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Spin-orbit effects in heavy-ion elastic scattering

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It is shown how spin-orbit interactions affect resonant backward angular distributions of strongly absorbed particles.

[NUCLEAR REACTIONS Heavy ions, backward angles, spin-orbit interactions, resonances.]

I. INTRODUCTION

The spin-orbit interaction for heavy ions is of current interest.[1-9] Estimates from analyses of transfer reactions[2-4] are more than 10 times larger than predictions.[1,5,6] An enhanced prediction has been reported for ^{19}F.[7] The scattering of polarized ^{6}Li ions has shown clear evidence for a spin-orbit interaction.[8] A significant spin-flip process has recently been observed for ^{13}C inelastic scattering.[9]

This work suggests looking for spin-dependent effects on the elastic backward angle distributions of *unpolarized* heavy ions. It is shown how such effects can alter the surface-diffraction patterns which are typical for strongly absorbed particles. Unlike earlier discussions based on the average properties of the S matrix,[10] the emphasis is on enhanced distributions due to particular poles of the S matrix.

The pole representation of the scattering amplitude provides an elegant way of describing surface waves which propagate around the scattering center. Although this representation is well known in general and has often been applied to the scattering of spinless particles (see Ref. 11 and references therein), it apparently has not been applied to the elastic scattering of particles with spin. It will be seen that interesting new features arise in this case due to the interference of poles corresponding to different spin orientations. For the sake of simplicity the discussion will be limited to spin-$\frac{1}{2}$ systems. Cases with higher spin can be studied in a similar way.

II. FORMALISM

The unpolarized elastic cross section for the scattering of spin-$\frac{1}{2}$ particles may be written as an incoherent sum of two parts,

$$\sigma = |A|^2 + |B|^2, \tag{1}$$

corresponding to whether or not the spin orientation is changed during the scattering. Choosing the z axis in the beam direction and letting ϑ be the polar angle in the center of mass frame, the amplitude for scattering without a change in orientation is given by

$$A = \frac{1}{2ik} \sum_{l=0}^{\infty} \{\exp(i2\sigma_l)[(l+1)S_l^+ + lS_l^-] - (2l+1)\} P_l^0(\cos\vartheta), \tag{2}$$

while

$$B = \frac{1}{2ik} \sum_{l=1}^{\infty} \exp(i2\sigma_l)(S_l^+ - S_l^-)P_l^1(\cos\vartheta). \tag{3}$$

In these equations k is the relative wave number, the σ_l are Coulomb phase shifts, the $S_l^\pm$ are nuclear partial wave S matrices corresponding to the total spin $j = l \pm \frac{1}{2}$ states, and the P_l^m denote associated Legendre polynomials. We note that the last term in the summand of Eq. (2) contributes only at $\vartheta = 0$ and will be dropped from now on.

Making use of

$$P_l^m(\cos\vartheta) = (-)^{l+m} P_l^m(-\cos\vartheta) \tag{4}$$

and the Watson-Sommerfeld transformation, we can separate A and B into "pole (p)" and "background (b)" contributions. Thus,

$$A_p = \frac{-\pi}{2ik}\left[\frac{\exp(i2\sigma_{l_p^+})}{\sin\pi l_p^+}(l_p^+ + 1)\beta_p^+ P_{l_p^+}^0(-\cos\vartheta) + \frac{\exp(i2\sigma_{l_p^-})}{\sin\pi l_p^-} l_p^- \beta_p^- P_{l_p^-}^0(-\cos\vartheta)\right] \tag{5}$$

and

$$A_b = \frac{1}{2\pi i}\int_{C_0} dl\, \frac{\pi \exp(i2\sigma_l)}{2ik\sin\pi l}[(l+1)S_l^+ + lS_l^-] \times P_l^0(-\cos\vartheta), \tag{6}$$

where $l_p^\pm$ are complex poles of $S_l^\pm$ with residues $\beta_p^\pm$. The contour C_0 encircles these poles, the origin, and the positive real l axis. Similarly,

