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Radial sensitivity of elastic scattering

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A new technique has been developed, employing a localized perturbation of the radial nuclear optical potential, which permits direct investigation of the sensitivities of optical model analysis of elastic scattering data to the details of the radial potential. It is found that both light- and heavy-ion scattering probe primarily the nuclear surface region. Higher energy scattering data probe further into the interior than lower energy data. The value of the potential at the center of the nucleus cannot be determined, but only inferred if a fixed parametrization such as Woods-Saxon geometry is specified. In addition, it is found that the region of radial sensitivity of the imaginary potential is systematically closer to the center of the nucleus than is that of the real potential.

[NUCLEAR REACTIONS Radial sensitivity of optical model calculations to elastic $\sigma(\theta)$ data analyzed for $p+{}^{12}C$, $\alpha+{}^{58}Ni$, ${}^{16}O+{}^{28}Si$ at low and intermediate incident energies.]

I. INTRODUCTION

The elastic scattering of heavy ions has been studied in a large number of experimental investigations, and a large body of data is available on elastic cross sections. Rather surprisingly, analysis of these data has produced little firm information on the interaction potential between a heavy ion and a nucleus. Little agreement is found among such analyses on such questions as the depth, the radius, the diffuseness, or the appropriate geometry for the nuclear potential, and potentials cannot be extrapolated with confidence from one energy to another or from one projectile or target to another.

A closer look at the dynamics of heavy-ion elastic scattering reveals the root cause of this situation: a veritable "iron curtain" of strong absorption hides most of the features of the nuclear potential from easy investigation through elastic studies. Particles which venture into the stronger parts of the nuclear potential are absorbed and never emerge, while particles further out are affected mainly by the strong Coulomb repulsion between a heavy ion and a nucleus. Only a very small fraction of the flux of elastically scattered particles carries information on the details of the nuclear potential, and this information bears on the potential only in a localized radial region.[1] Thus any derived potential which approximates the interaction in this region will give acceptable fits to elastic scattering data. With lighter ions, however, it is commonly considered that the interior region is more accessible.

It is therefore important to know what radial regions of the nuclear potential can be considered to be well mapped by the analysis of elastic scattering data and which regions are still *terra incognita*. In other words, one would like to be able to perform an elastic scattering cross section measurement, analyze the data with the optical model, and plot the resulting potential with confidence limits showing to what degree various portions of the potential have been determined with a given data set. Work by Satchler[2] and by the Brookhaven Group[3] has shown that by using radial cutoffs, the sensitive region of the potential can be bracketed. In the present work we proceed a step further, providing a semiquantitative measure of the sensitivity as a function of radius.

II. THE NOTCH PERTURBATION METHOD

The basic approach which has been used here is to introduce a localized perturbation into the radial optical potential (either the real or the imaginary part) and to observe the effect of such a perturbation on the predicted cross section as the perturbation is moved systematically through the potential. The perturbation employed has the effect of cutting a notch out of the potential, i.e., reducing it to zero in some localized region and then returning it to its normal value. Two slightly different techniques have been used to accomplish this. The first is a variant of the familiar Woods-

Saxon radial form factor

$$f(R, a, r) = [1 + \exp(r - R)/a]^{-1} .$$

We multiply the potential to be perturbed by the factor:

$$g(R, a, d, r) = \{1 - 4df(R, a, r)[1 - f(R, a, r)]\} . \quad (1)$$

Thus the potential itself, if it is of the Woods-Saxon form, becomes

$$V(r) = V_0 f(R_r, a_r, r)\, g(R', a', d, r) ,$$

where R_r and a_r are the familiar Woods-Saxon parameters, d is the fraction by which the potential is reduced, r' determines the position of the notch, and a' determines the width of the notch. The second technique simply reduces the nuclear potential by a factor of $(1-d)$ over a region of width a' centered at R'. These two techniques give essentially identical results (including the oscillations discussed below). Figure 1 shows a typical potential perturbed with the two different notch functions.

