PHYSICAL REVIEW C VOLUME 22, NUMBER 2 AUGUST 1980

Multiple scattering formalism for the effective interaction

Sadhan K. Adhikari
Departamento de Fisica, Universidade Federal de Pernambuco, 50,000 Recife, PE, Brasil

H. Bandō
*Departamento de Fisica, Universidade Federal de Pernambuco, 50,000 Recife, PE, Brasil and Division of Mathematical Physics, Fukui University, Fukui, Japan**

(Received 5 November 1979)

A general formalism is proposed for evaluating the effective interaction between valence nucleons, taking into consideration the low-lying contributions of two-particle two-hole (2p2h), two-particle one-hole (2p1h), three-particle one-hole (3p1h), three-particle two-hole (3p2h), and four-particle two-hole (4p2h) correlations. The formalism consists of solving a hierarchy of multiple scattering type equations with coupling between various correlations and is naturally combined with the Q-box approach for folded diagram theory. The energy dependence of the reaction matrix is included in the formalism. The effect of 4p2h correlations is studied for a simple two-level model for mass 42 by solving the hierarchy of coupled equations numerically by neglecting the coupling to 2p1h and 3p1h configurations. The 4p2h correlation with 4p coupled to isospin 0 has dominant contributions as expected. For a reasonable choice of the energy denominator, the 4p2h contribution to the two-body effective interaction is sizably attractive.

[NUCLEAR STRUCTURE Shell model, effective interaction between valence nucleons, particle-hole multiple scattering formalism for low-lying states.]

I. INTRODUCTION

An accurate description of the effective interaction between valence nucleons is one of the basic subjects in nuclear structure theory.[1] The starting point of such a calculation is essentially the formulation of Brueckner. The basic two-body vertex in the microscopic theory of effective interaction is the Brueckner reaction matrix G, which by construction takes into account short range (high-lying) two-body correlations. The G matrix alone has been found, however, to be a poor approximation for the effective interaction between valence nucleons especially for the low-lying states.[2] Various corrections to the G matrix needed for the calculation of such effective interaction can be properly incorporated using the folded diagram theory for the effective interaction.[3-6] We use a particular variant of this approach—the Q-box approach[6]—in the present formulation. The G matrix can thus in principle be used to take into account low-lying correlations between many particles and holes. Special care is needed to avoid double counting ambiguities in such treatment of low-lying states. Within the framework of the Brueckner theory the G matrix adequately treats the high-lying states. In the present formalism both high-lying and low-lying states are treated in a democratic way, following the idea of the double-partition approach.[7] The immediate space is divided into high and low parts. The high-lying space is adequately treated to yield the Brueckner G matrix with some important rearrangement effects. The effect of low-lying states is then included by a multiple scattering formalism introduced in earlier works.[8-11]

The renormalization of effective interaction through core polarization effects has been a long-standing problem.[1,2,11-14] In the case of two valence nucleons, previous works by Andō, Krenciglowa, and one of us (H.B.) have used multiple scattering formalism to study the contribution of low-lying three-particle one-hole (3p1h) intermediate states to the effective interaction between two valence nucleons.[10,11] The multiple scattering formalism efficiently sums up a series of connected diagrams that contribute to the effective interaction between valence nucleons. The resulting equation, which was obtained using the multiple scattering formalism, takes into account full three-particle one-hole correlation effects. Such an approach was a generalization of two-particle one-hole multiple scattering formalism which was employed to get a generalized core polarization vertex in the case of a one valence nucleon.[8,9] The energy dependence of the G matrix is properly taken into account in such an approach, although it is difficult to include such energy dependence in a shell-model type approach. An extensive study was performed for mass-16 and -40 regions by incorporating several important physical ingredients into the 3p1h multiple

scattering approach using different N-N potentials.[11]

It is well known that four nucleons will have strong correlation in a many nucleon problem. The two valence nucleons outside core can excite two nucleons in low-lying states and create four-particle two-hole (4p2h) intermediate states. Such four-particle states will have strong correlations and can really influence the effective interaction between valence nucleons. The lowest order 4p2h diagram has already been considered in such a context. Here in the present work we develop a multiple scattering formalism to take into account strongly correlated states of this type in such a way that it is consistent with the folded diagram theory. The present approach therefore takes into account only connected diagrams with proper enegy dependence of the G-matrix vertices. The present method consists in solving a hierarchy of equations of the multiple scattering type. At each stage of the hierarchy only connected diagrams are retained as this makes the implementation of the formalism much easier in practive. The 4p2h correlations under consideration are expected to have physical correspondence with the "intruder state" which can pose difficulties in "order-by-order" approaches. These difficulties are now largely overcome.[15-18] As our basic approach is not "order-by-order" in the interaction, "intruder" difficulties are not expected here *a priori*.

