

## Effective $M3$ operator and relevant transitions in $^{24}\text{Al}$ , $^{24}\text{Na}$ , $^{34}\text{Cl}$ , $^{38}\text{K}$ , and $^{38}\text{Cl}$

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Theoretical calculations are presented for the magnetic octupole electromagnetic matrix elements in light nuclei. Shell-model wave functions are used to calculate the  $M3$  matrix elements for the cases  $^{24}\text{Al}$   $1^+ \rightarrow 4^+$ ,  $^{24}\text{Na}$   $1^+ \rightarrow 4^+$ ,  $^{34}\text{Cl}$   $3^+ \rightarrow 0^+$ ,  $^{38}\text{K}$   $0^+ \rightarrow 3^+$ , and  $^{38}\text{Cl}$   $5^- \rightarrow 2^-$ . The radial matrix elements are calculated with harmonic-oscillator and spherical Hartree-Fock potentials. The comparison of the calculated and experimental matrix elements is expressed in terms of effective spin  $g$  factors. Theoretical calculations for the core-polarization corrections are presented and relations between the  $E2$  core-polarization charge and the  $M3$  effective spin  $g$  factors are derived.

[NUCLEAR STRUCTURE  $^{24}\text{Na}$ ,  $^{24}\text{Al}$ ,  $^{34}\text{Cl}$ ,  $^{38}\text{K}$ ,  $^{38}\text{Cl}$ ; calculations of  $M3$  decay strengths; extraction of effective  $M3$  operator; full  $d_{5/2}$ - $s_{1/2}$ - $d_{3/2}$  shell-model calculations with Chung-Wildenthal Hamiltonians.]

### I. INTRODUCTION

Recent electron scattering experiments have obtained interesting new information about the high ( $L \geq 3$ ) multipole moments of nuclei.<sup>1-3</sup> Previously, nuclear models have been predominantly tested by and designed to explain  $E0$ ,  $E1$ ,  $E2$ , and  $M1$  matrix elements. The higher moments will provide new and hopefully discriminating tests of the predictions of these models. The  $M1$  matrix elements for  $A = 17-39$  nuclei are quite well accounted for by shell-model calculations within a full  $(sd)^n$  basis using  $g$  factors very close to the free-nucleon values.<sup>4</sup> The large  $E2$  matrix elements in this region are also well accounted for by these same shell-model calculations if a constant isoscalar enhancement factor of  $1 + \delta e_p + \delta e_n = 1.7 \pm 0.1$  is used.<sup>5</sup> This enhancement can be understood as a core-polarization effect involving the rearrangement of particles in the orbits below and above the  $sd$  orbits.

In the context of these results for  $M1$  and  $E2$  phenomena, it was initially surprising that the  $M3$  matrix element observed in the electron scattering from  $^{17}\text{O}$  (Ref. 1) was much smaller than the value predicted for a  $d_{5/2}$  neutron single particle. However, subsequent calculations<sup>6-8</sup> have shown that a hindrance of the  $M3$  matrix elements can be understood by the same core-polarization mechanism that gives rise to the enhancement of

the  $E2$  matrix elements.

Unfortunately, it is difficult to extract a precise value for the  $M3$  matrix element in  $^{17}\text{O}$  because the magnetic electron scattering cross section is dominated by the  $M1$  and  $M5$  contributions.<sup>1</sup> The situation is similar for the elastic electron scattering on other nuclei with  $\frac{5}{2}^+$  ground states.<sup>2,3</sup> In the next few years much more information on the  $M3$  matrix element will become available from elastic electron scattering on the nuclei with  $\frac{3}{2}^+$  ground states, as well as inelastic excitation of  $3^+$  states in even-even nuclei, but at present the only other source of information on this topic comes from the few precisely measured half-lives of  $M3$  gamma decays in the  $sd$  shell. It is the purpose of the present work to concentrate on the shell-model predictions for these  $M3$  gamma decays.

In the  $sd$  shell nuclei only three  $M3$  gamma decay half-lives have been measured: in  $^{24}\text{Al}$ ,  $^{24}\text{Na}$ , and  $^{34}\text{Cl}$ . The experimental properties are given in Table I. Presented in addition in Table I is information on  $^{38}\text{Cl}$  which will be discussed briefly in Sec. V in terms of a  $d_{3/2}f_{7/2}$  configuration. The experimental situation for  $^{24}\text{Al}$  has been much improved by recent experiments<sup>9,10</sup> which have shown that the previous assignment<sup>11</sup> of a 439 keV gamma ray in  $^{24}\text{Al}$  was in error.

The configuration mixing among the  $d_{5/2}s_{1/2}$  and  $d_{3/2}$  orbits is large for the  $A = 24$  and  $A = 34$  nuclei,

TABLE I. Experimental properties of the  $M3$  transitions. The experimental values are taken from Ref. 11 unless noted.

Nucleus	$^{24}\text{Al}$	$^{24}\text{Na}$	$^{34}\text{Cl}$	$^{38}\text{Cl}$
$J_i \rightarrow J_f$	$1^+ \rightarrow 4^+$	$3^+ \rightarrow 0^+$	$3^+ \rightarrow 0^+$	$5^- \rightarrow 3^-$
$T_i \ T_f$	1 1	1 1	0 1	2 2
$\tau_{1/2}$	$129 \pm 4$ ms <sup>a</sup>	$20.18 \pm 0.10$ ms	$32.23 \pm 0.14$ m	$715 \pm 3$ ms
$E_\gamma$ (keV)	$425.8^b$	472.29	146.36	671.27
Branching ratio (%)	$80 \pm 3^c$	99.97	$46.9 \pm 1.0$	100
Conversion coefficient ( $\alpha$ )	0	0	$0.100 \pm 0.009$	0.0005
$B(M3)$ ( $\mu_N^2 \text{ fm}^4$ ) <sup>d</sup>	$269 \pm 13$	$1038 \pm 5$	$16.8 \pm 0.4$	$2.503 \pm 0.010$

<sup>a</sup> Reference 9,  $\tau_{1/2} = 128 \pm 6$  ms; Ref. 10,  $\tau_{1/2} = 127 \pm 6$  ms; Ref. 11,  $\tau_{1/2} = 130 \pm 4$  ms; Adopted,  $\tau_{1/2} = 129 \pm 4$  ms.

<sup>b</sup> Reference 10.

<sup>c</sup> Reference 9, BR =  $78 \pm 3\%$ ; Ref. 10, BR =  $82.5 \pm 3.0\%$ ; adopted, BR =  $80 \pm 3\%$ .

