Comments on two tests of partial conservation of axial vector current in ^{3}He

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Another look is taken at the transition amplitude for radiative muon capture. We conclude that the standard, or diagrammatic, amplitude first given by Opat for hydrogen is a good approximation to the systematic expansion of Adler and Dothan, and use it to calculate the angle-integrated photon spectrum in $^3\mathrm{He}\mu^- \to {}^3\mathrm{H}\gamma\nu$. The result is in fair agreement with Beder and Fearing. Also, the "soft-pion" prediction for $^3\mathrm{He}\pi^- \to {}^3\mathrm{H}\gamma$ is compared with experimental data.

[RADIOACTIVITY $\mu^- \, {}^3\mathrm{He} \to \nu_\mu {}^3\mathrm{H}\gamma$, $\pi^- \, {}^3\mathrm{He} \to {}^3\mathrm{H}\gamma$; theories of radiative muon capture; tests of PCAC and soft-pion predictions.]

I. INTRODUCTION

Interest in radiative muon capture in light nuclei has recently been aroused, since the relatively small cross sections should be measurable at high-flux meson beam facilities. At least one experiment, in ^{3}He, is currently planned at LAMPF.[1]

Besides being of intrinsic interest as a fundamental semileptonic weak process, radiative muon capture is expected to be particularly sensitive to the hadronic weak pseudoscalar coupling $g_p(Q^2)$. The photon in the final state allows the momentum transferred at the weak vertex to become timelike and attain a maximum value $Q^2 \cong m_\mu{}^2$, enhancing the pion pole which partially conserved axial vector current (PCAC) predicts will dominate g_p. This is in contrast with the nonradiative reaction where $Q^2 \cong -m_\mu{}^2$. The problem is to determine just how g_p enters the radiative transition amplitude. This question will be addressed in some detail in the next section.

Uncertainties in the theoretical interpretation of previous measurements of radiative muon capture in ^{40}Ca and ^{16}O arise from the complexity of these nuclei.[2-8] The cleanest experiment would be done in hydrogen, but it should be possible to avoid some of these problems in light nuclei such as ^{2}H and ^{3}He, where the nuclear structure and spectrum are fairly well understood and the final state is unique. In addition, the reaction rate in nuclei is enhanced by a factor of Z^3 with respect to hydrogen.

Numerical results for ^{3}He have recently been given by Beder,[9] Fearing,[10] and Hwang and Primakoff.[11] The first two authors base their calculations on the diagrammatic amplitude first given by Opat[12] for hydrogen and make no nonrelativistic approximation as has been done in the past. (Presumably, this feature does not add accuracy since the invariant amplitude almost certainly neglects terms explicitly the same size as the relativistic corrections.) Despite the fact that Beder and Fearing do what appear to be identical calculations, their results do not coincide. Partly to resolve this discrepancy the present calculation was undertaken.

Hwang and Primakoff derive their own amplitude from very general assumptions and, using the leading nonrelativistic term, obtain results markedly different from both Beder and Fearing. By comparing the diagrammatic amplitude with Hwang's and Primakoff's, Fearing has accounted for this disagreement as well as provided an excellent criticism of the Hwang-Primakoff method. (Recently Hwang has shown that a radiative capture experiment could both test PCAC and distinguish the correct theory.[13])

All three authors adopt an "elementary particle treatment" in which initial and final nuclei are represented by fundamental Dirac fields possessing observed charges and anomalous magnetic moments. This approach efficiently avoids uncertanties inherent in the impulse approximation. Nuclear properties, such as wave function correlations, are subsumed by the measured weak elastic form factors of the $^3\mathrm{He} \to {}^3\mathrm{H}$ transition. The prediction of PCAC for g_p for this case is probably less accurate then for the nucleon transition. For example, owing to an anomalous breakup threshold, continuum contributions to g_p begin before $(3m_\pi)^2$. However, such corrections are not expected to greatly affect the usual assumption of pole dominance,[14] which leads to

$$g_p(Q^2) = \frac{2M_{^3\mathrm{He}}\, g_A(Q^2)}{Q^2 - m_\pi{}^2} \quad (|Q^2| < m_\pi{}^2). \tag{1}$$

Other nuclear nonradiative form factors are measured in β decay, μ capture, and electron scattering, and determined by isospin rotation (conserved vector current hypothesis).[15]

The most obvious way to relate the radiative amplitude to these elastic form factors is via Low's theorem for soft bremsstrahlung.[16] However, the prescription loses accuracy as the threshold for an intermediate channel is approached. This is the case in radiative muon capture since the photon may have $k_0 \cong m_\mu \lesssim m_\pi$. We will argue that the generalization of Low's theorem to the weak, axial vertex by Adler and Dothan[17] successfully removes this problem. It should be observed that Low's theorem, and its generalization, gives the leading terms in a rigorous expansion of the amplitude in powers of the momentum transfers, and does not concern itself simply with a certain set of Feynman diagrams.

