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Retardation, quasipotential equations, and relativistic corrections to the deuteron charge operator

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Four different methods for handling the retardation of a single meson exchange in the nuclear force have been examined, together with the corresponding contributions to the deuteron charge operator. It is shown that to order $(v/c)^2$ these operators are all part of a single unitarily equivalent family. The Gross quasipotential equation is examined and relativistic corrections to the deuteron charge form factor are shown to be the same as those generated by the author's method, when converted to a common unitary representation. The same result has also been demonstrated for the folded diagram method. The asymmetric terms in Gross's Hamiltonian are shown to take the place of recoil graph contributions to the charge operator. These terms are necessary for Lorentz invariance and the "Gross correction" to the deuteron charge form factor.

[NUCLEAR STRUCTURE Deuteron, ρ, meson exchange currents, relativistic effects.]

I. INTRODUCTION

The extension of nuclear physics into new domains of energy following the advent of modern accelerators has reopened old questions left unanswered by a previous generation of physicists. One of these questions is: How does one introduce special relativity into nuclear physics consistent with general principles and with sufficient flexibility to allow model calculations of observables? It is understood that since nuclei are basically nonrelativistic, even $(v/c)^2$ corrections (first-order relativistic corrections) would suffice for most applications. Because it is light and *relatively* simple, much of the interest has centered on the deuteron.

Three basic methods have been used in studying relativistic effects. The genesis of the most common method is the Bethe-Salpeter[1] (BS) equation. This equation sums exchanges of bosons of arbitrary complexity between two fermions, producing a four-dimensional integral (or integro-differential) equation for the composite wave function of the two-fermion system. Transition probabilities can be calculated with these wave functions using the method of Mandelstam.[2] In addition to the usual three-dimensional relative momentum or relative space coordinates of the fermions found in the Schrödinger equation, the relative energy or relative time also occurs; this has no nonrelativistic analog.[3] Concomitantly, there may be an additional quantum number[4] [to the usual (n, l, m) for spinless meson exchange between spinless "nucleons," for example] for solutions of the BS equation which have no nonrelativistic analog.[4,5] Interpretation of solutions of the Bethe-Salpeter equation is therefore difficult, even if one could easily solve this equation.

Relatively few attempts to solve this equation have been made,[6,7] and almost all of these have used the ladder approximation, summing only the nonoverlapping Feynman ladder of boson exchanges. This neglect of crossed ladder graphs has serious consequences in atomic physics,[4] where it is known that if the mass of the heavier of the fermions m_t becomes infinite, the resulting two-fermion amplitude (including all photon exchanges) reduces to the ordinary Dirac amplitude for the light fermion.[8] This nontrivial result does *not* hold in the ladder approximation,[4] where spurious terms of order $(Z\alpha)^3 \log(Z\alpha)$ appear,[9,10] compared to the usual fine structure terms of order $(Z\alpha)^4$; these terms are exactly canceled by the contribution of the crossed graphs.[4] Consequently the Bethe-Salpeter equation is rarely solved, but rather is converted into a three-dimensional equation, whose lowest-order approximation is the Breit equation.[11] A variety of methods have been used, such as the procedure of Salpeter and that of Sucher and others.[5] The Breit equation, while approximate, has the correct limiting form as $m_t \to \infty$ and is simple in form; it is widely used.

The form of the Breit equation is what one would *naively* write for a two-particle system. If the Hamiltonians of the two free particles are defined as H_1 and H_2, respectively, the Breit equation[11,12] is given by

$$(E - H_1 - H_2 - V)\psi = 0, \tag{1}$$

where V is a function of $\vec{P}(=\vec{p}_1 + \vec{p}_2)$ the total mo-

momentum, $\vec{p}[=(\vec{p}_1-\vec{p}_2)/2]$ the relative momentum, and $\vec{x}(=\vec{x}_1-\vec{x}_2)$ the relative coordinate of the two nucleons, whose canonical coordinates and momenta are $\vec{p}_i$ and $\vec{x}_i$. In addition, V may depend on the (Pauli) spin operators $\vec{\sigma}_1$ and $\vec{\sigma}_2$ of the two fermions. The usual form of V in atomic physics is derived from one-photon exchange.[11,13] Note that $\vec{p}$ is so defined that $[p^\alpha, x^\beta]=-i\delta^{\alpha\beta}$. While the Breit equation has the correct limiting behavior, it does not have correct analytic properties. Indeed, the usual Breit interaction allows processes which are unphysical, such as one fermion making a transition into a negative energy state. This spurious behavior is called "Brown's disease" and must be corrected for.[13,14] Fortunately, for sufficiently low center-of-mass (binding) energies compared to the total mass this problem is not particularly important. Nevertheless it must always be borne in mind that the form (1) is basically an approximation which, even though it may yield binding energies virtually identical to the original (Bethe-Salpeter) equation, is not necessarily equivalent to it in all aspects.[4]

A more serious problem is the lack of uniqueness of the potential V (to order α). Historically, two atomic potentials were developed: the Breit[15] and Gaunt[16] interactions. They differ in that the retardation in one-photon exchange, calculated in Lorentz gauge, was ignored by Gaunt, while Breit ignored retardation in Coulomb gauge. For the hydrogen atom, the Gaunt interaction introduces recoil terms of order α^4/m_t, while the Breit interaction produces α^5/m_t corrections.[11] Thus, the Breit interaction is "better" in the sense that it produces smaller corrections to the Dirac spectrum. The reason is that Coulomb gauge automatically builds retardation into the interaction between charges; Gaunt's choice of gauge and his approximations neglected this. The *complete* set of all corrections to the energy must be the same, irrespective of gauge, and higher order (in α) parts of the potential must be introduced which are different in the two cases. Clearly the effective *Hamiltonian* due to photon exchanges can be gauge dependent, while the *energy* itself must be gauge independent. Because of the close connection between gauge transformations and unitary transformations, it should produce no surprises that the same retardation ambiguity will appear in the deuteron problem as a unitary ambiguity.

Nuclear physics, as well as atomic physics, has approached the Bethe-Salpeter formulation of the deuteron problem with the intention of molding an efficacious three-dimensional form from the original four-dimensional formulation. A variety of methods have been used[17-21]; the primary feature of these various methods is that they lead to different Hamiltonians. The process is often called a Blankenbecler-Sugar[21] reduction, while the various forms are often referred to as quasipotential equations.[3,4] These differences have been noted and studied by Yaes,[17] Woloshyn and Jackson,[18] and Klein and Lee,[20] among others. Basically, the different techniques generate equations of motion where the relative energy has been fixed by a specific prescription, while enforcing *elastic* unitary for the two-body amplitudes. Extrapolations away from the elastic cut are largely arbitrary; infinite classes of different results are possible. In addition, certain of the equations have the form of the *square* of the Schrödinger equation and others, in particular the Gross equation,[22,23] are not symmetric under intercharge of the two (identical) particles. The other ingredient is the fact that the potentials are not in general energy independent.

An energy-independent potential has several practical and conceptual advantages.[24] The usual interpretation of the probability density as the square of the wave function and the Hermiticity of the Hamiltonian rely upon the *effective* potential being independent of energy, as does orthogonality of different eigenstates in the usual fashion.[24-26] Unfortunately, most methods based on ordinary perturbation theory and the quasipotential methods previously described generate energy-dependent potentials. Special techniques have been developed, however, which allow a systematic generation of an energy-independent effective potential below the first inelastic threshold. The oldest method is perhaps the Fukuda-Sawada-Taketani (FST) formalism,[27] an algebraic method. Another is the folded diagram method popularized by Johnson.[24] A third method is less elegant and perhaps more transparent,[28,29] being analogous to the mass and wave function renormalization[30] used in relativistic quantum mechanics. All three methods for treating the two-body problem we will classify as *equations of motion* methods, as opposed to the quasipotential reformulations of the Bethe-Salpeter equation.

The FST method rearranges the equations of motion of the two-body system so that an effective potential is derived which is independent of energy. The basic problem can be stated schematically, as follows. Imagine that the Hilbert space of our nucleus can be divided into two sectors when the meson-nucleon interaction is turned off: free nucleons (upper) and free nucleons plus free mesons (lower). When the meson-nucleon interaction is turned back on, an effective potential is generated between nucleons but, unless some care is exercised, this potential will depend on the energy. Operators connecting upper to lower

components are called *odd*, while *even* operators do not connect upper and lower parts. The FST method eliminates this problem by guaranteeing that the two components of the original Hilbert space are totally decoupled below meson production threshold. Thus the orthonormality of the original noninteracting problem is retained in the more complicated interacting problem. This decoupling procedure is precisely what was accomplished by Foldy and Wouthuysen[31] (FW) in a different physical (though formally identical) problem using successive unitary transformations to eliminate all *odd* operators, leaving only *even* ones. Indeed, the most elegant formulation of FST was produced[32] using unitary transformations, since unitary transformations guarantee orthonormality at every step. A problem exists, however, that was originally overlooked: Since an even operator can be converted into an even operator by an even unitary transformation, the FW method for generating potentials is not unique.[28] Clearly, nuclear transition operators corresponding to external interactions are also not unique, being related to the various FW Hamiltonians mentioned above by the *same* set of unitary transformations. This lack of uniqueness is sometimes called the Barnhill ambiguity,[33] although it was known in various forms long before the work of Ref. 33. What was not appreciated, however, was that the transition *matrix elements* (as opposed to operators) must be the same, regardless of the formulation of the same *physical* problem.[34]

The folded diagram method, on the other hand, works with the individual components of Feynman graphs (propagators) and directly attacks the source of the energy dependence (the relative time) in the two-body problem. By averaging over the relative *energy* in a well-defined way, a nonstatic, energy-independent instantaneous potential is generated which is not unique. This method, described in detail in Ref. 24, has certain calculational advantages over other methods.

