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Boson expansion description of collective states in Ru and Pd isotopes

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We have applied the boson expansion method to describe the low-lying positive parity states of even-even Ru and Pd isotopes. Energy levels, $B(E2)$'s, branching ratios, and magnetic dipole and electric quadrupole moments have been calculated and are in good agreement with experiment. It is particularly emphasized that our theory describes rather well the properties of the three-phonon-like 3_1^+ state.

[NUCLEAR STRUCTURE $^{98-104}$Ru, $^{102-110}$Pd, energy levels, $B(E2)$'s, branching ratios, magnetic moments, static quadrupole moments, boson expansion.]

I. INTRODUCTION

In the past few years, we have been engaged in describing nuclear collective motions in terms of a boson expansion technique, and the results were reported in several publications.[1-4] The basic formulation of the calculation was first presented in Ref. 1, and Ref. 2 gave additional formulation as well as analyses of about ten collective nuclei in the $A = 100-200$ region. The work of Ref. 2 was further refined in Ref. 3, restricting our interest to the Sm isotopes. In Ref. 4, some of the more formal aspects of the boson expansion method as a whole were discussed, including the validity of the calculations that had been made. The purpose of the present paper is similar to that of Ref. 3, discussing in detail the properties of even Ru and Pd isotopes. (References to a number of earlier publications made by other authors concerning the boson expansion methods can be found in the above four papers.)

The nuclei considered in Ref. 2 were chosen from several regions of the periodic table by virtue of their having some distinctive feature in their spectrum, that is, harmonic, gamma-unstable, prolate or oblate deformation, or transitional nature. The calculations reproduced these features without invoking different assumptions for different regions.

The calculations of Ref. 3 corrected some errors in Ref. 2 and introduced sixth order terms in the Hamiltonian. It was found that the Sm isotopes which range from near spherical to well deformed shapes were well described this way. For a spherical nucleus (^{148}Sm) it was confirmed that the expansion converged at the fourth order, the results being virtually the same as those of sixth order calculations. Except for high-lying states, the same was true for deformed nuclei as well, demonstrating the convergence at the sixth order for practical purposes.

The problem of convergence was also discussed in more general terms in Ref. 4. It was emphasized that the boson expansion should be made only for collective superpositions of fermion pairs. We then demonstrated that the smallest parameter that justifies the power series expansion is essentially the inverse of the number of active (particle-hole) pairs that constitute the collective mode, and thus, can in fact be very small. (Compare this view with earlier work, quoted in Ref. 4, which considered the convergence of a boson expansion of a *pure* fermion pair, and thus resulted in a pessimistic view.) In Ref. 4, we further considered the boundary beyond which the convergence breaks down, and found that the calculations we had performed were very likely within this boundary. The numerical result of convergence as found in Ref. 3 is considered a confirmation of this fact.

The present paper deals with nuclei which are considered to be basically spherical, and the expected[3,4] convergence was again seen. Thus the sixth order results, which are presented, can also be regarded as the results of a fourth order calculation. With our computer program the sixth order calculation can be done nearly as fast as the fourth order calculation.

Since the formulas used for the present calculations are the same as those given in Ref. 3, we shall not repeat them here. We thus give in Sec. II the obtained results and compare them with experiment. Discussion of those results, and comparison with some other theoretical investigations, will be given in Sec. III.

II. RESULTS OF CALCULATIONS AND COMPARISON WITH EXPERIMENT

As in Refs. 2 and 3, calculations were made by permitting two parameters f_2 and g_2 to be

varied slightly, in order to obtain the best overall fit to experimental spectra of individual nuclei. They measure, respectively, the strengths of the particle-hole type and pairing type interactions, and their reasonable values are expected to lie[2] somewhere around 0.8. It was shown, in Ref. 3, that a good fit to data can be obtained, for $^{148\text{-}154}$Sm, by allowing f_2 and g_2 to vary by 4% from their respective median values. The variation needed in the present work was greater, but still within 10%. Note that the variation from the Sm region to this region is only 20%, and, furthermore, the use of a fixed pair of f_2 and g_2 does not change the basic nature of the spectra (see, e.g., Fig. 7 of Ref. 2). The purpose of varying these interaction strengths, from one nucleus to another, is to smooth out the effects of the errors inherent in the use of effective interactions, core-valence particle separation, and the underlying shell model procedure itself.