Hwang and Primakoff[11] obtain an amplitude in a systematic but, it seems, *ad hoc* way by introducing a "linearity hypothesis." The result, as pointed out by Fearing, violates Low's theorem. This will be briefly discussed in the next section.

The model employed by Beder and Fearing in their calculations is usually cast as a set of Feynman diagrams. As such, it may seem the least general and systematic approach. However, it will be shown to be a valid approximation to the Adler-Dothan amplitude, which contains some unknown structure terms.

In the next section, the transition amplitude will be discussed in some detail and we will outline an alternative approach to the result first obtained by Adler and Dothan. However, whereas Adler and Dothan were primarily interested in determining higher order effects, we will try to emphasize the use of PCAC to resolve the problem discussed above. This done, higher order terms will be dropped to arrive at the diagrammatic model which is employed in Sec. III to calculate the photon spectrum for radiative muon capture. Section IV is concerned with comparing the PCAC prediction for the radiative capture of soft pions on ^{3}He with experiments.

II. THE TRANSITION AMPLITUDE

The radiative capture rate can be written, to lowest order in the weak and electromagnetic couplings, as

$$\Gamma(^3\mathrm{He}\mu^- \to {}^3\mathrm{H}\gamma\nu) = \frac{|\phi_{1s}(0)|^2}{(2\pi)^5}\left(\frac{eG}{\sqrt{2}}\right)^2 \times \int \frac{M_{^3\mathrm{He}}}{E'}\frac{1}{2k_0}\tfrac{1}{2}\, d^3\vec{k}\, d^3\vec{q}'\, d^3\vec{p}'\,\delta^4 \times (p+q-p'-q'-k)\tfrac{1}{4}\sum_{\mathrm{spins}}|T|^2 \tag{2}$$

for $^3\mathrm{He}(p)+\mu^-(q)\to{}^3\mathrm{H}(p')+\nu_\mu(q')+\gamma(k)$. The T-matrix element is given by

$$T = T^{(\mathrm{lep})}+T^{(\mathrm{had})} = -\bar{u}(\vec{q}')\gamma^\mu(1+\gamma_5)\frac{\not{q}-\not{k}+m_\mu}{-2q\cdot k} \times \not{\epsilon}^* u(\vec{q})\langle{}^3\mathrm{H}|J_\mu^{\omega k}(0)|{}^3\mathrm{He}\rangle + \bar{u}(\vec{q}')\gamma^\mu(1+\gamma_5)u(\vec{q})(e^{-1}) \times \langle{}^3\mathrm{H}\gamma|J_\mu^{\omega k}(0)|{}^3\mathrm{He}\rangle\,. \tag{3}$$

Dirac matrix conventions and metric are those of Bjorken and Drell[18]; $\phi_{1s}(0)$ is the muonic wave function averaged over the ^{3}He nucleus:

$$|\phi_{1s}(0)|^2 = \frac{0.96}{\pi}\left(\frac{2\alpha m_\mu}{1+m_\mu/M_{^3\mathrm{He}}}\right)^3;\quad G=\frac{10^{-5}}{m_p^{\,2}}\,,$$

$e=(4\pi\alpha)^{1/2}$, and $J_\mu^{\omega k}=V_\mu+A_\mu$ is the weak hadronic current; state vectors are asymptotic; isotopic indices are suppressed. Also, define $Q=q'-q$ and $P=p-p'$. Equation (2) is valid for a statistical spin ensemble. The first term in Eq. (3) is the amplitude for radiation by the muon; the matrix element is

$$\langle{}^3\mathrm{H}|J_\mu^{\omega k}|{}^3\mathrm{He}\rangle = \bar{u}(\vec{p}')[f_V(P^2)\gamma_\mu + if_M(P^2)/2M\sigma_{\mu\lambda}P^\lambda + g_A(P^2)\gamma_\mu\gamma_5 + g_p(P^2)P_\mu\gamma_5]u(\vec{p}) = \langle{}^3\mathrm{H}|(V_\mu+A_\mu)|{}^3\mathrm{He}\rangle\,. \tag{4}$$