<sup>d</sup>  $B(M3) = \frac{0.1099 \times (\text{branching ratio})}{\tau_{1/2}(\text{sec}) \times [E_\gamma(\text{MeV})]^7} \left( \frac{1}{1 + \alpha} \right)$ .

and the lowest order spherical shell-model configurations ( $d_{5/2}$ )<sup>8</sup> and ( $d_{3/2}$ )<sup>-6</sup>, respectively, provide a very poor description of the energy levels. A Nilsson model with the particles in the lowest prolate deformed orbits is better since  $^{24}\text{Mg}$  is well deformed, but is inadequate because configuration mixing among various intrinsic configurations should be taken into account. We have calculated the  $M3$  matrix elements from wave functions (discussed further in Sec. II) which span the full  $d_{5/2}s_{1/2}d_{3/2}$  basis space. The experimental  $B(M3)$  values are all found to be hindered by about 30% relative to these calculated results. In Sec. III this hindrance is interpreted in terms of renormalized  $g$  factors. In Sec. IV the core-polarization corrections to the renormalized  $g$  factors are discussed and theoretical results outlined by Zamick<sup>6</sup> are developed. Simple relations between the  $E2$  effective charges and the  $M3$  effective  $g$  factors are obtained, and compared with the empirical values. In Sec. V calculations for the  $^{38}\text{K } 0^+ \rightarrow 3^+$  and  $^{38}\text{Cl } 5^- \rightarrow 2^-$  decays are presented, and in Sec. VI we present a summary together

with some comments on further theoretical and experimental work.

## II. SHELL-MODEL CALCULATIONS FOR THE $M3$ TRANSITIONS

The model wave functions which are utilized in our calculations for the  $sd$ -shell  $M3$  transitions were obtained in the course of a more general project<sup>12</sup> directed at producing wave functions for the positive-parity states of all spins ( $J$ ) and isospins ( $T$ ) in this same region. The model space assumed for the calculation of these wave functions was comprised by the single-nucleon orbits with quantum numbers  $0d_{5/2}$ ,  $1s_{1/2}$ , and  $0d_{3/2}$ , i.e., the conventional  $sd$  shell. The complete set of basis vectors allowed in this space by the Pauli principle was utilized in all cases, a fact which is particularly relevant in the context of calculations of matrix elements of the orbital and spin angular momentum operators, operators which play a key role in the physical processes which will be of interest in the present discussion. The calculations were carried out in a  $j$ - $j$  coupling,  $J$ - $T$  re-

TABLE II. One-body transition densities calculated for the  $1^+ \rightarrow 4^+$  decays in  $^{24}\text{Al}$  and  $^{24}\text{Na}$ , the  $3^+ \rightarrow 0^+$  decay in  $^{34}\text{Cl}$  and the  $0^+ \rightarrow 3^+$  decay in  $^{38}\text{K}$ .

Model	$J_i J_f$	$\Delta T$	$\frac{5}{2}-\frac{5}{2}$	$\frac{5}{2}-\frac{1}{2}$	$\frac{5}{2}-\frac{3}{2}$	$\frac{1}{2}-\frac{5}{2}$	$\frac{3}{2}-\frac{5}{2}$	$\frac{3}{2}-\frac{3}{2}$
$(sd)^8$	$4^+ 1^+$	0	-0.5292	0.0748	-0.1688	-0.1907	0.1193	0.0540
		1	0.3986	-0.0931	0.1491	-0.0643	-0.1021	-0.0289
$(d_{5/2})$	$4^+ 1^+$	0	-0.8369					
		1	0.7197					
$(sd)^{-6}$	$3^+ 0^+$	1	0.0318	-0.0323	-0.0630	-0.0156	-0.0133	-0.6122
		1						-0.7071
$(d_{3/2})^{-6}$	$3^+ 0^+$	1						
		1	-0.0004	0.0006	0.0385	0.0007	-0.0092	0.2477
$(sd)^{-2}$	$0^+ 3^+$	1						0.7071
		1						



$$O(M\Delta J)_{\Delta T} = \left( g_{\Delta T}^s + \frac{2g_{\Delta T}^i}{(\Delta J + 1)} \right) \nabla [r^L Y_{LM}(\hat{r})]_{\Delta T}, \quad (3)$$

where

$$g_{\Delta T}^s = \frac{g_p^s + (-1)^{\Delta T} g_n^s}{2}, \quad (4)$$

$$g_{\Delta T}^i = \frac{g_p^i + (-1)^{\Delta T} g_n^i}{2},$$

and  $g^i$  and  $g^s$  are the orbital and spin  $g$  factors. The subscript  $\Delta T$  on the operators means that the operator has associated with it a unit operator in isospin space when  $\Delta T = 0$  and the operator  $\tau$  in isospin space when  $\Delta T = 1$ . The single-particle isospin reduced matrix elements are  $\langle t || O(\Delta T) || t \rangle = [2(2\Delta T + 1)]^{1/2}$ . The single-particle matrix elements of  $Op(M\Delta J)_{\Delta T}$  are given by

$$\begin{aligned} \langle j || O(M\Delta J)_{\Delta T} || j' \rangle &= [2(2\Delta T + 1)\Delta J(2\Delta J + 1)]^{1/2} \\ &\times \left[ g_{\Delta T}^s \langle j || r^{\Delta J-1} [\underline{Y}_{\Delta J-1} \times \underline{s}]^{\Delta J} || j' \rangle \right. \\ &\left. + \left( \frac{2}{\Delta J + 1} \right) g_{\Delta T}^i \langle j || r^{\Delta J-1} [\underline{Y}_{\Delta J-1} \times \underline{l}]^{\Delta J} || j' \rangle \right]. \quad (5) \end{aligned}$$

The values of these matrix elements for the  $M3$  operator in terms of  $g^i$ ,  $g^s$ , and the  $r^2$  radial integrals are given in Table III. In terms of these isoscalar and isovector matrix elements, the proton and neutron matrix elements are given by

$$\begin{aligned} \langle j || O(M\Delta J)_{p/n} || j' \rangle &= \sum_{\Delta T} \frac{(+/-1)^{\Delta T}}{[2(2\Delta T + 1)]^{1/2}} \langle j || O(M\Delta J)_{\Delta T} || j' \rangle, \quad (6) \end{aligned}$$

where the subscript  $p/n$  means that the operator acts only on protons or neutrons, respectively.

For a first approximation for the radial matrix elements we will use harmonic-oscillator wave functions with the size constant adjusted to reproduce the rms point proton radii of the nearby

TABLE III.  $M3$  single-particle matrix elements.

