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Systematics of the (π^+, p) reaction with light nuclei

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Recent (π^+,p) angular distribution measurements for ground state transitions in several light nuclei exhibit energy independence in both shape and magnitude over a wide energy range. They also exhibit a simple exponential decrease with momentum transfer, $\exp(-q/\lambda)$. The parameter λ has a smooth variation with target mass A. The magnitudes of the cross sections are proportional to the spectroscopic strength of the transition except for $^{11}B(\pi^+,p)$, which is a factor of 3 high and $^{16}O(\pi^+,p)$, which is a factor of 10 low. A single neutron pickup distorted-wave calculation appears to be incapable of reproducing the experimental data and will require a major modification. A two nucleon model is briefly examined and shows potential for success.

[NUCLEAR REACTIONS (π,p) on ^{6,7}Li, ^{10}Be, ^{11}B, ^{12,13,14}C, ^{16}O; $5 < T_\pi < 200$ MeV compilation of data, theoretical study of mechanism.]

I. INTRODUCTION

In this paper we wish to describe some striking systematics among recent data taken for the (p, π^+) reaction with light nuclei at low energy. The (p, π^+) reaction has been the subject of many papers,[1] both because of its potentially informative possibilities and because of difficulties associated with the theory. It is potentially informative because of the large momentum transfer involved, giving hope to the possibility of understanding the short-range behavior of nuclear wave functions. The theory, on the other hand, is plagued by uncertainties and appears to be sensitive to every aspect of input, including relativistic corrections, questions of wave function nonorthogonality,[2] the choice of reactions mechanism, as well as the effect of distortions. Indeed, while it is possible to fit a given set of data, the large number of adjustable (or unknown) parameters makes it difficult to have convidence in the interpretation.

Given the complexity of the theory, the systematics presented here are surprisingly simple. Indeed, where interpretation is involved, the empasis will be placed upon determining whether the systematics of the theoretical predictions are as simple as those of the data. In our analysis, we examine both a distorted-wave neutron-pickup model and a plane-wave two nucleon process, primarily as a guide toward understanding the data. In addition , we also discuss some problems encountered in the energy dependence of the distorted-wave Born approximation (DWBA) calculations.

II. SYSTEMATICS

All of the experimental results are represented below as (π^+,p) data, with detailed balance used to invert the (p, π^+) data. The analysis is restricted to (π^+, p) transitions to the ground state of the final nucleus for the following target nuclei: ^{6}Li, ^{7}Li, ^{10}Be, ^{11}B, ^{12}C, ^{13}C, ^{14}C, and ^{16}O. Each of these transitions has a nearly maximal $1p$-shell spectroscopic strength for a neutron pickup reaction. We will ignore transitions to excited states with smaller spectroscopic strengths, because they are more likely to be dominated by multistep processes involving inelastic excitation.

Recent measurements[3-5] of $^{11}B(\pi^+, p)^{10}B$ at 12 pion energies between 6.7 and 393 MeV pion energy clearly demonstrate the exponential behavior of the cross section at forward angles,

$$\frac{d\sigma}{d\Omega} = C \exp(-q/\lambda) , \qquad (1)$$

with $q = |\vec{p} - \vec{k}|$. $\vec{p}$ and $\vec{k}$ are the proton and pion momentum in the center of mass and the parameters C and λ are adjusted to give excellent fits to the data. The results of such fits are shown in Fig. 1.

The striking feature of this data is that the parameters λ and C are nearly constant with pion energy. The energy independence of the cross section extends over the full range of measurements from 6.7 MeV, where the pion has a long mean free path in nuclear matter, up through the 3-3 resonance, where the nucleus becomes essentially black to the pion.

The energy independence of C becomes even better if the angle variable is converted from $q = |\vec{p}-\vec{k}|$ to $q = |\vec{p}-(A-1)/A\vec{k}|$, which is the momentum transferred in the laboratory system or the momentum of the bound neutron in a simple one-step neutron pickup model. Using this new definition of q, which will be used in the remainder of this paper, the energy dependences of λ and C are demonstrated in Fig. 2 and 3, respectively. The parameter λ increases slowly with pion energy up to $T_\pi = 50$ MeV, then levels off and decreases slightly at higher energy. The total variation in λ is less than a factor of 2. The curves in Figs. 2 and 3 are from the DWBA calculation described in Sec. III below.

The magnitude of the cross section at $q = 475$ MeV/c, proportional to the parameter C, is plotted in Fig. 3. The points above $T_\pi = 200$ MeV are an extrapolation since $q = 475$ MeV/c at these energies represents and unphysical angle, $|\cos\theta| > 1$. Even at lower pion energies $q = 475$ MeV/c corresponds to angles less than 40 degrees. The cross section at fixed q changes by only a factor of 2 over the full pion energy range. Part of this change may possibly be attributable to inconsistent normalization between different experimental groups.