The present formalism takes into account all possible interactions between four particles and two holes and is in fact a six-body theory. To study the effect of four-particle correlation we neglect the coupling with 3)1h correlation and the particle-hole interaction from the present formalism. This makes the numerical calculation simpler. Of course a rigorous calculation must take into consideration particle-hole interactions at every stage of calculation. We carry out a numerical calculation for a simple two-level model which simulates the mass-42 region. The basic interaction vertex is obtained by simulating the realistic G matrix. Our model calculation takes into account pure 4p2h, while 3p1h correlation was investigated in detail.[8-11] The calculation we perform is a model calculation but through this calculation we can learn about the effect of strong four-particle correlation on the effective interaction between valence nucleons.

The plan of the paper is as follows. In Sec. II we develop the hierarchy of equations. In Sec. III we represent explicit angular momentum representation of such equations. Section IV reports our numerical calculation and finally Sec. V gives a brief summary and concluding remarks.

II. MULTIPLE SCATTERING EQUATIONS

The model space projection operator P will have the form

$$P \equiv P_{2p} = \sum |ij\rangle\langle ij|, \tag{2.1}$$

where the summation extends over certain specified orbits. The Q space projection operator

$$Q = 1 - P \tag{2.2}$$

is divided in the following way

$$Q = Q^l_{2p} + Q^h_{2p} + Q^l_{3p1h} + Q^l_{4p2h} + \cdots, \tag{2.3}$$

where Q^l_{2p} has the form of (2.1) with the summation extending over another major shell and Q^h_{2p} refers to high-lying two-particle space. Q^l_{3p1h} and Q^l_{4p2h} are confined to several of the lowest orbits. The Q space is truncated to the terms exhibited in Eq. (2.3) and the high-lying terms in Q are treated by the G matrix which with orthogonalized plane wave intermediate states satisfies

$$G(\omega) = V + VQ^h_{2p}\frac{1}{\omega - Q^h_{2p}TQ^h_{2p}}Q^h_{2p}G(\omega), \tag{2.4}$$

where T, V, and ω are, respectively, the kinetic energy, nucleon-nucleon potential, and G-matrix starting energy. Essentially "exact" methods[19,20] for calculating the G matrix of Eq. (4) have been developed so that no double counting problems arise.

According to the folded diagram theory the effective interaction between two valence nucleons will have a contribution from all connected diagrams with two particles in both initial and final states. Now we propose a multiple scattering formalism for summing a subset of the relevant diagrams. For this purpose we propose a hierarchy of equations of the multiple scattering type whose driving terms are always connected. Such formalism takes into account a large family of diagrams. In this section we represent the formalsim diagrammatically and explain their physical content.

To find out the effect of 4p2h correlations we have to introduce various correlation operators between subclusters. The idea is similar to that which we need in the study of nonrelativistic few-body problems. For example, the formulation of a six-body problem becomes easier when expressed in terms of t matrices of various subclusters. So we construct correlations of the following types: two particle two hole (2p2h), two particle one hole (2p1h), and three particle two hole (3p2h), in addition to 4p2h and 3p1h correlations. There is one interesting difference between the nonrelativistic few-body problem and the

present formalism in that particle-hole configurations can be created or annihilated in the present formalism.

First we introduce the 2p2h correlation through the diagrammatic equation in Fig. 1(a). The point vertex indicates a G-matrix vertex and we include all possible interactions between the particles and holes on the right hand side, and the summation runs over all their interactions. The unknown T_{2p2h} in Fig. 1(a) is a renormalized G-matrix vertex.

In a nonrelativistic few-body problem only subcluster t matrices appear in the driving term of a bigger problem but the possibility of spontaneous creation or destruction of a particle-hole pair correlation involving a larger number of particles and holes may contribute to the driving term of correlation involving a smaller number of particles and holes. This will happen in the 2p1h type correlation explicitly demonstrated in the equations in Fig. 1(b) and 1(c). The equation Fig. 1(c) defines the driving term which includes the effect of 2p2h and 3p3h correlations. Such contributions did not appear in Refs. 8 and 9 because the Q space was truncated to exclude such correlations. The