$j$	$j'$	$\langle j    O(M3)_{\Delta T}    j' \rangle / [2(2\Delta T + 1)]^{1/2}$
$\frac{5}{2}$	$\frac{5}{2}$	$-(81/5\pi)^{1/2} (g_{\Delta T}^s + g_{\Delta T}^i) \langle d_{5/2}   r^2   d_{5/2} \rangle$
$\frac{5}{2}$	$\frac{1}{2}$	$(147/8\pi)^{1/2} g_{\Delta T}^s \langle d_{5/2}   r^2   s_{1/2} \rangle$
$\frac{5}{2}$	$\frac{3}{2}$	$(6/5\pi)^{1/2} (2g_{\Delta T}^s - 3g_{\Delta T}^i) \langle d_{5/2}   r^2   d_{3/2} \rangle$
$\frac{1}{2}$	$\frac{5}{2}$	$(147/8\pi)^{1/2} g_{\Delta T}^i \langle d_{5/2}   r^2   s_{1/2} \rangle$
$\frac{3}{2}$	$\frac{5}{2}$	$-(6/5\pi)^{1/2} (2g_{\Delta T}^s - 3g_{\Delta T}^i) \langle d_{5/2}   r^2   d_{3/2} \rangle$
$\frac{3}{2}$	$\frac{3}{2}$	$(9/20\pi)^{1/2} (g_{\Delta T}^s - 4g_{\Delta T}^i) \langle d_{3/2}   r^2   d_{3/2} \rangle$
		$g_{\Delta T} \equiv [g_p + (-1)^{\Delta T} g_n] / 2$

stable nuclei. The relationship between the measured charge radius  $r_{ch}$  and the point proton radius is<sup>19</sup>

$$r_{ch}^2 = r_p^2 + r_{proton}^2 + \frac{N}{Z} r_{neutron}^2 + \frac{3}{4} \left( \frac{\hbar}{mc} \right)^2, \quad (7)$$

where

$$r_{proton}^2 = (0.86)^2,$$

$$r_{neutron}^2 = -(0.35)^2.$$

In Eq. (7) we have ignored the relativistic spin orbit correction<sup>19</sup> which is not important for  $N = Z$  nuclei.

For a harmonic oscillator potential  $\frac{1}{2}m\omega^2 r^2$  the rms radius  $r_p$  for  $sd$ -shell nuclei is

$$r_p^2 = \left( \frac{18 + (Z - 8)\frac{7}{2}}{Z} \right) b^2 - \frac{3b^2}{2A}, \quad (8)$$

where  $b^2 = \hbar/m\omega$ . The last term is the correction for center of mass motion. The relevant data<sup>20-22</sup> for  $r_{ch}$  and the extracted values of  $b$  are given in Table IV. The harmonic-oscillator radial matrix elements in the  $sd$  shell are given by

$$\begin{aligned} \langle d | r^2 | d \rangle &= 7b^2/2, \\ \langle d | r^2 | s \rangle &= -(10)^{1/2} b^2. \end{aligned} \quad (9)$$

The final results for  $B(M3)$  values [see Eq. (1)] are obtained by combining the one-body transition densities (Table II) with the single-particle matrix elements of Table III using the radial matrix elements of Eq. (9) and the free-nucleon  $g$  factors

$$\begin{aligned} g_p^i &= 1, \\ g_n^i &= 0, \\ g_p^s &= 5.58, \\ g_n^s &= -3.82. \end{aligned} \quad (10)$$

The results are given in Table V. The calculations have succeeded in qualitatively explaining

TABLE IV. Rms charge radii and extracted oscillator length parameters.

Nucleus	$r_{ch}$ (fm)	$b$ (fm)	$\hbar\omega^d$ (MeV)
<sup>24</sup> Mg	3.035 <sup>a</sup>	1.813	12.61
<sup>32</sup> S	3.254 <sup>a</sup>	1.875	11.79
<sup>36</sup> Ar	3.396 <sup>b</sup>	1.936	11.06
$A = 34$		1.905	11.42
<sup>38</sup> Ar	3.414 <sup>c</sup>	1.948	10.93

<sup>a</sup> From a model independent analysis of the data in Ref. 20.

<sup>b</sup> Reference 21.

<sup>c</sup> Reference 22.

<sup>d</sup>  $\hbar\omega = 41.465/b^2$ .

TABLE V.  $B(M3)$  values (in units of  $\mu_N^2 \text{ fm}^4$ ) compared with the  $sd$ -shell-model predictions using free-nucleon  $g$  factors.

Nucleus $J_i \rightarrow J_f$	$^{24}\text{Al}$ $1^+ \rightarrow 4^+$	$^{24}\text{Na}$ $1^+ \rightarrow 4^+$	$^{34}\text{Cl}$ $3^+ \rightarrow 0^+$
Exp <sup>a</sup>	$269 \pm 13$	$1038 \pm 5$	$16.8 \pm 0.4$
Th <sup>b</sup>	344	1538	26.0
Exp/Th	0.78	0.68	0.65

<sup>a</sup> Table I.

<sup>b</sup> Harmonic-oscillator wave functions from  $^{24}\text{Mg}$  and  $A = 34$  (Table IV).

the relative  $B(M3)$  values, which vary by nearly an order of magnitude, but the experimental values are seen to be smaller (about 30%) than those calculated. In the next section we will investigate how this reduction might be understood in terms of empirical effective  $g$  factors.

### III. EMPIRICAL EFFECTIVE $g$ FACTORS FOR THE $M3$ TRANSITIONS

Although the  $sd$ -shell-model calculations discussed in the last section encompass many degrees of freedom, several important types of excitation are not included. The most significant of these involves the lifting of nucleons from one major oscillator shell to another, for example, the  $2\hbar\omega$  excitations,  $0s \rightarrow 1s0d$ ,  $0p \rightarrow 1p0f$ , and  $1s0d \rightarrow 2s1d0g$ . The meson degrees of freedom and the degrees of freedom internal to the nucleon also have been ignored. All of these omitted components have unperturbed excitation energies which are large ( $\geq 2\hbar\omega = 24 \text{ MeV}$ ) compared with the excitation energies of the states participating in the  $M3$  transitions, and thus it is reasonable to expect that they may modify the  $sd$ -shell results in some smooth and average manner. In general, such modifications can possibly be represented in terms of a state dependent (i.e., a dependence on  $j$  and  $j'$ ) renormalization of the single-particle matrix elements. In a more restricted view, consistent with the very limited amount of available experimental data, we will explore in this and the following section the possibility that the effects of these extra degrees of freedom upon the  $M3$  matrix elements can be incorporated into the present calculations by the use of renormalized  $g$  factors, i.e., a state-independent renormalization.