The equations of motion technique of the present author[28,29] has its genesis in the individual equations of motion of nucleons and mesons. By means of an FW transformation, a nonrelativistic reduction of the equations of motion is made. This reduction generates meson-nucleon vertices corresponding to nonrelativistic and relativistic correction terms. These vertices are connected by means of ordinary Feynman diagrams using time-dependent perturbation theory. Because perturbation theory generates an energy-dependent potential, a renormalization procedure was used to remove the energy dependence by means of an expansion technique formally identical to the expansion of the self-mass function during mass renormalization in field theory. The expansion is *not* an adiabatic *approximation*, but rather an expansion about this approximation. This "mass renormalization" simply defines an energy-independent meson-exchange potential to be used in the calculation of the nuclear wave functions. A similar procedure was used by Drake[35] in his derivation of the Breit interaction. The mass renormalization necessitates a wave function renormalization which *exactly cancels* the *static part* of the recoil graph, a result which also follows from the FST formalism and was noted by Woloshyn[2] for the BS equation. The results for the potential and the nonstatic residue of the recoil graph were not unique. In this case it was possible to demonstrate that the separate Hamiltonian and recoil graph (charge) operators were unitarily equivalent. These results were later extended to the isoscalar current operator, as well. The main result of this series of calculations was that certain operators were not uniquely defined, but that matrix elements were unique for a given physical model. Furthermore, the ambiguities occurred in relativistic corrections and not in the static, nonrelativistic operators.

The third approach to relativistic corrections eschewed dynamical assumptions of the type inherent in the calculations described above and examined carefully the requirements of the Poincaré group inherent in *any* calculation. Many different people have worked on this approach, but for our purposes the work of Foldy[36] and Krajcik and Foldy[37] are central. In order that the total energy E of a system of particles with total momentum $\vec{P}$, rest mass $\hat{M}$, and rest-frame wave function ψ_0 have the form $E=(\vec{P}^2+\hat{M}^2)^{1/2}$, the wave function must have the form[38] [to order $(v/c)^2$]

$$\psi_{\vec{P}} \cong [1-i\chi(\vec{P})]\psi_0 e^{i\vec{P}\cdot\vec{R}}\ , \tag{2}$$

where $\vec{R}$ is the *usual* center-of-mass coordinate corresponding to $\vec{P}$. The function χ has been chosen to vanish when $\vec{P}=0$ and does not depend on $\vec{R}$; it can depend on the relative momenta and coordinates. Foldy[36] has prescribed a condition for $\chi(\vec{P})$ such that E will have the aforementioned form. Note that $\chi(\vec{P})$ has the form of a unitary transformation and is also not unique. Nevertheless the parts of χ independent of potential (χ_0) are essentially unique and ambiguities are restricted to the potential-dependent parts χ_v. Later, Close and Osborn[39] used and extended the above results by examining the conditions under which the matrix elements of the charge and current operators have the correct Lorentz transformation properties. They showed that χ played a cen-

tral role in this important condition. The present author then demonstrated[29] how a specific series of model calculations fit the conditions of Foldy and Close and Osborn and illustrated the model dependence of χ_v. Thus, the work of Foldy and Close and Osborn established *constraints* on the results obtained from any dynamical theory.

Somewhat earlier work by the present author,[40] unconnected to exchange currents, showed that the χ_0 term defined above was the means by which Lorentz contraction, the Thomas precession, and several other relativistic phenomena were incorporated into charge form factors. The extremely complex results of Gross,[41,55] obtained using his quasipotential method, were simplified and shown to fit within the framework of the Krajcik-Foldy χ_0 correction (i. e., purely kinematic) except for a single potential-dependent term which was called the "Gross correction." It was speculated that the Gross correction originated in a χ_v term connected with retardation of the nuclear force.[38] The original Gross work[41] and the original work of the present author[40] did not include meson-exchange currents.

In a separate vein from considerations of retardation and energy-independent potentials which apply to all meson exchanges, a special set of considerations apply to pion exchanges because of the equivalence theorem. This theorem, which is only approximate, relates the physics of pseudovector and pseudoscalar couplings of pions to nucleons. Effected by means of the canonical transformation of Dyson[42] on field theories, it is more general than the approximate nonrelativistic version which is useful in discussing exchange currents. Basically, the theorem states[29,43] that the one-pion-exchange currents, together with matrix elements of the impulse charge operator calculated with a potential including OPEP (one-pion-exchange potential), must be the same for both PS (pseudoscalar) and PV (pseudovector) couplings to order g^2 in *certain limiting cases*. Although this may appear trivial, it is actually a severe constraint on approximate calculations. Deleting either the nonstatic parts of OPEP or the recoil graph (i. e., relativistic corrections) is sufficient to spoil the equivalence. In an operational sense the equivalence is proven by demonstrating that the Hamiltonians and transition operators corresponding to the two different couplings are unitarily equivalent in the nuclear Hilbert space. As an interesting byproduct of this work, the unitary transformation involved can change the deuteron D-state percentage, demonstrating that this quantity is not measurable.[44]

It should be abundantly obvious at this point that unitary transformations abound which are associated with the relativistic aspects of the nuclear Hamiltonian and charge-current operators. Recently, three criteria were proposed[45] for a successful calculation of nuclear charge and current operators: (1) The current must be conserved. (2) The current and Hamiltonian must satisfy the constraints of special relativity; that is, the model must provide a realization of the Poincaré group. (3) The current and Hamiltonian must satisfy the (approximate) constraints of the equivalence theorem. Although the first two requirements are obvious, the third is also important in order to establish internal consistency and to probe any model dependence.

In this rather lengthy introduction we have several times appealed to the literature of atomic physics for supporting arguments and insight. In what follows, we will continue to refer to this literature where it is relevant. Because the basic interaction mechanism in atoms is known, calculational sophistication is advanced. In addition, some experiments can be performed with accuracy unattainable in other fields, further pushing theoretical development. The lessons of atomic physics are relevant for nuclear physics.

In the body of this work we will discuss the effect of pion exchange, as well as vector- and scalar-meson exchange, on the charge form factor of the deuteron, keeping terms of order $(v/c)^2$. In Sec. II we will sketch the results of the equations of motion method as derived by the present author for the case of the deuteron charge form factor. In Sec. III the results of the FST method for the recoil graph will be discussed, while in Sec. IV the folded diagram approach will be examined. In Sec. V the quasipotential method, with emphasis on Gross's approach to the deuteron problem, will be discussed. Finally, in Sec. VI the results of the previous sections will be compared and unified. Detailed formulas for various matrix elements will be relegated to an appendix. Throughout this work, the crucial importance of the various unitary transformations which forbid uniqueness will be emphasized. Indeed, using these transformations it will be shown that correct calculations based on these diverse techniques are unitarily equivalent to the order these calculations have been performed; that is, matrix elements for form factors and other observables are identical. Thus, very different techniques yielding different *appearing* results generate identical *physics*.

II. FW APPROACH

Since the results of the present author's FW calculation of the elastic deuteron charge form factor

were calculated in some detail elsewhere,[29] we will present only the results. Fortunately, although calculations of individual contributions can be lengthy, there are only three form factor forms which arise from all the (isoscalar) processes we will discuss. The charge form factor $F(\vec{q})$, defined so that $F(0)=1$, has the form

$$F(\vec{q}) = F^0(q) - \frac{S_{12}(\hat{q})}{\sqrt{8}} F^2(q) , \tag{3}$$

where F^0 and F^2 are the monopole and quadrupole parts of F, respectively, and $S_{12}(\hat{q})$ is the usual tensor operator constructed by replacing $\hat{x}$ by $\hat{q}$, the unit vector corresponding to the momentum transfer $\vec{q}$.

The three form factors which will arise and comprise F are the impulse approximation contribution F_0, the pion exchange term D_π, and the "Gross" term D_G. The latter form differs somewhat for one-pion exchange, denoted by D_G, and for scalar and vector exchange, where it is denoted by D_G'. Only linear combinations of F_0, D_π, D_G, and D_G' with factors of G_E^S and G_M^S, the isoscalar electric and magnetic nucleon form factors, will occur. Detailed expressions are given in the Appendix.

There are three basic types of isoscalar processes which contribute to F, if we restrict ourselves to ordinary one-boson exchange; that is, we ignore isobar contributions, $\rho\pi\gamma$ processes, and the like. The first is the impulse approximation, which includes motional relativistic corrections generated by χ. The terms generated by χ_0 have been discussed in detail elsewhere[40] and are included at the end of the Appendix; we will ignore them and concentrate on χ_v. The second type of process is the "seagull," of which the pair contribution, from virtual nucleon-antinucleon pairs, is typical. The last type is from the renormalized recoil graph, where only the nonstatic residue of this graph survives the exact cancellation of the static part by the renormalized disconnected graphs.[29] All of our results can be so categorized.