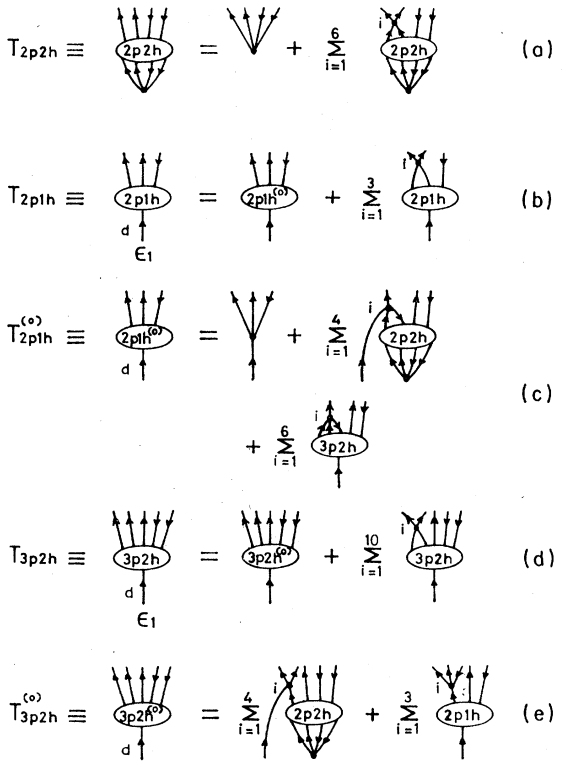

FIG. 1. Diagrammatic equations for the 2p2h, 2p1h, and 3p2h correlation operators T, with $T^{(0)}$ being the driving terms. The point vertex indicates a G matrix and the summation $\sum_i$ runs over all possible interaction pairs.

unknown T_{2p1h} of Fig. 1(b) is also in a sense a renormalized G-matrix vertex.

Next we define the 3p2h correlation. This again satisfies an equation similar to the equation in Fig. 1(b), namely the equation in Fig. 1(d) with the driving term given by the equation in Fig. 1(e). The driving term now contains both 2p2h and 2p1h correlations.

The 2p2h correlation given by the equation in Fig. 1(a) contains no particles in the initial state. The 2p1h and 3p2h correlations contain one particle in the initial state. Next we consider another type of term which contains two particles in the initial state.

The 3p1h correlation can be summed to T_{3p1h} and satisfies the equation of Fig. 2(a) with the driving term given by the equation of Fig. 2(b). Then we have the 4p2h correlation denoted by T_{4p2h} which satisfies the equation of Fig. 2(c) with the driving term given by the equation in Fig. 2(d). In the equations in Figs. 2(b) and 2(d), two coexisting 2p1h blobs implicitly represent a sum of alternatively repeated two 2p1h correlations. Note that these can be made on-energy-shell for nonfolded diagram contributions. Finally the effective two-body interaction between valence nucleons is calculated using $\hat{Q}_C$ defined by the equations of Figs. 3(a) and 3(b), where the con-

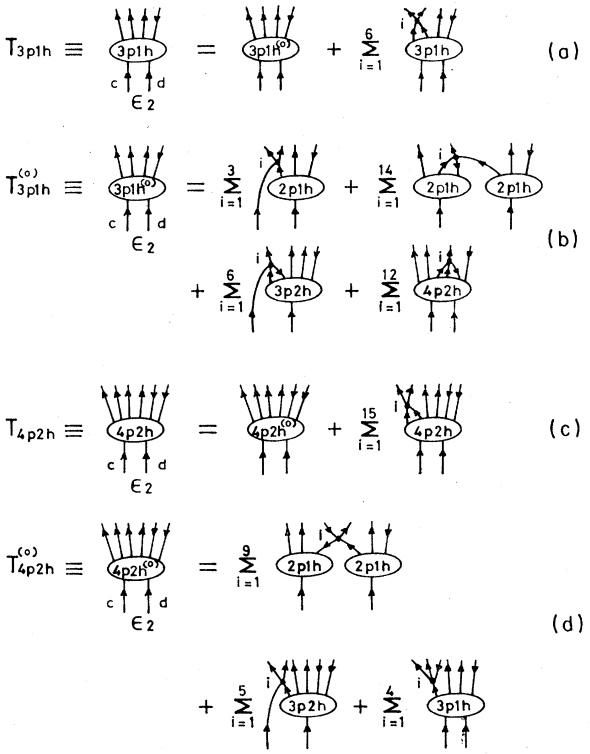

FIG. 2. Diagrammatic equations for the 3p1h and 4p2h correlation operators.

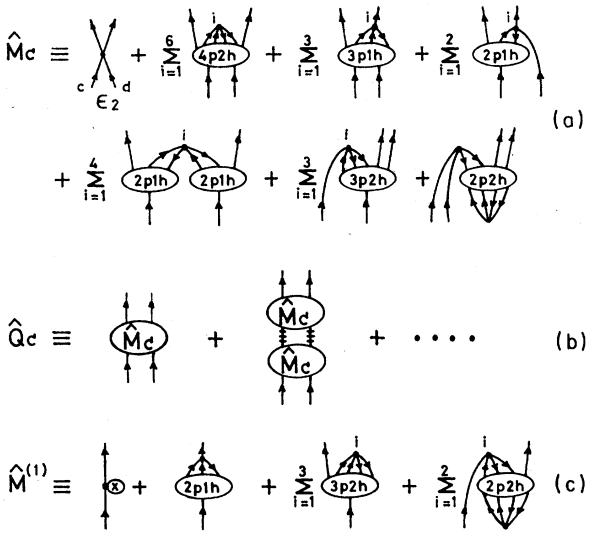

FIG. 3. Diagrammatic expressions of the connected two-body M box $\hat{M}_C$, connected two-body Q box $\hat{Q}_C$, and one-body M box $\hat{M}^{(1)}$. The railed line in (b)) indicates a particle state in the $\hat{Q}^I_{2p}$ space.