This simple exponential behavior persists in the other ground state transitions within the $1p$ shell. All of the recent data which are currently available to us are consistent with a smooth exponential of Eq. (1) at forward angles. Table I lists the slope parameter λ and the value $d\sigma/d\Omega$ (475 MeV/c) obtained from the exponential fits to this data. The table also gives the experimental group or location

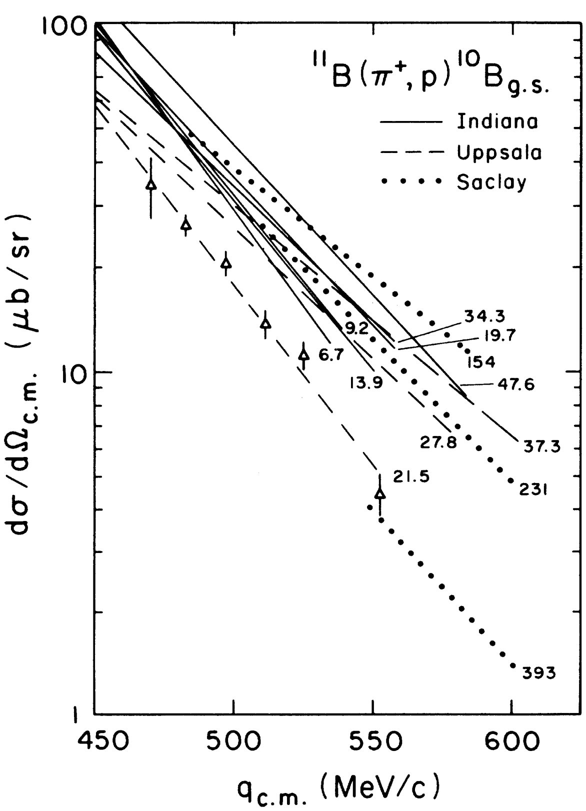

FIG. 1. The differential cross sections (Refs. 3–5) of the $^{11}B(\pi^+,p)^{10}B$(g.s.) reaction are plotted against momentum transfer. The lines are fits of Eq. (1) to the data, and the numbers by the line give the pion center of mass energy in MeV. A typical fit is shown for the Uppsala data at $T_\pi = 21.5$ MeV.

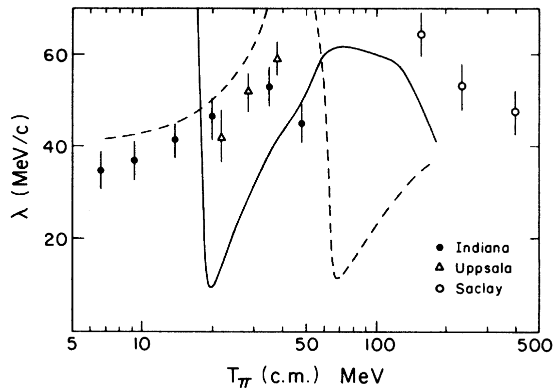

FIG. 2. The parameter λ obtained from fits to the $^{11}B(\pi^+,p)^{10}B$(g.s.) data are plotted against pion kinetic energy. The solid (dashed) lines are DWBA calculations using a Kisslinger (local Laplacian) pion potential (see text).

TABLE I. Forward angle (π^+,p) cross sections.

Group	$E_{c.m.}$ (MeV)	$\frac{d\sigma}{d\Omega}(475)\ \frac{\mu b}{sr}$	λ (MeV/c)
	$^{16}O(\pi^+,p)$		
Orsay (6)	65	4.1 ± 1	29 ± 5
CMU (7)	64	3.5 ± 1	
	$^{14}C(\pi^+,p)$		
Uppsala (8) [a]	37.5	27.9 ± 2	25 ± 2.5
Indiana (9)	51	22.5 ± 3	26 ± 3
	$^{13}C(\pi^+,p)$		
CMU (7)	32	14.6 ± 1.5	26 ± 5
Uppsala (10) [a]	32	14.2 ± 1.5	33 ± 3
Triumf (11)	47	17.0 ± 2.0	36 ± 4
Indiana (9)	47	15.5 ± 2	31 ± 3
EPICS (12)	87	13.5 ± 2	35 ± 5
EPICS (12)	164		53 ± 10
	$^{12}C(\pi^+,p)$		
CMU(13)	48	46.3 ± 2	42 ± 3
CMU (7)	63	51 ± 4	
EPICS (12)	87	51 ± 3	33 ± 3
EPICS (12)	173	105	36 ± 3
	$^{10}Be(\pi^+,p)^9Be$		
Indiana (14)	5	37 ± 3	53 ± 5
Uppsala (15) [a]	30	18 ± 3	79 ± 10
Triumf (16)	43	20 ± 3	72 ± 10
Saclay (4)	216		32 ± 10
	$^7Li(\pi^+,p)^6Li$		
EPICS (17)	71	19.5 ± 3	48 ± 5
EPICS (17)	163	19.2 ± 4	48 ± 5
Saclay (4)	345		46 ± 5
	$^6Li(\pi^+,p)^5Li$		
EPICS (17)	70	33.5 ± 4	53 ± 4
EPICS (17)	161	32.5 ± 3	50 ± 3

[a]Cross sections multiplied by 1.7.