The actual values used for $^{98\text{-}104}$Ru were $f_2 = 0.774$, 0.762, 0.784, and 0.862 and $g_2 = 0.820$, 0.802, 0.846, and 0.834, showing indeed a very weak dependence on the mass number except that f_2 for ^{104}Ru is somewhat too large. The corresponding values used for $^{102\text{-}110}$Pd were $f_2 = 0.853$, 0.810, 0.808, 0.910, and 0.882 and $g_2 = 0.753$, 0.810, 0.873, 0.880, and 0.729. It is seen that we have a larger variation here than in the Ru isotopes; still it is within 10% of the median values, as we noted above.

It was emphasized in Ref. 3 that the single-particle energies, where our numerical calculations start from, must be taken properly. For the mass region of present interest, a careful study of these energies was made by Lie and Holzwarth,[5] who took, first, Nilsson's levels[6] and then modified them slightly by taking into account experimental information that was available. We used their single-particle energies as they stand, except that they were augmented by adding the $h_{11/2}$ neutron and $f_{5/2}$ proton orbits. For ^{110}Pd, we further included two more neutron orbits $f_{7/2}$ and $h_{9/2}$, due to the rising Fermi energy. These two orbits were insignificant for lighter elements.

We show in Fig. 1 the theoretical spectra, obtained for $^{102\text{-}110}$Pd, along with experiment. Similar presentation is made for $^{98\text{-}104}$Ru in Fig. 2. As is seen, good overall agreement with experiment has been achieved for both sets of isotopes.

Going into some detail, the agreement appears particularly good for the Pd isotopes. For example, the two-phonon-like 0_2^+, 2_2^+, and 4_1^+ states are almost degenerate in ^{104}Pd, and the theory reproduces this fact. In ^{106}Pd, the 0_2^+ and 2_2^+ states are almost degenerate, while the 4_1^+ state lies somewhat high. In ^{108,110}Pd on the other hand, the

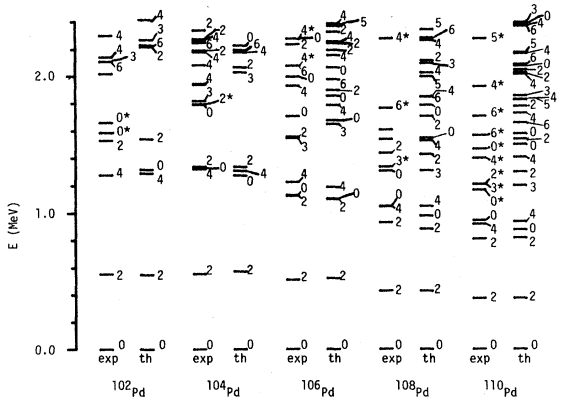

FIG. 1. Comparison of theoretical and experimental (Refs. 7–14) energy levels for $^{102\text{-}110}$ Pd. Starred experimental levels are tentatively assigned.

0_2^+ and 4_1^+ states are nearly degenerate, while the 2_2^+ states lie somewhat low. All these features are again well reproduced by theory. It is also seen that the positions of the (three-phonon-like) 3_1^+ states are predicted very well, although the theory has a tendency to locate these states slightly too high. The predicted energies of the 6_1^+ states are also in good agreement with experiment.

The two-phonon-like 2_2^+ and 4_1^+ states in all the Ru isotopes, except ^{100}Ru, are nearly degenerate. As seen in Fig. 2, the theory explains the energies of the two-phonon states very well, except for predicting the 0_2^+ states slightly too high in ^{100,102}Ru and the 2_2^+ state slightly too low in ^{100}Ru. As for the three-phonon-like 3_1^+ and 6_1^+ states, the experimental information is less definite here than it