nected M box $\hat{M}_C$ is appropriate for the $P+Q^I_{2p}$ space and the Q box $\hat{Q}_C$ for the P space is evaluated by summing the $\hat{M}$ ladders. The expression corresponding to the equation of Fig. 3(b) is given by

$$P\hat{Q}_C P = P\left[\hat{M}_C + \hat{M}_C Q^I_{2p} \frac{1}{\epsilon_2 - Q^I_{2p}(H_0+\hat{M})Q^I_{2p}} Q^I_{2p}\hat{M}_C\right]P,$$

$$\hat{M} = \hat{M}_C + \hat{M}^{(1)}.$$

The one-body M box $\hat{M}^{(1)}$ here can also be evaluated by using the correlation operators as in the equation of Fig. 3(c), which is a generalization of Refs. 8, 9, and 21.

The coupling scheme is described in a diagrammatic way in Fig. 4. The arrow indicates to which quantities a particular correlation contributes. The diagram is self-explanatory. For example,

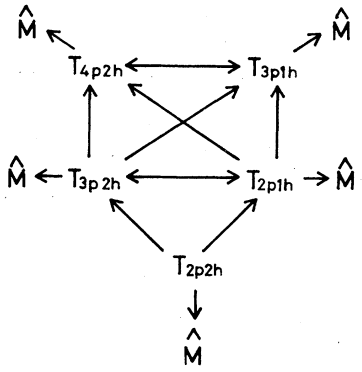

FIG. 4. The coupling scheme of the various correlation operators.

T_{2p1h} contributes to $\hat{M}$ and also to the driving terms T_{3p2h}, T_{3p1h}, and T_{4p2h}.

III. ALGEBRAIC EQUATIONS FOR SPECIFIC 4p2h CORRELATIONS

In this section we present explicit algebraic equations for specific 4p2h correlations which are considered to be important among a hierarchy of mutually dependent multiple scattering equations diagrammatically given in the previous sections. The connected series of equations, $T_{2p2h} \rightarrow T_{3p2h} \rightarrow T_{4p2h}$ of Fig. 4, correspond physically to the conventional shell model which takes into account the coupling between 2p and 4p2h configurations.[22] The path $T_{2p1h} \rightarrow T_{3p1h}$ has been already investigated in detail.[10,11] The approximation which arises from taking only the path $T_{2p2h} \rightarrow T_{3p2h} \rightarrow T_{4p2h} \rightarrow \hat{Q}$ is the neglect of the coupling between 4p2h and 3p1h configurations.

We introduce antisymmetrizing operators defined by

$$\mathcal{A}_{p_1p_2}\phi(p_1p_2\Gamma) = \tfrac{1}{2}[\phi(p_1p_2\Gamma) - \theta(p_1p_2\Gamma)\phi(p_2p_1\Gamma)], \tag{3.1}$$

$$\mathcal{A}_{p_1p_2p_3}\phi(p_1,p_2p_3(\Gamma_2)\Gamma)$$
$$= \tfrac{1}{3}\mathcal{A}_{p_2p_3}\{\phi(p_1,p_2p_3(\Gamma_2)\Gamma) - \theta(p_2p_3\Gamma_2)\sum_{\Gamma'_2}\theta(p_2\Gamma'_2\Gamma)[p_1p_3\Gamma p_2;\,\Gamma'_2\Gamma_2]2\mathcal{A}_{p_1p_3}\phi(p_2,p_1p_3(\Gamma'_2)\Gamma)\}, \tag{3.2}$$

$$\mathcal{A}_{p_1p_2p_3p_4}\phi(p_1p_2(\Gamma_1)p_3p_4(\Gamma_2)\Gamma)$$
$$= \tfrac{1}{6}\mathcal{A}_{p_1p_2}\mathcal{A}_{p_3p_4}\Big(\phi(p_1p_2(\Gamma_1)p_3p_4(\Gamma_2)\Gamma) + \theta(\Gamma_1\Gamma_2\Gamma)\phi(p_3p_4(\Gamma_2)p_1p_2(\Gamma_1)\Gamma)$$
$$- 4\sum_{\Gamma'_1\Gamma'_2}[p_1p_2\Gamma_1, p_3p_4\Gamma_2, \Gamma'_1\Gamma'_2\Gamma]\mathcal{A}_{p_1p_3}\mathcal{A}_{p_2p_4}\phi(p_1p_3(\Gamma'_1)p_2p_4(\Gamma'_2)\Gamma)\Big), \tag{3.3}$$