$$B_p = \frac{\pi}{2ik}\left[\frac{\exp(i2\sigma_{l_p^+})}{\sin\pi l_p^+}\beta_p^+ P^1_{l_p^+}(-\cos\vartheta) - \frac{\exp(i2\sigma_{l_p^-})}{\sin\pi l_p^-}\beta_p^- P^1_{l_p^-}(-\cos\vartheta)\right] \tag{7}$$

and

$$B_b = \frac{-1}{2\pi i}\int_{C_1} dl\,\frac{\pi\exp(i2\sigma_l)}{2ik\sin\pi l}(S_l^+ - S_l^-)P_l^1(-\cos\vartheta). \tag{8}$$

Here C_1 is the same as C_0 except that the origin is not enclosed. The sums of Eq. (5) with (6) and Eq. (7) with (8) give exact representations of A and B, respectively. However, their usefulness depends on the pole structure of $S_l^\pm$ and on the scattering angle considered.

As mentioned above and discussed more fully in Ref. 11 and references therein, the pole terms describe the tendency of partial waves to diffract around the scattering center. They will be particularly important at the far backward angles if there are poles close to the real l axis in the surface region and if the classical backscattering included in the background terms is relatively small; that is, if the absorption is weak for the grazing partial waves and strong for the lower ones. We will assume this to be the case and will only consider the pole terms at backward angles. We will also assume that the spin dependence of the interactions is of the nature of a perturbation so that $l_p^\pm$ lie close to a pole $l_p = L + i\lambda$ which is determined by the spin-independent part of the interaction, with L being close to the grazing partial wave and $L \gg \lambda$. For the same reason we will assume that $\beta_p^+ = \beta_p^- = \beta$, where β is the residue of l_p. It is an additional simplification to consider only one pole l_p. Generally, a sum of such pole contributions may be of interest.

Before leaving aside the background terms in Eqs. (6) and (8), it is interesting to note that B_b will be much smaller than A_b at the backward angles when considering a spin-orbit interaction. This is because the spin-orbit interaction has little effect for low partial waves. The distinction between the important values of S_l^+ and S_l^- for the classical backscattering becomes small. Thus the two parts of B_b tend to cancel while they add together in A_b. It might be possible to exploit this natural "background subtraction" mechanism when searching for resonances in the scattering of spin-$\frac{1}{2}$ particles.

Returning to the preceding considerations, we neglect some terms of order $1/L$ and λ/L and obtain

$$A_p = \frac{-\pi}{2ik}\left[\frac{\exp(i2\sigma_{l_p^+})}{\sin\pi l_p^+} + \frac{\exp(i2\sigma_{l_p^-})}{\sin\pi l_p^-}\right]\beta L P_L^0(-\cos\vartheta) \tag{9}$$

and

$$B_p = \frac{\pi}{2ik}\left[\frac{\exp(i2\sigma_{l_p^+})}{\sin\pi l_p^+} - \frac{\exp(i2\sigma_{l_p^-})}{\sin\pi l_p^-}\right]\beta P_L^1(-\cos\vartheta). \tag{10}$$

These equations are the main results of this paper. They show explicitly how the cross section will depend on the scattering angle and on the spin-dependent splitting of $l_p^\pm$ in the limit that a single pole dominates the scattering.

For instance, the squares of P_L^0 and P_L^1 oscillate out of phase with each other as ϑ varies with a period close to $\Delta\vartheta = \pi/L$. The angular dependence can be simplified using an approximation which is valid for large L and for angles at or near $\vartheta = \pi$:

$$P_L^0(-\cos\vartheta) = P_L^0(\cos\bar\vartheta) \simeq h(\bar\vartheta)J_0(L\bar\vartheta). \tag{11}$$

Here $\bar\vartheta = \pi - \vartheta$, J_m denotes Bessel functions, and

$$h(\bar\vartheta) = (\bar\vartheta/\sin\bar\vartheta)^{1/2}. \tag{12}$$

Note that $h = 1$ at $\vartheta = \pi$ and varies slowly with ϑ. Thus we also can use

$$P_L^1(\cos\bar\vartheta) = \frac{d}{d\bar\vartheta}P_L^0(\cos\bar\vartheta) \simeq -h(\bar\vartheta)LJ_1(L\bar\vartheta). \tag{13}$$

