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# Goldberger-Treiman discrepancy and the momentum variation of the pion-nucleon form factor and pion decay constant

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We suggest that the observed 6% Goldberger-Treiman discrepancy is due in part to a 3% variation in the pion-nucleon form factor and in part due to a 3% variation in the pion decay form factor from $q^2 = m_\pi{}^2$ to $q^2 = 0$.

[NUCLEAR REACTIONS Goldberger-Treiman discrepancy, $\pi NN$ form factor, pion decay form factor, chiral symmetry breaking, current quark masses.]

The deviation of the experimental parameters in the Goldberger-Treiman (GT) relation from the $q^2=0$ limiting value of[1] $f_\pi g = m_N g_A(0)$ is of significance for the theory of chiral symmetry and the nuclear physics programs involving the $\pi NN$ vertex and one-pion exchange (OPE) potentials. The various parameters in this relation have the experimental values[2]

$$m_N = \tfrac{1}{2}(m_p + m_n) = 938.9264 \pm 0.0027 \text{ MeV}\,, \tag{1a}$$

$$g_A(0) = 1.254 + 0.007\,, \tag{1b}$$

along with[3]

$$g \equiv g_{\pi^0 pp} = 13.4 \pm 0.1\,, \tag{1c}$$

determined most accurately from low energy $\pi N$ data and

$$f_\pi = 93.2 \pm 0.1 \text{ MeV} \tag{1d}$$

found from $\pi \to \mu\nu$ decay[2] including a 0.1% radiative correction[4] coupled with a Cabibbo angle of $\theta_C \approx 13.2°$.

What is usually done is to compute the GT discrepancy as

$$\Delta = 1 - m_N g_A(0)/f_\pi g = 0.06 \pm 0.01 \tag{2}$$

and then try to explain this difference in various chiral breaking models.[5] Alternatively, one assumes that the GT relation is an identity at $q^2 = 0$ and then tries to blame the complete discrepancy on the variation of the pion form factor,[5,6]

$$g(q^2) = gF_{\pi NN}(q^2)\,, \quad F_{\pi NN}(m_\pi{}^2) = 1\,. \tag{3}$$

Then (2) suggests that $F_{\pi NN}(0) = 1 - \Delta \approx 0.94$.

If the complete discrepancy is indeed due to $F_{\pi NN}(q^2)$, the effect on the nucleon-nucleon interaction and the nuclear many-body problem is not insignificant. The attractive central potential due to OPE is converted to repulsion at short distances (the change of sign is at $r \approx 1.4$ fm) and the tensor part is appreciably weakened there.[7] A 6% change in $F_{\pi NN}(q^2)$ induces a "pionic radius" for a nucleon $\langle r^2\rangle^{1/2} \approx 0.87$ fm, even larger than the charge radius $\approx 0.84$ fm of the proton. It is the former radius, for example, which is presumably of interest in estimates of the density of the transition from neutron star matter to quark matter. The $\pi NN$ vertex with the nucleons on the mass shell and the pion off shell also plays a role in three-nucleon forces, pion condensates, and pion absorption and production from the $2N$ system. Thus the conclusion that $F_{\pi NN}(0) \approx 0.94$ has strong implications for nuclear physics.

However, there are many other independent ways of estimating $F_{\pi NN}(0)$ and now they *all* appear to yield $F_{\pi NN}(0) \approx 0.97$. In particular:

(i) A covariant diagrammatic estimate of $F_{\pi NN}(0)$, saturating the subtracted dispersion relation with $\rho\pi$ and $\sigma\pi$ intermediate states, gives[5] $F_{\pi NN}(0) \approx 0.97$.

(ii) A data analysis substituting $\pi\pi \to N\bar{N}$ $p$-wave[8] and $s$-wave[9] phase shifts in lieu of the above $\rho\pi$ and $\sigma\pi$ intermediate states again finds[9] $F_{\pi NN}(0) \approx 0.97$.