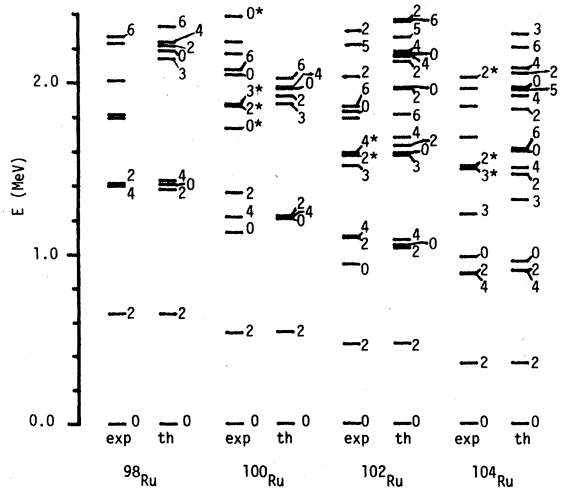

FIG. 2. Comparison of theoretical and experimental (Refs. 7, 8, 15–17) energy levels for $^{98\text{-}104}$Ru. Starred experimental levels are tentatively assigned.

is in Pd. When data are available, the theory predicts their energies better than it did for the Pd isotopes.

Predictions concerning the electromagnetic properties are summarized in Tables I–IV, and are compared with experiment. The $B(E2)$ values, the static quadrupole moments (Q), and the magnetic g factors (g_R) for Ru and Pd isotopes are presented in Tables I and II, respectively, while Tables III and IV give branching ratios. Tables I and II also give the effective charge,[2] $e_{\rm eff}$, used for the individual elements. Their values were fixed so as to obtain the best overall fit to data, but with the restriction that the predicted $B(E2; 2_1^+ \to 0_1^+)$ agrees with experiment within the experimental uncertainty.

The entries of these tables speak for themselves, and we shall give only a few remarks. In general, the obtained theoretical values agree very well with experiment. If we take, as a measure of the obtained agreement, the ratio R, say, of the theoretical entry to the nearest end of the experimental error bar (or its inverse if this ratio is less than unity), it is seen in Table I that R exceeds 1.15 only for the following two quantities: $B(E2; 4_1 \to 2_1)$ and $B(E2; 2_2 \to 0_1)$ for ^{98}Ru, the values of R being 1.16 and 1.23, respectively. As for Table II, there are five cases which give $R > 1.15$. They are the $B(E2; 2_2 \to 2_1)$ for ^{108}Pd, the $B(E2; 0_2 \to 2_1)$'s for ^{106}Pd and ^{108}Pd, the $B(E2; 2_2 \to 0_1)$ for ^{110}Pd, and the $Q(2_1)$ for ^{102}Pd. The corresponding values of R are 1.41, 1.24, 1.41, 2.96, and 1.48, respectively. Note that the entries in these two tables include the $B(E2)$'s for the crossover transition $2_2^+ \to 0_1^+$, whose absolute value is very small. We were able to predict correctly even these small values, except for ^{110}Pd.

When experimental values are given in terms of branching ratios, rather than of $B(E2)$ values, it normally involves transitions pertaining to higher states and also those which are rather weak. Therefore, it is often harder to fit the branching ratio data. It is then pleasing to see in Tables III and IV that almost every theoretical value agrees with experiment within a factor of 2.

In these two tables it is worthwhile to note that there are several branching ratios for transitions originating from the 3_1^+ state. If this 3_1^+ state is of dominantly three-phonon character, the $3_1^+ \to 2_1^+$ is a crossover transition and we expect a small ratio for $B(E2; 3_1^+ \to 2_1^+)/B(E2; 3_1^+ \to 2_2^+)$. Experiment shows that this is indeed the case, and the theory gives values that agree very well with experiment. Compared with this ratio, however, the ratio $B(E2; 3_1^+ \to 2_1^+)/B(E2; 3_1^+ \to 4_1^+)$, which is also expected to be small, is 5 to 10 times larger; nevertheless the theory succeeds in predicting this ratio also. [This fact is further seen in the agreement of theory with experiment with regard

TABLE I. $B(E2; I_i \to I_f)$ in units of $10^{-2} e^2 b^2$, quadrupole moments in units of eb, and magnetic moment g_R factor in units of μ_N, for $^{98-104}$Ru. Experimental $B(E2)$ taken from Refs. 18–20, 22. The superscripts a through i refer to Refs. 20–28, respectively. An asterisk after the theoretical $B(E2)$ denotes that the matrix element is negative.