With Eqs. (11) and (13) we see that on the average A_p and B_p have the same order of magnitude, since this is true of J_0 and J_1 and is also true for the spin-dependent factors in Eqs. (9) and (10). It is also useful to note that the first zero of J_0 is at 2.4, close to the first maximum of J_1 at 1.8. Thus spin-dependent effects will be pronounced at an angle of about $\vartheta = \pi - 2/L$.

Considering the factors in brackets in Eqs. (9) and (10), it is seen that $|A_p|^2$ and $|B_p|^2$ oscillate out of phase with each other as functions of the spin dependence. Thus the shape of the total angular distribution depends characteristically on the splitting between l_p^+ and l_p^-. For no splitting, $B_p = 0$ and the normal Fraunhofer diffraction pattern of J_0^2 is obtained, just as for the case of spinless particles.[11] As the splitting increases from zero, A_p decreases in magnitude while B_p increases. When they are equal, the angular distribution is given by $J_0^2 + J_1^2$ and is therefore smooth, without pronounced diffraction oscillations. For stronger spin dependence, B_p dominates A_p so that an oscillatory J_1^2 distribution appears which is out of phase with the normal Fraunhofer pattern, having a minimum at $\vartheta = \pi$. As the splitting increases further, these types of patterns repeat until one of the poles, say l_p^+, lies much closer to the real axis than l_p^-. In this limit A_p and B_p have equal magnitudes, the cross section is smooth, and it is also enhanced with respect to the spin-independent scattering.

It is useful to consider the perturbation limit. To this end we use the approximation

$$\sin\pi l_p^\pm \simeq -\exp(-i\pi l_p^\pm)/2i, \tag{14}$$

which is valid for physical poles which lie above the real axis but not extremely close to it ($\lambda \gtrsim 0.5$). Such poles are typical of potential resonances (see Ref. 11 and references therein). Other "nonphysical" poles lying below the real axis can be of interest when the S matrix is parametrized in terms of functions whose pole structure is known, such as is done in Ref. 10. Summarizing, we now write

$$A_p = \pi[\exp(i\phi_{l_p^+}) + \exp(i\phi_{l_p^-})]\beta L h(\mathfrak{J}) J_0(L\mathfrak{J})/k \qquad (15)$$

and

$$B_p = \pi[\exp(i\phi_{l_p^+}) - \exp(i\phi_{l_p^-})]\beta L h(\mathfrak{J}) J_1(L\mathfrak{J})/k, \qquad (16)$$

where $\phi_l \equiv 2\sigma_l + \pi l$. Then in the perturbation limit where

$$l_p^\pm = l_p \pm \Delta l, \qquad (17)$$

we expand

$$\phi_{l_p^\pm} \simeq \phi_{l_p} \pm (\theta_c(l_p) + \pi)\Delta l \equiv \phi_{l_p} \pm \psi, \qquad (18)$$

where

$$\theta_c(l) = 2\tan^{-1}(\eta/l) \qquad (19)$$

is the Coulomb deflection function and η is the Sommerfeld parameter. Thus we obtain

$$A_p = 2\pi \exp(i\phi_{l_p})\cos\psi \beta L h(\mathfrak{J}) J_0(L\mathfrak{J})/k \qquad (20)$$

and

$$B_p = i2\pi \exp(i\phi_{l_p})\sin\psi \beta L h(\mathfrak{J}) J_1(L\mathfrak{J})/k. \qquad (21)$$

Thus the minimum splitting required to produce a smooth angular distribution occurs when $\psi = \pi/4$ or

$$(\Delta l)_{\min} = \frac{1}{4}\,\frac{1}{[1+\theta_c(l_p)/\pi]}. \qquad (22)$$

For estimates it is useful to note that

$$\theta_c(l_p) \simeq \theta_c(L) \approx \theta_g, \qquad (23)$$

where θ_g is the grazing or quarter-point angle. Thus in any case $(\Delta l)_{\min} \lesssim \frac{1}{4}$.