In this section we will determine the empirical effective  $g$  factors, denoted by  $\tilde{g}^l$  and  $\tilde{g}^s$ , which are needed to explain the  $B(M3)$  values. Unfortunately there are four unknowns  $\tilde{g}_p^l$ ,  $\tilde{g}_n^l$ ,  $\tilde{g}_p^s$ , and  $\tilde{g}_n^s$  and only three experimental data. Thus we will make the additional assumption based on the results to be discussed in Sec. IV that  $\tilde{g}_p^l = 1$  and

$$\tilde{g}_n^l = 0.$$

In order to obtain the effective  $g$  factors in a transparent manner, we first write out the reduced matrix elements in terms of  $\tilde{g}^l$  and  $\tilde{g}^s$  using harmonic-oscillator wave functions with the  $b^2$  parameters from Table IV:

$$[B(M3)]^{1/2;^{24}\text{Al}} = 2.03\tilde{g}_p^s + 7.90\tilde{g}_n^s + 0.27\tilde{g}_p^l + 2.91\tilde{g}_n^l,$$

$$[B(M3)]^{1/2;^{24}\text{Na}} = 7.90\tilde{g}_p^s + 2.03\tilde{g}_n^s + 2.91\tilde{g}_p^l + 0.27\tilde{g}_n^l,$$

$$[B(M3)]^{1/2;^{34}\text{Cl}} = 0.88\tilde{g}_p^s - 0.88\tilde{g}_n^s - 3.22\tilde{g}_p^l + 3.22\tilde{g}_n^l.$$

(11)

Of course the relative sign of  $[B(M3)]^{1/2}$  between  $^{24}\text{Al}$  and  $^{24}\text{Na}$  is not measured, but if we use free-nucleon  $g$  factors we obtain

$$[B(M3)^{24}\text{Na}]^{1/2}/[B(M3)^{24}\text{Al}]^{1/2} = -2.11.$$

Thus, to obtain effective  $g$  factors which are not drastically different from the free-nucleon values, the isoscalar  $g$  factors are related to the *difference* in the two  $[B(M3)]^{1/2}$  values, whereas the isovector  $g$  factors are related to the *sum* of the two. Setting  $\tilde{g}_p^l = 1$  and  $\tilde{g}_n^l = 0$  we thus obtain

$$[B(M3)]^{1/2} \frac{^{24}\text{Na} - ^{24}\text{Al}}{2} = 9.93\tilde{g}_0^s + 1.59 = 7.9 \pm 0.2,$$

$$[B(M3)]^{1/2} \frac{^{24}\text{Al} + ^{24}\text{Na}}{2} = 5.87\tilde{g}_1^s + 1.32 = 24.3 \pm 0.2, \quad (12)$$

$$[B(M3)]^{1/2;^{34}\text{Cl}} = 1.76\tilde{g}_1^s - 3.22 = 4.10 \pm 0.05,$$

where  $g_0^s$  and  $g_1^s$  are defined by Eq. (4). The deviations from the free-nucleon  $g$  factors which are needed to explain the experimental data are

$$A = 24: \delta_0^s = -0.28, \quad \delta_1^s = -0.17, \quad (13)$$

$$^{34}\text{Cl}: \delta_1^s = -0.12, \quad (14)$$

where

$$\delta^s \equiv \frac{\tilde{g}_s}{g_s} - 1. \quad (15)$$

Since the single-particle binding energies of the valence orbits in  $A = 24$  are relatively small, it is important to investigate the effects of using more realistic radial wave functions. Recently, spherical Hartree-Fock calculations<sup>23</sup> have been carried out for the  $sd$  shell using the Skyrme interactions<sup>24</sup> together with shell-model occupation numbers. For  $^{24}\text{Mg}$  these calculations with a Skyrme III interaction gave an rms charge radius of 3.086 fm and a charge radius of 3.038 fm if a small effective mass adjustment is made in order to reproduce the  $^{24}\text{Mg}$  binding energy. Since the latter value is in agreement with the experimental value of 3.04 fm, we have used the radial wave functions from similar calculations in  $^{24}\text{Al}$  and  $^{24}\text{Na}$  to compare with the previous harmonic-oscillator re-

sults. The  $r^2$  radial matrix elements are given in Table VI.

The radial matrix elements are different for the protons and neutrons because of the Coulomb potential. In addition, the radial matrix elements in  $^{24}\text{Na}$  and  $^{24}\text{Al}$  differ because of the symmetry potential (the radial matrix elements in  $^{24}\text{Mg}$  are given to a good approximation by the average of those in  $^{24}\text{Al}$  and  $^{24}\text{Na}$  and thus these are not listed separately in Table VI). Hence the  $M3$  matrix elements no longer display the mirror symmetry apparent in Eqs. (11) and we can no longer take the sum and difference to obtain the isoscalar and isovector matrix elements. Combining the radial matrix elements of Table VI with the single-particle matrix elements and one-body transition densities we obtain [compare with Eqs. (11)]

$$\begin{aligned} [B(M3)]^{1/2; 24}\text{Al} &= 1.98\bar{g}_p^s + 7.56\bar{g}_n^s + 0.26\bar{g}_p^i \\ &\quad + 2.55\bar{g}_n^i, \\ [B(M3)]^{1/2; 24}\text{Na} &= 7.79\bar{g}_p^s + 1.92\bar{g}_n^s + 2.58\bar{g}_p^i \\ &\quad + 0.25\bar{g}_n^i. \end{aligned} \quad (16)$$

Again assuming  $\bar{g}_p^i = 1$  and  $\bar{g}_n^i = 0$  and equating these to the experimental values of  $-16.40$  and  $32.22$ , respectively, we obtain

$$A = 24: \delta_0^s = -0.30, \quad \delta_1^s = -0.14. \quad (17)$$

The differences between the harmonic-oscillator results [Eq. (13)] and the Hartree-Fock results [Eq. (17)] are not large. We expect an even smaller difference for  $^{34}\text{Cl}$  since the orbits are more deeply bound.

The experimental errors in the  $\delta^s$  values are very small. Much more important is the theoretical error arising from uncertainties in the  $sd$ -shell-model wave functions. As a rough estimate of this theoretical error we consider the comparison of the large number of  $E2$  transitions throughout the  $sd$  shell. The  $E2$  matrix elements require an isoscalar effective charge of  $1 + \delta e_p + \delta e_n = 1.7 \pm 0.1$ , where the  $\pm 0.1$  represents changes in the ratio of experiment to theory as a function of mass as well as for different transitions within a given nucleus.

TABLE VI. Radial matrix elements  $\langle j|r^2|j' \rangle$  obtained from spherical Hartree-Fock calculations<sup>a</sup>.

$j$	$j'$	$^{24}\text{Na}$		$^{24}\text{Al}$	
		$r_p^2$ (fm <sup>2</sup> )	$r_n^2$ (fm <sup>2</sup> )	$r_p^2$ (fm <sup>2</sup> )	$r_n^2$ (fm <sup>2</sup> )
$d_{5/2}$	$d_{5/2}$	11.02	10.87	11.17	10.82
$d_{5/2}$	$s_{1/2}$	-10.15	-10.05	-10.49	-9.88
$d_{5/2}$	$d_{3/2}$	11.54	11.42	11.84	11.31
$d_{3/2}$	$d_{3/2}$	12.09	12.45	13.12	11.94

<sup>a</sup> Reference 23. A Skyrme III interaction was used.

