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Compressibility and the monopole collective field

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We show the relation between conventional random phase approximation and the collective field description of monopole vibrations by Hamamoto and Mottelson. When surface compression is properly included, the collective treatment yields the same vibrational frequency as conventional random phase approximation, for a given compressibility.

[NUCLEAR STRUCTURE Giant monopole vibrations, surface effects, relation to compressibility.]

The compressibility of nuclear matter is an important parameter which, unlike the binding energy and saturation density, cannot be obtained directly from static properties of finite nuclei. The giant monopole resonance is currently the main source of empirical information on the compressibility. There has been, however, some controversy over the value of the bulk compressibility K, implied by the monopole frequency. Blaizot *et al.*[1] have performed self-consistent calculations of the giant monopole resonance in spherical nuclei in the random phase approximation (RPA). This analysis[2] indicates that a compressibility $K=210\pm 30$ MeV successfully reproduces the data. However, Hamamoto and Mottelson[3] found that a macroscopic model of collective fields for the monopole vibration in ^{208}Pb required that $K=400$ MeV.

One suggested explanation for this discrepancy was a difference in effective mass m^*, used in the two calculations. This is not the case. We present below RPA calculations with Skyrme-type interactions having $m^*/m=1$ which are consistent with the analysis of Ref. 2. The origin of the discrepancy is instead in the functional form assumed for the surface field in the macroscopic model. It is necessary to include the effect of compression in the surface field. We show this by tracing the connection between the models, and examining the effect on the energy of different assumptions for the surface field.

Our microscopic calculations employ the response function formalism with coordinate-space Green's functions.[4] We sketch here the important points. The particle-hole Green's function, $G(r,r',\omega)$, is given in the RPA by

$$G^{\mathrm{RPA}}=G^0\left(1-\frac{\delta V}{\delta\rho}G^0\right)^{-1}, \tag{1}$$

where the bare Green's function, G^0, may be written as

$$G^0(r,r',\omega)=\sum_{p,h}\phi^0_p(r)\phi^0_h(r)\left[\frac{1}{\omega-(\epsilon_p-\epsilon_h)}-\frac{1}{\omega+(\epsilon_p-\epsilon_h)}\right]\times\phi^0_p(r')\phi^0_h(r'). \tag{2}$$

The poles of G identify the natural resonances of the system. The density perturbation induced by an external field of the form $V_{\mathrm{ext}}(r)\exp(-i\omega t)$ is given by

$$\delta\rho(r)=\int G^{\mathrm{RPA}}(r,r',\omega)V_{\mathrm{ext}}(r')d^3r'. \tag{3}$$

If V_{ext} excites a single eigenstate ψ_n, as it would near a resonance, this is proportional to the transition density $\delta\rho=\langle n|\hat\rho|0\rangle$. Equations (1)–(3) are solved as matrix equations on a coordinate space mesh.

The calculations start with the Hartree-Fock ground state of ^{208}Pb. We use a velocity-independent Skyrme interaction,[5] with parameters $t_0=-1100$ MeV fm^3, $t_3=16\,000$ MeV fm^6, $t_1=t_2=x=0$. The absence of momentum dependence implies that $m^*/m=1$. At saturation density $\rho_0=0.16$ fm^{-3}, the compressibility $K=418$ MeV, with contributions of 221 MeV from the kinetic energy and 197 MeV from the potential. This particular interaction gives $\langle r^2\rangle^{1/2}=5.3$ fm for ^{208}Pb and a single-particle spectrum which is somewhat overbound, but is adequate for the purposes of our demonstration.

The resonance in the isoscalar monopole response is at $\omega = 22$ MeV, pushed down from the free response peak at $E_0 = 23$ MeV by the residual interaction. These values are both somewhat high due to the overbinding of the ground state, but the energy shift is still significant. In particular, it is much smaller than that found by Hamamoto and Mottelson, using $m^*/m = 1$ and a similar compressibility. Our calculation is consistent with those of Refs. 1 and 2 for comparable values of K, thus ruling out the effective mass as the source of disagreement on the compressibility.