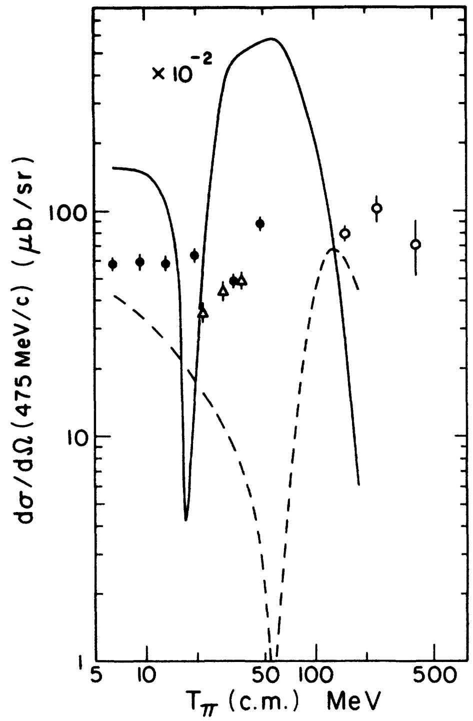

FIG. 3. The differential cross section of the $^{11}B(\pi^+,p)^{10}B$(g.s.) reaction at q=475 MeV/c obtained from fits to the data. The solid (dashed) lines are DWBA calculations using a Kisslinger (local Laplacian) pion potential (see text).

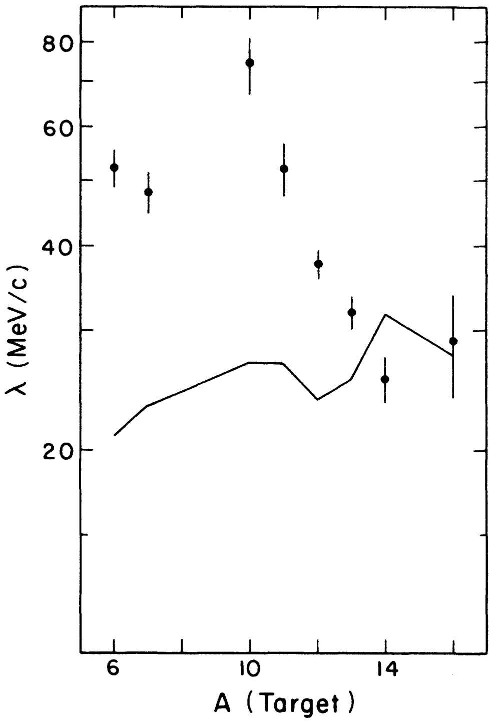

FIG. 4. The parameter λ obtained from fits to the (π^+,p) data for ground state transitions in the energy region $30 < T_\pi < 72$ MeV. The solid line is a DWBA calculation at T_π=49 MeV using a Kisslinger pion potential.

where the measurement was made and the pion center of mass kinetic energy. We ignore transitions to excited states which usually have much smaller neutron spectroscopic strengths and can be populated strongly through two step processes which involve collective inelastic excitation.[14] It should also be noted that this exponential behavior, Eq. (1) is limited to targets with $A<16$ and a momentum transfer region with an upper bound q_{max} which decreases as A increases: $q_{max} > 800$ MeV/c for ^{6}Li and $q_{max} < 520$ MeV/c for ^{16}O. The (π^+, p) reaction with heavier targets has a dramatically different angular distribution.[1] Also, all of the older Uppsala data[8,10,15] (but not the new data[5]) have been renormalized upwards by a factor of 1.7 to agree with more recent measurements.

From Table I we observe that for the targets ^{6}Li, ^{7}Li, ^{12}C, and ^{13}C the slope parameter stays nearly constant to within the 10–20% errors over all pion energies extending over a range of 55 to 270 MeV. In fact, λ stays more constant for these four targets than for ^{11}B.

Next, we examine the A dependence of $d\sigma/d\Omega$ (475 MeV/c). The experimental numbers are again taken from the data for $30 \leq T_\pi \leq 72$ MeV and are shown in Fig. 5 divided by the spectroscopic strength C^2S=1.18, 1.0, 2.8, 0.8, 1.5, 2.0 for ^{10}Be, ^{11}B, ^{12}C, ^{13}C, ^{14}C, ^{16}O, respectively. The lowest three A involve a $1p_{3/2}$ neutron and the highest three A involve a $1p_{1/2}$ neutron. The Li nuclei involve a combination of $1p_{3/2}$ and $1p_{1/2}$ and, to avoid this complication, are not shown. In a low-momentum-transfer neutron pickup model, the value of $(d\sigma/d\Omega)/C^2S$, as plotted, is nearly the same for each of these nuclei. These data show this behavior for ^{10}Be, ^{12}C, ^{13}C, and ^{14}C. The $(d\sigma/d\Omega)/C^2S$ is the same to within 10% for these four target nuclei although their spectroscopic strength and cross sections vary together by more than a factor of 3. However, it must be remembered that these four cross sections do not have the same slope λ, as shown in Fig. 4. This means that $(d\sigma/d\Omega)/C^2S$ is only constant at forward angles, $\theta < 40°$, and differs significantly at larger angles.