In addition to these categories, the results may be further subdivided according to pion exchange, and scalar- or vector-meson exchange. The latter category is much simpler; no seagull terms occur to order (v^2/c^2), for example. Moreover, two classes of unitary equivalence are generated for pion exchange and only one for vector or scalar exchange.

In FII (Ref. 29) a general representation was adopted which allowed a single calculation to subsume the special cases of PS and PV pion-nucleon couplings to manifest the constraints of the equivalence theorem. An arbitrary parameter μ was introduced which determined the strength of the Dyson transformation of the PS-coupling Hamiltonian. The value $\mu=0$ corresponds to PS coupling, and ignoring nucleon anomalous magnetic moment terms, $\mu=1$ corresponds to PV coupling. The (isoscalar) charge operator is simplest for $\mu=3$, while $\mu=-1$ is the most common (implicit) choice and tends to *maximize* exchange corrections. These values of μ correspond to the FW representation we use and *not* to the free spinor representation used by those who follow the method of Chemtob and Rho,[46] which corresponds to $\mu=-1$. We will display our results using this parameter after a brief discussion of the other contributions.

It was shown in FII that the particular χ_v arising in one-pion exchange which is necessary for internal consistency and Lorentz invariance depended on μ. Denoting this by χ_μ, we demonstrate in the Appendix that it vanishes for the deuteron *elastic* charge form factor. Thus, for deuteron elastic scattering there are no contributions from χ_v in our formalism for single pion, scalar-, or vector-meson[26] exchange.

In addition to generating μ-dependent pion-exchange terms related to the equivalence theorem, the recoil graph generates a special type of term for all types of exchange. The form is determined by the fact that it builds into the formalism the effect of retardation of the nuclear force (i.e., finite meson propagation time). A single-meson-exchange potential has the schematic form $V_1V_2/(\vec{q}'^2+m^2-q_0'^2)$. The problem that we discussed in the Introduction concerned the interpretation and treatment of q_0', the fourth component of the momentum q' carried by the meson and depicted in Fig. 1(a). For free particles q_0' is given by the appropriate kinetic energy difference of particle 1 (ΔE_1) or particle 2 (ΔE_2) in the initial and final states. In an interacting system ΔE_1 and ΔE_2 are different and it is not clear which one to use or which linear combination is appropriate. Neglecting for a moment the vertex functions V_1 and V_2, we expand the propagator to first order in $q_0'^2$ and obtain

$$V \cong \frac{V_1V_2}{\vec{q}'^2+m^2} + \frac{q_0'^2V_1V_2}{(\vec{q}'^2+m^2)^2} \equiv V + \Delta V_{\rm RET} . \tag{4}$$

The formalism of FII produced the following prescription: If h_0 is the nonrelativistic two-body Hamiltonian (except for V, but including other meson exchanges), replace q_0' by commutators of V_1 and V_2 with h_0:

$$\Delta V_{\rm RET} = \frac{1}{2(\vec{q}'^2+m^2)^2}\left([h_0,[h_0,V_1V_2]] - \{[h_0,V_1],[h_0,V_2]\}\right). \tag{5}$$

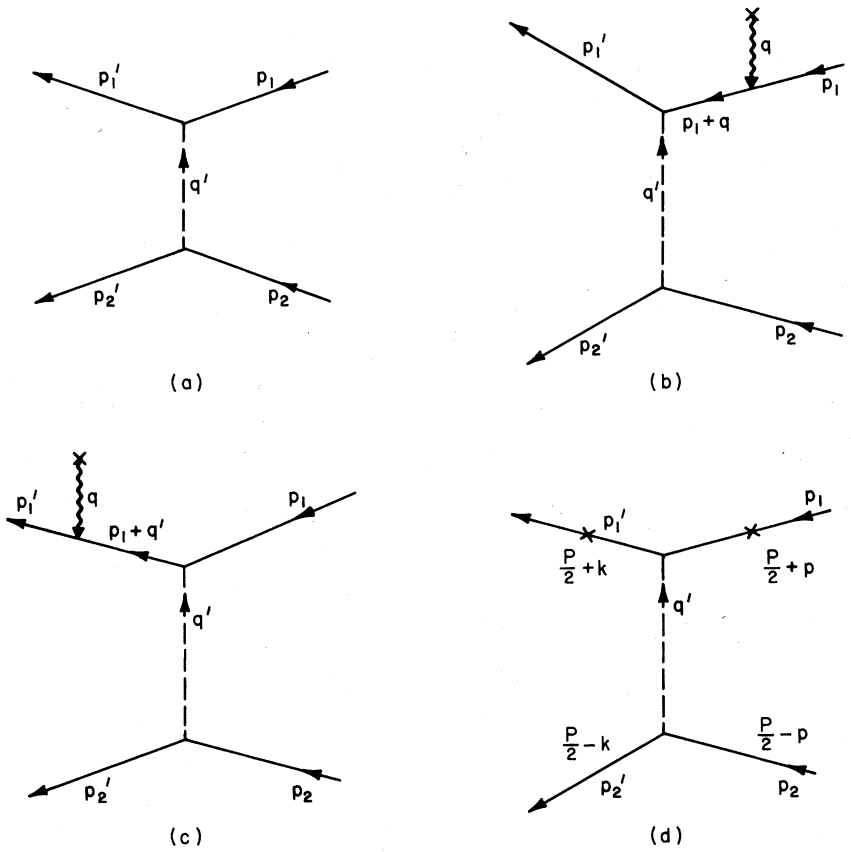

FIG. 1. Kinematics for meson exchange including an external electromagnetic interaction in (b) and (c), and (d) Gross's kinematics with nucleon 1 on-shell (indicated by cross).

Neglecting any isospin or momentum dependence of the potential part of V, only the kinetic energy parts of h_0 fail to commute with V_1 and V_2 in the second term in Eq. (5). However, the potential in the second commutator of the first term will not commute with the first commutator. This will generate three-body forces in a many-body system, and a special class of two-body forces as well. Thus three-body forces are inevitable except possibly in the "soft" representation discussed below. It is possible to eliminate the entire first term, with its complicated potential terms, by means of a unitary transformation ($U_{\rm RET}$) of $h' = h_0 + \Delta V_{\rm RET}$ to first order in $U_{\rm RET}$:

$$U_{\rm RET} \cong -\frac{i\nu}{2}\left[h_0, \frac{V_1 V_2}{(\vec{q}^2 + m^2)^2}\right], \tag{6a}$$

$$h'' \cong h' - i[h_0, U_{\rm RET}] \equiv h_0 + \Delta V'_{\rm RET} \tag{6b}$$

with $\nu = 1$. The remaining (last) term in Eq. (5) then has a simpler structure than what we started with. For reasons that will become apparent later, we will label $\nu = 0$ the "standard" representation and $\nu = 1$ the "soft" representation. In order to investigate further the properties of $\Delta V_{\rm RET}$, we will drop the potential terms in h_0 and replace the kinetic energy commutators by the differences of energies ΔE_i before and after the meson is exchanged. We find that $\Delta V'_{\rm RET}$ can be represented by

$$\Delta V'_{\rm RET} = \frac{V_1 V_2}{2(\vec{q}'^2 + m^2)^2}[(\Delta E_1 + \Delta E_2)^2(1-\nu) - 2\Delta E_1 \Delta E_2]. \tag{7}$$

In the *center-of-mass frame* $\Delta E_1 = \Delta E_2$ and the choice $\nu = \frac{1}{2}$ eliminates $\Delta V'_{\rm RET}$; this is the "no-retardation" representation, which has obvious advantages, at least as far as the potential is concerned.

Corresponding to the various representations of the potential are isoscalar recoil graph contributions to the charge operator ρ_R. These may be obtained from Eqs. (79) and (80) of FII and have the schematic form

$$\rho_R = \frac{(1-\nu)\vec{q} \cdot \vec{q}' V_1 V_2 G_E^S}{2M(\vec{q}'^2 + m^2)^2}. \tag{8}$$

This vanishes in the "soft" representation and is

nonvanishing in all others, which is another advantage of the soft representation. We note further that the one-photon-exchange potential in atomic physics is traditionally calculated in Coulomb gauge. This choice of gauge leads immediately to a *static* Coulomb potential and smaller nonstatic corrections from the transverse components of the current. Simply neglecting the retardation in Feynman gauge leads to nonstatic corrections involving the complete current; the difference is a contribution from the longitudinal current component, which has the form of Eq. (7) with $\nu=1$ (i.e., $\sim\Delta E_1\Delta E_2$). Retardation is included in Coulomb gauge, and using this gauge corresponds to the soft representation. It was demonstrated recently[47] that transition operators for spin-flip electric dipole transitions in heliumlike atoms calculated by traditional methods[48] are characterized by $\rho_R=0$ (i.e., $\nu=1$). Thus, atomic physics prefers to work in Coulomb gauge (the soft representation) where recoil corrections to energy levels are known to be smaller than in other gauges, as we discussed in the Introduction while contrasting the Breit and Gaunt interactions. Also, three-body forces are smaller.