Second class currents are assumed absent. Since the leptonic currents in Eq. (3) are exact, radiative muon capture probes the two hadronic matrix elements. In contention is an approximate expression for $\langle{}^3\mathrm{H}\gamma|J_\mu^{\omega k}|{}^3\mathrm{He}\rangle$ in terms of known amplitudes and g_p. Any such expression will be constrained to satisfy current conservation and the hypotheses of conserved vector current (CVC) and partially conserved axial vector current (PCAC), at least in some approximation. Making the decomposition

$$\langle{}^3\mathrm{H}\gamma|J_\mu^{\omega k}|{}^3\mathrm{He}\rangle = \langle{}^3\mathrm{H}\gamma|V_\mu|{}^3\mathrm{He}\rangle + \langle{}^3\mathrm{H}\gamma|A_\mu|{}^3\mathrm{He}\rangle \equiv V_{\mu\lambda}\epsilon^{\lambda *} + A_{\mu\lambda}\epsilon^{\lambda *}\,.$$

Standard reduction formulas give

$$k^\lambda V_{\mu\lambda} = -\langle{}^3\mathrm{H}|V_\mu|{}^3\mathrm{He}\rangle\,, \tag{5a}$$

$$k^\lambda A_{\mu\lambda} = -\langle{}^3\mathrm{H}|A_\mu|{}^3\mathrm{He}\rangle\,, \tag{5b}$$

$$Q^\mu V_{\mu\lambda} = \langle{}^3\mathrm{H}|V_\lambda|{}^3\mathrm{He}\rangle\,, \tag{6a}$$

$$Q^\mu A_{\mu\lambda} = \langle{}^3\mathrm{H}|A_\lambda|{}^3\mathrm{He}\rangle + D_\lambda\,, \tag{6b}$$

with

$$D_\lambda = -\int d^4x\, e^{-ik\cdot x}\langle{}^3\mathrm{H}|T(\partial^\mu A_\mu(0)J_\lambda^{\mathrm{EM}}(x))|{}^3\mathrm{He}\rangle$$

satisfying $k^\lambda D_\lambda = -\langle{}^3\mathrm{H}|(Q+k)^\lambda A_\lambda|{}^3\mathrm{He}\rangle$. In Eq. (6) "seagull" and Schwinger terms[19] are assumed to cancel as they do in Eq. (5).[20]

The low energy expansions of Low[16] and Adler and Dothan[17] are based on the fact that, in an expansion of the amplitude in powers of Q and k, the leading behavior comes entirely from the graphs in which currents couple to external particle lines (i.e., the Born graphs). These graphs, in turn, are determined by known elastic amplitudes. Higher order terms are then obtained by requiring that the divergence equations be satisfied to that order. For example, Eqs. (5a) and (6a) uniquely determine $V_{\mu\lambda}$ up to terms $O(Q,k)$:

$$V_{\mu\lambda}=\bar{u}(\vec{p}')\Big\{\Gamma_\lambda^{^3\mathrm{H}}(k)S(p'+k)\Big[f_V(Q^2)\gamma_\mu+i\frac{f_M(Q^2)}{2M}\sigma_{\mu\nu}Q^\nu\Big]+\Big[f_V(Q^2)\gamma_\mu+i\frac{f_M(Q^2)}{2M}\sigma_{\mu\nu}Q^\nu\Big]$$
$$\times S(p-k)\Gamma_\lambda^{^3\mathrm{He}}(k)-i\frac{f_M(Q^2)}{2M}\sigma_{\mu\lambda}-2f_V'(0)(Q_\lambda\gamma_\mu+k_\mu\gamma_\lambda-g_{\mu\lambda}\not{k})$$
$$-2i\frac{f_M'(0)}{2M}(Q_\lambda\sigma_{\mu\nu}Q^\nu-k_\mu\sigma_{\lambda\nu}k^\nu)+O(Q^2,k^2,Qk)\Big\}u(\vec{p})\,, \tag{7}$$

with $S(p)=(\not{p}-M)^{-1}$,

$$\Gamma_\lambda^{^3\mathrm{H},\,^3\mathrm{He}}(k)=e^{^3\mathrm{H},\,^3\mathrm{He}}\gamma_\lambda+\frac{\mu^{^3\mathrm{H},\,^3\mathrm{He}}}{2M}\sigma_{\lambda\nu}k^\nu\,;$$