This represents deviations from unity on the order of 5% and could be an estimate of the theoretical error in the  $M3$  matrix elements also. Thus for  $\delta_1^s$  we have  $\delta_1^s = -0.12 \pm 0.05$  from  $^{34}\text{Cl}$  and  $\delta_1^s = -0.14 \pm 0.05$  from  $A = 24$ ; these are consistent with an average value of  $\delta_1^s = -0.13 \pm 0.05$ .  $\delta_0^s$  is obtained only from  $A = 24$ ;  $\delta_0^s = -0.30 \pm 0.05$ .

To summarize the results obtained in this section, the comparison of the calculated and experimental  $B(M3)$  values has been made in order to extract effective  $g$  factors. Both harmonic-oscillator and Hartree-Fock radial wave functions have been used for the  $A = 24$  calculations. If we assume that the orbital  $g$  factors have the free-nucleon values we obtain the renormalization to the spin  $g$  factors given by

$$\delta_0^s = -0.30 \pm 0.05$$

and

$$\delta_1^s = -0.13 \pm 0.05, \quad (18)$$

where  $\delta^s$  is defined by Eq. (15) and the errors are an estimate of the theoretical uncertainty in the  $sd$ -shell-model wave functions.

#### IV. CORE-POLARIZATION CORRECTIONS TO THE $M3$ SINGLE-PARTICLE MATRIX ELEMENTS

In this section we calculate the core-polarization corrections to the  $M3$  operator using a delta-function nuclear interaction. The results obtained here for the relationships between the corrections to  $E2$  and  $M3$  matrix elements are the same as those obtained by Zamick.<sup>6</sup> However, instead of expressing the results in terms of proton-neutron properties, we concentrate on the isoscalar-isovector properties which can be directly related to the well-determined experimental isoscalar properties of nuclei. The present results with a delta-function interaction are compared with the results of Horikawa *et al.*<sup>3</sup> obtained from a realistic  $G$  matrix interaction.

Core-polarization calculations can be easily carried out for the restricted cases of one particle outside  $^{16}\text{O}$  or one hole outside  $^{40}\text{Ca}$ . We will assume that these results apply even to the present situations when there are many valence particles or holes; this is an additivity assumption. The additivity assumption is verified empirically by the observed constancy of the isoscalar  $E2$  effective charge in the region  $A = 20-36$ .<sup>17</sup> In addition, we note that the results we obtain do not take into account the binding energy of the valence orbits, and if they are used to compare with experimental results for a few nucleons outside  $^{16}\text{O}$ , binding energy corrections<sup>25</sup> should be made.

The core-polarization corrections to the sep-

arate terms of Eq. (5) are calculated by first re-writing it in the form

$$\langle j ||| O(M\Delta J)_{\Delta T} ||| j' \rangle = [\Delta J (2\Delta J + 1)]^{1/2} [g_{\Delta T}^s \langle j ||| O(\Delta J)_{\Delta T}^s ||| j' \rangle + (2/\Delta J + 1)g_{\Delta T}^l \langle j ||| O(\Delta J)_{\Delta T}^l ||| j' \rangle], \quad (19)$$

where

$$O(\Delta J)^s \equiv r^{\Delta J - 1} (\underline{Y}_{\Delta J - 1} \times \underline{s})^{\Delta J}, \\ O(\Delta J)^l \equiv r^{\Delta J - 1} (\underline{Y}_{\Delta J - 1} \times \underline{l})^{\Delta J}, \quad (20)$$

$$M(jj' \Delta J \Delta T) = \sum_{j_a j'_a J T} \frac{(-1)^{j'+j'_a+J+T+1} (2J+1)(2T+1)}{\epsilon(j') - \epsilon(j) - |\epsilon(j_a) - \epsilon(j'_a)|} \begin{Bmatrix} j & j' & \Delta J \\ j'_a & j_a & J \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & \Delta T \\ \frac{1}{2} & \frac{1}{2} & T \end{Bmatrix} [(1 + \delta_{j j_a})(1 + \delta_{j' j'_a})]^{1/2} \\ \times \langle j j_a J T | V | j' j'_a J T \rangle \langle j_a ||| O(\Delta J)_{\Delta T} ||| j'_a \rangle. \quad (22)$$

The index  $j_a$  runs over all empty orbits and  $j'_a$  over all filled orbits. For the single-hole matrix element the correction can be evaluated by the same expression if the index  $j_a$  is taken to run over all filled orbits and the index  $j'_a$  to run over all empty orbits.

In the case where  $\langle j ||| O(\Delta J) ||| j' \rangle$  is nonvanishing (as is the case here) it is convenient to define a reduction factor  $\delta(jj' \Delta J \Delta T)$  by the relation

$$\langle j ||| O(\Delta J) ||| j' \rangle + M(jj' \Delta J \Delta T) = \langle j ||| O(\Delta J) ||| j' \rangle [1 + \delta(jj' \Delta J \Delta T)]. \quad (23)$$

Now we evaluate the  $E2$  and  $M3$  core-polarization corrections by making some simplifying assumptions. Since all of the operators contain  $r^2$ , the only particle-hole configurations which contribute to the sum in Eq. (22) are those with  $\Delta E = |\epsilon(j_a) - \epsilon(j'_a)| = 2\hbar\omega$  in the harmonic oscillator. We will assume that all of these particle-hole states are degenerate in the unperturbed case and that the residual interactions push the  $2^+$  and  $3^+$   $T=0$  states down to a value  $\Delta E_0$  and push the  $2^+$  and  $3^+$   $T=1$  states up to a value of  $\Delta E_1$ . We will also ignore the difference between  $\epsilon(j')$  and  $\epsilon(j)$  in Eq. (22) since they are small relative to  $2\hbar\omega$ .