In calculating the coordinate space form of ρ_R, it is convenient to integrate by parts after multiplying by $e^{i\vec{q}'\cdot\vec{x}}$ and using the identity $\vec{q}'/(\vec{q}'^2+m^2)^2 = -\frac{1}{2}\vec{\nabla}_q' 1/(\vec{q}'^2+m^2)$. This produces the $\vec{q}\cdot\vec{x}$ factor which is characteristic of retardation (i.e., it looks like the expansion of a plane wave). For *pions* the V_1V_2 factors have the form $\vec{\sigma}_1\cdot\vec{q}'\vec{\sigma}_2\cdot\vec{q}'$ so that additional terms are generated by the integration by parts which are not present for scalar and vector exchanges to the order we work. Performing all the necessary manipulations we find that the deuteron form factor contribution F_R arising from ρ_R has the form

$$F_R = G_E^S(D_G - D_\pi)(1-\nu) \quad \text{(one-pion exchange)}, \tag{9a}$$

$$F_R = G_E^S D'_G(1-\nu)$$

(scalar-, vector-meson exchange). (9b)

The extra term in the pion result arises from the spin-dependent V_1V_2 factor. Several remarks are in order. In an effort to simplify the two-nucleon potential in FII as far as possible, we *chose* to work with $\nu=\frac{1}{2}$, the no-retardation representation. For that case F_R was $\frac{1}{2}$ the "Gross" deuteron correction, which follows from Eq. (9). Earlier, in FI (Ref. 28), the standard representation was used; although it was not pointed out at that time, this was the first instance that the Gross correction actually appeared in any form outside Gross's work as the result of a dynamical calculation. The first term in Eq. (3) of FI, when evaluated with deuteron wave functions, is equal to $G_E^S(D_G - D_\pi)$. The Gross term was "derived" in Ref. 51 but this was not the result of a dynamical calculation. For this reason the statement in Ref. 49 that retardation corrections had not previously been calculated is erroneous. Moreover, the consequences of the equivalence theorem for pion exchange were also pointed out in FI; this was the first correct and complete [to order $(v/c)^2$] calculation of the one-pion-exchange contribution. Although the full Gross correction is realized only in standard representation, in view of our previous discussion the relationship of the Gross correction to retardation is exactly as hypothesized in Ref. 38; this will be made even clearer in Sec. V.

The remaining terms which contribute to the deuteron form factor are all proportional to D_π for one-pion exchange. To the order we calculate, the isoscalar seagull terms for scalar and vector exchange are of order V/m^3 and we neglect them[26]; these contributions are therefore complete. The seagull and remaining pieces of the recoil graph can be obtained from Eqs. (73a) and (82) of FII:

$$F_{SG} = [2G_M^S - G_E^S(1+\mu)]D_\pi, \tag{10a}$$

$$F'_R = (\mu+1)G_E^S D_\pi/2\,. \tag{10b}$$

The total potential-dependent correction to the form factor, F_v, is the sum of Eqs. (9) and (10):

$$F_v = G_E^S D'_G(1-\nu)$$

(scalar-, vector-meson exchange), (11a)

$$F_v = \{2G_M^S - G_E^S[(1+\mu)/2 + 1 - \nu]\}D_\pi + G_E^S D_G(1-\nu) \quad \text{(one-pion exchange)}. \tag{11b}$$

For pseudovector coupling replace G_M^S by G_E^S.

It is clear that *any* linear combination of D_π and D_G is possible for one-pion exchange. It may be shown that the wave functions contain μ- and ν-dependent terms which, when used in F_0, exactly cancel the corresponding terms in F_v; the *matrix elements* have no ambiguities. It is crucial, however, to use the wave functions *corresponding* to a given operator when calculating matrix elements. This was emphasized by Woloshyn[2] and in FI and FII. In actual calculations this has not been done. Note that $\nu=1$, $\mu=3$ eliminates F_v completely if we neglect the difference of G_M^S and G_E^S.

The specific way in which the unitary equivalence is realized for various observables was considered in Ref. 44. Basically, some of the contents of the impulse approximation matrix elements Eqs. (A1) and (A2) are "dialed" out by means of the transformation and generate D_G, D_π, etc., in the same way Eqs. (A19) and (A20) were manipulated. Thus, another way in which the

peculiar problems discussed here can be stated is that the impulse approximation is not unique. The interested reader should refer to Ref. 44.

The calculation which led to Eq. (11) was more complicated than others which had the same goal, because the FW transformation generates a large number of $\vec{p}$- (momentum-) dependent terms, each of which must be incorporated into Feynman diagrams in order to obtain amplitudes. This procedure is a natural way to investigate unitary equivalence which arise, however. It also allows us to calculate substantial parts of some crossed meson-exchange graphs without additional work. This feature, unique to our formalism, is analogous to the calculation of single-photon recoil corrections in atomic physics where all Coulomb ladder graphs are summed in intermediate states.[11] A virtually identical method has been used by Lin[50] to calculate relativistic corrections to atomic potentials and transition operators for use in the problem of forbidden transitions in heliumlike atoms.

The momentum-dependent terms referred to above play a dominant role in proving the Lorentz covariance of the transition matrix elements of the current. This was demonstrated in FII. The proof and subsequent work[45] refuted the claim of Coester[51] that the relativistic corrections of Ref. 40 were incorrect. The χ_v term found by Coester and needed by him to prove the covariance of Gross's formalism did not arise (and in fact was not needed) in FI. We will see in Sec. V that this term arises in a natural way in Gross's work because his formalism is not symmetric under (identical) particle interchange, while ours is symmetric. It was the *assumption* by Coester of Gross's asymmetry that led to his result and the unwarranted conclusion that Ref. 40 was incorrect. However, the latter work *explicitly* neglected meson-exchange currents and was therefore incomplete.

III. FST METHOD

The FST method is a method of projection which allows one to construct energy-independent potentials and orthonormal wave functions. Gari and Hyuga[52,53] used this method and confirmed the results of FI. A straightforward evaluation of Eqs. (19)–(21) of Ref. 49 to order V/m for π, ρ, and ω exchange in the point π-N form factor limit yields

$$F_R^{FST} \cong G_E^S(D_G - D_\pi) \quad \text{(one-pion exchange)}, \tag{12a}$$

$$F_R^{FST} \cong G_E^S D_G' \quad \text{(scalar-, vector-meson exchange)}, \tag{12b}$$

which agrees with the standard representation. If the corresponding potential terms are evaluated from Eq. (23) of Ref. 53, those terms which do not vanish in the two-nucleon center-of-mass frame are identical to Eq. (5) for point nucleons. The FST method generates the same results as this author's renormalization method in *standard* representation. The existence of various retardation representations and the relationship of the potential form to recoil contributions to F was not pointed out in Refs. 49 and 53.

The results of Gari and Hyuga do *not* agree with ours if their pion-nucleon form factor is used. The reason is that their introduction of a form factor was made in a fashion inconsistent with relativity. The vertex factors V_i are proportional to the pion-nucleon form factor $F_{\pi N}$. These form factors are expected to be functions of $q^2 = \vec{q}^2 - q_0^2$, just as the propagator was. When integrating by parts to simplify the coordinate space form of recoil operators [see discussion above Eq. (18)] the actual expression one works with becomes[29] $-\frac{1}{2}\vec{\nabla}_{\vec{q}}^{\,i}(F_{\pi N}{}^2(\vec{q}'^2)/(\vec{q}'^2 + m^2))$. After integrating by parts, the form factor enters in the *transformed* operator in the usual way, rather than in the original form, which generates derivatives of the form factor while integrating by parts. The derivative terms are spurious.

Riska and Radomski[54] have developed a scheme for extracting the appropriate parts of the recoil plus disconnected diagrams. This scheme appears to lack the $\vec{q}\cdot\vec{x}$ terms which are so important and lead to the Gross correction. Similar terms of the form $\vec{P}\cdot\vec{x}$ arise in the same way and are absolutely crucial for relativistic consistency. Few details were given in Ref. 54, however.

The Feynman diagram or FST approach of Refs. 52–54 generates pair terms with $\mu = -1$. That is, these terms are porportional to G_M^S only, which follows for any approach which treats the nucleons as free particles. This contribution is then identical in form to Eq. (36).

IV. FOLDED DIAGRAMS

As mentioned in the Introduction, the folded diagram technique[24] allows one to construct an *effective* energy-independent, nonstatic, Hermitian, instantaneous, nucleon-nucleon potential and *corresponding* effective transition operators. Wave function orthonormality is ensured. This method averages over relative energy (q_0') in Feynman diagrams and generates a class of "equivalent" results. Folded diagrams therefore offer another method for handling retardation. The strength of this technique is that it relies on time-dependent perturbation theory, is diagram-

matic (pictorial), very general, and systematic. It has unfortunately been almost completely ignored in work on exchange currents.