With this method we adjust the width $a'$and/or the cutdown fraction d to a suitable size (as will be described below), and move the notch radially through the potential. At radial positions where the calculation has no sensitivity to the details of the potential, the presence of the notch will have no effect on the calculation. At positions where the calculation strongly depends on the details of the potential, the results of the calculation will be strongly altered by the perturbation. Thus the degree to which, in whatever way, the results of the calculation are altered gives us a semiquantitative measure of the potential sensitivity as a function of radial position. Since such calculations are ultimately to be compared with experimental data, however, it would seem to be more

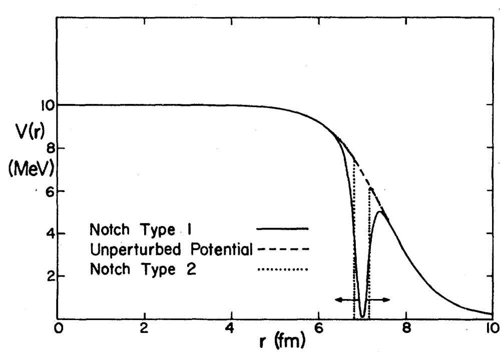

FIG. 1. Comparison of unperturbed potential with the two forms of perturbation described in text. The dashed curve shows the derivative Woods-Saxon notch (type 1), while the dotted curve shows the step function notch (type 2). The potential shown is the global $E18$ potential (Ref. 10).

meaningful to compare perturbed calculations with experimental data than with an unperturbed calculation. In particular, we have used the χ^2 value of a fit to data to measure the effect of perturbing the fit potential.

In the performance of these calculations, it was necessary to establish a (somewhat arbitrary) criterion for the width a', and the cutdown fraction d of the perturbation used at each energy and with each data set investigated. It is well known that $R_{\text{turn}}(l_g)$, the semiclassical distance of closest approach for the grazing partial wave,[4] is a point of particular sensitivity in heavy-ion optical model calculations, so we have established the criterion for a notch width at $R' = R_{\text{turn}}(l_g)$. At the lower energies we set d equal to 1 and adjust a' until the χ^2 for the perturbed calculation becomes roughly 100 times worse than its unperturbed value. At higher energies this procedure cannot be used because of problems arising when a' becomes comparable to the integration step size dr. Therefore at higher energies we set a' equal to $2dr$ and vary d until a similar χ^2 criterion is achieved.

It was found that in some situations, particularly those involving relatively high bombarding energies, a "beating" between the R' steps and the integration steps could be observed. It was therefore decided that R' would always be made an exact multiple of the radial integration step. This eliminated the beating effect and permitted better investigation of the radial sensitivities of the potential. An additional problem was encountered when the very high energy data sensitivity was probed. Perturbations of the potential in the sensitive radial region caused shifts in the predicted oscillatory structure of the angular distributions. In fact, the phase between the oscillatory structure of the data and the predicted cross section was found to vary fairly smoothly with the position of the notch. Therefore, the mathematical quality of the fits to the data (χ^2) can show spurious oscillations in the sensitive region. Such oscillations are apparent in the analysis of oxygen scattering at 1 and 2 GeV, as will be discussed below.

Standard heavy-ion optical model programs HOP-TWO[5] and GENOA[6] were modified to permit iterated calculations at successive values of R'. These calculations proved to be fairly lengthy, since each sensitivity plot requires on the order of 30 optical model calculations. It is clear that the calculation time could be greatly improved by storing the unperturbed optical model wave function and handling the perturbed calculations with first order perturbation theory, but this approach has not been used up to now.