(iii) A recent Regge analysis of charge-exchange $d\sigma/dt|_{np}$ minus $d\sigma/dt|_{\bar{p}p}$ which isolates OPE along the lines of Ref. 7, but includes the pion Regge pole factor $(s/s_0)^{\alpha_\pi(t)}$ evaluated at the appropriate invariant squared energy $s$, claims that[10] $F_{\pi NN}(0) \approx 0.975$.

Other estimates of $F_{\pi NN}(0)$ are more model dependent than (i)–(iii), but they too appear to yield about the same result:

(iv) Assume that the shape of $F_{\pi NN}(q^2)$ is the same as $g_A(q^2)$, at least for low $q^2$, as suggested by the link with the GT relation itself. Then recent determinations of the dipole mass $m_A$ in $g_A(q^2) \propto (1 - q^2/m_A{}^2)^{-2}$ as found from neutrino scattering,[11] threshold pion photoproduction,[12] and electroproduction[13] all yield $m_A \approx 1.3$ GeV. This in turn implies that $F_{\pi NN}(0)=1-2m_\pi^2/m_A^2\approx 0.977$.

(v) Further assume the Veneziano model for the axial current divergence form factor.[14] Then "reasonable" choices of the free parameter $\beta$, which can be checked in the asymptotic region,[10,15] yield $F_{\pi NN}(0)\approx 0.97$.

(vi) Even further assume that the Veneziano model also applies to the $\gamma\pi\pi$ vertex in a Regge analysis of charged-pion photoproduction.[16] Then the value of $\beta$ which fits $d\sigma/dt|_{\gamma p}$ for unpolarized and polarized photons yields $F_{\pi NN}(0)\approx 0.98$.

(vii) The most realistic models of the two-nucleon interaction, incorporating a dispersion-theoretic treatment of $2\pi$ exchange, indicate that a fit to the $D$-wave phase shifts demands[17] $F_{\pi NN}\approx 0.97$. Also one-boson exchange potentials indicate that a value of $F_{\pi NN}(0)\approx 0.97$ fits higher partial wave phase shifts.[18] Although a larger value of $F_{\pi NN}(0)\approx 0.99$ appears to be needed to fit low partial waves,[18] $\rho$ exchange presumably accounts for some of this difference. Inclusion of $\rho$ exchange has a similar effect on the value of $F_{\pi NN}(0)$ obtained in some model calculations of[19] $\pi^+ d\to pp$.

Thus, given this almost universal consensus that the pion form factor varies by at most 3% from $q^2=m_\pi^2$ to $q^2=0$, we must again reconsider the larger GT discrepancy of 6%. One possible resolution of this apparent conflict is to allow for $q^2$ variation of the pion decay constant $f_\pi\to f_\pi(q^2)$. This $q^2$ variation arises naturally in the *constituent* quark picture of the vacuum-pion matrix element of the axial vector hadronic current. We envision the graph of Fig. 1 and recall the definition of $f_\pi$,

$$\langle 0|j^i_{\mu 5}(x)|\pi^j(q)\rangle = if_\pi(q^2)q_\mu\,\delta^{ij}e^{-iq\cdot x}\,. \tag{4}$$

In the spirit of the dispersion-theoretic calculations[5,6] of $F_{\pi NN}(q^2)$, we assume a once-subtracted dispersion relation for $f_\pi(q^2)$ to arrive at

$$f_\pi(q^2)-f_\pi(0)=\frac{q^2}{\pi}\int_0^\infty \frac{dq'^2\,\mathrm{Im}f_\pi(q'^2)}{q'^2(q'^2-q^2)}\ . \tag{5}$$

From unitarity for two-particle intermediate quark "states" which couple in an *on-shell* constituent-quark manner to pions as $g_{\pi qq}\bar{q}\gamma_5\tau^j q$, we can write

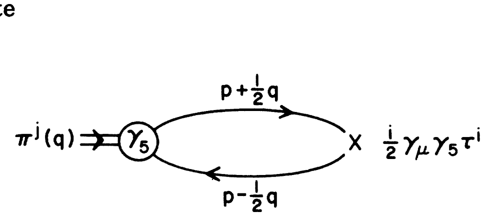


FIG. 1. The constituent quark graph for the pion-vacuum matrix element of the hadronic axial vector current (represented by the symbol X).