We will not describe the derivation of the folded diagram techniques, but merely quote the necessary results.[24] Corresponding to a meson-exchange propagator $V_1V_2/(\vec{q}^2+m^2-q_0^2)$, there exists an effective potential:

$$V_{FD}=V_1V_2\int_{-1}^{1}d\lambda' f(\lambda')/\{-[\Delta E_1(1-\lambda')/2 + \Delta E_2(1+\lambda')/2]^2 + (\vec{q}^2+m^2)\}. \quad (13)$$

The function $f(\lambda)$ is symmetric $[f(\lambda)=f(-\lambda)]$ and normalized to one $[\int_{-1}^{1}f(\lambda)=1]$. It is obvious by inspection that one is averaging over q_0. The quantities ΔE_1 and ΔE_2 were defined before and need not be nonrelativistic approximations. We wish to calculate only the leading-order relativistic corrections, so we expand the integrand in Eq. (13) to order $\Delta E^2\sim 1/M^2$. Defining $e_q=(\vec{q}^2+m^2)^{1/2}$ and

$$\int_{-1}^{1}f(\lambda')\lambda'^2 d\lambda'\equiv\lambda, \quad (14)$$

we find

$$V_{FD}\cong\frac{V_1V_2}{e_q^2}+\frac{V_1V_2}{2e_q^4}\left[(\Delta E_1+\Delta E_2)^2\left(\frac{1+\lambda}{2}\right) - 2\Delta E_1\Delta E_2\right]. \quad (15)$$

It follows that $\lambda\equiv 1-2\nu$. The folded diagram method generates the same potential we found before. The preferred representation of Ref. 24 corresponds to $\lambda=0$ or $\nu=\frac{1}{2}$, which, of course, eliminates retardation in one-boson-exchange potentials.

The effective charge operator is the sum of four terms: The first has the form

$$\rho_{FD}^{(1)}=-e_1V_1V_2\int_{-1}^{1}d\lambda' f(\lambda')[X_a(\lambda')-X_b(\lambda')], \quad (16)$$

where

$$X\binom{a}{b}(\lambda)=\frac{1-\lambda}{2}\left[\frac{1}{-e_q\mp\Delta E_2}\right]\frac{1}{2e_q} \times\left[\frac{1}{-e_q\pm\Delta E_1(1-\lambda)/2\mp\Delta E_2(1-\lambda)/2}\right]. \quad (17)$$

Expanding to the appropriate order, one finds

$$\rho_{FD}\cong-\frac{e_1V_1V_2}{4e_q^4}[\Delta E_1-3\Delta E_2+\lambda(\Delta E_1+\Delta E_2)]. \quad (18)$$

The virtual photon has landed on the initial leg of particle 1 (charge e_1) in the exchange diagram 1(b). In each case ΔE is the energy of the nucleon after the meson arrives minus the energy before it arrives. The corresponding diagram with the photon on the outgoing leg [Fig. 1(c)] can be obtained from the previous result by multiplying by -1 and modifying the definitions of ΔE_1 (to $\Delta E_1'$) in terms of momenta accordingly. After expanding the ΔE's to order $1/M$, this yields

$$\rho_{FD}^{(1+2)}=-\frac{e_1V_1V_2}{4e_q^4}(1+\lambda)(\Delta E_1-\Delta E_1') \cong\frac{e_1V_1V_2(1+\lambda)\vec{q}\cdot\vec{q}'}{4Me_q^4}. \quad (19)$$

The corresponding graphs with $1\leftrightarrow 2$ are identical with $e_1\to e_2$; therefore

$$\rho_{FD}=\frac{(e_1+e_2)V_1V_2\vec{q}\cdot\vec{q}'(1-\nu)}{2Me_q^4}. \quad (20)$$

When e_1+e_2 is replaced by G_E^S, this result is seen to be identical to Eq. (8).

The confirmation that the retardation ambiguity of FII and the folded diagram ambiguity[24] are identical lends considerable confidence to our result. We note that the $1/e_q^3$ (leading) term in Eq. (17), which is the static recoil graph contribution, *cancels identically* in each set of folded graphs. We have also verified that to the order we have calculated, the folded diagram ambiguity is actually a unitary equivalence. The equivalence of the various folded diagram results was noted in Ref. 24. Although it was never specifically claimed that this equivalence was of unitary type, it was implied.

V. QUASIPOTENTIAL METHOD

The wide variety of quasipotential equations is discussed in several places. The treatment of these equations by Woloshyn and Jackson[18] (WJ) fulfills most of our requirements, and we will follow it, with modification by Gross[23] to allow the introduction of nucleon spin. We begin with the BS equation for the scattering amplitude of two nucleons written in schematic form:

$$T=V+VGT, \quad (21)$$

where V is the sum of all connected irreducible (two-body) diagrams. This is a four-dimensional equation. Replacing G by $g+(G-g)$, where g is three dimensional, allows the equation for T to be rearranged in the form

$$T=W+WgT, \quad (22a)$$

$$W=V+V(G-g)W. \quad (22b)$$

The Green's function g is chosen to have the same *elastic* unitarity cut as G, which does not uniquely specify it, however. Nevertheless, Eq. (22a)

looks like the Schrödinger (or Breit) equation with W playing the role of the potential. Since g is not unique, W is also not unique. In view of our emphasis on unitary equivalences, it is worth noting how different three-dimensional equations can lead to the same T, or, in the case of the analogous bound state equation, to the same binding energy. It is instructive to examine Table I of WJ, which lists six different quasipotential forms. As noted by Johnson,[24] several of these forms will generate *energy-dependent potentials*. In addition, several equations have the form of the square of the Schrödinger equation, that is, they do not necessarily correspond to first-order differential equations in time. For example $(E-H)\psi=0$ becomes $(E^2-H^2)\psi=0$ after multiplying by $E+H$. Finally, parts of the effective potentials are related by unitary equivalences. Thus the main differences of the equations and their effective potentials are (a) energy dependence in V, (b) variance of the form of the equation, and (c) unitary equivalence. There is also no consensus that any one equation is "best" in any sense.

We will examine two different equations, and one, the Gross equation, in some detail. The kinematics appropriate to this equation are displayed in Fig. 1(d). The system proceeds from a state of total momentum P, which is conserved, and relative momentum p to a state with relative momentum k after interacting by means of meson exchange. For our purposes V and W are the same, since we will restrict ourselves to one-boson exchange. According to Gross and WJ, the three-dimensional Gross Green's function g_G has the form

$$g_G=2\pi i\int ds'\,\frac{\sqrt{s'}}{\sqrt{s}}\,\frac{\delta^{(+)}(p_1'^2-M^2)}{(\sqrt{s'}-\sqrt{s}-i\epsilon)}\times\delta^{(+)}((P'-p_1')^2-M^2)(M+\not{p}_1')(M+\not{p}_2'),\tag{23}$$

where $s=P^2$ is the square of the total energy, $P'\equiv\sqrt{s'}P/\sqrt{s}$, and $\delta^{(+)}(Q^2-M^2)\equiv\delta(Q^2-M^2)\theta(Q_0)$. For the moment we will ignore the spin projection operators $(M+\not{p})$, which we denote by S; our spinor conventions are those of Ref. 30.

Gross's work is characterized by putting one particle, the spectator, on its mass shell. This introduces asymmetry in the wave function and complicates the use of Pauli principle, for example. The on-shell particle is denoted by the cross in Fig. 1(d). It is a simple matter to evaluate the integral over s' in Eq. (23); because relativistic corrections depend on the center-of-mass (c.m.) momentum $\vec{P}$, we do not restrict ourselves to the c.m. frame, which is conventional. We find

$$g_G=\frac{\pi i}{\sqrt{s}E_1'}\,\frac{\delta(p_1'^0-E_1')S}{2(P\cdot p'/\sqrt{s})-\sqrt{s}-i\epsilon},\tag{24}$$

where we have defined $E_1'=(\vec{p}_1'^{\,2}+M^2)^{1/2}$, and $p_1=(p_1^0,\vec{p}_1)$, etc. This bears little resemblance to the Schrödinger equation and, even worse, contains extra energy-dependent factors which could force the potential to be energy dependent. The spinor factor S compensates for this, however. In order to transform g_G to a usable form we will expand all the kinematical quantities in powers of $1/M$, keeping terms of order $1/M^2$ beyond the nonrelativistic limit for both nucleons. Because particle 1, the spectator, is on-shell, its spin projection operator can be represented by $2Mu_1\bar{u}_1(\vec{p}_1)$, where a sum over helicities is implied. Particle 2, the target, is off-shell and its spin projection operators are linear combinations of positive energy $(u_2\bar{u}_2)$ and negative energy $(v_2\bar{v}_2)$ spinors. We therefore divide g_G into two parts: $g_G^{(+)}$ (positive energy) and $g_G^{(-)}$ (negative energy). Defining $E\equiv P^0$, we find after a tedious calculation

$$g_G^{(+)}\cong-2\pi i\,\frac{\delta(p_1'^0-E_1')}{E-E_1'-E_2'+i\epsilon}u_1\bar{u}_1(\vec{p}_1')u_2\bar{u}_2(\vec{p}_2')\tag{25a}$$

and

$$g_G^{(-)}\cong-\frac{2\pi i\delta(p_1'^0-E_1')}{2M}u_1\bar{u}_1(\vec{p}_1')v_2\bar{v}_2(-\vec{p}_2'),\tag{25b}$$

where the spinors are normalized according to the invariant convention, $u^\dagger u=1$, instead of the usual covariant convention $\bar{u}u=1$. The additional spin factor S has removed the apparent energy dependence $(\sqrt{s})$ in $g_G^{(+)}$ in Eq. (24) and the resulting Green's function denominator has the Schrödinger or Breit form, obtained from $(E-E_1-E_2-V)\psi=0$. The Green's function also has spin projectors and the δ function which allows us to reduce the original BS equation to a three-dimensional equation. If Eq. (22) is iterated, the *effective potential* in the positive energy subspace corresponding to the exchange in Fig. 1(d) becomes

$$V_{\rm eff}=\frac{\bar{u}(\vec{p}_1')\bar{u}(\vec{p}_2')\Upsilon u(\vec{p}_1)u(\vec{p}_2)}{\vec{q}'^{\,2}+m^2-q_0^2}\bigg|_{q_0=E_1'-E_1},\tag{26}$$

where Υ is the product of spinor factors pertaining to a given meson exchange which may depend on q_0.