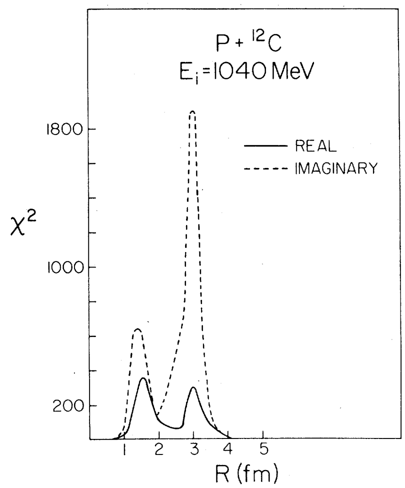

FIG. 2. Radial sensitivity of relativistic optical model calculations for 1040 MeV $p+{}^{12}C$ elastic scattering. The solid curve shows the result of perturbing the real potential, while the dashed curve indicates the same for the imaginary potential.

III. LIGHT-ION ELASTIC SCATTERING RADIAL SENSITIVITY

One might expect the nuclear interior to be most intimately probed by the scattering of very energetic protons from light nucleus, e.g., the scattering 1 GeV protons from ^{12}C as shown in Fig. 2. The sensitivity test shown was obtained using data of the Saclay group[7] and a potential similar to one employed by Rost.[8] As is shown in Fig. 2, even this extreme case indicates little sensitivity in the scattering to the details of the potential in the interior.

We have applied the above method to the elastic scattering of alphas on ^{58}Ni at 139 MeV, using the data and potential of Goldberg *et al.*[9] The resultant plots of χ^2 vs radius are shown in Fig. 3. It has been shown[9] that such high energy data are capable of resolving the optical model parameter ambiguities normally associated with a six parameter Woods-Saxon form, i.e., a unique value of V_0 has been obtained. Nevertheless, it is clear from Fig. 3 that $V(r=0)$ has not been determined. Thus we observe that (a) even light-ion scattering does not probe into the nuclear interior, and (b) with a fixed parametrization of the potential, for example Woods-Saxon geometry, it is not necessary to measure $V(r=0)$ in order to determine V_0.

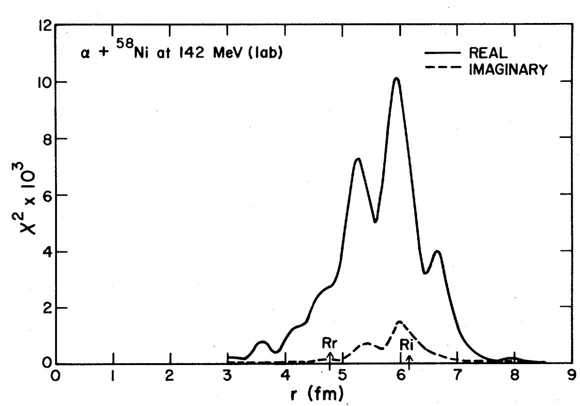

FIG. 3. Radial sensitivity of optical model calculations for 142 MeV $\alpha+{}^{58}Ni$ elastic scattering. The optical model real and imaginary radius parameters are shown.

IV. HEAVY-ION ELASTIC SCATTERING RADIAL SENSITIVITY

Figure 4 shows some sensitivity functions calculated for $^{16}O+{}^{18}Si$ elastic scattering at several energies, using potential $E18$, a global potential which has been found to fit $^{16}O+{}^{18}Si$ data between 33 and 215.2 MeV.[10] The near-Coulomb-barrier (33 MeV) sensitivity function for the real potential forms a rather broad peak centered at about 9 fm with full width at half maximum (FWHM) of about 2 fm, which provides excellent sensitivity to the nuclear potential in the tail region at this bombarding energy. The imaginary potential sensitivity function is doubled peaked, with a broad peak corresponding to that of the real potential in the tail region and a narrower peak with its maximum at 7.4 fm, which is actually inside the real potential radius R_0 of 7.5 fm, but outside the imaginary potential radius R_i of 6.8 fm.