$$\mathrm{Im}f_\pi(q^2)=\frac{3g_{\pi qq}}{2}\,\frac{4\hat{m}}{8\pi}\left(1-\frac{4\hat{m}^2}{q^2}\right)^{1/2}\theta(q^2-4\hat{m}^2)\,, \tag{6}$$

where $\hat{m}=\frac{1}{2}(m_u+m_d)$ is the constituent nonstrange quark mass, and the factor of 3 comes from the sum over color. Inserting (6) into (5) and neglecting $q^2$ in the denominator of (5), as it is of higher order in $q^2$, we observe that the integral is finite and (5) becomes, upon dividing both sides by $f_\pi$,

$$\frac{f_\pi(q^2)-f_\pi(0)}{f_\pi}=\frac{q^2}{8\pi^2}\,\frac{g_{\pi qq}}{\hat{m}f_\pi}\,, \tag{7}$$

which we rewrite as a discrepancy for $q^2=m_\pi$:

$$1-f_\pi(0)/f_\pi=m_\pi^2/8\pi^2f_\pi^2\approx 0.03\,. \tag{8}$$

We have invoked the GT relation at the quark level $g_{\pi qq}=\hat{m}/f_\pi$ to eliminate the two unknowns $g_{\pi qq}$ and $\hat{m}$ in (7) in favor of the measured on-shell $f_\pi(m_\pi^2)=f_\pi$. Such a GT relation at the quark level only makes sense for *constituent* quark masses as it does for the nucleon mass with $m_N\approx 3\hat{m}$ being related in a weak binding, "on-shell" manner. Furthermore, it is consistent with hadron phenomenology in that it predicts[20] $m_s/\hat{m}=2f_K/f_\pi-1\approx 1.4$, the accepted[21] constituent quark mass ratio.

We note that the formal loop integral for $f_\pi$ in Fig. 1 diverges for $p^2$-independent quark propagators, but the once-subtracted relation (5) for $f_\pi(q^2)-f_\pi(0)$ does not. This quark loop calculation is similar to the Adler quark loop calculation of $\pi^0\to 2\gamma$ through the triangle anomaly[22] and to the Tarrach calculation for meson charge radii,[23] e.g., $\propto F'_{\pi\pi\gamma}$. Gluon corrections to "free" constituent quarks in the $f_\pi$ and $\langle r_{\pi^+}{}^2\rangle$ loop calculations are of higher order in the spontaneous generation of Nambu-Goldstone mesons.[24,25] In all three cases, color enters in a multiplicative fashion and the nonstrange constituent quark mass $\hat{m}$ traverses the loop and cancels out of the final answer.[24] Thus the result (8) implying 3% change in $f_\pi$ is quark-mass independent and is *not* cutoff dependent in contrast to the "chiral perturbation theory" calculation[26]

$$1-f_\pi(0)/f_\pi=\frac{m_\pi^2}{16\pi^2f_\pi^2}\ln\frac{\Lambda^2}{4m_\pi^2}\sim 0.02\text{ to }0.04 \tag{9}$$

for cutoffs $\Lambda^2=m_\rho^2$ to $4m_N^2$. In either case, however, it would appear that $f_\pi$ does vary with $q^2$, and in the right direction to account for part (~3%) of the GT discrepancy.

Therefore, we can understand the complete 6% GT discrepancy (2) in terms of a 3% increase in the $\pi NN$ form factor $F_{\pi NN}(q^2)$ and a 3% increase in the pion decay constant $f_\pi(q^2)$ from $q^2=0$ to $q^2=m_\pi^2$. It is only $F_{\pi NN}(q^2)$ which enters into the nuclear physics problems involving one-pion-exchange, however, so one should use a form which incorporates a 3% increase as suggested by estimates (i)–(iii). A convenient parametrization is

$$F_{\pi NN}(q^2)=\frac{\lambda^2-m_\pi^{\ 2}}{\lambda^2-q^2} \tag{10}$$

with $\lambda\approx 5.8m_\pi\approx 800$ MeV. Most nuclear problems require that the pion $q^2$ be off shell by more than one pion (mass)$^2$, but often the low $q^2$ region dominates so that the choice of a particular form for $F_{\pi NN}(q^2)$ does not change the results significantly (Table III and Figs. 4 of Ref. 27 demonstrate this form invariance in the pion condensate problem).