We will divide the relativistic corrections in $V_{\rm eff}$ into two parts: the retardation part in the denominator which is the same for all exchanges, and the part arising from the spinors in the numerator. The retardation part will be dealt with first. Expanding Eq. (26) in powers of $1/M$ we find

$$V_{\rm eff}\cong\frac{V_1V_2}{\vec{q}'^{\,2}+m^2}+\frac{V_1V_2\Delta E_1^{\,2}}{(\vec{q}'^{\,2}+m^2)^2}+\frac{\Delta(V_1V_2)}{\vec{q}'^{\,2}+m^2},\tag{27}$$

where V_1V_2 depicts the nonrelativistic numerator factors and $\Delta(V_1V_2)$ represents the relativistic corrections to them. Note that the retardation term ($\Delta V_{\rm RET}^G$) is not symmetric under interchange of 1 and 2. The retardation potential can be separated into c.m. parts and frame-dependent terms which depend on $\vec{\rm P}$. We find

$$\Delta V_{\rm RET}^G = \Delta V_{\rm RET}^S + \Delta V_{\rm RET}(\vec{\rm P}) + \Delta V_G(\vec{\rm P}), \tag{28}$$

where $\Delta V_{\rm RET}^S$ is the standard-representation retardation potential in the c.m. frame and $\Delta V_{\rm RET}(\vec{\rm P})$ is the set of $\vec{\rm P}$-dependent terms obtained from Eq. (98f) of FII for pion exchange or Eq. (26) of Ref. 26 for scalar and vector exchange. These terms are the "standard" frame-dependent retardation terms which are crucial for Lorentz invariance. The remaining term has the following form for pion exchange:

$$\Delta V_G(\vec{\rm P}) = \frac{if^2}{4m}[T, \vec{\sigma}_1\cdot\vec{\nabla}\vec{\sigma}_2\cdot\vec{\nabla}\vec{\rm P}\cdot\vec{\rm x}h]. \tag{29}$$

Use of Foldy's relation, Eq. (14) of FII, produces a contribution to χ_v denoted by $\chi_G^{\rm T}$:

$$\chi_G^{\rm T} = \frac{f^2}{4M}(\vec{\sigma}_1\cdot\vec{\rm P}\vec{\sigma}_2\cdot\vec{\nabla}h + \vec{\sigma}_2\cdot\vec{\rm P}\vec{\sigma}_1\cdot\vec{\nabla}h) + \chi_G, \tag{30a}$$

$$\chi_G = \frac{\vec{\rm P}\cdot\vec{\rm x}}{4M}V_0, \tag{30b}$$

where V_0 is the nonrelativistic potential. For scalar- and vector-meson exchange, the spin-dependent factors are missing, and only χ_G survives. The term χ_G is precisely the term in the wave function deduced by Casper and Gross,[55] which leads to the "Gross correction."

Let us summarize the derivation of $\Delta V_{\rm RET}$. The retardation potential can be separated into three parts, two of which are symmetric under particle interchange and are included in the standard-representation results derived by a variety of methods in Secs. II–IV. The third term is new because it is antisymmetric under particle interchange; it leads directly to the "Gross correction" using Eq. (A20). For pion exchange there is an additional term in $\chi_G^{\rm T}$ which can be shown to produce a form factor contribution $-G_E^S D_\pi$. The terms in F which arise from χ_v we will denote F_M (M for motion); thus far we have

$$F_M = G_E^S(D_G - D_\pi) \quad \text{(one-pion exchange)}, \tag{31a}$$

$$F_M = G_E^S D_G' \quad \text{(scalar-, vector-meson exchange)}. \tag{31b}$$

In order to complete the calculation we must evaluate the $\Delta(V_1V_2)$ factors. For scalar and vector exchange they are identical to those found in Ref. 26 and do not lead to any additional χ_v terms. For pseudoscalar-coupling pion exchange, the result corresponds exactly to the representation $\mu=-1$. This leads to χ_μ, which was shown earlier not to contribute to elastic scattering to order $1/M^2$.

Pseudovector coupling requires a special discussion. Gross[23,59] defines a pion-exchange potential which is a linear combination of pseudovector and pseudoscalar couplings

$$\Upsilon = \vec{\tau}(1)\cdot\vec{\tau}(2)g^2\Lambda_1(q)\,\Lambda_2(-q), \tag{32a}$$

where

$$\Lambda_1(q) = \lambda\gamma_5{}^1 + \frac{(1-\lambda)}{2M}\gamma_1\cdot q\gamma_5{}^1. \tag{32b}$$

This depends on q_0. The remaining q_0-independent terms are symmetric under particle interchange. Manipulation of the spinors in Λ leads to the following results:

$$\bar{u}_1\Lambda_1 u_1 = \bar{u}_1\gamma_5{}^1 u_1, \tag{33a}$$

$$\bar{u}_2\Lambda_2 u_2 \cong \bar{u}_2\gamma_5{}^2 u_2 + \frac{(\lambda-1)}{4M^2}\langle\vec{\sigma}_2\cdot(\vec{\rm p}_2' + \vec{\rm p}_2)\rangle \times \frac{(\vec{\rm p}_1'^2 + \vec{\rm p}_2'^2 - \vec{\rm p}_1^{\,2} - \vec{\rm p}_2^{\,2})}{2M}, \tag{33b}$$

where $\langle\,\rangle$ indicates an expectation value with respect to two-component spinors. Equation (33) indicates that with the exception of a small term of relative order $(1/M^2)$, Gross's PS and PV forms are identical in the positive energy subspace. We find, furthermore, that Eq. (33) leads to

$$V_{\rm eff} = V_{\rm eff}^{\rm PS} - i\frac{(1-\lambda)f^2}{2M}[T, \{\vec{\sigma}_2\cdot\vec{\rm p}_2, \vec{\sigma}_1\cdot\vec{\nabla}h\}], \tag{34a}$$

which leads to a χ term for PV coupling, if we insert $\vec{\rm p}_2 = -\vec{\rm p} + \vec{\rm P}/2$,

$$\chi_{\rm PV} = -\frac{f^2}{2M}\vec{\sigma}_2\cdot\vec{\rm P}\vec{\sigma}_1\cdot\vec{\nabla}h, \tag{34b}$$

and an additional term independent of $\vec{\rm P}$ which can be removed by a unitary transformation $u_{\rm PV}$,

$$u_{\rm PV} = \frac{f^2}{2M}\{\vec{\sigma}_2\cdot\vec{\rm p}, \vec{\sigma}_1\cdot\vec{\nabla}h\}. \tag{34c}$$

Eliminating both these terms from the PV potential and therefore from the deuteron wave function generates two additional contributions to F which are identical; their sum has the form

$$\Delta F_{\rm PV} = \Delta F_M^{\rm PV} + \Delta F_X^{\rm PV} = 2G_E^S D_\pi. \tag{35}$$

The $\chi_{\rm PV}$ term is a motional term and has been labeled appropriately. With this transformation the wave functions for both PS and PV couplings should be the $\mu=-1$ choice.

The remaining process which contributes to the charge operator to order V/M is the pair or seagull process. For isoscalar transitions there is

no such contribution from vector, scalar, and PV-coupled pion exchanges. There is a contribution from PS-coupled pion exchange, which may be calculated using $g_G^{(-)}$. This negative energy channel is the origin of Gross's deuteron "p states," and produces a contribution

$$\Delta F_{\mathrm{SG}} = 2G_M^S D_\pi . \tag{36}$$

This completes the catalog of deuteron form factor contributions, which will be tabulated momentarily.

Although we have evaluated only isoscalar matrix elements, it is instructive to calculate those isovector contributions which depend on the *total* momentum of the two-body system in an arbitrary frame of reference. These contributions determine the behavior of the matrix elements under a Lorentz transformation and have a prescribed form [see Eq. (96) of FII]. In Gross's formalism, contributions for PS coupling come from χ_μ and χ_G alone and can be shown to have the correct form. For PV coupling additional contributions arise from χ_{PV} and the usual PV-gauge term, which exactly cancel. For scalar and vector exchange, the situation is identical to the one discussed in Ref. 26. Lorentz invariance in Gross's formalism is therefore verified, which is no surprise, since his formalism can be formulated covariantly.