where Γ denotes angular momentum and isospin collectively ($\Gamma \equiv JT$) and $\theta(p_1p_2\Gamma) \equiv (-)^{j_1+j_2-J}(-)^{1/2+1/2-T}$, $[p_1p_3\Gamma p_2;\,\Gamma'_2\Gamma_2] \equiv ([\Gamma'_2][\Gamma_2])^{1/2}W(p_1p_3\Gamma p_2.\,\Gamma'_2\Gamma_2)$,

$$[p_1p_2\Gamma_1,p_3p_4\,\Gamma_2,\,\Gamma_1'\,\Gamma_2'\Gamma]\equiv([\Gamma_1][\,\Gamma_2][\,\Gamma_1'][\,\Gamma_2'])^{1/2}\begin{pmatrix}p_1 & p_2 & \Gamma_1\\ p_3 & p_4 & \Gamma_2\\ \Gamma_1' & \Gamma_2' & \Gamma\end{pmatrix}$$

with $[\Gamma]=(2\Gamma+1)$.

In the following we use simpler notations:

$$T_{2p2h}\to\mathfrak{G},\quad T_{3p2h}\to T_{3p},\quad T_{4p2h}\to T_{4p}. \tag{3.4}$$

The equation of Fig. 1(a) for the renormalized 2p2h vertex $\mathfrak{G}$ is given by

$$\begin{aligned}\mathfrak{G}(p_3p_4h_1h_2\Gamma_2)=G(p_3p_4h_1h_2\Gamma_2;\omega_{2p})&+\sum_{p_3'p_4'}G(p_3p_4p_3'p_4'\,\Gamma_2;\omega_{2p})\frac{1}{e_{2p}}\mathfrak{G}(p_3'p_4'h_1h_2\Gamma_2)\\&+\sum_{h_1'h_2'}G(h_1h_2h_1'h_2'\Gamma_2;\omega_{2h})\frac{1}{e_{2h}}\mathfrak{G}(p_3p_4h_1'h_2'\,\Gamma_2)\\&+2\theta(p_3p_4\Gamma_2)\mathfrak{A}_{h_1h_2}\sum_{\Gamma_2'}[\,\Gamma_2']\,W(p_3p_4h_1h_2;\,\Gamma_2\Gamma_2')\sum_{p'h'}\Big[F(p_3h_1p'h'\,\Gamma_2';\omega_{ph})\frac{2}{e_{ph}}\,\mathfrak{F}(p'h'h_2p_4\Gamma_2')\\&\qquad+F(h_2p_4h'p'\,\Gamma_2';\omega_{ph})\frac{2}{e_{ph}}\,\mathfrak{F}(p_3h_1h'p'\,\Gamma_2')\Big],\end{aligned} \tag{3.5}$$

where $F(\mathfrak{F})$ is a p-h coupling of $G(\mathfrak{G})$ given by

$$F(php'h'\,\Gamma_2')=\sum_{\Gamma_2}[\,\Gamma_2]\,W(ph'hp';\,\Gamma_2\Gamma_2')\theta(ph'\,\Gamma_2)G(ph'hp'\,\Gamma_2), \tag{3.6}$$

where G is an antisymmetrized but non-normalized matrix element. The G-matrix starting energies are given by

$$\begin{aligned}&\omega_{2p}=\omega_{2h}=E_{h_1}+E_{h_2},\\&\omega_{ph}=E_{h_1}+E_{h_2}+E_{h'}-\varepsilon_{p_s},\end{aligned} \tag{3.7}$$

where p_s denotes a spectator particle not involved in the relevant vertex. The energy denominators are

$$\begin{aligned}&e_{2p}=E_{h_1}+E_{h_2}-E_{p_3'}(\omega_{p_3'})-E_{p_4'}(\omega_{p_4'}),\ \omega_{p_i'}=E_{h_1}+E_{h_2}-\varepsilon_{p_j'},\\&e_{2h}=E_{h_1'}+E_{h_2'}-E_{p_3}(\omega_{p_3})-E_{p_4}(\omega_{p_4}),\quad \omega_{p_i}=E_{h_1'}+E_{h_2'}-\varepsilon_{p_j},\\&e_{ph}=E_{h_s}+E_{h'}-E_{p_s}(\omega_p)-E_{p'}(\omega_{p'}),\quad \omega_p=E_{h_s}+E_{h'}-\varepsilon_{p'}.\end{aligned} \tag{3.8}$$

In Eqs. (3.7) and (3.8), ε_p is the unperturbed sp energy, while E_h and E_p include self-energy G-matrix insertions. Hole energies E_h can be determined self-consistently on the energy shell, while particle energies E_p are to be evaluated off the energy shell.[9(b),10(b)] Determination of Eqs. (3.7) and (3.8) are based on the "downward-projecting core" argument[3,23] and thus the vertex $\mathfrak{G}$ does not depend on the diagram starting energy ϵ_2.