III. POTENTIAL SCATTERING

The discussion up to now has been general. It remains to relate Δl to models for the scattering. This can be done roughly for potential scattering as follows. Consider a central potential $U_0(r)$ and a spin-orbit potential

$$V_{so}(r)\langle \vec{l}\cdot\vec{s}\rangle = V_{so}(r)\begin{Bmatrix} l/2 & j=l+\frac{1}{2} \\ -(l+1)/2 & j=l-\frac{1}{2} \end{Bmatrix}. \qquad (24)$$

Assume that the pole l_p is due to the nuclear part of U_0 and that the absorptive part is strong in the interior. Then the semiclassical (Bohr-Sommerfeld type) condition for the position of l_p is given by[12]

$$\int_{r_2}^{r_3} dr\, k[1-(l_p+\tfrac{1}{2})^2/(kr)^2 - U_0(r)/E]^{1/2} = c. \qquad (25)$$

Here the integrand is the local radial wave number in the effective potential, r_2 and r_3 are the outer turning points determined by the barrier of the effective potential, $E = \hbar^2 k^2/2\mu$ is the center of mass energy, and c is a constant. The same condition will apply when a small spin-orbit potential is introduced. The integral is to be evaluated in the complex r plane. However, the integrand vanishes at the end points and peaks for values of r in the vicinity of the barrier position R of the effective potential. Thus we can estimate $l_p^\pm$ in terms of l_p by requiring the local wave number to have a fixed value at R. That is, ignoring terms of order $1/l_p$, we require

$$(l_p/kR)^2 = (l_p^\pm/kR)^2 \pm V_{so}(R) l_p^\pm/2E. \qquad (26)$$

Then with Eq. (17) we solve for Δl to leading order in V_{so}:

$$\Delta l = -\mu R^2 V_{so}(R)/2\hbar^2. \qquad (27)$$

Note that according to Eqs. (20) and (21) the sign of Δl is irrelevant. This result together with Eqs. (22) and (23) give an estimate of the minimum spin-orbit potential required to produce a smooth angular distribution

$$[V_{so}(R)]_{\min} = \frac{\hbar^2}{2\mu R^2}\,\frac{1}{(1+\theta_g/\pi)}. \qquad (28)$$

IV. SPECULATIONS

The only direct evidence for spin-dependent interactions in heavy-ion scattering for projectiles heavier than ^{6}Li has been reported recently by Dünnweber *et al.*[9] This group measured the polarization of the γ ray emitted in the inelastic scattering ^{24}Mg(^{13}C, ^{13}C′)^{24}Mg (2^+, 1.368 MeV) at $E_{lab} = 35$ MeV. It was determined that there is about 1% probability for the spin-$\frac{1}{2}$ ^{13}C projectile to flip its spin during the reaction.[9] It was noted that a spin-orbit potential with a value of $V_{so}(R) = 0.07$ MeV at the grazing distance of $R = 7.85$ fm could account for the observed effects.[9] Using $\theta_g = 50°$ ($L = 15$), one finds from Eq. (28) that $(V_{so})_{\min} = 0.04$ MeV at this distance. Thus the spin-orbit potential of Ref. 9 would be strong enough to cause significant effects on the backward angle elastic ^{13}C + ^{24}Mg scattering if this scattering were basically due to surface diffraction.

An indication that this might be the case is given by the recent ^{12}C + ^{24}Mg measurements reported by Ford *et al.*[13] It is seen that the back scattering cross section has gross structure in the excitation function with pronounced Legendre polynomial squared angular distributions.[13] These features are similar to those observed for $^{12}C + ^{28}Si$ and $^{16}O + ^{28}Si$ which are discussed in Ref. 11 and references therein. Thus it would be interesting to see if the same features appear in the $^{13}C + ^{24}Mg$ case but modified by a spin-orbit interaction which is consistent with the spin-flip measurements.