Figure 4 also shows the sensitivity function for the potential $E18$ at a bombarding energy of 55 MeV. Here the sensitivity function is sharply peaked at $R_{\mathrm{turn}}(l_g)$. This condition of sharp localization of the real potential sensitivity at this point is found to be present in all data examined between 38 and 81 MeV, and thus all data sets in this region measure the real potential at essentially the same point. We note, however, that the sensitivity function for the imaginary potential has three distinct peaks, none of which corresponds to the peak of the real potential. This provides evidence that data in this region is actually sensitive to the imaginary potential over a larger radial region than that of the real potential. Thus, the imaginary potential is better determined in the surface region than is the real potential from these data.

Figure 4 shows the sensitivity function for a bombarding energy of 215.2 MeV. Here we see the real potential function has two prominent peaks, a small peak just beyond $R_{\mathrm{turn}}(l_g)$, and a larger peak at essentially the real potential radius of 7.5 fm. Again the imaginary potential is triple peaked and

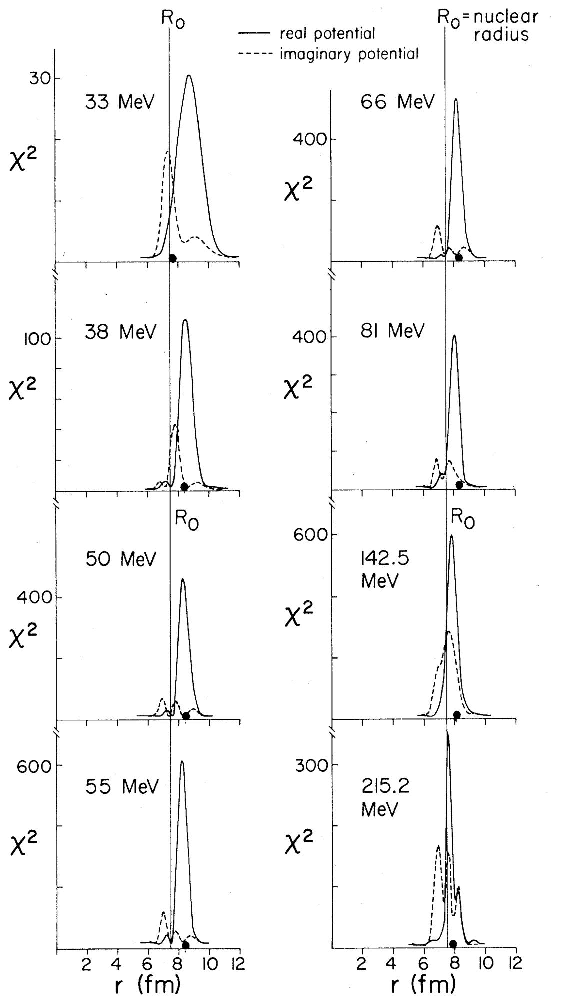

FIG. 4. Sensitivity functions for the system $^{16}O+^{28}Si$ at laboratory bombarding energies between 33 and 215.2 MeV, as evaluated using global potential $E18$ and the experimental data from Ref. 10. The solid curve shows the result of perturbing the real potential, while the dashed curve shows the same for the imaginary potential. For reference, the radius R_0 of the potential is indicated by a vertical line, and the values of $R_t(l_g)$ are shown by a black dot.

sensitive over a broader region than the real potential function. The inner peak in the real potential sensitivity function may be associated with far-side interference effects which are expected at these energies, and which should lead to sensitivities deeper in the potential.

Figure 5 shows a summary of the results of the analyses shown in Fig. 4, plotting the central position and half-maximum values of the real potential sensitivity peaks as a function of bombarding energy, for the $^{16}O+^{28}Si$ system. Also shown on the figure are R_o, the value of the real radius parameter, and the position of the Coulomb barrier.