This discrepancy is all the more puzzling if one considers that the collective coordinate theory of Bohr and Mottelson[6] is mathematically equivalent to the RPA with separable interactions that are constrained by consistency. We will now demonstrate this connection. They solve the equation

$$\delta\rho(r) = \int G^0(r, r')\alpha\,\delta V_0(r')d^3r' \qquad (4)$$

with the dependence of the transition potential δV_0 on the ground state potential V_0 assumed to be the same as the dependence of $\delta\rho_0$ on ρ_0. Consistency also demands that the resulting magnitude

$$\alpha_{\text{out}} \equiv \frac{\int \delta V_0 \delta\rho\, d^3r}{\int \delta V_0 \delta\rho_0 d^3r} \qquad (5)$$

be the same as the input α. Substituting (4) into (5), the final equation to be solved is

$$1 = \frac{\int\int \delta V_0(r) G^0(r, r')\delta V_0(r')d^3r\, d^3r'}{\int \delta V_0(r)\delta\rho_0(r)d^3r}. \qquad (6)$$

In conventional RPA, we obtain this result by approximating $\delta V/\delta\rho$ with a separable potential. If a transition density $\delta\rho_0$ induces a transition potential δV_0, the separable potential must be

$$\frac{\delta V}{\delta\rho}(r, r') = \frac{\delta V_0(r)\delta V_0(r')}{\int \delta V_0 \delta\rho_0 d^3r}. \qquad (7)$$

We can insert this potential in Eq. (1) and find the pole using the operator identity

$$[1 - f(r)g(r')]^{-1} = 1 + f(r)\frac{1}{1 - \int f(r)g(r)d^3r}g(r') \qquad (8)$$

with $f(r) = \delta V_0(r)$ and

$$g(r') = \frac{\int \delta V_0(r'')G^0(r', r'')d^3r''}{\int \delta V_0 \delta\rho_0 d^3r}. \qquad (9)$$

The condition for a pole is $1 - \int fg = 0$, which is identical to Eq. (6).

We now must examine the collective field in detail and compare with RPA. The numerical $\delta\rho$ from our RPA calculation is plotted in Fig. 1, along with the ground state density ρ_0. If we think

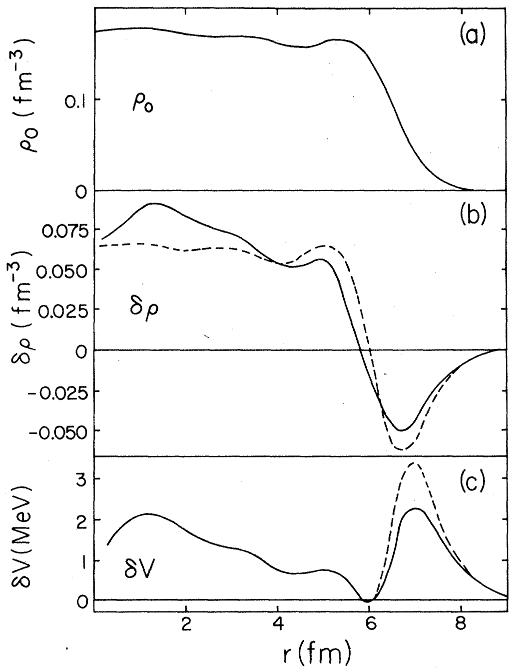

FIG. 1. (a) Hartree-Fock ground state density for ^{208}Pb. (b) Transition densities, as given by the full numerical RPA calculation (solid curve), and by the Tassie model (dashed curve). (c) Transition potential, as given by full RPA calculation (solid line). Also shown is the quantity $\beta Y^0 R dV_0/dr$ (dashed line).