The corrections are then calculated with a delta-function residual interaction.

$$V(\vec{r}_{12}) = -[V_0 + V_\sigma \vec{\sigma}_1 \cdot \vec{\sigma}_2] \delta(\vec{r}_{12}). \quad (24)$$

The physical explanation of the results in this case has been given by Zamick.<sup>6</sup> The results can be expressed in the form

$$\delta e_0 \equiv \delta(E2, \Delta T = 0) = \frac{(3V_0 - 3V_\sigma)}{\Delta E_0(2^+)} C(jj'),$$

and in addition we will define the operator associated with the electric multipole operator of rank  $\Delta J - 1$  by

$$O(\Delta J - 1) = r^{\Delta J - 1} \underline{Y}_{\Delta J - 1}. \quad (21)$$

For a particle outside a closed shell the core-polarization corrections due to one-particle one-hole excitations of the core contribute to the total matrix element by the addition of a term  $M(jj' \Delta J \Delta T)$  which is given by<sup>26</sup>

$$\delta e_1 \equiv \delta(E2, \Delta T = 1) = \frac{(-V_0 - 3V_\sigma)}{\Delta E_1(2^+)} C(jj'), \quad (25)$$

$$\delta_0^s \equiv \delta(M3, \Delta T = 0)^s = \frac{(-V_0 + 5V_\sigma)}{\Delta E_0(3^+)} C(jj'),$$

$$\delta_1^s \equiv \delta(M3, \Delta T = 1)^s = \frac{(-V_0 + V_\sigma)}{\Delta E_1(3^+)} C(jj'),$$

and

$$\delta(M3, \Delta T = 0)^l = \delta(M3, \Delta T = 1)^l = 0. \quad (26)$$

All of the orbit dependence is contained in the coefficients  $C(jj')$  which depend on  $l$  and  $l'$ ; for one particle outside  $^{16}\text{O}$

$$C(d-d) = 0.0109 \text{ fm}^{-3}, \quad (27)$$

$$C(d-s) = 0.0072 \text{ fm}^{-3},$$

and for one hole outside  $^{40}\text{Ca}$

$$C(d-d) = 0.0123 \text{ fm}^{-3}, \quad (28)$$

$$C(d-s) = 0.0109 \text{ fm}^{-3}.$$

Thus, in the limit of a delta-function residual interaction there are relationships between the first-order core-polarization corrections to the  $M3$  and  $E2$  operators. The simplest of these relates the effective isovector spin  $g$  factor to the  $E2$  isoscalar effective charge

$$\delta_1^s = -\left(\frac{\Delta E_0(2^+)}{\Delta E_1(3^+)}\right) \frac{\delta e_0}{3}. \quad (29)$$

Another relation could take the form

$$\delta_0^s = -\left(\frac{\Delta E_1(2^+)}{\Delta E_0(3^+)}\right) \delta e_1 - \frac{2}{3} \left(\frac{\Delta E_0(2^+)}{\Delta E_0(3^+)}\right) \delta e_0. \quad (30)$$

It is important to compare these results obtained with a delta-function interaction with those ob-

tained from more realistic interactions. Horikawa *et al.*<sup>8</sup> have calculated the corrections to the  $E2$  and  $M3$  matrix elements using the two-body  $G$  matrix elements of Kuo.<sup>8</sup> They used the empirical single-particle energies of Ref. 27 which amount to putting  $\Delta E_0(2^+) \approx \Delta E_1(2^+) \approx \Delta E_0(3^+) \approx \Delta E_1(3^+) \approx 40$  MeV. Thus we might expect  $\delta_1^s \approx -(\frac{1}{3})\delta e_0$  and  $\delta_0^s \approx -\delta e_1 - (\frac{2}{3})\delta e_0$  for these calculations. These quantities are compared in Table VII and it is seen in most cases that these equalities hold fairly well, even with a realistic interaction.

Secondly, it is important to use realistic values for  $\Delta E$ . The isoscalar modes represent the oscillation of neutrons and protons in phase and thus  $\Delta E_0 < 2\hbar\omega$ . Theoretical calculations<sup>28</sup> for the energy of the giant quadrupole state have found  $\Delta E_0(2^+) = \sqrt{2}\hbar\omega$  and this seems to be in agreement with experimental observations for at least  $A \geq 24$ .<sup>29</sup> The other three modes have not yet been observed experimentally. The isovector quadrupole resonance has been estimated in macroscopic models to have  $\Delta E \approx 3.3\hbar\omega$ .<sup>28</sup> Thus it seems reasonable to use in terms of round numbers,

$$\begin{aligned}\Delta E_0(2^+) &= \Delta E_0(3^+) = \sqrt{2}\hbar\omega, \\ \Delta E_1(2^+) &= \Delta E_1(3^+) = 2\sqrt{2}\hbar\omega.\end{aligned}\quad (31)$$

Putting these values into Eqs. (29) and (30) give

$$\begin{aligned}\delta_1^s &\approx -\frac{1}{6}\delta e_0, \\ \delta_0^s &\approx -2\delta e_1 - \frac{2}{3}\delta e_0.\end{aligned}\quad (32)$$

The  $E2$  isoscalar effective charge has been accurately determined by a comparison of experimental and theoretical  $B(E2)$  values and quadrupole moments in the region  $A = 20-36$ .<sup>17</sup> The empirical isoscalar effective charge is remarkably constant over this mass range with a value  $e_p + e_n = 1.7 \pm 0.1e$  or  $\delta e_0 = 0.7 \pm 0.1$ . Thus the relation  $\delta_1^s = -\delta e_0/6$  is satisfied with the empirical value

TABLE VII. Core-polarization corrections calculated with the two-body  $G$  matrix elements of Kuo. The values were obtained from Table I of Ref. 8. The empirical single-particle energies of Ref. 27 were used for these calculations; the average  $\Delta E$  is about 40 MeV.

	$j-j'$		
	$\frac{5}{2}-\frac{5}{2}$	$\frac{5}{2}-\frac{3}{2}$	$\frac{1}{2}-\frac{5}{2}$
$\delta e_0$	0.295	0.42	0.28
$\delta e_1$	-0.145	-0.12	-0.10
$-(\frac{1}{3})\delta e_0$	-0.10	-0.14	-0.09
$\delta_1^s$	-0.08 <sup>a</sup>	-0.14 <sup>a</sup>	-0.06
$-\delta e_1 - (\frac{2}{3})\delta e_0$	-0.05	-0.14	-0.09
$\delta_0^s$	-0.015 <sup>a</sup>	-0.14 <sup>a</sup>	-0.09

<sup>a</sup> These values were extracted from Table I of Ref. 8 by assuming that  $\delta_0^s = \delta_1^s = 0$ .

of  $\delta_1^s = -0.13 \pm 0.05$ .

The second relationship, involving  $\delta_0^s$ , would be satisfied with  $\delta_0^s = -0.30 \pm 0.05$  if the correction to the isovector  $E2$  charge is small, i.e.,  $\delta e_1 = -0.08 \pm 0.04$  ( $e_p - e_n = 0.92 \pm 0.04e$ ). The  $B(E2)$  values and quadrupole moments in the region  $A = 20-36$  are not very sensitive to the isovector effective charge since the transition densities (the matrix elements) are typically 90% dominated by the isoscalar component. However, a recent analysis<sup>31</sup> of transitions in mirror nuclei in this region indicates that the isovector effective charge is indeed near its free-nucleon value. In contrast, an analysis<sup>25</sup> of the nuclei with one or two holes or particles outside  $^{16}\text{O}$  and  $^{40}\text{Ca}$  indicated the need for a large correction to the isovector effective charge,  $e_p - e_n \approx 0.6e$  or  $\delta e_1 \approx -0.4$ . However, in this analysis the isovector effective charge could be made closer to the free-nucleon value if it could be shown (1) that the Woods-Saxon method as used in Ref. 25 consistently *overestimates* the rms radii of *valence particles* (this is the same direction that is needed to resolve the Nolen-Schiffer anomaly), and (2) that the reported value for the experimental quadrupole moment of  $^{39}\text{K}$  (see Table IX in Ref. 25) is in error, perhaps due to uncertainty in the Sternheimer corrections.