$f_V'(0)=[df_V(Q^2)/dQ^2]_{Q^2=0}$, etc. A detailed derivation of $V_{\mu\lambda}$ is given by Christillin and Servaido.[21,22]

It is important to note that the accuracy of this expression depends upon the Taylor expanded amplitudes being slowly varying in the interesting regions of Q and k, i.e., that $f_V(Q^2)+2Q\cdot kf_V'(0)\cong f_V(Q^2+2Q\cdot k)$ be an acceptable approximation. This is the case for f_V and f_M, which should be dominated by the propagators of heavy particles; in fact, the diagrammatic expansion neglects all variation of f_V and f_M and drops the last two terms of Eq. (7).

Determination of $A_{\mu\lambda}$, however, is hampered by just this problem since expansion of $g_p(Q^2)\sim(Q^2-m_\pi^2)^{-1}$ is impractical for $Q^2\lesssim m_\pi^2$. It is necessary to treat these "pion-pole" terms exactly, that is, to all orders in Q and k. Fortunately, the PCAC condition allows such an exact expansion to be done.[23] In general, that is, regardless of the mechanism of its realization, the PCAC hypothesis specifies how pion-pole terms enter a matrix element of the axial current. For example, Adler and Dothan use PCAC to relate $Q^\mu A_{\mu\lambda}$ to a matrix element of the pion current through the operator identity $\partial_\mu A^\mu(x)\sim\phi_\pi(x)$, with ϕ_π the pion interpolating field. D_λ is then interpreted as an off-shell radiative pion capture amplitude. Alternatively, we can obtain Adler and Dothan's result without resorting to an off-shell amplitude and in a way which explicitly shows that the Adler-Dothan expression is the leading term (expanded in Q and k) in a perturbation series in the small parameter ϵ characterizing the strength of the chiral symmetry breaking piece of the hadron Hamiltonian.[24] In this approach, PCAC is taken as an expression of chiral ($SU_2\otimes SU_2$) symmetry realized in the Goldstone mode by a triplet of massless pseudoscalar pions. The axial current is conserved, but care must be taken to extract poles due to couplings to the massless Goldstone bosons. For $A_{\mu\lambda}$, we write

$$\epsilon^{\lambda *}A_{\mu\lambda}=\frac{Q_\mu}{Q^2}f_\pi\langle{}^3\mathrm{He}\,\gamma\,|\,{}^3\mathrm{He}\,\pi(Q)\rangle+\epsilon^{\lambda *}\bar{A}_{\mu\lambda} \tag{8}$$

with $f_\pi=0.94m_\pi$ the pion decay constant, and $\bar{A}_{\mu\lambda}$ regular at $Q^2=0$. D_λ vanishes, and Eqs. (5) and (6) become

$$k^\lambda A_{\mu\lambda}=k^\lambda\bar{A}_{\mu\lambda}=-\langle{}^3\mathrm{H}\,|A_\mu(0)|\,{}^3\mathrm{He}\rangle\,, \tag{5$'$}$$
$$Q^\mu A_{\mu\lambda}=f_\pi T_{\pi\lambda}+Q^\mu\bar{A}_{\mu\lambda}=\langle{}^3\mathrm{H}\,|A_\lambda(0)|\,{}^3\mathrm{He}\rangle\,, \tag{6$'$}$$

since $T_{\pi\lambda}$ is on-shell. [Note Eq. (6′) is just what would be obtained by extracting from D_λ a pion pole and neglecting the remainder.] Proceeding as, for example, do Christillin and Servaido, and ultimately replacing the pion mass in Eq. (8) and in g_p, the result is precisely that of Adler and Dothan:

$$A_{\mu\lambda}=\bar{u}(\vec{p}')\{\Gamma_\lambda^{^3\mathrm{H}}(k)S(p'+k)[g_A(Q^2)\gamma_\mu\gamma_5+Q_\mu g_p(Q^2)\gamma_5]+[g_A(Q^2)\gamma_\mu\gamma_5+Q_\mu g_p(Q^2)\gamma_5]S(p-k)\Gamma_\lambda^{^3\mathrm{He}}(k)\}u(\vec{p})$$
$$-\bar{u}(\vec{p}')\gamma_5u(\vec{p})g_p[(Q+k)^2]g_{\mu\lambda}-\bar{u}(\vec{p}')\gamma_5u(\vec{p})\frac{(2Q+k)_\lambda}{2Q\cdot k}Q_\mu\{g_p[(Q+k)^2]-g_p(Q^2)\}$$
$$+2g_A'(0)\bar{u}(\vec{p}')(k_\mu\gamma_\lambda-g_{\mu\lambda}\not{k}-Q_\lambda\gamma_\mu)\gamma_5u(\vec{p})+\frac{Q_\mu f_\pi\bar{T}_{\pi\lambda}}{Q^2-m_\pi^2}+f_\pi R_{\mu\lambda}+O(Q^2,k^2,Qk)\,. \tag{9}$$

In Eq. (9), $\bar{T}_{\pi\lambda}$ is the non-Born part of $T_{\pi\lambda}$ and may be written in terms of photoproduction amplitudes[25]; $R_{\mu\lambda}$ is equivalent to the Adler-Dothan $O_{\mu\lambda}$.[26] Also, departures from pion-pole dominance of g_p have been neglected.[27] With $A_{\mu\lambda}$ written in this form, it is clear that, whereas Low's theorem would have approximated $\{g_p[(Q+k)^2]-g_p(Q^2)\}(2Q\cdot k)^{-1}$ by $g_p'(Q^2)$, PCAC determines the full pole structure.

Now, since we have found that the Adler-Dothan result corresponds to the chirally symmetric limit, i.e., a world in which the pion is massless, the higher order terms they neglect must either be independent of the pion mass, or vanish as $m_\pi^2 \to 0$. We are, therefore, justified in considering Q and k to be soft compared to the mass scale of these terms.

One further approximation can be made by dropping terms $O(Q,k)$ which survive the $m_\pi = 0$ limit, but retaining Q/m_π and k/m_π to all orders. This amounts to neglecting f_V', f_M', g_A' and $\bar{T}_{\pi\lambda}$ in Eqs. (7) and (9). What remains is just the diagrammatic model, which we therefore conclude is correct through $O(Q^0,k^0)$ and through all orders in $Q/m_\pi, k/m_\pi$; terms neglected by the diagrammatic model should be $\sim(m_\mu/m_p)$. (This explains our earlier comment regarding relativistic corrections.) Also, in this approximation, the diagrammatic model satisfies all of the divergence constraints.

Finally, we would like to comment on the work of Hwang and Primakoff.[11] As mentioned above, the amplitude they obtain by adopting a "linearity hypothesis" yields numerical results which differ considerably from those of the usual treatment. Fearing[10] pins down the primary source of this discrepancy after enumerating how Hwang and Primakoff differ with the diagrammatic amplitude. He correctly points out that the Hwang-Primakoff expression violates Low's theorem by changing the sign on certain $O(k^0)$ terms of $A_{\mu\lambda}$. The amplitude remains current conserving since the sign change amounts to the addition of terms separately gauge invariant; however, Low's theorem does not admit such ambiguity. On this basis alone, the Hwang-Primakoff result is dubious.

We might also point out that the Hwang-Primakoff result is at odds with PCAC; it appears the linearity hypothesis is simply not rich enough to make prudent use of this constraint. (While Fearing concludes that the hypothesis is too general, in this context it is more likely too limiting.) Thus, Hwang and Primakoff make no specification of D_λ in Eq. (6b), but determine it along with $A_{\mu\lambda}$ purely from gauge invariance, Eq. (5.6), the gauge condition on $k^\lambda D_\lambda$, and the strict limitations of the linearity hypothesis; PCAC is never employed [note Eq. (6b) contains no information unless an assumption is made about D_λ]. This is in great contrast to the interpretation of D_λ by Adler and Dothan or the alternative given here ($D_\lambda \equiv 0$). But such a course was necessary for Hwang and Primakoff. Any meaningful model of D_λ would be inconsistent with the linearity hypothesis, since the structure of the radiative form factors dictated by the hypothesis is insufficient to do justice to such a model.[28] Thus, it is not surprising that the result differs violently from the low energy expansion precisely in how g_p enters the amplitude.

