Experimental test of virtual photon theory for pion electroproduction on light nuclei

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The virtual photon spectrum shape and intensity within several MeV of the kinematic limit was measured for ${}^{9}\mathrm{Be}(e,\pi^{+}){}^{9}\mathrm{Li},e'$ and ${}^{16}\mathrm{O}(e,\pi^{+}){}^{16}\mathrm{N},e'(T_{\pi}=28$ MeV, $\theta_{\pi}=90°)$. The intensity over this interval is 1.25 ± 0.10 times plane-wave virtual photon theory predictions; the shape agrees with theory within errors. Measurements on the proton 30 to 55 MeV below the end point agree with the intensity predictions of virtual photon theory within the errors ($\pm 8\%$).

NUCLEAR REACTIONS ${}^{16}\mathrm{O}(e,\pi^{+})$, $E=180.4$ MeV; ${}^{9}\mathrm{Be}(e,\pi^{+})$, $E=184.8$ MeV; measured $\sigma(E_{\pi},90°_{\mathrm{lab}})$ relative to (γ,π^{+}), near $E_{\pi}=28$ MeV. ${}^{1}\mathrm{H}(e,\pi^{+})$, $E=230$ MeV; measured $\sigma(E_{\pi},90°_{\mathrm{lab}})$ relative to (γ,π^{+}), near $E_{\pi}=18$, 30 MeV. Compared to PWBA virtual photon theory.

Recent experimental studies of pion electroproduction have included several measurements of pion spectra for light nuclei.[1,2] In these experiments the double differential cross section $d^{2}\sigma/dT_{\pi}d\Omega_{\pi}$ is obtained, and the scattered electron is undetected. There exist no detailed theoretical calculations for these reactions, and these experimental results have therefore had to be compared to theoretical results on the closely related (γ,π) process, for which a number of calculations exist.[3] To make this comparison, the experimental data have been analyzed using virtual photon theory,[4-6] in which the electromagnetic interaction of the incident electron with the target nucleus is represented by the interchange of a virtual photon. This has permitted the extraction from the experimental data of values for $d\sigma/d\Omega$ for virtual photons, which can then be compared to the theoretical results for real photons, it if is assumed that $d\sigma/d\Omega$ for virtual photons is equal to that for real photons. This procedure has been previously tested for electronuclear and photonuclear reactions in the low energy (giant dipole) region.[7]

Most theoretical treatments of the virtual photon spectrum are based on first-order Born approximation with plane waves for incident and outgoing electrons [plane-wave Born approximation (PWBA)] and assume a point nucleus and the forward peaking approximation. Such treatments are especially suspect in the region near the high energy limit of the virtual photon spectrum (tip region) for which the outgoing electron has low energy. Previous measurements[8,9] of virtual photon intensity in (e,π) reactions obtained agreement with PWBA treatments within errors, but these were total cross section measurements which sampled a large energy slice of the virtual spectrum and were insensitive to the spectrum tip region which is in fact that part of the spectrum which has been utilized in the recent (e,π) experiments.[1,2] Thus experimental studies of the virtual spectrum in the region near the kinematic limit are particularly needed. We report here the first such measurements within several MeV of the end point, for the cases ${}^{9}\mathrm{Be}(e,\pi^{+}){}^{9}\mathrm{Li},e'$ and ${}^{16}\mathrm{O}(e,\pi^{+}){}^{16}\mathrm{N},e'$.

In the experiment, pions of kinetic energy near 28 MeV emitted at 90° were detected by a magnetic spectrometer system, described in detail elsewhere.[10] Overall system resolution was about 600 keV. The reactions on ${}^{9}\mathrm{Be}$ and ${}^{16}\mathrm{O}$ are experimentally favorable since the first excited state of ${}^{9}\mathrm{Li}$ is at 2.69 MeV above the ground state, and the first state in ${}^{16}\mathrm{N}$ above the close-lying ground state quartet of states is at an excitation energy of 3.36 MeV. Therefore in these cases a measurement of the pion spectrum within several MeV of its high energy end point involves only transitions to the ${}^{9}\mathrm{Li}$ ground state and to the ${}^{16}\mathrm{N}$ ground state quartet of levels, and directly reflects the virtual photon spectrum shape within several MeV of its upper limit, provided $d\sigma/d\Omega$ for virtual photons does not vary over this limited energy range. This has been checked theoretically.[11]