Finally we remark on the parameters of the delta interaction as related to the  $E2$  and  $M3$  corrections in Eqs. (25). The delta function is an extreme simplification of the two-body interaction which has been used historically as well as here to derive analytic results which may provide some physical insights. In finite nuclei a renormalized interaction must be used and this interaction is not *a priori* accurately known. The delta  $V(\vec{r}_{12}) = -A_T\delta(\vec{r}_{12})$ , surface delta (SDI)  $V^{\text{SDI}}(\vec{r}_{12}) = -A_T\delta(\vec{r}_{12})\delta(r_1 - R)$ , and modified surface delta (MSDI)  $V^{\text{MSDI}}(\vec{r}_{12}) = V^{\text{SDI}}(\vec{r}_{12}) + B_T$ , interactions<sup>26</sup> provide minimal parametrizations for effective interactions which are remarkably successful in reproducing observed spectra.

The parameters  $V_0$  and  $V_\sigma$  are related to the delta interaction in specified isospin channels,  $-A_T\delta(\vec{r}_{12})$ , by  $A_0 = V_0 + V_\sigma$  and  $A_1 = V_0 - 3V_\sigma$ . For the two-nucleon system we know that the  $nm$  and  $np$  potentials have about the same volume integrals, but that the  $np$  potential is a little larger since it has one bound state whereas the  $nm$  potential does not, thus  $A_0 > A_1 > 0$  or  $V_0 \gg V_\sigma > 0$ . If the valence particle spectra in nuclei are fitted with a delta or SDI interaction, then the same conclusion as above is reached, namely  $V_0 \gg V_\sigma > 0$ . However, if an effective long (infinite) range component is allowed for by using the MSDI, one finds in contrast  $A_0 < A_1$  (Ref. 26 p. 116) or  $V_\sigma < 0$ . [Note that the MSDI  $B_T$  parameter does not enter into the



off-diagonal matrix elements in Eq. (22)].

Typical values of the delta interaction parameters which have been used previously<sup>9, 25</sup> to estimate the  $E2$  and  $M2$  core-polarization corrections are  $V_0 = 500 \text{ MeV fm}^3$  and  $V_\sigma = 60 \text{ MeV fm}^3$  (in Zamick's notation<sup>6</sup>  $V(\vec{r}_{12}) = -A[1 + (-1)^{T_x}]\delta(\vec{r}_{12})$ , and these parameters correspond to  $A = 472 \text{ MeV fm}^3$  and  $x = 0.273$ ; in Ref. 6 Zamick used  $x = \frac{1}{3}$ ). Using  $\Delta E = 2\hbar\omega = 28 \text{ MeV}$  for  $^{16}\text{O}$  we find  $\delta e_0 = 0.51$  and  $\delta e_1 = -0.27$  for the  $d$ - $d$  matrix element. These are fairly consistent with the empirical results<sup>25</sup> for  $A = 17$  and  $18$  as well as with the values obtained using the Kuo  $G$  matrix; from Table VII, using  $\Delta E = 28 \text{ MeV}$  rather than  $\Delta E = 40 \text{ MeV}$ , we obtain  $\delta e_0 \approx 0.42$  and  $\delta e_1 \approx -0.21$  for the  $d_{5/2}$ - $d_{5/2}$  matrix element. For  $A = 24$ , if we take a linear interpolation of the  $C$  coefficients between  $^{16}\text{O}$  [Eq. (27)] and  $^{40}\text{Ca}$  [Eq. (28)] and use  $\Delta E = 2\hbar\omega = 24 \text{ MeV}$ , then from Eqs. (25) we obtain  $\delta_0^s = -0.09$  and  $\delta_1^s = -0.20$  for the  $d$ - $d$  matrix elements. (These are the results of the calculations discussed in Ref. 9, however, the value of  $\delta_0^s = -0.314$  given in Table III of Ref. 9 is incorrect, it should be  $-0.09$  as above.)

However, if we are to understand the empirical result  $\delta_0^s \approx -0.30$  ( $\delta e_1 \approx -0.08$ ) we require  $V_\sigma \approx -30 \text{ MeV fm}^3$  rather than the previously assumed value of  $+60 \text{ MeV fm}^3$ . Thus we require  $V_\sigma < 0$ , which is consistent with the empirical MSDI effective interactions for valence particles.

#### V. $M3$ GAMMA DECAYS IN $^{38}\text{K}$ AND $^{38}\text{Cl}$

In this section we present the results of calculations for the as yet unobserved  $0^+ \rightarrow 3^+$  gamma decay in  $^{38}\text{K}$  and the observed<sup>11</sup> but very weak decay strength of the  $5^- \rightarrow 2^-$  transition in  $^{38}\text{Cl}$ .

The  $sd$ -shell-model transition density for the  $^{38}\text{K}$   $0^+ \rightarrow 3^+$  matrix element is given in Table II. With free-nucleon  $g$  factors the result  $B(M3) = 0.048 \mu_N^2 \text{ fm}^4$  is obtained, whereas with effective spin  $g$  factors of  $\delta_1^s = -0.13$  and  $\delta_0^s = -0.30$  the calculated  $B(M3)$  value is quenched to  $0.025 \mu_N^2 \text{ fm}^4$ . We have used a harmonic-oscillator parameter  $b = 1.948 \text{ fm}$  obtained from the experimental  $^{38}\text{Ar}$  rms charge radius of  $3.414 \text{ fm}$  (see Table IV).

The  $^{38}\text{K}$   $0^+$  ( $T = 1$ ) level is observed<sup>11</sup> to beta decay to the  $0^+$  level in  $^{38}\text{Ar}$  with a 100% branch and with  $T_{1/2} = 924.6 \pm 1.5 \text{ ms}$ . The above calculated  $B(M3)$  values give branching ratios for the  $0^+ \rightarrow 3^+$  gamma decay of  $2.6 \times 10^{-5}\%$  and  $1.4 \times 10^{-5}\%$ , respectively, with the free and effective  $g$  factors. Thus, because the  $M3$  matrix element is small it will be difficult to observe this gamma-decay branch.

