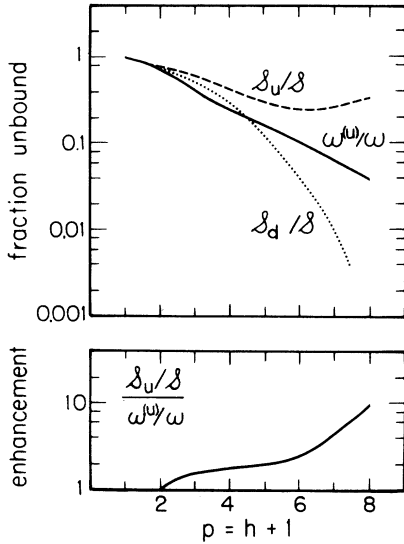


FIG. 5. Fraction of strength in unbound states compared with the fraction of the states which are unbound. Calculations are for the same sample system as in Fig. 2.

(2, 1) states if the starting configuration were taken to be (1, 0).] At higher particle numbers, $S_u(p, h)/S(p, h)$ becomes a factor of 1.5 to 1.9 larger than the fraction of unbound states. Even the fraction of possible MSD strength, which drops steadily with increasing particle number, is higher than the fraction of unbound states for $p=3$ and 4. At still higher particle numbers the relative enhancement of the unbound state occupation probability begins to rise sharply and the probability of being in an unbound state actually increases.

The second abrupt rise, in this case at $p=6$, seems to result from a breakdown in the validity of the closed form method and not, primarily, of the equal probabilities assumption in the physical system. First, as p approaches \bar{p} , pair annihilation rates become significant. More importantly, it has been assumed that the strength represented in Fig. 1 by arrows ending at unbound states specified by $(p-1, h-1)$ and (p, h) will all pass to unbound states specified by $(p+1, h+1)$. This assumption is violated more seriously as p approaches \bar{p} and unbound states become an ever smaller part of the total. More accurate and complicated closed form expressions could be used, but fortunately very little of the particle emission occurs from these states so that the present expressions should be adequate.

The more moderate enhancements of unbound state occupation probabilities at particle numbers just above p_0 , however, do seem to reflect a failure of the equal probabilities assumption. Most of the strength arriving in a particular class

of states does so by pair creation from states with fewer degrees of freedom and a resulting higher fraction of unbound states. Since pair creation from unbound states leads to an enhanced probability for populating other unbound states (and likewise from bound states to bound states), the initially large unbound state occupation probability tends to propagate in the system. Since the effect compounds with each additional pair creation, the ratio of ratios in the lower part of Fig. 5 increases steadily.

The enhanced occupation of the unbound states cannot be removed by using different M^2 values for the bound and unbound starting configurations since it depends on the branching ratios $\Gamma^{(uu)}$ and $\Gamma^{(bu)}$ which are nearly independent of M^2 . Changing the relative sizes of the matrix elements for $\lambda_+^{(uu)}$ and $\lambda_+^{(ub)}$ or alternatively $\lambda_+^{(bu)}$ and $\lambda_+^{(bb)}$ can be used to remove the enhancement, but the changes needed are quite large and without physical basis. For example, it is necessary to change the ratio of the matrix elements for $\lambda_+^{(uu)}$ and $\lambda_+^{(ub)}$ by a factor of 6 in order to remove the enhancement in Fig. 5 for $p=3, 4, 5, 6$. This must be done in such a way as to preserve the branching ratio for particle emission from the initial $p=2$ states. Alternatively, the matrix element for $\lambda_+^{(bb)}$ can be increased by a factor of 2.5. Such changes are unrealistic and well outside the roughly 30% uncertainty in the normalization of M^2 for a given pre-equilibrium model formulation. Thus the enhanced occupation of unbound states seems to be a real effect not foreseen in earlier model calculations.

The practical importance of this violation of the equal occupation probability assumption can only be estimated by examining the calculated particle emission spectra.

D. Particle emission spectra

Figure 6 shows examples of the preequilibrium components of particle emission spectra. The upper part of the figure compares the MSD+MSC contributions from the newer formalism with both the closed form and master equation results in the older formalism. The same two-body matrix element normalization is used. At higher emission energies, the agreement between MSD+MSC and the older results for this and other systems examined is generally comparable to the level of agreement between the closed form and master equation results in the earlier formalism. At lower emission energies the MSD+MSC spectrum shows consistently higher preequilibrium cross sections than the other two. This is due in part to the less stringent use of the equal probabilities