Next we would like to present the contribution of $\mathfrak{G}$ to the driving term of the 3p2h correlation defined by $T_{3p}^{(0)}$. This contribution which is diagrammatically represented by the first term on the right of the equation of Fig. 1(e) is explicitly given by

$$\begin{aligned}T_{3p}^{(0)}&(p_2,p_3p_4(\Gamma_2)\Gamma_3;h_1h_2(\Gamma_h);\,\Gamma_d;\epsilon_1)\\&=\frac{2}{e_3}\sum_{\Gamma_2'}[p_2p_3\Gamma_3p_4;\,\Gamma_2'\,\Gamma_2]\,\mathfrak{A}_{p_2p_3p_4}\Big\{\sum_{p_2'p_3'}G(p_2p_3p_2'p_3'\,\Gamma_2')\sum_{\Gamma_2''}[p_2'p_3'\,\Gamma_3p_4;\,\Gamma_2'\,\Gamma_2'']\\&\qquad\times r^{(0)}(p_2',p_3'p_4(\Gamma_2'')\Gamma_3;h_1h_2(\Gamma_h);\,\Gamma_d)\Big\}\\&+\frac{2}{e_3}\mathfrak{A}_{p_2p_3p_4}\mathfrak{A}_{h_1h_2}\Big\{\sum_{\gamma_1\gamma_2}[p_3\Gamma_2\Gamma_3,h_1h_2\Gamma_h,\gamma_1\gamma_2\Gamma_d]\sum_{p_2'h_1'}2F(p_2h_1p_2'h_1'\gamma_1)\sum_{\Gamma_3'\Gamma_h'}[p_2'\,\Gamma_2\Gamma_3',h_1'h_2\Gamma_h',\gamma_1\gamma_2\Gamma_d]\\&\qquad\times r^{(0)}(p_2',p_3p_4(\Gamma_2)\Gamma_3';h_1'h_2(\Gamma_h');\,\Gamma_d)\Big\},\end{aligned} \tag{3.9}$$

where

$$r^{(0)}(p_2,\, p_3p_4(\Gamma_2)\Gamma_3;h_1h_2(\Gamma_h);\,\Gamma_d)=\theta(\Gamma_d\,\Gamma_h\,\Gamma_3)\left(\frac{[\,\Gamma_3]}{[\,\Gamma_d]}\right)^{1/2}\frac{1}{e_{2p}}\mathcal{G}(p_3p_4h_1h_2\Gamma_2)\delta_{\Gamma_2\Gamma_h}\delta_{p_2d}. \tag{3.10}$$

Now in explicit angular momentum representation T_{3p} satisfies

$$\begin{aligned}
&T_{3p}(\,p_2,p_3p_4(\Gamma_2)\Gamma_3;h_1h_2(\Gamma_h);\,\Gamma_d;\epsilon_1)\\
&\quad=T^{(0)}_{3p}+\frac{3}{e_3}\,\mathcal{A}_{p_2p_3p_4}\Big\{\sum_{p_3'p_4'}G(p_3p_4p_3'p_4'\,\Gamma_2)T_{3p}(p_2,p_3'p_4'(\Gamma_2)\Gamma_3;h_1h_2(\Gamma_h);\,\Gamma_d;\epsilon_1)\Big\}\\
&\qquad+\frac{1}{e_3}\Big\{\sum_{h_1'h_2'}G(h_1h_2h_1'h_2'\,\Gamma_h)T_{3p}(p_2,p_3p_4(\Gamma_2)\Gamma_3;h_1'h_2'(\Gamma_h);\,\Gamma_d;\epsilon_1)\Big\}\\
&\qquad+\frac{6}{e_3}\,\mathcal{A}_{p_2p_3p_4}\mathcal{A}_{h_1h_2}\Big\{\sum_{\gamma_1\gamma_2}[\,p_2\Gamma_2\Gamma_3,h_1h_2\Gamma_h,\gamma_1\gamma_2\Gamma_d\,]\sum_{p_2'h_1'}2F(p_2h_1p_2'h_1'\gamma_1)\\
&\qquad\qquad\times\sum_{\Gamma_3'\Gamma_h'}[\,p_2'\,\Gamma_2\Gamma_3',h_1'h_2\Gamma_h',\gamma_1\gamma_2\Gamma_d\,]\,T_{3p}(p_2',p_3p_4(\Gamma_2)\Gamma_3';h_1'h_2(\Gamma_h');\,\Gamma_d;\epsilon_1)\Big\}.
\end{aligned} \tag{3.11}$$