The greatest anomaly, however, is that $(d\sigma/d\Omega)/C^2S$ is a factor of 10 smaller than expected for ^{16}O and a factor of 3 larger for ^{11}B. This enhancement of the $^{11}B(\pi^+, p)^{10}B$(g.s.) and suppression of the $^{16}O(\pi^+,p)^{15}O$(g.s.) occurs at all pion energies. The anomalously small cross section for the $^{16}O(\pi^+,p)^{15}O$ reaction was measured by two separated groups,[6,7] giving consistent results at T_π= 65 MeV. A recent $^{16}O(\pi^+,p)$ measurement at EPICS (Ref. 18) at T_π =

60, 110, and 170 MeV failed to show the ^{15}O ground state transition and has set upper limits of a few μb/sr for the cross section near $q = 475$ MeV/c at all these energies.

III. DWBA ANALYSIS

We have attempted to understand the systematics described above by employing the DWBA code PUPASYM written by Tsangarides.[19] The model is a simple neutron pickup process in which the pion is absorbed on a *single* neutron. The pion annihilation operator has the standard form $\sigma \cdot \nabla_\pi$.

The proton optical potential is based upon the analysis of Comfort and Karp[20] to elastic scattering from ^{12}C,

$$U(r) = Vf(r) + iWf'(r) + 1/r V_{so} \frac{d}{dr} f_{so}(r) \vec{l} \cdot \vec{s}, \quad (2a)$$

where the $f(r)$ are Woods Saxon shapes with $r_0 = 1.2$, $a = 0.68$, $r' = 1.2$, $a' = 0.61$, $r_{so} = 0.9$, and $a_{so} = 0.47$ fm.

$$V = -13 + 0.065\ [T_p(\text{lab}) - 180]\ \text{MeV},$$
$$W = -13 - 0.04\ [T_p(\text{lab}) - 180]\ \text{MeV}, \quad (2b)$$
$$V_{so} = -16.5 + 0.025\ [T_p(\text{lab}) - 180]\ \text{MeV}.$$

We have also tried other proton optical potentials[21] and found that the results presented below are insensitive to reasonable changes in the proton potential except for a variation of 2 in the magnitude of the cross section.

In fact, there are a multitude of uncertainties in the *calculated magnitude* of the cross section. For example, the standard nonlocality correction to the proton optical potential[22] used for low-energy reactions can cause a factor of 4 decrease in the (π^+, p) cross section. Although we have not used this correction term in our calculations, the general problem remains, and we have concentrated instead on the *relative* cross section predictions as a function of energy, momentum transfer, or mass number.

The primary sensitivity and uncertainty in the calculation comes from the pion optical potential. We have used both a Kisslinger form with an angle transformation,

$$V_N(r) = -Ak^2 b_0 \rho(r) + A b_1 \vec{\nabla} \cdot \rho \vec{\nabla} - \frac{\epsilon_\pi}{2MN} \nabla^2 \rho(r), \quad (3a)$$

and a local Laplacian form

$$V_N(r) = -Ak^2(b_0 + b_1)\rho(r) - A\left(\frac{\epsilon_\pi}{2MN} + 1\right) \frac{b_1}{2} \nabla^2 \rho. \quad (3b)$$

The calculations for $^{11}B(\pi^+, p)^{10}B$ are shown in Fig. 2 and 3. Concentrating first on Fig. 3 we observe that the Kisslinger potential gives cross sections more than two orders of magnitude larger than the local Laplacian because of its well known singular off-shell behavior.[1]

We discuss here an unexpected difficulty related to the pion wave functions. The calculations predict a deep minimum for the cross section at fixed q and, in the angle-integrated cross section, at 18 MeV for the Kisslinger pion potential and at 65 MeV for the local Laplacian potential. These minima are evidently *anomalous*, since there is no evidence for any such behavior in the data.

The anomalous minimum occurs when the forward angle part of the angular distribution is suppressed by more than two orders of magnitude. Part of the suppression is characterized by a shallow minimum in the angular distribution, moving forward from its "normal" location beyond 120° to less than 40°. Examining the properties of the DWBA calculation, we find that the forward angle part of the angular distribution is dominated by the overlap integrals $I^{l_n j_n}_{l_\pi j_p}$ for which $j_p - l_\pi = j_N = \frac{3}{2}$ in the $^{11}B(\pi^+, p)^{10}B$ reaction. j_p is the total angular momentum of the proton partial wave, l_π specifies the pion partial wave, and j_n specifies the neutron bound state. Specifically[19]

$$I^{l_n j_n}_{l_\pi j_p} = \int_0^\infty dr\, \alpha_{l_\pi}\left(\frac{A}{A+1} r\right) \chi_{l_p j_p}(r) R_{l_n j_n}(r), \quad (4)$$

where χ and R are the proton and neutron wave functions and α_{l_π} is the result of the gradient in the pion annihilation operator acting on the pion wave function, $\phi_{l_\pi}(r)$,

$$\alpha_{l_\pi}(r_\pi) = \frac{d\phi_{l_\pi}(r)}{dr_\pi} - (l_\pi + 1) \frac{\phi_{l_\pi}(r)}{r_\pi}. \quad (5)$$