in terms of a velocity field for the vibration, the transition density may be expressed

$$\delta\rho = (\nabla \cdot \vec{u})\rho_0 + \vec{u}\cdot\nabla\rho_0. \qquad (10)$$

Here the velocity field is a set of displacement vectors $\vec{u}(\vec{r})$ with a harmonic time dependence. The $\nabla\cdot\vec{u}$ term represents the change in density due to the stretching or squeezing of the wave function under a nonuniform displacement. The Tassie model for the transition density[7,8] is obtained by assuming that a single coherent mode, with the smoothest possible velocity field, exhausts a sum rule. The result is

$$\delta\rho = \begin{cases} \beta r^{L-1}\dfrac{Y^L(\Omega)}{(2L+1)^{1/2}}\dfrac{d\rho_0}{dr}, & L \neq 0 \qquad (11a) \\ \beta\left(3\rho_0 + r\dfrac{d\rho_0}{dr}\right)Y^0(\Omega), & L \neq 0. \qquad (11b) \end{cases}$$

For the monopole, this is simply radial scaling of the ground state density: $\vec{u}(\vec{r}) \propto \vec{r}$. We plot (11b) in Fig. 1, with $\beta = 0.043$ fixed to exhaust the energy weighted sum rule (EWSR) with a single resonance at 22 MeV. We find that this simple model gives the RPA transition density to within 20%, except for the central bump due to shell structure.

For a potential which depends only on the local density the transition potential is simply $\delta V = (\delta V/\delta\rho)\delta\rho$. For our Skyrme interaction $V = (\frac{3}{4}t_0\rho + \frac{3}{16}t_3\rho^2)$, so that

$$\delta V = (\tfrac{3}{4}t_0 + \tfrac{3}{8}t_3\rho)\delta\rho \,. \tag{12}$$

The quantity in parentheses changes sign at the surface. Knowing $\delta\rho$, we can compute δV. The RPA result is shown in Fig. 1. The appropriate collective field potential, assuming a Tassie transition density, is now

$$\delta V_0 = \left[3\beta\rho_0\left(\frac{\partial V}{\partial\rho}\right) + \beta r\frac{dV_0}{dr}\right]Y^0(\Omega)\,. \tag{13}$$

Hamamoto and Mottelson use an essentially equivalent form, but set the first term equal to zero outside the core. Thus the contribution to the collective field potential from compression of the surface is neglected. The point we wish to make is that this term is *not* negligible. We plot in Fig. 1 the quantity $\beta Y^0 R dV_0/dr$ in the surface. We see that for a "small" nucleus such as ^{208}Pb the change in surface thickness decreases δV by about one-third. This contrasts with the situation for $L \neq 0$ collective states, Eq. (11a), where $\nabla \cdot \vec{u} = 0$ and consequently there is no surface compression at all.

We can show the effects of each term in δV by examining the energy shift from the free response peak for various descriptions of the surface coupling and core compression. A simple expression for this shift is obtained by using Eq. (8) to estimate the pole in G^{RPA} under the assumption that a single particle-hole configuration dominates near resonance. We find

$$\omega^2 = E_0{}^2 + 2E_0 \int (\delta\rho)^2\left(\frac{\delta V}{\delta\rho}\right)d^3r\,. \tag{14}$$

We divide the integral in Eq. (14) into core ($R < 6$ fm) and surface ($R > 6$ fm) contributions. Using only the second term in Eq. (13) for δV in the surface, Eq. (14) gives a surface shift $\Delta E_s = -9$ MeV, in agreement with Hamamoto and Mottelson's calculation for the effect of the surface potential.[9,10] We find that the inclusion of the surface compression raises this energy by 3 MeV.[11] Then only a small contribution from the core potential is needed to reproduce the empirical position of the state. Thus the RPA conclusion, that the compressibility is about 210 MeV, is supported by the macroscopic treatment when surface compression is properly included.

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