In Eqs. (3.9)–(3.11) and the following, the energy dependence of F and G have been suppressed but implicit. This energy dependence and the energy denominator e_3 can be identified following the standard diagram rule.[3]

Next we give the contribution of such 3p2h correlations to the driving term of 4p2h correlations denoted by $T^{(0)}_{4p}$. $T^{(0)}_{4p}$ is given by

$$\begin{aligned}
&T^{(0)}_{4p}(p_1p_2(\Gamma_1)p_3p_4(\Gamma_2)\Gamma_4;h_1h_2(\Gamma_h);\,\Gamma_{cd};\epsilon_2)\\
&\quad=\frac{3}{e_4}\,\mathcal{A}_{p_1p_2p_3p_4}\Big\{\sum_{p_1'p_2'}G(p_1p_2p_1'p_2'\,\Gamma_1)\,t_1^{(0)}(\,p_1'p_2'(\Gamma_1)p_3p_4(\Gamma_2)\Gamma_4;h_1h_2(\Gamma_h);\,\Gamma_{cd};\epsilon_2)\Big\}\\
&\qquad+\frac{2}{e_4}\,\mathcal{A}_{p_1p_2p_3p_4}\mathcal{A}_{h_1h_2}\Big\{\sum_{\Gamma_3}[\,p_1p_2\Gamma_4\Gamma_2;\,\Gamma_1\Gamma_3]\sum_{\gamma_1\gamma_2}[\,p_1\Gamma_3\Gamma_4,h_1h_2\Gamma_h,\gamma_1\gamma_2\Gamma_{cd}]\sum_{p_1'h_1'}2F(p_1h_1p_1'h_1'\gamma_1)\\
&\qquad\qquad\times\sum_{\Gamma_4'\Gamma_h'}[\,p_1'\,\Gamma_3\Gamma_4',h_1'h_2\Gamma_h',\gamma_1\gamma_2\Gamma_{cd}]\,t_2^{(0)}(p_1',\{p_2,p_3p_4(\Gamma_2)\}\,\Gamma_3\,\Gamma_4';h_1'h_2(\Gamma_h');\,\Gamma_{cd};\epsilon_2)\Big\},
\end{aligned} \tag{3.12}$$

where

$$\begin{aligned}
&t_2^{(0)}(p_1,\{\,p_2,p_3p_4(\Gamma_2)\}\,\Gamma_3\Gamma_4;h_1h_2(\Gamma_h);\,\Gamma_{cd};\epsilon_2)\\
&\qquad=2\mathcal{A}_{cd}[\,p_1\Gamma_3\Gamma_{cd}\Gamma_h;\,\Gamma_4\Gamma_d]\,T_{3p}(p_2,p_3p_4(\Gamma_2)\Gamma_3;h_1h_2(\Gamma_h);\,\Gamma_d;\epsilon_2-\epsilon_c)\,\delta_{p_1c}
\end{aligned} \tag{3.13}$$

and

$$\begin{aligned}
&t_1^{(0)}(p_1p_2(\Gamma_1)p_3p_4(\Gamma_2)\Gamma_4;h_1h_2(\Gamma_h);\,\Gamma_{cd};\epsilon_2)\\
&\qquad=\sum_{\Gamma_3}[\,p_1p_2\Gamma_4\Gamma_2;\,\Gamma_1\Gamma_3]\,t_2^{(0)}(p_1,\{\,p_2,p_3p_4(\Gamma_2)\}\,\Gamma_3\Gamma_4;h_1h_2(\Gamma_h);\,\Gamma_{cd};\epsilon_2).
\end{aligned} \tag{3.14}$$

Equation (3.12) is diagrammatically represented by the equation of Fig. 2(d). Finally T_{4p} for 4p2h correlation given by the equation of Fig. 2(c) satisfies