Calculations for  $^{38}\text{Cl}$  which include all  $sd$ -shell orbits for the protons and all  $fp$ -shell orbits for

the neutrons have not been carried out. However, it is clear that the wave functions for the  $2^-$  and  $5^-$  states have predominantly a  $\pi d_{3/2} \nu f_{7/2}$  configuration and it is instructive to calculate the  $B(M3)$  value with this simple configuration. The reduced matrix element is

$$\begin{aligned} [B(M3), 5^- \rightarrow 2^-]^{1/2} &= \frac{1}{(11)^{1/2}} \langle \pi d_{3/2} \nu f_{7/2} 2^- || M3 || \pi d_{3/2} \nu f_{7/2} 5^- \rangle \\ &= [-0.0165 \langle \nu f_{7/2} || (M3)_n || \nu f_{7/2} \rangle \\ &\quad + 2.370 \langle \pi d_{3/2} || (M3)_p || \pi d_{3/2} \rangle] \\ &= (-0.652 + 0.625)b^2, \end{aligned} \quad (33)$$

where free-nucleon  $g$  factors and harmonic-oscillator wave functions have been used to obtain the last line. Using  $b = 1.948 \text{ fm}$  (appropriate for  $^{38}\text{Ar}$ ) we obtain the nearly vanishing result of  $B(M3) = 0.010 \mu_N^2 \text{ fm}^4$  which results from cancellation between neutron and proton components. This cancellation would be extremely sensitive to other  $sd$  and  $fp$  components in the wave function. Thus, we can understand why the experimental value of  $B(M3) 5^- \rightarrow 2^- = 2.38 \mu_N^2 \text{ fm}^4$  is small, but until more complete shell-model calculations are carried out we cannot discuss this transition on the same level of sophistication concerning the effective  $g$  factors as is possible for the  $M3$  transitions in  $^{24}\text{Al}$ ,  $^{24}\text{Na}$ ,  $^{34}\text{Cl}$ , and  $^{38}\text{K}$ .

#### VI. SUMMARY AND CONCLUSIONS

Shell-model calculations have been carried out for the measured  $M3$  transitions in light nuclei. Wave functions for the valence particles are adequate to explain the order-of-magnitude variation in  $B(M3)$  values which range from  $9.1 \text{ W.u.}$  in  $^{24}\text{Na}$  to  $0.011 \text{ W.u.}$  in  $^{38}\text{Cl}$ . The relatively large transition strengths in  $^{24}\text{Al}$ ,  $^{24}\text{Na}$ , and  $^{34}\text{Cl}$  are found to be hindered by about 30% relative to the shell-model calculations. In Sec. IV the core-polarization contributions to the  $M3$  matrix elements were calculated and related to the well known  $E2$  core-polarization effects. Relationships between the effective  $M3$  spin  $g$  factors and the effective  $E2$  core-polarization charges were derived. The empirical values are found to satisfy these relationships if the isovector  $E2$  effective charge is near the free-nucleon value,  $e_p - e_n \approx 0.92e$ . Further experiments are needed to directly determine the isovector effective charge, such as comparative  $\pi^*$  inelastic scattering.<sup>30</sup>

For a better understanding of the  $M3$  matrix element, what is greatly needed are experimental values of the  $M3$  moments of stable nuclei with

$\frac{3}{2}^+$  ground states in the  $sd$  shell; these can be determined from electron scattering experiments. The discussion in this paper has been focused on the gamma-decay matrix elements which correspond to the electron scattering matrix elements at a momentum transfer of  $q=0$ . For finite momentum transfer the core-polarization contributions depend on  $q$ , and calculations of this  $q$  dependence will be discussed in a separate paper.<sup>31</sup> Nevertheless it is interesting to quote the values for the  $^{17}\text{O}$  and  $^{39}\text{K}$  moments based on the theory at  $q=0$  presented in Sec. IV. If we use  $\delta_1^s = -0.13$  and  $\delta_0^s = -0.30$ , then the ratio of the effective  $\nu d_{5/2}$  matrix element to the single-particle value is

$$\frac{\tilde{g}_n^s}{g_n^s} = \frac{[g_0^s(1 + \delta_0^s) - g_1^s(1 + \delta_1^s)]}{g_n^s} = 0.91, \quad (34)$$

or the electron scattering cross section should be reduced by 20% relative to the single-particle value. This is not in agreement with the experimental cross section for  $^{17}\text{O}$  (Ref. 1) which is reduced by about a factor of 3 from the single-particle value. This discrepancy may be due to a strong  $q$  dependence of the effective  $g$  factors, or an anomaly in the neutron  $d_{5/2}$  radial wave function.

For the  $\pi d_{3/2}^{-1}$  ground state of  $^{39}\text{K}$ , the ratio of the effective matrix element to the single-particle value is

$$\frac{\tilde{g}_p^s - 4}{g_p^s - 4} = \frac{[g_0^s(1 + \delta_0^s) + g_1^s(1 + \delta_1^s)] - 4}{g_p^s - 4} = 0.45. \quad (35)$$

That is, the cross section is only 20% of the single-particle value. Clearly the  $^{39}\text{K}$   $M3$  moment is a very sensitive measure of  $\tilde{g}_p^s$ . Further predictions for the moments of all  $sd$ -shell nuclei will be presented in a subsequent paper.<sup>31</sup>

Finally some comments about the more general aspects of the calculations are presented here. Our view of nuclear structure is that the variety of patterns of motion exhibited in nuclear spectra

are governed by the interaction of valence nucleons. The core nucleons participate only to the extent that they tend to follow in a self-consistent fashion the motions set up by the valence nucleons. It is essential to treat the valence nucleons as completely and as consistently as possible. For the  $sd$  shell we have done this by always using a complete  $(d_{5/2}, s_{1/2}, d_{3/2})$  basis; for heavy nuclei the interacting boson model<sup>32</sup> may provide a useful semicomplete basis. The core-polarization effects of the core nucleons give rise to renormalized single-particle and two-particle matrix elements which vary slowly with mass and excitation energy, in contrast to the features of the spectra which vary rapidly with mass and excitation energy.

The core-polarization effects can be parametrized by using effective charges and effective  $g$  factors. However, these  $g$  factors depend on multipolarity (and perhaps momentum transfer). For instance, the  $M1$  matrix elements in the  $sd$  shell require essentially the free-nucleon  $g$  factors, whereas as we have seen in the present work the  $M3$  spin factors are substantially quenched. However, there is a relationship between the effective  $M3$  operator and the effective  $E2$  operator as shown in Sec. IV. It would be very interesting to extend this relationship to more general ones concerning the  $M(L)$  and  $E(L-1)$  operators. Mesonic exchange effects and effects due to the internal excitation of the nucleons have been neglected and presumed small in this work. Microscopic calculations of these effects along the lines of Refs. 7 and 33 are needed.

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