III. NUMERICAL RESULTS

An amplitude corresponding to the diagrammatic model or to the Adler-Dothan expansion minus the terms mentioned above was used to calculate the photon spectrum in radiative muon capture on ^{3}He. The invariant amplitude and phase space were expanded in powers of $m_\mu/M_{^3\text{He}}$; the lowest order result[29] is shown in Fig. 1 for a statistical spin distribution, along with the results of Beder, Fearing, and Hwang and Primakoff. Figure 2 shows the high energy tail of the spectrum for several values of g_p. f_V, f_M, and g_A were evaluated at the phase space average of P^2, $P^2 = -\frac{3}{5}m_\mu^2$.[30] The total integrated rate can be given as a quadratic function of the scale factor $\xi = g_p/g_p$ (PCAC):

$$\Gamma(\text{sec}^{-1}) \cong \int_0^{m_\mu} dk_0 \frac{d\Gamma}{dk_0} = 0.50 + 0.06\xi + 0.05\xi^2 .$$

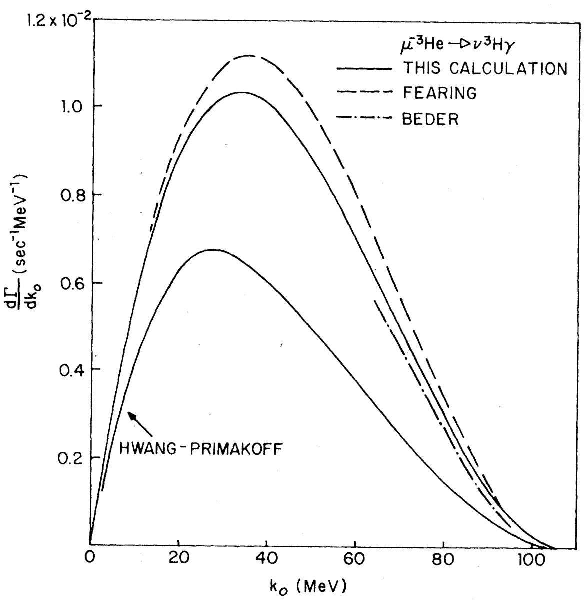

FIG. 1. Angle-integrated photon spectra for statistical spin average.

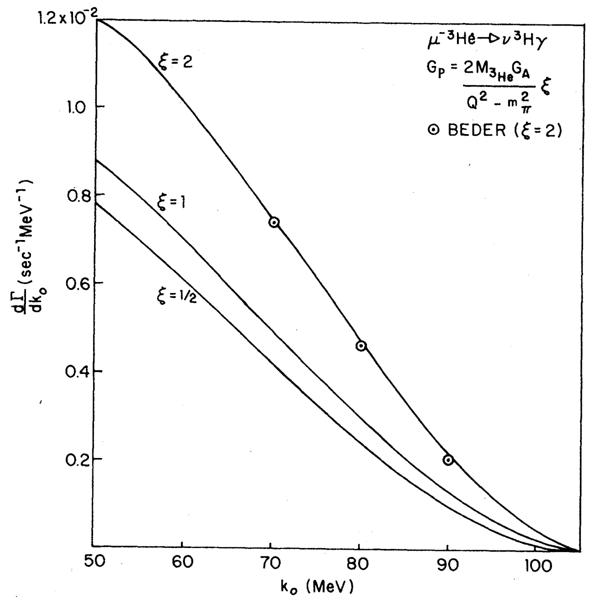

FIG. 2. High energy tail of photon spectrum for several values of g_p.

This result does not distinguish between the results of Beder and Fearing, and is probably consistent with both. A rough estimate of lowest order nuclear recoil contributions, fully included by Beder and Fearing, suggested suppression of the differential rate by between 2% and 15%. Both authors also allow f_V, f_M, and g_A to vary, though in slightly different ways. This conflicts somewhat with truncating the low energy expansion; however, we found that if true form factor variation [i.e., with Q^2 or $(Q+k)^2$ as the case might be] was allowed, corrections were negligible ($\lesssim 3\%$). On the other hand, the spectrum suffers a uniform 15% increase if all form factors are evaluated at zero momentum transfer. We expect to be very safe in assuming our result is accurate to 15%, which should easily account for the neglect of higher order effects as well as uncertainty in the form factors other than g_p.

In the next section, a very different approach to testing PCAC in ^{3}He is discussed.