Transition	^{98}Ru exp.	^{98}Ru th.	^{100}Ru exp.	^{100}Ru th.	^{102}Ru exp.	^{102}Ru th.	^{104}Ru exp.	^{104}Ru th.
$2_1 \to 0_1$	8.0 ±0.6	8.1	10.4 ±0.7	10.0	13.0 ±0.9	12.7	16.4 ±1.2	15.7
$4_1 \to 2_1$	10.77±1.22	14.0	14.6 ±1.5	17.1	20.1 ±3.8	21.3*	21.7 ±0.4	25.1*
$6_1 \to 4_1$		17.9*		21.5		25.7*		29.6*
$8_1 \to 6_1$		20.2*		23.9		26.2*		31.5
$2_2 \to 2_1$	14.7 ± 2.5	11.0	9.10 ± 1.55	12.0	11.7 ± 1.5	14.5	12.3 ±1.9	9.9
$2_2 \to 0_1$	0.15 ± 0.02	0.21	0.37 ± 0.05	0.33	0.33 ± 0.04	0.40	0.55 ± 0.06	0.69
$3_1 \to 2_1$		0.46*		0.71*		0.82		1.20*
$2_3 \to 0_1$		0.07*		0.12*		0.13*		0.09
$4_2 \to 2_1$		0.13*		0.15		0.19		0.18*
$0_2 \to 2_1$		9.9	9.55 ± 1.45	11.6	9.95 ± 1.45	12.7*	7.6 ±0.8	9.4*
$0_3 \to 2_1$		0.51		0.55*		1.3*		2.3*
$3_1 \to 4_1$		3.8		4.2		5.0		3.9*
$Q(2_1)$	−0.33 ± 0.14 [b] −0.20 ± 0.09 [c]	−0.33	−0.43 ± 0.07 [b] −0.43 ± 0.07 [c] −0.40 ± 0.12 [d]	−0.42	−0.57 ± 0.07 [c] −0.4 ±0.1 [d] −0.37 ± 0.24 [e] −0.68 ± 0.06 [f]	−0.48	−0.76 ± 0.19 [a] −0.66 ± 0.05 [b] −0.70 ± 0.08 [c] −0.84 ± 0.21 [e] −0.63 ± 0.20 [g]	−0.69
$Q(2_2)$		0.23		0.31		0.38		0.60
$Q(4_1)$		−0.50		−0.62		−0.69		−0.90
$g_R(2_1)$	0.30 ± 0.17 [h]	0.39	0.42 ± 0.03 [i]	0.36	0.34 ± 0.06 [i]	0.34	0.29 ± 0.04 [h]	0.30
$e_{\rm eff}$		0.96		1.06		1.22		1.13

TABLE II. Same as Table I except for $^{102-110}$Pd. Experimental $B(E2)$ taken from Refs. 19, 29, and 34. The superscripts a through h refer to Refs. 27–34, respectively, while i and j refer to Refs. 21 and 44.