There are some elastic scattering data available for $^{13}C + ^{28}Si$ at the far backward angles (which apparently are the only such data for heavy ions with nonzero spin). These were measured by Ost *et al.*[14] in parallel with back-angle $^{12}C + ^{28}Si$ experiments. It was found that the ^{12}C scattering shows resonance behavior.[14] However, the ^{13}C distributions do not show pronounced diffraction structures. It may be that this is due to the spin-orbit potential for $^{13}C + ^{28}Si$. Using a surface transparent optical potential to reproduce the magnitude of the ^{13}C data, it was found that a spin-orbit potential close to that of Ref. 9 is indeed strong enough to smooth out the typical surface diffraction pattern.[15]

There have been a number of attempts to learn about spin-dependent interactions using heavy-ion transfer reactions.[2-4] In particular, Bayman *et al.*[3] introduced a spin-orbit potential in order to explain anomalies in the $^{40}Ca(^{13}C, ^{14}N)^{39}K$ reaction. The elastic scattering for $^{13}C + ^{40}Ca$ calculated in Ref. 3 (parameter set *B*) is shown in Fig. 1 together with the available data[16] and including the backward angle region. Also shown for comparison is the calculation which results when the spin-orbit potential is switched off. It is clear that the spin-orbit effects dominate the backward scattering. The spin-orbit potential is seven times that of Ref. 9 in the surface region. This case is indicative of the limit noted above where one pole is pulled down due to the attraction of the spin-orbit potential and the other is pushed away.

The size of the $^{13}C + ^{40}Ca$ backward cross section in Fig. 1 is especially remarkable when considered in the light of the $^{12}C + ^{40}Ca$ data taken by Renner *et al.*[17] This group observed backward cross sections of about 10^{-4} times the Rutherford value at center of mass energies up to $E = 34$ MeV. Normally one would expect a smaller cross section for $^{13}C + ^{40}Ca$ because of the increased absorption. This is the case for the $^{12,13}C + ^{28}Si$ scattering.[14] If the $^{13}C + ^{40}Ca$ backward cross section were

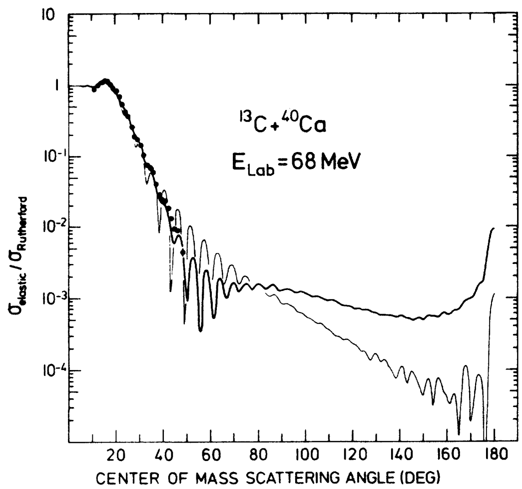

FIG. 1. Elastic scattering angular distribution for $^{13}C + ^{40}Ca$ at $E_{lab} = 68$ MeV. The data points are from Ref. 16. The thick curve is calculated using the potential of Ref. 3 (parameter set *B*). The thin curve results when no spin-orbit interaction is present.

smooth and enhanced with respect to $^{12}C + ^{40}Ca$ it could not be explained conventionally and the strong spin-orbit potential of Ref. 3 would be supported. On the other hand, if such a cross section is not observed, then limits can be set on the role played by the spin-orbit potential in the $^{40}Ca(^{13}C, ^{14}N)$ reaction.

In summary, there is very little direct evidence for spin-dependent effects in heavy-ion collisions. There is also very little data available on the backward angle scattering for heavy-ions with spin. The present work shows how the backward angle elastic cross section is sensitive to spin-dependent effects when it is dominated by surface diffraction scattering. It is suggested that further measurements using ^{13}C with ^{24}Mg, ^{28}Si, and ^{40}Ca should be carried out in parallel with ^{12}C experiments in an effort to learn more about the nature of the spin-orbit potential for heavy ions.

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