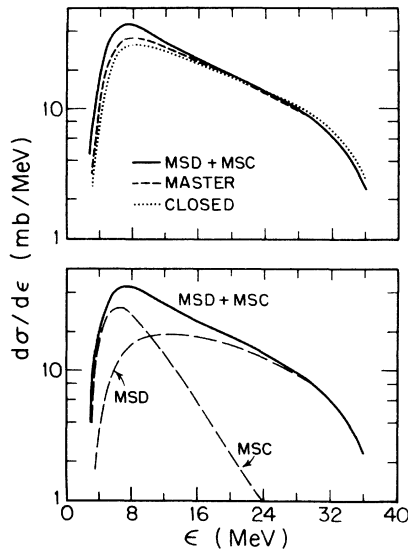


FIG. 6. Comparison of calculated preequilibrium emission spectra. The upper set of curves compares the preequilibrium components calculated in the present formalism with those obtained in the closed form and master equation methods in the old formalism. In the lower set of curves the MSD + MSC preequilibrium spectrum is shown along with the separated MSD and MSC contributions. Calculations are for the same sample system as in Fig. 2 which is now assumed to be formed by proton bombardment of ^{54}Fe . The proton spectra are shown.

assumption and in part to the failure of the closed form equations in the new formalism. In spite of the differences in the preequilibrium components, the total (preequilibrium + equilibrium) energy spectra are virtually unchanged from the earlier formalism. Since the distinction between equilibrium and preequilibrium is somewhat arbitrary in any event, its slight dependence on the validity of various model assumptions should not be disturbing while the constancy of the total spectra should be encouraging.

The reasons for the success of the Griffin model in the presence of violated assumptions can be understood fairly simply. Preequilibrium spectra are dominated by emission from the very simple states where the closed form approach is still valid. The contributions from the states near $p = \bar{p}$ are much smaller and are evaporationlike in shape. Since the total reaction cross section is an input quantity and any cross section not used in the preequilibrium phase is included in the evaporation components, uncertainties in the magnitude of the preequilibrium contributions from these more complex states alter only the preequilibrium/equilibrium division but not the total spectrum.

The equal probabilities assumption, on the other

hand, breaks down well before p reaches \bar{p} , and could be expected to cause disagreement between results from the old and new formalisms. The additional compensating factor seems to be that the average lifetime of an unbound state in the present formalism, (45), is less than the average overall lifetime of a (p, h) state in the old formalism. The principal factor shortening the lifetimes for the unbound states is the higher average particle emission rates given in (20). Other contributions are from exciton scattering and pair annihilation to bound states. The pair creation rates from the old and new formalisms are in good agreement with one another.

E. Relative MSD and MSC contributions

In the new formalism designed to facilitate the study of continuum angular distributions, the division of the cross section into MSD and MSC components is more fundamental than the preequilibrium/equilibrium division. As discussed in (I), the MSD and MSC preequilibrium cross sections considered are not the only contributions. Direct reactions such as nucleon transfer or cluster knockout, which are not included in the model, can contribute to the MSD cross section for reactions involving complex particles. For all reactions the compound nucleus evaporation components must be added to the MSC cross section.

The lower part of Fig. 6 shows the MSD + MSC preequilibrium spectrum broken down into its two components. The MSD part is the largest fraction and totally dominates at high emission energies. The smaller MSC part contributes mainly at the lower emission energies. The MSD contribution is intermediate in spectral shape between the full preequilibrium component and the contribution from the $(p_0, h_0) = (2, 1)$ states. The MSC contribution has a shape which looks almost like an evaporation component but is significantly more abundant in higher energy particles. This also implies that the MSD/MSC division of the cross section will, like the total calculated spectrum, be insensitive to failures in the model assumptions. Figure 5 shows that contributions from states with p near \bar{p} are MSC in nature whether they are equilibrium or preequilibrium so that uncertainties in the preequilibrium fraction due to failure of the closed form equations will not effect the MSD fraction.

V. SUMMARY AND CONCLUSIONS

In this work a revised Griffin (or exciton) model formalism has been developed and included in the computer code PRECO-D.^{2c} It classifies the states of the system by their bound or unbound character

as well as by the numbers of particle and hole degrees of freedom they contain. It thus avoids the extreme equal probabilities assumption for all states with a given number of degrees of freedom, replacing it with a similar assumption applied to the bound and unbound states independently.

Numerical calculations indicate that after the first step of the reaction unbound states are populated with a greater probability than would be predicted from their relative abundance. This represents a failure of the equal probabilities assumption that lies at the heart of this and other preequilibrium models. Although the enhancement of unbound state occupation can amount to 50–90%, the emission spectra are relatively unaffected. This is attributed to the dominance of emission from the initial (p_0, h_0) states, where the assumption is believed to be valid and to the compensating factor of shorter lifetimes for the unbound states in the new formalism.

Possible failure of the closed form equations near $p = \bar{p}$ causes some uncertainty in the division of the cross section into preequilibrium and equilibrium parts, but the total spectra calculated and

the division of the cross section into MSD and MSC components are not effected.

It is found that at high emission energies the preequilibrium cross section is almost completely MSD, while MSC processes dominate at low emission energies close to the evaporation peak (which is also MSC). This is in qualitative agreement with experimental continuum angular distributions which are observed to be most forward peaked at high emission energies. In paper I this description has been shown to be useful in a quantitative analysis of continuum angular distributions.

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