In the present author's formalism[29] the recoil graph contributions make an important contribution to Lorentz invariance, because this is the only place the frame-dependent, isovector $\vec{P}\cdot\vec{x}V$ terms can arise. Isoscalar terms of the same general form which lead to the Gross correction in the deuteron can only arise in the "standard"-recoil correction. The motional factor χ_G generates the *same* set of terms in Gross's formalism. As noted by Woloshyn[2] and Gross,[59] the recoil graph is included in the BS formalism. The motional corrections χ_G and χ_{PV} arose because of asymmetry introduced into Gross's equation through the on-mass-shell condition applied to *one* nucleon. As indicated in WJ an alternative and symmetric quasipotential method exists which accounts for the relative time according to the original prescription of Blankenbecler and Sugar. The Green's function then has the schematic form

$$g \sim \int ds' \delta^{(+)}((P'/2+k)^2 - M^2)\delta^{(+)}((P'/2-k)^2 - M^2)$$
$$\sim \delta^{(+)}(2P\cdot k) = \delta^{(+)}(P\cdot(p_1' - p_2')) = \delta^{(+)}(p_1'^2 - p_2'^2), \tag{37}$$

which places both particles off-shell symmetrically. If the retardation potential is calculated, only the retardation potential $\Delta V_{\mathrm{RET}}(\vec{P})$ appears. Since this potential is important for purposes of Lorentz invariance and appears in everyone's method, its appearance is hardly surprising. However, the fact that this potential vanishes for $\vec{P}=0$ tells us that the above prescription corresponds to the "no-retardation" representation ($\nu = \frac{1}{2}$), favored by Johnson[29] and used in FII.

Finally we compile and add all the contributions to the form factor corresponding to our result in the $\mu=-1$ and standard representations, and the Gross results that we have deduced and *converted* to the *same* representation. These are shown in Table I for all the various cases we have treated: scalar- and vector-meson exchange, PS and PV-coupled pion exchange, and Gross's linear combination of pion couplings. We have denoted the contributions from Gross's nonsymmetric operators with an asterisk (*). The second seagull contribution arises from u_{PV}. The first observation is that although the separate columns are different for the two cases, the *totals are the same*. This illustrates what we have stated many times before: Different representations spread the total result differently over separate "physical" processes, such as recoil graphs, gauge-term graphs, pair graphs, etc. It therefore makes no more sense to try to separate these different, coherent physical processes experimentally, than to "measure" the gauge in electrodynamics. It also appears possible, as we surmised in FII, to remove Gross's asymmetry by means of unitary transformations, at least for the processes we have examined in this work. The wave functions that we must use in calculating the matrix elements D_G and D_π are then *symmetric* under interchange. Our previous discussion has also clearly confirmed the relationship of the Gross correction to retardation in the potential, which was previously speculated. This correction is known to be numerically important.[51,57]

Although we have tried to deduce what Gross's results should be to order $(v/c)^2$, only a limited calculation by Gross exists.[58] This calculation of the *charge* form factor part of the PS result established that the sum of the "Gross correction" and the pair term was equal to previously published results by others. The monopole part of the pair term was given in the form $(2G_M^S - G_E^S)D_\pi^0$. It is possible to verify from Gross's equations that the complete result is actually $(2G_M^S - G_E^S)D_\pi$. This disagrees with our previous calculation of Gross's pair term $(2G_M^S D_\pi)$, although the sum of this correction and the "Gross correction" is the same as what we found. Three obvious possibilities for this discrepancy exist: (1) We have incorrectly dealt with the asymmetry in Gross's equations, although this would imply that there are additional uncalculated processes which contribute. (2) Be-

TABLE I. Gross and Friar results for the deuteron form factor converted to a common representation. The columns labeled with an * denote contributions to Gross's results which arise from nonsymmetric operators. The two forms D_G and D'_G have slightly different forms because of the spin dependence of the pion-nucleon vertex.

	Motion*	Seagull	Vector scalar Recoil	Seagull*	Total
Gross	$G_E^S D'_G$	0	0	0	$G_E^S D'_G$
Friar	0	0	$G_E^S D'_G$	0	$G_E^S D'_G$
			PS		
Gross	$G_E^S(D_G - D_\pi)$	$2G_M^S D_\pi$	0	0	$G_E^S D_G + (2G_M^S - G_E^S)D_\pi$
Friar	0	$2G_M^S D_\pi$	$G_E^S(D_G - D_\pi)$		$G_E^S D_G + (2G_M^S - G_E^S)D_\pi$
			PV		
Gross	$G_E^S D_G$	0	0	$G_E^S D_\pi$	$G_E^S(D_G + D_\pi)$
Friar	0	$2G_E^S D_\pi$	$G_E^S(D_G - D_\pi)$	0	$G_E^S(D_G + D_\pi)$
			λ(PS) + $(1-\lambda)$PV		
Gross	$G_E^S D_G - \lambda G_E^S D_\pi$	$2G_M^S D_\pi \lambda$	0	$(1-\lambda)G_E^S D_\pi$	$G_E^S(D_G + D_\pi) + 2\lambda(G_M^S - G_E^S)D_\pi$
Friar	0	$2G_E^S D_\pi + 2\lambda D_\pi(G_M^S - G_E^S)$	$G_E^S(D_G - D_\pi)$	0	$G_E^S(D_G + D_\pi) + 2\lambda(G_M^S - G_E^S)D_\pi$

cause of the way Gross separates out the Gross correction term, which is common to all meson-exchange models, any additional terms from motion are *forced* to appear elsewhere. (3) Gross showed many years ago in his earlier work that errors in his *approximate* calculation of the deuteron form factor could be as large as relative order $1/M^2$. This is just the order we are now calculating In fact it was by neglecting most of the singularities in a relative energy integral for the form factor that the asymmetry was introduced. We do not believe, however, that (3) is the case, and (2) is the most likely possibility. The fact that the charge form factor has correct Lorentz transformation properties to order $1/M^2$, at least as we have calculated it, would indicate that the corrections to Gross's form factor are probably no larger than order $(1/M)^4$, at least for one-boson exchange.

Regardless of the reason for this minor discrepancy, the fact that our total results agree to the order we have calculated has to be regarded as a major success. In view of the large number of common representations that exist in the literature for handling retardation and the equivalence theorem, Gross's agreement of his PS corrections with one other calculation which uses *different* representations must be regarded as fortuitous. No agreement with PV coupling would have been found, for example. Nevertheless, with appropriate care the agreement has been demonstrated and the equivalence theorem for PS and PV couplings may be shown, by deleting the equivalence-breaking nucleon anomalous magnetic moment terms in Table I.

The Dyson transformation, which established the limits of validity of this theorem, relates "equivalent" field theories. Using perturbation theory, the equivalence of matrix elements may then be established. This is completely nontrivial, since we are not making nonrelativistic approximations, etc. In the previous discussion we pretended that the entire Hamiltonian was the kinetic energy. To be consistent we should have included the potential terms as well. We could not consistently do this in the FST, folded diagram, and Gross approaches because we had restricted ourselves to one-boson exchange. In FII, however, additional couplings between isoscalar scalar or vector bosons and pions were introduced that allowed the complete Hamiltonian (kinetic energy plus scalar and vector boson potentials) to be used in establishing the unitary equivalence. These additional terms included three-body forces and contributions from direct and *crossed* boson-plus-pion forces. The inclusion of crossed graphs is unique to FII.

In addition, the Dyson transformation changes the PS theory, with its large pair terms, into a modified PV theory, with small pair terms. The latter theory has substantial gauge terms (true seagull terms) which take the place of the "pairs." For this reason no particular attention should be paid to the amount of negative energy states[61]; this quantity, like the deuteron D-state percen-

tage, is easily changed by a unitary transformation. For the same reason it is pointless to separate physical processes which have identical structure and coupling constants into "relativistic" and "nonrelativistic" parts according to which Feynman graph provides their genesis. As an example of this, we note that the $e^2\vec{A}^2/2M$ term in the nonrelativistic Hamiltonian with electromagnetism $(\vec{p}-\vec{e}A)^2/2M$ is due to a pair term. It would make little sense to call the $\vec{p}^2/2M$ term, and the $e\vec{p}\cdot\vec{A}/M$ term nonrelativistic while labeling the $\vec{A}^2/2M$ term relativistic.

Gross's model, unlike ours, includes some processes to all orders. In particular, the negative energy components are carried along with the positive energy ones at each step of the wave function calculation. In traditional (nonrelativistic) calculations they contribute to two-boson exchange potentials, for example. Nevertheless, these components can be calculated, in principle, using perturbation theory, assuming that the perturbation expansion converges. This is particularly true of the equations of motion method, which removes the negative energy states and generates effective operators which reproduce the effect of such states.

VI. DISCUSSION AND CONCLUSIONS

We have discussed a wide variety of methods, based upon perturbation expansions of various types. It has been assumed that the expansions make sense, although it is certain that there are conditions under which the expansions do not converge. We have also restricted ourselves to the deuteron problem in order to show the equivalence of various methods. The simplicity of the two-body problem does not extend to the many-body problem, and it is by no means clear that these techniques have *equal* applicability to the more complicated problem. For example, the BS approach to the many-body problem is likely to be intractible.

We have stressed the lack of uniqueness of operators, as opposed to matrix elements, and the importance of the equivalence theorem and the Dyson transformation. This has been misinterpreted,[53] unfortunately, because of the *accidental* equality of the nonrelativistic limits of Dyson's transformation[42] and the one Barnhill[33] used. The latter transformation is quite far removed from the physics of exchange currents; what is important is the relationship of PS to PV couplings.