	^{102}Pd		^{104}Pd		^{106}Pd		^{108}Pd		^{110}Pd	
Transition	exp.	th.	exp.	th.	exp.	th.	exp.	th.	exp.	th.
$2_1 \rightarrow 0_1$	9.0 ±1.0	8.2	10.4 ±0.4	10.1	12.4 ±0.8	12.7	14.6 ±1.0	15.3	17.2 ±1.2	18.3
$4_1 \rightarrow 2_1$	14.7 ±0.09	14.0	15.2 ±0.09	17.5*	22 ±3	21.6*	28 ±4	24.8*	31 ±4	29.6
$6_1 \rightarrow 4_1$		17.3*		21.9		26.2*		30.0*		35.3
$8_1 \rightarrow 6_1$		19.3*		24.2		25.0		32.0*		37.2
$2_2 \rightarrow 2_1$	3.9 ±1.1	5.6*	8.4 ±1.8	10.0	13.6 ±0.2	14.1*	24.8 ±5.5	13.7	18 ±3	14.1
$2_2 \rightarrow 0_1$	0.50 ± 0.03	0.44*	0.42 ± 0.03	0.45	0.36 ± 0.03	0.44*	0.34 ± 0.03	0.41	0.26 ± 0.02	0.83
$3_1 \rightarrow 2_1$		0.94		0.92*		0.83		0.59*		1.46
$2_3 \rightarrow 0_1$		0.13*		0.11*		0.07		5.4E-4*		0.057*
$4_2 \rightarrow 2_1$		0.15*		0.19*		0.24		0.12*		0.36*
$0_2 \rightarrow 2_1$		9.2*		12.1*	9.0 ±2.0	13.6*	13.5 ±3.0	7.51	11.5 ±2.0	9.3*
$0_3 \rightarrow 2_1$		0.24		0.67*		1.7		5.5*		2.9*
$3_1 \rightarrow 4_1$		2.0*		3.4*		4.8		5.2*		5.1*
$Q(2_1)$	−0.20 ± 0.15 [c]	−0.52	−0.47 ± 0.10 [c]	−0.49	−0.52 ± 0.12 [d]	−0.50	−0.66 ± 0.18 [f]	−0.58	−0.83 ± 0.19 [h]	−0.70
	−0.21 ± 0.07 [i]		−0.25 ± 0.12 [d]		−0.56 ± 0.08 [e]		−0.51 ± 0.06 [e]		−0.72 ± 0.08 [g]	
			−0.21 ± 0.07 [i]				−0.65 ± 0.13 [g]		−0.55 ± 0.08 [e]	
							−0.48 ± 0.05 [i]		−0.72 ± 0.14 [f]	
$Q(2_2)$		0.36		0.37		0.40		0.57		0.62
$Q(4_1)$		−0.71		−0.71		−0.74		−0.83		−0.95
g_R		0.36	0.40 ± 0.05 [j]	0.34	0.29 ± 0.17 [a]	0.33	0.30 ± 0.04 [a]	0.30	0.25 ± 0.03 [a]	0.30
					0.34 ± 0.01 [b]		0.42 ± 0.05 [j]		0.37 ± 0.03 [j]	
e_{eff}		0.72		0.96		1.20		1.05		1.14

TABLE III. Branching ratios for $^{98-104}$Ru. Superscripts a, b, and c refer to Refs. 19, 35, and 17, respectively.

		^{98}Ru		^{100}Ru		^{102}Ru		^{104}Ru	
I_i	I_f/I_f'	exp.	th.	exp.	th.	exp.	th.	exp.	th.
2_2	$0_1/2_1$	0.02 [a]	0.019	0.041 [a]	0.028	0.038 [b]	0.028	0.054 [c]	0.069
2_3	$0_1/2_1$		1400		35.3	1.10 [b]	3.02		0.89
2_3	$0_1/0_2$		0.0099		0.015	0.003 [b]	0.014		0.0084
2_3	$0_1/2_2$		0.036		0.054		0.081		0.13
2_3	$0_1/4_1$		0.018		0.028		0.032		0.033
4_2	$2_1/2_2$		0.015		0.015	0.007 [b]	0.016		0.015
4_2	$4_1/2_2$		0.75		0.71	0.54 [b]	0.66		0.51
0_2	$2_1/2_2$		1.36		1.18		1.02		0.60
0_3	$2_1/2_2$		0.044		0.044	0.160 [b]	0.12		0.35
3_1	$2_1/2_2$		0.040		0.053	0.04 [a] 0.037 [b]	0.048	0.038 [c]	0.057
3_1	$2_1/4_1$		0.12		0.17	0.170 [a] 0.16 [b]	0.16		0.31
3_1	$4_1/2_2$		0.33		0.31	0.25 [a]	0.29	0.15 [a] 0.37 [c]	0.19
4_1	$2_1/2_2$		62.2		68.4		95.1		579.2

to the ratio $B(E2; 3_1^+ \rightarrow 4_1^+)/B(E2; 3_1^+ \rightarrow 2_2^+)$. This ratio is redundant, but we retain it in the table, because some experimental values were presented in terms of this ratio.] Combined with the good agreement obtained for the energies of the 3_1^+ state, as seen in Figs. 1 and 2, we can conclude that our calculation describes the (deviation from the pure) three-phonon character of the 3_1^+ state very well.