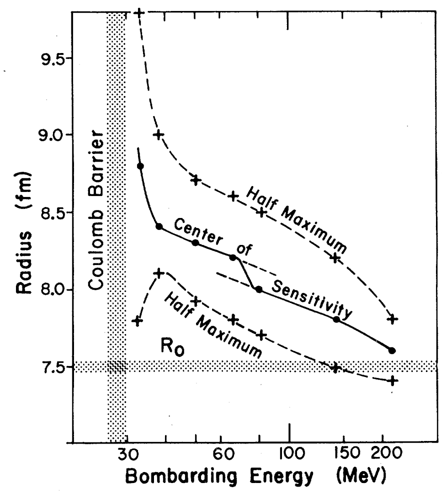

FIG. 5. Summary of sensitivity functions of the system $^{16}O+^{28}Si$, as presented in Fig. 4. The radial positions of sensitivity function maxima (solid curve) and half maxima (dashed curves) are plotted against laboratory bombarding energy. The radius R_0 of potential $E18$, and the energy location of the Coulomb barrier for the system are shown for reference by the shaded regions.

The strong dependence of the sensitive radius on bombarding energy is apparent, as is the sensitivity inside the strong absorption radius which occurs at higher energies.

We should emphasize that the sensitivity functions presented here are specific to a particular potential, in this case $E18$, and that other potentials may show different sensitivities. However, investigations of the $^{16}O+^{28}Si$ systems with global potentials[10] other than $E18$ have shown very similar sensitivity functions.

V. HIGH ENERGY HEAVY-ION SENSITIVITIES

The calculations with the $^{16}O+^{28}Si$ data set discussed above indicate that even at $E=215$ MeV the data are not sensitive to radii inside 6.5–7.0 fm. It has been shown that an energy dependent folding model calculation[11] is capable of reproducing the complete data set essentially as well as the Woods-Saxon potential $E18$. The folding model, however, ignores Pauli exclusion effects, while potential $E18$ is exceptionally shallow in the interior region. Both of these objections may not be relevant to the existing data set, however, since only the surface region is probed. Thus it is of interest to consider going to even higher energies in order to (hopefully) probe even further into the nucleus. We have made nonrelativistic calculations using dummy data generated with potential $E18$ for incident energies of 1 and 2 GeV as shown in Fig. 6. It appears that one can push into the interior only an additional Fermi or two, even at

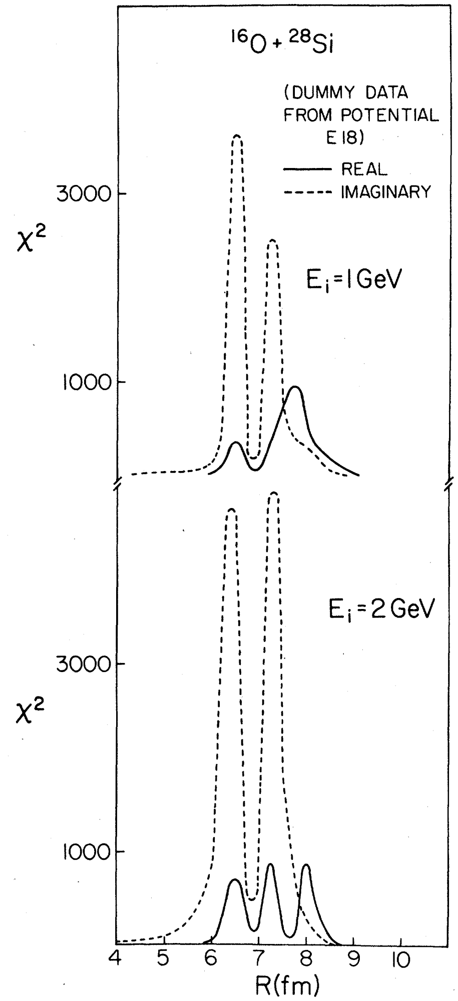

FIG. 6. Radial sensitivity of optical model calculations for 1 and 2 GeV (total energy) $^{16}O+^{28}Si$ elastic scattering using "data" generated from potential $E18$.