Although we have dealt only briefly with Lorentz invariance, this principle is central to the entire discussion of relativistic corrections. We have seen that various dynamical paths to realizing this principle exist; no unique prescription exists, and the fact that one method works does not mean that others are wrong.

The various representations used are by no means equally distributed throughout the literature. We have seen that the standard representation for retardation was naturally generated by the FW, FST, and Gross methods. The free spinor ($\mu=-1$) representation is also the most common. By an amusing coincidence these representations, among the ones we have discussed, *maximize* exchange contributions to the isoscalar charge operator.

A number of different one-photon-exchange potentials have appeared recently, derived using various quasipotential equations. Some surprise is usually expressed that certain of these diverse potentials do not agree with the Breit interaction. These differences are generally traceable to an energy-dependent potential or the use of Feynman gauge in the photon propagator and different treatments of the retardation.[62] For each separate quasipotential formulation of the two-nucleon potential is a corresponding treatment of the electromagnetic problem. As we discussed in the Introduction, however, this does not mean that all are equally "good," and if one is restricted to one-photon exchange, there will be an (electromagnetic) gauge dependence of the potential which results.

We would suggest therefore that the various methods which are used in calculating relativistic corrections be carefully examined for internal consistency in order to determine any energy dependence in the potential, as well as the representations. That is, a nonrelativistic reduction of the equations of motion should provide enough information to determine these specifics. Each technique will lead to different transition operators, Hamiltonians, and wave functions. It remains to be seen which method, if any, will prove superior to the others in the various applications which will arise.

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APPENDIX

We list here explicit forms for the various contributions to F, the deuteron charge form factor. The impulse approximation to F, denoted by F_0,

has the form

$$F_0^0 = G_E^S \int_0^\infty C(x) j_0(qx/2)dx\,, \tag{A1}$$

$$F_0^2 = 2G_E^S \int_0^\infty Q(x) j_2(qx/2)dx\,, \tag{A2}$$

where G_E^S is the isoscalar nucleon form factor $[G_E^S(0)=1]$ and j_n is the usual spherical Bessel function. In addition

$$C(x) = u^2(x) + w^2(x) \tag{A3}$$

and

$$Q(x) = u(x)w(x) - w^2(x)/\sqrt{8} \tag{A4}$$

are the charge and quadrupole combinations of the usual deuteron reduced wave function components u (S state) and w (D state). The two most common physical quantities calculated from F in the small-q limit are the mean-square radius (assuming point nucleons) $\langle r^2\rangle_D$ and the quadrupole moment Q_D. For the impulse approximation we find

$$\langle r^2\rangle_0 = \tfrac{1}{4}\int_0^\infty x^2 C(x)dx\,, \tag{A5}$$

$$Q_0 = \frac{1}{\sqrt{50}}\int_0^\infty x^2 Q(x)dx\,. \tag{A6}$$

An additional pion-exchange contribution to F, denoted by $D_\pi(\vec{q})$, can arise and can be decomposed according to Eq. (3):

$$D_\pi^0 = \frac{f_0^2}{2Mm_\pi}\int_0^\infty dx\, h'(x) q j_1(qx/2)[C(x) + 4\sqrt{2}Q(x)]\,, \tag{A7}$$

$$D_\pi^2 = -\frac{f_0^2\sqrt{8}}{2Mm_\pi}\int_0^\infty dx\, h'(x)\left[q j_1(qx/2)(C - Q/\sqrt{2}) - \frac{27w^2}{2x} j_2(qx/2)\right]. \tag{A8}$$

This contribution leads to

$$\langle r^2\rangle_\pi = -\frac{f_0^2}{2Mm_\pi}\int_0^\infty dx\, x h'(x)\,(C + 4\sqrt{2}Q)\,, \tag{A9}$$

$$Q_\pi = -\frac{f_0^2}{Mm_\pi}\int_0^\infty dx\, x h'(x)[(C - Q/\sqrt{2}) - \tfrac{27}{20}w^2]\,, \tag{A10}$$

where we have defined

$$h(x) = \frac{4\pi}{m_\pi}\int \frac{d^3q}{(2\pi)^3}\frac{F_{\pi N}^2(\vec{q}^2)e^{i\vec{q}\cdot\vec{x}}}{\vec{q}^2 + m_\pi^2} \to \frac{e^{-m_\pi x}}{m_\pi x}\,, \tag{A11}$$

which is dimensionless. The last relationship holds if the pion-nucleon form factor $F_{\pi N}$ is identically one. In addition, the rationalized pion-nucleon coupling constant $f_0^2 = f^2 m_\pi^2/4\pi \cong 0.079$ is defined in terms of $f = g/2m$ and the usual pion-nucleon coupling constant g, the pion mass m_π, and the nucleon mass M.

The remaining type of exchange contribution D_G has the form

$$D_G^0 = -\frac{f_0^2}{4Mm_\pi}\int_0^\infty dx\, qx j_1(qx/2)[C(h'' + 2h'/x) + 4\sqrt{2}Q(h'' - h'/x)]\,, \tag{A12}$$

$$D_G^2 = \frac{f_0^2\sqrt{8}}{2Mm_\pi}\int_0^\infty dx\, q\frac{d}{dq} j_2(qx/2)\left[C(h'' - h'/x) - \frac{Q}{\sqrt{2}}(h'' - 4h'/x)\right] \tag{A13}$$

for pion exchange, which leads to

$$\langle r^2\rangle_G = \frac{f_0^2}{4Mm_\pi}\int_0^\infty dx\, x^2[C(h'' + 2h'/x) + 4\sqrt{2}Q(h'' - h'/x)] \tag{A14}$$

and

$$Q_G = \frac{f_0^2}{5Mm_\pi}\int_0^\infty dx\, x^2\left[C(h'' - h'/x) - \frac{Q}{\sqrt{2}}(h'' - 4h'/x)\right]. \tag{A15}$$

It is demonstrated in the text that all our deuteron results can be expressed as linear combinations of F_0, D_π, and D_G. We restrict ourselves to contributions of order V_π/M where V_π is the static OPEP; thus our corrections to F_0 have the dimensional form $f_0^2 m_\pi/M$ which is approximately 0.01 and sets the scale of the corrections.

For scalar- and vector-meson exchanges, D_G' has a simpler form (denoted by the prime) than for pions, because the nonrelativistic potential V_0 has no spin dependence. The form of D_G' is given by

$$D_G'^0 = -\frac{q^2}{M}\frac{d}{dq^2}\int_0^\infty C(x) j_0(qx/2) V_0 dx\,, \tag{A16}$$

$$D_G'^2 = -\frac{q^2}{M}\frac{d}{dq^2}2\int_0^\infty Q(x) j_2(qx/2) V_0 dx\,, \tag{A17}$$

where V_0 contains the necessary isospin factors and appropriate Yukawa form in coordinate space.

The transformation χ_μ which arises from pion exchange has the form

$$\chi_\mu = \frac{f_0^2}{8Mm_\pi}(\mu - 1)(\vec{\sigma}_1\cdot\vec{P}\vec{\sigma}_2\cdot\vec{\nabla}h - \vec{\sigma}_1\cdot\vec{\nabla}h\vec{\sigma}_2\cdot\vec{P})\vec{\tau}(1)\cdot\vec{\tau}(2)\,, \tag{A18}$$

where $\vec{\tau}(i)$ is the isospin operator of the ith nucleon; the isospin factor has the value -3 for the deuteron. Any χ_v term in the wave function can be converted into an effective charge operator using Eqs. (2). We find

$$\langle\psi|\hat{\rho}_0(\vec{q})|\psi\rangle \cong \langle\psi_0|\hat{\rho}_0(\vec{q})+\Delta\rho(\vec{q})|\psi_0\rangle, \tag{A19}$$

$$\Delta\rho(\vec{q}) = i[\chi_v, \hat{\rho}_0(\vec{q})] \tag{A20}$$

with

$$\rho_0(\vec{q}) = \sum_i \hat{e}_i e^{i\vec{q}\cdot\vec{x}_i} \tag{A21}$$

and

$$\hat{e}_i = \left(\frac{1+\tau_3(i)}{2}\right)G_E^p + \left(\frac{1-\tau_3(i)}{2}\right)G_E^n \rightarrow \frac{G_E^S}{2}. \tag{A22}$$

For the specific χ_v given above, namely χ_μ, we find that $\langle\Delta\rho\rangle$ vanishes because χ_μ is antisymmetric under spin interchange.

The remaining relativistic corrections ($F \equiv F_v + F_0 + \Delta F_0$) can be obtained from Ref. 40 in the form (to order $1/M^2$)

$$\Delta F_0 = (1-\vec{q}^2/8M^2)G_E^S F_0(q^2) - \frac{\vec{q}^4}{16M^2}G_E^S\frac{d}{d\vec{q}^2}F_0 + (2G_M^S - G_E^S)F_{so}, \tag{A23}$$

where the Darwin-Foldy, Lorentz contraction, and spin-orbit contributions are the three separate terms. In addition, the arguments of G_E^S and F_0 must be q^2 (the four-momentum transfer) rather than $\vec{q}^2$. These four kinematical corrections are common to the early work of Gross[41] and the present author.[19] Until recently, pair and other potential-dependent contributions had not been considered.

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