The 4_2^+ and 0_3^+ are also considered to be states of primarily three-phonon character. However, it is seen in Tables I–IV that only a limited quantity of data[19,35] is available for their transition properties. Accumulation of such data is still needed. (See also Ref. 37 for preliminary data for ^{110}Pd.)

Hasselgren and co-workers[20,32,38] have determined the sign of the Coulomb interference term (normally called P_3) in ^{102,104}Ru and ^{108,110}Pd to be consistently negative. Our calculated matrix elements predict this sign to be negative for all the isotopes considered in this study.

We have evaluated the magnetic moment (μ) for the first time using the boson expansion method and have found excellent agreement with experiment. No free parameter is introduced in obtaining this quantity. It is given in lowest order as

TABLE IV. Branching ratios for $^{102-110}$Pd. All experimental entries are taken from Ref. 19, except for a, which denotes Ref. 36.

		^{102}Pd		^{104}Pd		^{106}Pd		^{108}Pd		^{110}Pd	
I_i	I_f/I_f'	exp.	th.	exp.	th.	exp.	th.	exp.	th.	exp.	th.
2_2	$0_1/2_1$	0.12	0.079	0.05	0.045	0.027	0.031	0.014	0.030	0.014	0.059
2_3	$0_1/2_1$		10^5		18.0		1.23		0.041		0.84
2_3	$0_1/0_2$		0.023		0.014		0.0075		0.000 26		0.0040
2_3	$0_1/2_2$		0.050		0.042		0.050		0.007 3		0.11
2_3	$0_1/4_1$		0.042		0.026		0.018		0.001 9		0.0021
4_2	$2_1/2_2$		0.020	0.03	0.019		0.021		0.009 0		0.024
4_2	$4_1/2_2$		0.48	0.41	0.60		0.66		0.50		0.55
0_2	$2_1/2_2$		0.84		1.05		0.98		0.42		0.41
0_3	$2_1/2_2$		0.032		0.064		0.23		1.44		0.91
3_1	$2_1/2_2$		0.10	0.03	0.070	0.03	0.047	0.026	0.026	0.014	0.057
3_1	$2_1/4_1$		0.47		0.27		0.17		0.15		0.29
3_1	$4_1/2_2$		0.22	0.40	0.26	0.33	0.27		0.21	0.50	0.20
4_1	$2_1/2_2$		58.6		63.0		77.1	>170 [a]	406.6		311.6

$$\mu(I) = -10 \sum_{j_1 \leq j_2} D^{-2}_{j_1 j_2} \{(g_s - g_l)\hat{j}_1 \hat{j}_2 W(\tfrac{1}{2} l_1 1 j_1; j_2 \tfrac{1}{2}) \delta_{l_1 l_2}/\sqrt{2} + g_l \hat{j}_1 [j_1(j_1+1)]^{1/2}/3\delta_{j_1 j_2}\}$$
$$\times (u_1 u_2 + v_1 v_2) \sum_j \Lambda^0_{j j_2} \Lambda^0_{j_1 j} W(j_2 j_1 22; 1j) \langle II | [\alpha^+ \tilde{\alpha}]_1 | II \rangle ,$$

where all quantities were defined and appeared in Refs. 1–3, with the exception of g_s and g_l which are given, e.g., in Ref. 39. The quantity g_R, in Tables I and II, is defined by $\mu(I) = g_R I$. Regarding $M1$ transitions, in lowest order, they all vanish identically, since $[\alpha^\dagger \tilde{\alpha}]_1$ is essentially the angular momentum operator. Finally we note that the dominant contribution to the dipole moment is given by large-j proton orbits close to the Fermi surface (in this mass region the $g_{9/2}$).