Although only $F_{\pi NN}(q^2)$ enters into OPE nuclear calculations, nevertheless it is the *entire* discrepancy $\Delta$ which enters into chiral-symmetry breaking considerations for the quark model. As an aside, we close by commenting on the *current* quark mass ratios implied by the 6% GT discrepancy. First defining the quark pseudoscalar density as $v_\pi=\bar{q}\tau_3\gamma_5 q$, we then remove the pion "tadpole" from the nucleon matrix element $\langle N|v_\pi|N\rangle$ as shown in Fig. 2:

$$m_N g_A=f_\pi g+\hat{m}^{\text{curr}}\langle N|\bar{v}_\pi|N\rangle\,. \tag{11}$$

It is the chiral-breaking background $\hat{m}^{\text{curr}}\langle N|\bar{v}_\pi|N\rangle$ which is proportional to the *entire* GT discrepancy[5,28]; such chiral breaking in hadron language corresponds to $f_\pi(m_\pi^{\ 2})-f_\pi(0)$ and $F_{\pi NN}(m_\pi^{\ 2})-F_{\pi NN}(0)$ departures from the exact GT relation at $q^2=0$, $m_N g_A(0)=f_\pi(0)g(0)$. If we express the background in terms of the hyperon-kaon-nucleon transitions $\langle\Lambda,\Sigma|\bar{v}_K|N\rangle$ and then apply SU(3) symmetry, the result at $q^2=0$ is a constraint on the *current* quark mass ratios occurring in the tadpole residue $\langle 0|\partial\cdot j_5^{\ \pi}|\pi\rangle=f_\pi m_\pi^{\ 2}=-\hat{m}^{\text{curr}}\langle 0|v_\pi|\pi\rangle$, etc.[5,28]:

$$\frac{m_s^{\text{curr}}}{\hat{m}^{\text{curr}}}+1=\frac{2}{\Delta}\left[\frac{f_K}{f_\pi}-(1-\Delta)\frac{m_\Sigma+m_\Lambda+2m_N}{4m_N}\right], \tag{12}$$

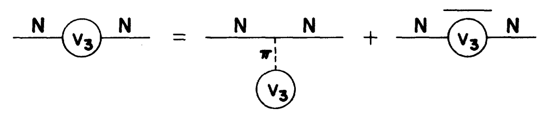


FIG. 2. Nucleon-nucleon matrix element of the quark pseudoscalar density $v_\pi$.

where $m_s^{\text{curr}}$ is the strange current quark mass and $\hat{m}^{\text{curr}}$ the mass of the nonstrange current quark. In the SU(3) limit $(m_s/\hat{m})_{\text{curr}}\to 1$, $f_K/f_\pi\to 1$, $m_\Sigma$, $m_\Lambda\to m_N$, so that (12) is seen to be an identity. Corrections to it are $O(10\%)$ in the particular combinations of baryon-meson and axial-vector coupling constants[5] which can occur in (12). For the SU(3)-broken (physical) masses and $f_K/f_\pi\approx 1.2$, the GT discrepancy converts (12) to roughly[5,28]

$$(m_s/\hat{m})_{\text{curr}}\sim 5\pm 3\,, \tag{13}$$

presumably valid for *any* model of chiral symmetry breaking. Only if $\Delta$ were 1% rather than 6% would (13) be consistent with the conventional "strong" partially conserved axial-vector current (PCAC) prediction[29] of $(m_s/\hat{m})_{\text{curr}}\approx 25$. Nevertheless, the "neutral" PCAC prediction[5,28,30] of $(m_s/\hat{m})_{\text{curr}}\approx 5$ is perfectly consistent with the experimental GT discrepancy yielding (13).

In any case, we suggest the following as the resolution to the GT discrepancy problem: The discrepancy is 6% as measured—half of which is due to the variation in $F_{\pi NN}(q^2)$ and half due to the variation of $f_\pi(q^2)$.

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