To obtain the intensity of the virtual photon spectrum relative to that for real bremsstrahlung, a ratio experiment was performed in which the pion spectrum from the target was measured with and without a radiator (0.187 g/cm^2 Ta) inserted 5 cm upstream in the electron beam, and the ratio R/V

was extracted from these data. Here R is the pion yield due to real photons from the radiator per radiation length of radiator, obtained by integrating the pion yield over the pion energy interval ΔT_π extending back from the spectrum end point, and V is the corresponding pion yield directly due to the electrons. Thus the ratio R/V is an average over ΔT_π. In obtaining this ratio, the data were corrected for pions due to bremsstrahlung photons produced in the target itself and for the reduced effectiveness for electroproduction by those electrons which lose energy by radiation in the radiator. The ΔT_π values used and the values for $(R/V)_{\text{expt}}$ obtained in this way from the data are given in Table I. The ratio can be written

$$\frac{R}{V}=\frac{\int_{T_0-\Delta k}^{T_0}(d\sigma/d\Omega)_R(k,\theta)N_R(T_0,k)dk}{\int_{T_0-\Delta k}^{T_0}(d\sigma/d\Omega)_V(k,\theta)N_V(T_0,k)dk}\,,$$

where N_R is the number of real photons at energy k per radiation length per electron per MeV and N_V is the number of virtual photons per electron per MeV for an incident electron kinetic energy T_0. $(d\sigma/d\Omega)_R$ and $(d\sigma/d\Omega)_V$ are the differential cross sections for the (γ,π) process for real and virtual photons, respectively. The values given by $(R/V)_{\text{theor}}$ in Table I are obtained from this expression assuming $(d\sigma/d\Omega)_R$ and $(d\sigma/d\Omega)_V$ are equal, and using the following $E1$ PWBA expression for the virtual spectrum N_V, as given, for example, in Table 2 of Ref. 6,

$$N_V=\frac{\alpha}{\pi k}\left(\frac{E_0^{\,2}+E_f^{\,2}}{p_0^{\,2}}\ln\frac{E_0E_f+p_0p_f+m_0^{\,2}}{m_0k}-\frac{2p_f}{p_0}\right). \tag{1}$$

E_0, E_f, p_0, and p_f are the total energies and momenta for the incident and outgoing electrons, k is the virtual photon energy, and m_0 is the electron rest energy. For the bremsstrahlung spectrum N_R the expression given by Matthews and Owens[12] was used. This is based on Bethe-Heitler extreme relativistic theory with Coulomb correction, modified in the end-point region to give a finite tip cross section. The results for ^{16}O and ^{9}Be are consistent within their uncertainties and imply that in the energy region studied the virtual spectrum intensity is 1.25 ± 0.10 times the theoretical $E1$ value. The uncertainty quoted only includes statistical errors in the R/V measurements and does not include, for example, any uncertainty in N_R. In this comparison, the choice of the $E1$ multipole is not critical since the experiment is insensitive to multipolarity in the near-tip region. (For $E_0=180$ MeV, PWBA calculations of N_V give values for $M1$ and $E2$ which exceed the $E1$ value by less than 2% at energies within 3 MeV of the virtual spectrum end point.)

The spectrum shape results are shown in Fig. 1 together with calculated curves which use Eq. (1) for N_V, and which also take account of the contribution (~25%) from real bremsstrahlung photons produced in the target itself, and in addition include the effects of electron and pion energy losses in the target and of finite spectrometer resolution. It can be seen that the calculated shape is consistent with the data, within experimental errors.

We have also made similar measurements of R/V for the elementary process ^{1}H(e,π^+) at virtual photon energies well back from the end point. In this situation a PWBA treatment should be valid, and nuclear size effects should be unimportant. There is a limited amount of earlier work on ^{1}H(e,π^+) including an R/V measurement of Panofsky *et al.*[13] ($\theta_\pi=75°$, $T_\pi=60$ and 170 MeV, and $E_0=600$ MeV) and a cross section measurement near the end point by Shoda *et al.*[14] ($\theta_\pi=30°$, $E_0=180$ MeV). Reasonable agreement with plane-wave theory was obtained in both experiments. The model-dependent calculations of Dalitz and Yennie[5] give a good fit to the data of Panofsky *et al.* and imply that the virtual photon's longitudinal components contribute only a few percent to the pion production. In the present experiment on hydrogen, pions emitted at 90°, of energy 18 and 30 MeV, were observed with an incident electron energy of

TABLE I. Results for R/V, the real-to-virtual ratio as defined in the text. The quantity E_0 is the incident electron total energy, T_π the pion kinetic energy, k the virtual photon energy, and ΔT_π the averaging interval over pion energy in the experiment (corresponding to the range of k). Note that k is close to the end point for ^{16}O and ^{9}Be but 30 to 55 MeV below for ^{1}H.

	^{16}O(e,π^+)	^{9}Be(e,π^+)	^{1}H(e,π^+)	
E_0(MeV)	180.4	184.8	230	230
T_π(MeV)	28	28	18	30
k(MeV)	176.8–179.9	182.2–184.3	174.7–181.7	188.3–199.4
ΔT_π(MeV)	3.0	2.0	4.9	7.6
$(R/V)_{\text{expt}}$	78 ± 9	72 ± 8	71 ± 6	80 ± 6
$(R/V)_{\text{theor}}$	91	96	70	77
$(R/V)_{\text{theor}}/(R/V)_{\text{expt}}$	1.17 ± 0.14	1.33 ± 0.15	0.99 ± 0.08	0.96 ± 0.08