The unit angular momentum of the gradient operator permits proton and pion angular momenta such that $\frac{1}{2} \leq |j_p - l_\pi| \leq \frac{5}{2}$, but only the overlap integrals with $(j_p, l_\pi) = (\frac{3}{2}, 0)$, $(\frac{5}{2}, 1)$ and $(\frac{7}{2}, 2)$ actually have a significant effect on the forward angle cross section. Table II gives the pion energies where these overlap integrals pass through zero (using the local Laplacian potential). All of these zeros lie in the vicinity of the anomalous minimum, and the $(\frac{5}{2}, 1)$ integral dominates the cross section. Surprisingly, both the real and

TABLE II. Pion energies T_π (MeV) at zeros of $I^{l_n j_n}_{l_\pi j_p}$.

j_p	l_π	Real	Imag.
$\frac{3}{2}$	0	35	
$\frac{5}{2}$	1	65	65
$\frac{7}{2}$	2	72	

imaginary parts of this integral change sign at 65 MeV, although the real and imaginary pion wave functions have quite different r dependences. Nevertheless, an anomalous minimum is still produced in this energy region even when only one of the real or imaginary part of the $(\frac{5}{2},1)$ integral is set equal to zero.

Figure 2 illustrates the behavior of the slope parameter λ, which is extracted from the calculation as the average inverse slope of log $(d\sigma/d\Omega)$ for $q<510$ MeV/c. In the region of the anomalous minimum, the smooth exponential behavior of the cross section is lost, and the extracted value of λ is no longer a meaningful quantity.

Our attempts to eliminate these anomalous minima by modifying the pion optical potential have been discouraging. Standard correction terms such as ρ^2 absorption contributions or Pauli bocking factors have little effect, as does the addition of an off-shell damping factor in the local Laplacian potential. Recoil corrections to the static pion absorption operator also do not change these conclusions. The energy location of the anomalous minimum is relatively insensitive to the strengths b_0 and b_1, whether they are taken from πN phase shifts or adjusted phenomenologically to fit pion-nucleus elastic scattering. One can shift the minimum downward in energy either by increasing Imb_1 in the Kisslinger potential by a factor of 10, or can eliminate it entirely by giving Reb_0 an extremely strong attractive value of 10 fm^3.

This anomalous behavior can be seen only by examining the *energy* dependence of the cross section in the low-energy region, and not by calculating angular distributions at a few selected energies, as has generally been the case in the literature to date. Thus, apart from the overall parameter sensitivity which one finds at all energies in the DWBA model, there is an added sensitivity in the region of this anomalous minimum, wherein one can calculate essentially an arbitrary angular distribution. Henceforth, we therefore avoid this energy region in our discussion of the DWBA results.

With a phenomenological Kisslinger potential obtained from fits to pion elastic scattering data for ^{12}C at $T_\pi=50$ MeV (Ref. 23), the calculation avoids the region of the anomalous minimum. The results are shown in Fig. 4. These calculations at 50 MeV predict reasonable behavior as a function of q, where the exponential slope changes by only 10% over a range $\Delta q=75$ MeV/c. However, this slope parameter λ has the wrong A dependence, increasing with mass number, whereas the experimentally extracted λ decreases with A.

One particular concern of this exercise is that the fitted pion optical potential parameters b_0 and b_1 have a significant A dependence, whereas first-order multiple-scattering theory relates b_0 and b_1 to the pion-*nucleon* scattering amplitude, regardless of target. The origin of this phenomenological A dependence is not clear. For example, the measured pion elastic differential cross sections[24] for ^{6}Li and ^{12}C are nearly identical in shape, but the best-fit values of b_0 and b_1 for ^{12}C are very poor for reproducing the ^{6}Li data. Most changes to the pion optical potential to fit the ^{6}Li elastic scattering data have little effect on the slope parameter λ for ^{6}Li(π^+,p). However, it is possible to obtain any desirable value for λ and also fit the elastic scattering by moving the anomalous minimum closer to $T_\pi=50$ MeV. This is achieved by setting Imb_1 close to zero, but we discount this anomalous minimum as being unphysical.