III. DISCUSSION

We have shown that our theory reproduces the experimental energies, transition properties, and moments for Ru and Pd very well in general. Nevertheless, a few problems remain. It is seen in Table I that the experimental $Q(2^+_1)$ for ^{102}Pd is substantially smaller than those in other isotopes, and we failed in reproducing this. Another problem concerns the mass-number dependence of $B(E2; 2_2 \rightarrow 2_1)$ for Pd. We were unable to explain the anomalous peak at ^{108}Pd, which has been a long-standing problem.[34]

Possible solutions to these problems might involve the activation of a few of the noncollective modes which have been truncated.[2,3] Such may be particularly important for the lighter elements, where, as seen in Figs. 1 and 2, even the two-phonon-like states lie fairly close to the lowest quasiparticle-pair states. Another recourse is to consider a mass number dependence of a few of the single-particle energies close to the Fermi surface. This may not only be justified, but also mandatory, because the proton-neutron pairing interaction is known to cause such a mass number dependence.[40] It would be expected that such refined calculations would also reduce the variations experienced for the parameters f_2, g_2, and e_{eff}.

A significant success of our theory has been the prediction of the lowering of the 3^+_1 state. The experimental dropping of the 3^+_1 state is probably related to the increasing deformation of the isotopes as the mass increases, and signals the start of the formation of a gamma band. The structure of our wave functions shows that a gradual shift away from a pure three-phonon character (89% to 73% for ^{98}Ru to ^{104}Ru, and 86% to 75% for ^{100}Pd to ^{110}Pd) is occurring. For Ru most of this shift goes to the five and six phonon states while for Pd the strength is spread over these and several other boson states. This is clearly a consequence of the third and fourth order anharmonic terms H_{30} and H_{31}, the three- and two-boson-number-changing operators.[2] Their coefficients increase by factors 2 and 2.5, e.g., in going from ^{98}Ru to ^{104}Ru, as a consequence of the changing position of the Fermi surface with respect to the underlying single-particle spectrum. Arbitrarily setting these terms equal to zero, while still calculating the other boson terms microscopically, destroys the lowering of the 3^+_1 state, the one- and two-phonon states retaining their energies almost unchanged. (Actual calculations made for ^{104}Ru and ^{110}Pd showed the rise of the 3^+_1 state to be 350 and 200 keV, respectively.)

A microscopic attempt to describe the positive parity states of ^{100,102}Ru was made by Holzwarth and his co-workers.[5,41] Their calculations suffered from too large spacings between states. They needed to use a scale factor as well as a mass number dependence of the single-particle states. Our calculations in Figs. 1 and 2 are presented in absolute scale. This was possible because of the inclusion of collective-noncollective coupling as discussed previously.[2,3]

Even with the above adjustment, Lie and Holzwarth[5] alluded to the necessity of extending their fourth order calculations to sixth order. In a more recent work, Holzwarth *et al.*[41] noticed that the fifth order terms are rather large. As remarked, we find little difference between our fourth and sixth order calculations. This may mean that our boson expansion has better convergence properties than does that of Ref. 5.

Regarding purely parametrized theories, Lie and Holzwarth[5] obtained extremely accurate fits to ^{100,102}Ru if they let all boson coefficients of their Hamiltonian be free parameters. Since our Hamiltonian [based on SU(5)] has basically the same form, their work shows that in principle we could obtain a perfect fit to experiment. To achieve that from a microscopic approach might, however, require, e.g., using more realistic nucleon-nucleon interactions and is beyond the scope of the present work.

There is available another parametrized theory, the interacting boson approximation (IBA).[42] It

is based on SU(6) and was used for ^{102}Ru and ^{104}Ru, respectively, in Refs. 42 and 17. However, the SU(5) theory, either ours or the parametrized version of Lie and Holzwarth,[5] fits data better than does IBA, in particular in the sense that the latter fails to predict the 3_1^+ states sufficiently low. Recalling our previous remarks, this failure of IBA is probably related to the absence of the H_{30} and H_{31} terms in its Hamiltonian. If more parameters such as the coefficient of d-boson changing operators are introduced into the formalism of Refs. 42 and 17, then the above problem will undoubtedly disappear.

Finally Hsu *et al.*[43] considered a rotation-vibration model and fitted quite well the spectra of the Pd isotopes. Their calculation, however, introduces seven free parameters, and an *ad hoc* degree of freedom.

To summarize, we have calculated energies and transition probabilities microscopically for a number of isotopes of Pd and Ru and found very good agreement with experiment for the low-lying collective states. In particular, the two-phonon triplet trends, the decrease of the 3_1^+ energy, and the details of the $B(E2)$'s have been well described using the boson expansion method.

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