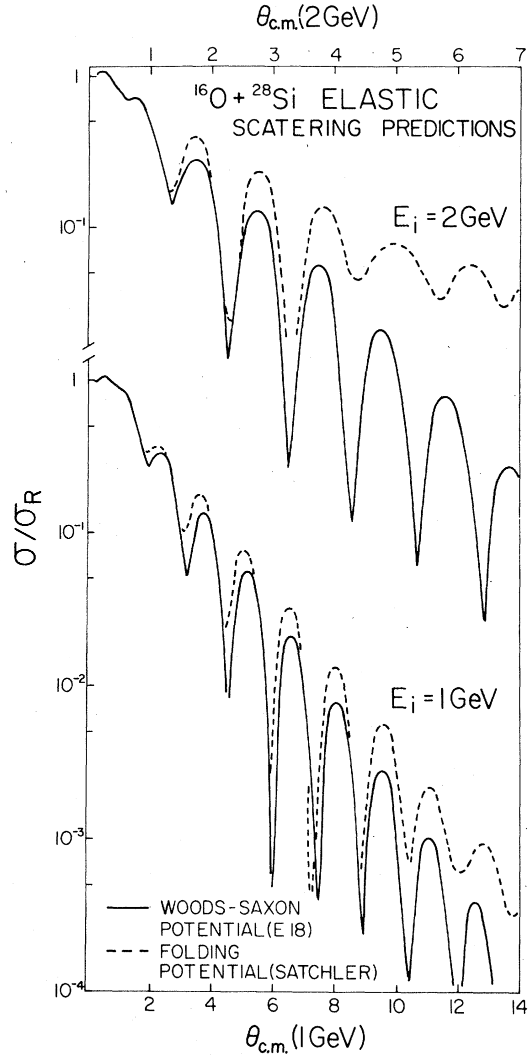

FIG. 7. Optical model predictions for 1 and 2 GeV elastic scattering using the phenomenological potential $E18$ and the microscopic folding model potential due to Love and Satchler. The two approaches predict nearly identical angular distributions at lower incident energies.

these high energies. The validity of both potential $E18$ and the folding potential at these energies is questionable, however, because the opening of pion production channels might be expected to increase the absorption even further, restricting the sensitive radial region to larger radii.

The same notch parameters were used in probing the real potential and the imaginary potential. Notice that the increase in χ^2 for perturbation of the imaginary potential, as seen in Fig. 6, is much larger than the increase arising from the perturbation of the real potential. This implies that the shape of the angular distribution at these energies is more sensitive to the details of the imaginary potential than the real potential. This behavior is opposite that seen in the analysis of data at lower energies (Fig. 4) where the excursions of χ^2 are at least a factor of 2 smaller for the imaginary potential than for the real potential. We note that the $p+^{12}C$ study at 1 GeV shown in Fig. 2 exhibits the same strong sensitivity in the imaginary potential. This dominance in sensitivity of the imaginary potential over the real potential will strongly affect what can be learned from high energy elastic scattering data.

Fortunately, however, this additional Fermi or two may be enough to gain considerably more information about the interior potential. Figure 7 displays predictions at 1 and 2 GeV for $^{16}O+^{28}Si$ elastic scattering using the folding potential[11] and potential $E18$. Strong differences are observed, particularly at 2 GeV.

VI. CONCLUSION

We have demonstrated a technique for finding the sensitive radial region of a potential fit to experiment elastic scattering data. We find that neither light- nor heavy-ion scattering at any energy probes the interior of the nucleus; however, higher energy data do probe further into the surface region than do low energy data. The sensitivities to the real and imaginary potential are quite different, with the imaginary potential being studied over a broader range of radius than the real potential.

It would appear fruitful to extend these studies to reaction data. If a quantum-mechanical description of fusion processes were to be developed, it would be interesting to compare the radial sensitivities of elastic scattering and fusion reactions.

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