Also shown in Fig. 5 are the DWBA predictions of $(d\sigma/d\Omega)/C^2S$ vs A. These calculations show a slight decrease in $(d\sigma/d\Omega)/C^2S$ with increasing A. Most of this decrease can be attributed to the fact that $A=10$, 11, 12 corresponds to $1p_{3/2}$ neutron pickup, whereas $A=13$, 14, 16 corresponds to $1p_{1/2}$ neutron pickup. The larger j_n for the $1p_{3/2}$ neutron allows more pion and proton partial waves

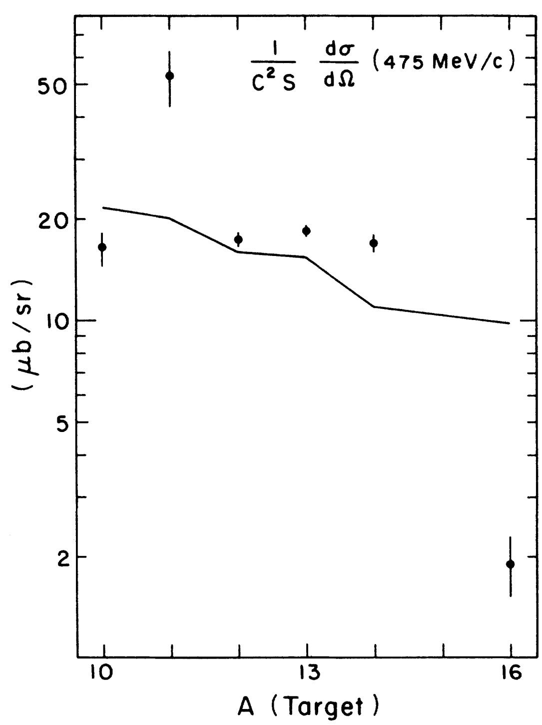

FIG. 5. The differential cross section at $q=475$ MeV/c for various (π^+,p) ground state transitions in the energy region $30<T_\pi<72$ MeV. The solid line is a DWBA calculation at $T_\pi=49$ MeV using a Kisslinger pion potential. The DWBA calculation is renormalized downward by a factor of 40.

to couple and gives about a factor of 1.6 increase in the cross section relative to the $1p_{1/2}$ neutron pickup. The remaining A dependence is due to binding energy and Q value effects. If we ignore ^{16}O and ^{11}B no such A dependence is observed experimentally at forward angles. However, because of the A dependence in the slope λ, the experimental data give values close to the predicted A dependence at larger angles.

The theory completely fails to predict the ^{11}B and ^{16}O anomaly. Interestingly ^{11}B$(\pi^+,p)^{10}$B is the only one of these reactions with both a nonzero spin target and final nucleus. It is conceivable that some spin coupling effect could be enhancing its cross section. (See the next section.)

IV. ZERO-RANGE SECOND BORN APPROXIMATION

The two-nucleon model which we will consider involves an explicit rescattering of the initial pion (or the final proton) from a second nucleon in addition to the pion-absorbing nucleon. While the one-nucleon DWBA approach includes pion and proton rescattering in the distorted waves, the presumption here is that it is more important to evaluate one rescattering explicitly than to generate it approximately using an optical potential. Ignoring higher order distortion effects, we have essentially the second Born approximation to the (π^+,p) amplitude.

In the present analysis, we assume further that the rescattering takes place over zero range and has no kinematic dependence other than via the momentum transfer q. With these assumptions, the differential cross section has the form

$$d\sigma/d\Omega \propto \left| \int d^3r\, \phi_n(r)\rho(r)\exp(-i\vec{q}\cdot\vec{r}) \right|^2 . \qquad (6)$$

We have evaluated this expression for neutron wave function $\phi_n(r)$ calculated from Woods-Saxon potentials with radius $R=1.2A^{1/3}$ fm and a diffuseness $a=0.55$ fm, along with a standard spin-orbit potential and a central well depth which is adjusted to reproduce the experimental separation energy. The matter distribution also has a Woods-Saxon shape with $R=1.1\,A^{1/3}$ fm and a diffuseness between 0.54 and 0.37 fm, which is adjusted to give the correct rms charge radius.[25] A Woods-Saxon shape rather than a harmonic oscillator model is used because we believe that it better represents the large momentum part of the form factor.

Equation (6) does not predict a perfect exponential decrease in the cross section. Specifically, it predicts a minimum in the angular distribution between 495 and 560 MeV/c, depending upon the nucleus. This is generally more forward than observed experimentally. Also, the slope parameter of the exponential falloff, λ, decreases with increasing q by about 25% every 50 MeV/c, and changes more rapidly near the minimum. To avoid the minimum, we evaluate λ at $q=460$ MeV/c and plot it in Fig. 6 along with the experimental data. Apart from the ^{10}Be and ^{11}B targets both the magnitude and A dependence are approximately correct. Specifically, the decreasing of λ with increasing A is very different than the prediction of the DWBA calculation and is in better agreement with the data. However, there is a significant discrepancy in this model with the data for the ^{10}Be and ^{11}B targets. Interestingly, these two targets, along with ^{7}Li, have a sizable quadrupole deformation. By including the quadrupole deformation in $\rho(r)$, but not $\phi_n(r)$ in Eq. (6), we obtain a 10% larger value for λ. This modification (not shown in Fig. 6) gives better but still poor agreement with the data.