IV. RADIATIVE PION CAPTURE

The PCAC constriant, Eq. (5′),

$$Q^\mu\langle {}^3\mathrm{H}\gamma | A_\mu(0) | {}^3\mathrm{He}\rangle = -f_\pi \langle {}^3\mathrm{H}\gamma | {}^3\mathrm{He}\pi(Q)\rangle + \langle {}^3\mathrm{H} | \epsilon^*_\mu A^\mu(0) | {}^3\mathrm{He}\rangle$$

relates the amplitude for the radiative capture of a zero mass pion to axial current matrix elements in a chirally symmetric world. Letting $Q_\mu \to 0$ while retaining $M_{^3\mathrm{He}} - M_{^3\mathrm{H}} \neq 0$ gives the "soft-pion" amplitude[31]

$$\langle {}^3\mathrm{H}\gamma | {}^3\mathrm{He}\pi^-(0)\rangle = f_\pi^{-1}\langle {}^3\mathrm{H} | \epsilon^{\mu *} A_\mu(0) | {}^3\mathrm{He}\rangle = \frac{g_A(0)}{f_\pi}\bar{u}(\vec{p}\,')\not\epsilon\gamma_5 u(\vec{p}) \,. \tag{10}$$

We endeavor to test this relation by extracting the matrix element on the left-hand side from the measured strong interaction width of the 1s level of pionic ^{3}He, the Panofsky ratio, and other measured branching ratios and comparing it with $g_A(0)$ measured in β decay of ^{3}He.

The most recent value for the strong width of the pionic 1s state is $\Gamma_{1s} = 36.7 \pm 7$ eV.[32] Also,[33]

$$P_3 = \frac{\sigma({}^3\mathrm{He}\pi^- \to {}^3\mathrm{H}\pi^0)}{\sigma({}^3\mathrm{He}\pi^- \to {}^3\mathrm{H}\gamma)} = 2.68 \pm 0.13\,,$$

$$B_3 = \frac{\sigma({}^3\mathrm{He}\pi^- \to dn\gamma + pnn\gamma)}{\sigma({}^3\mathrm{He}\pi^- \to {}^3\mathrm{H}\gamma)} = 1.12 \pm 0.05\,,$$

$$C_3 = \frac{\sigma({}^3\mathrm{He}\pi^- \to dn + pnn)}{\sigma({}^3\mathrm{He}\pi^- \to {}^3\mathrm{H}\gamma)} = 10.3 \pm 1.3\,.$$

If, for the moment, we consider these ratios to reflect absorption from the 1s level only,

$$\sigma({}^3\mathrm{He}\pi^- \to {}^3\mathrm{H}\gamma) \equiv \sigma(\mathrm{RPC}) = (\hbar)^{-1}\Gamma_{1s}(1 + P_3 + B_3 + C_3)^{-1} = 3.82\times 10^{15}(1 \pm 0.19)(\mathrm{sec}^{-1})\,. \tag{11}$$

On the other hand, σ(RPC) can be computed from Eq. (10). The rate is given by

$$\sigma = \frac{|\phi^\pi_{1s}(0)|^2}{(2\pi)^2} e^2 \int \frac{M_{^3\mathrm{He}}}{E'}\,\frac{1}{4km_\pi}\, d^3\vec{k}\, d^3\vec{p}\,'\delta^4(Q + p - k - p') \times \sum_{\mathrm{spins}} |\langle {}^3\mathrm{H}\gamma | {}^3\mathrm{He}\pi(0)\rangle|^2 = 2.40\times 10^{15}\left|\frac{g_A(0)}{f_\pi/m_\pi}\right|^2 (\mathrm{sec}^{-1})\,.$$

Comparing this with Eq. (11) gives

$$|g_A(0)| = 1.19(1 \pm 0.10)$$

in remarkable agreement with the measured value $|g_A(0)| = 1.22$.

It must be pointed out, however, that while the quoted Γ_{1s} is for the 1s level, the ratios P_3, B_3, and C_3 likely include some fraction of pions captured from the 2p state.[33] For such a light nucleus, the effect on the Panofsky ratio is expected to be small[34]; also, impulse approximation calculations produce values for B_3 in the 1s state which are compatible with the measured value.[35]

Thus, it is not improbable that the σ(RPC) extracted above is very close to the actual 1s rate (it is also in fair agreement with impulse approximation calculations[33,35]). In light of this, the success of the soft-pion relation represents a pleasing verification, within about 10%, of a "Goldberger-Treiman relation" for ^{3}He $\rightarrow$ ^{3}H.