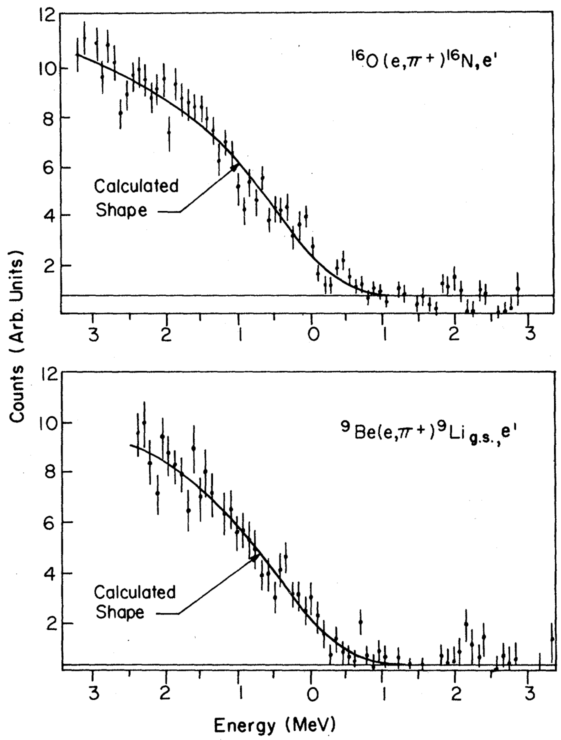

FIG. 1. Number of pions as a function of virtual photon energy. The zero on the abscissa corresponds to the virtual photon spectrum end point, 179.9 MeV for ^{16}O and 184.3 MeV for ^{9}Be. The solid curves were obtained as described in the text and normalized to the experimental data.

230 MeV. A hydrogen gas target (3500 kPa) was used. The results are given in Table I. Good agreement with the $E1$ PWBA calculated values is obtained for both pion energies (we expect $E1$ to be the dominant multipole at these energies).

The present results on ^{9}Be and ^{16}O represent the first near-tip virtual spectrum measurements for (e,π) reactions. While we find N_V to be higher than Eq. (1) by $25 \pm 10\%$ as stated earlier, this difference does not appear strikingly large when we consider the limitations of PWBA theory as applied to the spectrum tip region. In addition, Coulomb distortion of the electron waves might be expected to be important here, since the outgoing electron energy is low. The distorted-wave calculations of Onley and co-workers[15] for electron scattering at lower incident energies suggest, however, that the increase in N_V due to distortion is only at the few percent level for $Z < 10$. As regards the theory's neglect of nuclear size effects, in our experiment $qR > 1$, where q is the momentum transfer, so that a simple first-order treatment of size effects[16] cannot be made here. Shotter[17] has made PWBA Helm model calculations for $E1$ and $E2$ multipoles for electron scattering at lower incident energies and momentum transfers than for our case. At $E_0 = 100$ MeV and $A = 10$, for example, the N_V value for $E1$ is calculated to be reduced some 10% for the longitudinal component and significantly less for the transverse, as compared to the point nucleus value, with larger reductions predicted for $E2$. One might expect for $E1$, however, that the longitudinal component would be relatively unimportant near the end point. We cannot make a more quantitative assessment of the importance of such effects in the present work until further, more directly applicable, calculations are made.

In a recent (e,π) calculation, Furui[18] attempted to treat the dependence on outgoing electron angle more carefully than in the forward peaking approximation (FPA), and found N_V in the near-tip region to be about 0.4 times the value given by Eq. (1). This result is clearly not consistent with the present experiment. Asai and Tomusiak[19] have considered the accuracy of the FPA for $^3He(e,\pi^+)$ and find it yields results for that case within 10% of results obtained by integrating over all outgoing electron directions.

In the future, direct theoretical calculations of $d^2\sigma/dT_\pi d\Omega_\pi$ for (e,π) reactions which utilize the full amplitude for electroproduction, such as that of Fubini *et al.*,[20] may be available in place of analyses of the experiments using the virtual photon spectrum. Such calculations are now underway for $^1H(e,\pi^+)$.[21,22] The present results will in any case provide a good test of electroproduction theory as used in such calculations.

We conclude by noting that the PWBA formula for the virtual photon spectrum works much better than anticipated, considering its neglect of Coulomb final state and finite nuclear size effects. (However, a recent calculation[23] indicates the distortion due to Coulomb effects are smaller than about 1% within a few MeV of the tip.) The present results have enabled us to obtain (γ,π) cross sections from single-arm electroproduction experiments,[2] with moderate confidence. It should also be pointed out that the real bremsstrahlung spectrum near the tip is also not experimentally well verified. Our $(R/V)_{\text{theor}}/(R/V)_{\text{expt}}$ results (Table I) are not good enough to indicate whether there is any trend with Z in these values. Therefore, both a more accurate experiment and measurements over a larger range would be desirable. Caution should be used in comparing the proton results with those for ^{16}O and ^{9}Be as given in Table I since the former are far from the end point. The agreement with theory for the proton may be due to its low Z (and therefore small Coulomb corrections) or to the large distance from the end point, or both. In this connection we note that previous virtual photon (γ, π) experiments[8,9] performed far from the end point in ^{12}C, ^{27}Al, and ^{51}V are in agreement (within the errors of ~30%) with virtual photon theory. An accurate systematic study which examines the dependence both on Z and on the distance from the end point is therefore needed.

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