The A dependence of the magnitude (with arbitrary overall normalization) of $d\sigma/d\Omega$ at $q=475$ MeV/c is plotted with the data in Fig. 7. Surprisingly, this model predicts the factor of 10 decrease in $(d\sigma/d\Omega)/C^2S$ for the ^{16}O(π^+,p) reaction. This occurs because the minimum of $d\sigma/d\Omega$, in the calculation, is at $q=495$ MeV/c, which is only 20

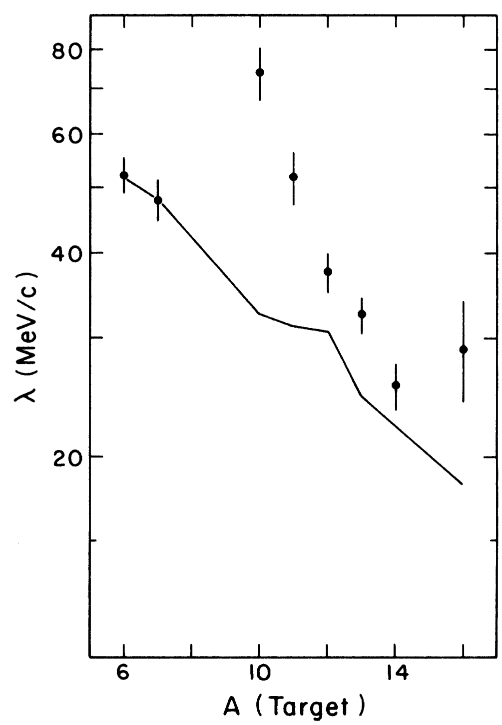

FIG. 6. The parameter λ obtained by the zero range second Born approximation, Eq. (6), is shown as a solid line. The data points are identical to those in Fig. 4.

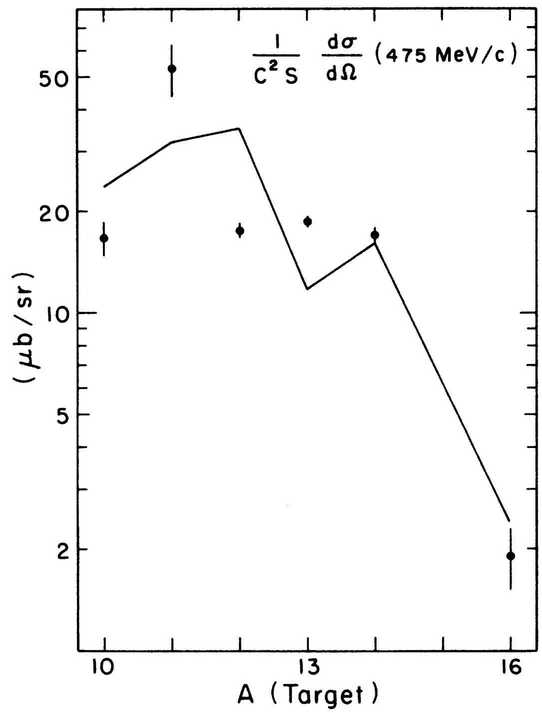

FIG. 7. The data points are identical to those in Fig. 5. The solid line is the zero range second Born approximation with arbitrary normalization.

MeV/c away.

This oversimplified two-nucleon model works surprisingly well and is suggestive of further success with a more sophisticated calculation. First, not only does the model predict the very small cross section for the $^{16}O(\pi^+,p)^{15}O$ reaction (cf. Fig. 7), but it also has the promise of predicting the larger $^{11}B(\pi^+,p)^{10}B$ cross section as well. In a one-neutron pickup model, the only allowed momentum transfer to the nucleus is $\Delta J=\frac{3}{2}$ or $\frac{1}{2}$, which is adequate for those p-shell reactions with 0^+ initial or final ground states, but not for boron, where $\Delta J_{max}=\frac{9}{2}$. A two-step mechanism can thus provide angular momentum coupling to larger values of ΔJ.

Second, while we have made no attempt to predict absolute cross sections, we refer to the recent work of Dillig *et al.*,[4] who predict successfully the cross-section magnitudes of (π^+,p) reactions with ^{2}H, ^{3}He, ^{6}Li, ^{9}Be, and ^{10}B at energies $T_1>300$ MeV. For input to the two-nucleon calculation, they use the experimental $pN\rightarrow NN\pi^+$ cross sections.

Finally, the shape of the angular distribution and the extracted slope λ should be examined in more detail. While our simplified calculation does not predict a linear exponential dependence upon q, as implied by the data, we speculate that distortions might effectively average our results at each angle over a finite region of momentum transfer, and thus flatten the shape of the momentum distribution. Also, it is interesting to note that the *experimental* exponential behavior of the $^{16}O(\pi^+,p)$ angular distribution[6] stops at a smaller value of q than in the lighter targets. This suggests that the $^{16}O(\pi^+,p)^{15}O$ reaction form factor is close to a minimum, as predicted by our model.