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[20]Equation (5) is quickly obtained by replacing ϵ_λ^* by k_λ in Eq. (3). Absence of Schwinger and "seagull" terms in the result indicates that they cancel in the reduction treatment.

[21]P. Christillin and S. Servaido, Nuovo Cimento 42A, 165 (1977).

[22]The result given here is an amalgam of expressions given by Adler and Dothan, and Christillin and Servaido. As the latter point out, Adler and Dothan incorrectly omit $2f'_V(0)g_{\mu\lambda}\not{k}$; Christillin and Servaido omit the term proportional to $2if'_M(0)/2M$ which is, in fact, determined.

[23]For example, application of Low's theorem gives rise to a term proportional to $g'_p(Q^2) \sim (Q^2-m_\pi{}^2)^{-2}$, whereas consideration of PCAC gives, correctly, $(Q^2-m_\pi{}^2)^{-1}(Q^2+2Q\cdot k-m_\pi{}^2)^{-1}$; the difference is negligible only for $k_0 \ll m_\pi$.

[24]See, e.g., R. Dashen and M. Weinstein, Phys. Rev. 183, 1261 (1969); H. Pagels, Phys. Rep. 16, 221 (1975) and references therein.

[25]In obtaining Eq. (9), the pion amplitude has been modeled as

$$T_{\pi\lambda}=g_{\pi,\,{}^3\mathrm{He},\,{}^3\mathrm{H}}\,u(\vec{\mathrm{p}}')\,[\Gamma_\lambda^{{}^3\mathrm{H}}(k)S(p'+k)\gamma_5 + \gamma_5 S(p-k)\Gamma_\lambda^{{}^3\mathrm{He}}(k)]u(\vec{\mathrm{p}}) + g_{\pi,\,{}^3\mathrm{He},\,{}^3\mathrm{H}}\frac{(2Q+k)_\lambda}{2Q\cdot k}\bar{u}(\vec{\mathrm{p}}')\gamma_5 u(\vec{\mathrm{p}}) + \bar{\boldsymbol{T}}_{\pi\lambda}.$$

[26]Compare Ref. 16, Eq. (58).

[27]Compare Ref. 16, Eq. (82).

[28]For example, the form required by the linearity hypothesis precludes the pion current term $[\sim g_p(P^2) - g_p(Q^2)]$. In Ref. 12 Hwang makes a somewhat disingenuous choice for D_λ and observes that it is in "remarkable" agreement with the $A_{\mu\lambda}$ of Hwang and Primakoff. It disagrees, however, almost precisely to the extent that the pion current should enter $A_{\mu\lambda}$ (and D_λ). Since inclusion of this term, which is infeasible through the linearity hypothesis, leads inevitably to a reconcilation with Low's theorem [i.e., forces the correct sign in $O(k^0)$], this disagreement with PCAC is significant.

[29]The calculation retained f_V and g_A to zeroth order in (m_μ/M); f_V, g_p, and $\mu^V=\mu^{{}^3\mathrm{He}}-\mu^{{}^3\mathrm{H}}$ to first order. Compare Ref. 3, p. 427.

[30]Form factors were assumed to vary as $\exp[+12Q^2\,(\mathrm{GeV}^2)]$. Compare Ref. 1, p. 454 (there appears to be a sign error here).

[31]See, e.g., Erikson and Rho, Phys. Rep. 5, 57 (1972). The authors are primarily concerned with corrections to the $m_\pi=0$, limit which are beyond the scope of the present discussion. Equation (10) is equivalent to the dominant electric dipole amplitude.

[32]G. R. Mason *et al.*, TRIUMF Report No. 79-17.

[33]H. W. Baer, J. A. Bistirlich, K. M. Crowe, N. de Bottom, and J. A. Helland, Phys. Rev. Lett. 32, 1263 (1974).

[34]M. Ericson and A. Figureau, Nucl. Phys. B3, 609 (1967); B11, 621 (1979).

[35]A. C. Phillips and F. Roig, Nucl. Phys. A234, 378 (1975). The authors seem to discard the possibility that capture from the 2p state is important in radiative breakup. They also calculate the 2p rates for radiative capture and charge exchange and find them negligible compared to 1s rates.