Our predictions of the slope parameter λ are poorest for the two nuclei beryllium and boron. There is an unexpected increase in λ between ^{7}Li and (^{10}Be, ^{11}B) in the energy region $T_\pi=50$ MeV (cf. Fig. 6). Although the data are incomplete, there is an indication that this unexpected behavior is occurring only in this energy region (cf. Fig. 2 and Table I). At lower and higher pion energies the slope parameter λ, for ^{10}Be and ^{11}B, in addition to the other nuclei, are in better agreement with this simple two-nucleon model. The energy dependence of λ is noticeably larger for ^{10}Be and ^{11}B than for the other nuclei and we speculate that this may be related to their sizable quadrupole deformation.

V. DISCUSSION

A standard (π^+,p) one nucleon pickup DWBA calculation has many uncertainties. When the model fails to agree with experimental data it is difficult to identify the source or even the severity of the error. The severity of an error in a model can be lost if small reasonable changes in the input parameters can give good agreement with the data and mask major deficiencies of the model. With large amounts of data it is easier to judge the basic validity of this DWBA calculation, and more difficult to mask its deficiencies. In this study, the DWBA model clearly has serious problems. Even with the great flexibility of changing the input parameters over the full range of uncertainty, it has not been possible for us to reproduce even crudely the systematic behavior of the data. The model appears to be incapable of reproducing the energy independence of the (π^+,p) differential cross sections. The energy independence of the differential cross section coupled with the strong energy dependence of the pion optical potential suggests that many features of the potential have considerably less effect on the reaction than the model predicts.

It is also interesting to observe that (p,d) angular distributions[26] in the same q region have the same exponential decrease (same slope λ) as the identical (π^+,p) transitions. Recently Wilkin[27] has suggested that the dominant mechanism in the (p,d) reaction consists of the incoming proton emitting an off-shell pion and capturing the nucleon from a (π^+,p) or (π^0,n) subprocess. Using this model, he has derived equations relating the (p,d) and (π^+,p) differential cross sections for the same nuclear transitions. The model works very well in the $1p$ shell.[12,27] For this model to be successful, Wilkin must assume that the off-shell (π^+,p) amplitude is slowly varying with pion momentum, when the total energy and scattering angle are held fixed. With this assumption, he can relate the off-shell amplitude to the measured on-shell cross section. The success of his model is added evidence that the (π^+,p) reaction cross section is only weakly dependent on the pion momentum, in contrast to the anomalous minimum predicted by the single-nucleon pickup DWBA calculation.

To conclude that the one-nucleon pickup model has serious deficiencies is only a first step to understanding the reaction mechanism. In the previous section we have seen that the basic mathematical features of a two nucleon model show excellent potential for explaining the (π^+,p) data in the $1p$ shell. Below, we summarize the evidence as it applies to the $1p$ shell.

(1) There is the serious problem of resolving the strong energy dependence of the pion optical potential with the energy independence of the data. It is possible that an accurate treatment of the off-shell pion dynamics[28] in the one-nucleon model could reduce the strong energy dependence. However, this problem also is likely solved with a two-nucleon model which is apparently less sensitive[29] to the pion optical potential than the one nucleon DWBA approach.

(2) The angular dependence and also the magnitude of the $(\vec{p},\pi^+)$ asymmetry data show simple uniformity[11] within the $1p$ shell. Specifically the asymmetry for all measured transitions has the same laboratory angle dependence as the elementary $pp\to\pi d$ reaction, suggesting that the reaction takes place in an effective πNN system. Any center of mass system other than this two-nucleon system would not exhibit the uniformity of the asymmetry. Furthermore, the asymmetries predicted by the computer code PUPASYM[19] do not reproduce the simple systematic behavior of the asymmetry data.

(3) The smooth decrease in the slope parameter λ with increasing A is reasonably well predicted from the basic feature of a two nucleon model. In contrast, the one-nucleon DWBA predicts the wrong A dependence. The sensitivity of the two-nucleon model to the matter density lends a plausible explanation for the larger λ observed in (π^+,p) reactions involving deformed nuclei, $^{10}Be(\pi^+,p)$ and $^{11}B(\pi^+,p)$.

(4) The abnormally large $^{11}B(\pi^+,p)^{10}B$ ground state cross section at all pion energies is an unexplained problem of the one nucleon DWBA model, whereas the two nucleon model can, in principle, account for this large cross section in a natural way by allowing larger angular momentum transfers.

(5) The very small $^{16}O(\pi^+,p)$ cross section at all pion energies is a serious problem of the one-nucleon DWBA model. By contrast, the two-nucleon model predicts this small cross section as caused by a node of $\phi_n\,\rho(q)$.

The evidence given above is, of course, only a suggestion that the two-nucleon model provides the correct picture. That is to say, the reaction appears to be dominated by the dynamics of rescattering, regardless of the actual method of computation. Unfortunately, one-nucleon DWBA calculations account for rescattering only in an average sense, and the only existing two-nucleon calculations to date contain approximations which are very difficult to justify. Furthermore, we note here that in the isobar doorway model, which accounts dynamically for intermediate Δ dynamics, the (π^+,p) reaction receives contributions both from one-nucleon and two-nucleon processes.[29] While more detailed calculations clearly must be preformed, it should also be kept in mind that the data, at least in the $1p$ shell, suggest ultimately a simple physical picture.

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