$$\begin{aligned}
&T_{4p}(p_1p_2(\Gamma_1)p_3p_4(\Gamma_2)\Gamma_4;h_1h_2(\Gamma_h);\,\Gamma_{cd};\epsilon_2)\\
&\quad=T^{(0)}_{4p}+\frac{6}{e_4}\,\mathcal{A}_{p_1p_2p_3p_4}\Big\{\sum_{p_1'p_2'}G(p_1p_2p_1'p_2'\,\Gamma_1)T_{4p}(p_1'p_2'(\Gamma_1)p_3p_4(\Gamma_2)\Gamma_4;h_1h_2(\Gamma_h);\,\Gamma_{cd};\epsilon_2)\Big\}\\
&\qquad+\frac{1}{e_4}\Big\{\sum_{h_1'h_2'}G(h_1h_2h_1'h_2'\,\Gamma_h)T_{4p}(p_1p_2(\Gamma_1)p_3p_4(\Gamma_2)\Gamma_4;h_1'h_2'(\Gamma_h);\,\Gamma_{cd};\epsilon_2)\Big\}\\
&\qquad+\frac{8}{e_4}\,\mathcal{A}_{p_1p_2p_3p_4}\mathcal{A}_{h_1h_2}\Big\{\sum_{\Gamma_3}[\,p_1p_2\Gamma_4\Gamma_2;\,\Gamma_1\Gamma_3]\sum_{\gamma_1\gamma_2}[\,p_1\Gamma_3\Gamma_4,h_1h_2\Gamma_h,\gamma_1\gamma_2\Gamma_{cd}]\sum_{p_1'h_1'}2F(p_1h_1p_1'h_1'\gamma_1)\\
&\qquad\qquad\times\sum_{\Gamma_4'\Gamma_h'}[\,p_1'\,\Gamma_3\Gamma_4',h_1'h_2\Gamma_h',\gamma_1\gamma_2\Gamma_{cd}]\sum_{\Gamma_1'}[\,p_1'p_2\Gamma_4'\,\Gamma_2;\,\Gamma_1'\,\Gamma_3]\\
&\qquad\qquad\times T_{4p}(p_1'p_2(\Gamma_1')p_3p_4(\Gamma_2)\Gamma_4';h_1'h_2(\Gamma_h');\,\Gamma_{cd};\epsilon_2)\Big\}.
\end{aligned} \tag{3.15}$$

The contribution of T_{4p} to the M box $\hat{M}_{C4p2h}$ will be given by

$$\hat{M}_{C4h2h}(abcd\Gamma;\epsilon_2) = 6 \sum_{p_3 p_4 h_1 h_2} \sum_{\Gamma_4 \Gamma_h} ([\Gamma_4]/[\Gamma])^{1/2} \theta(\Gamma \Gamma_h \Gamma_4) \times G(p_3 p_4 h_1 h_2 \Gamma_h)\, T_{4p}(ab(\Gamma) p_3 p_4 \times (\Gamma_h)\Gamma_4; h_1 h_2(\Gamma_h); \Gamma;\epsilon_2). \quad (3.16)$$

This completes the presentation of the simplified 4p2h equations. Here we neglect the coupling to correlations of type 2p1h and 3p1h. The energy dependence of the G matrix is dropped for simplicity after Eq. (3.8) but it is supposed to be there and can be included in numerical calculation. Furthermore, the center-of-mass treatment[9(b),24] and the high-lying space rearrangement effect can be easily incorporated by adding terms to the G-matrix mertex as was done in Refs. 10(b) and 11.

IV. TWO-LEVEL MODEL CALCULATION

In order to illustrate the feasibility of our treatment and also to get an idea of the importance of the 4p2h state contribution to the two-body valence interaction, we employ a two-level model with a $j_p = \frac{1}{2}^-$ particle orbit and a $j_h = \frac{3}{2}^+$ hole orbit which simulates mass-42 systems. We use the equations presented in Sec. III but with the neglect of the p-h and h-h interactions, considering that the p-p interaction among four particles is principally important and that the p-h and h-h interactions could be absorbed in the p-h single particle energy difference in an average sense. With this approximation, two holes enter only into the initial and final interactions. We do not evaluate the standard second-order 4p2h diagram as it is well known. The G-matrix elements that are used in this calculation are $G(j_p^4 JT) = -1.6, -1.1, -1.5, -2.3$ MeV for $J = 1, 3, 5, 7$, $T = 0$, and $-1.4, -0.9, -0.4, -0.2$ MeV for $J = 0, 2, 4, 6$, $T = 1$; $G(j_p^2 j_h^2 JT) = -1.3, -0.3$ MeV for $J = 1, 3$, $T = 0$, and $1.9, 0.4$ MeV for $J = 0, 2$, $T = 1$. These matrix elements simulate the energies obtained by diagonalizing a G matrix with full $(0f1p)^2$ configurations.[11] Energy dependence of the G-matrix vertex is not taken into consideration and a common value D is used for all energy denominators.

We solve a series of multiple scattering equations with a successive displacement iteration method so that each order of iteration includes far more than the corresponding order of perturbation series. The renormalized 2p2h vertex $\mathcal{G}$ of Eq. (3.5) can be obtained trivially and is put into the driving term $T_{3p}^{(0)}$, Eq. (3.10), for the 3p multiple scattering equation (3.11) for $T3_p$. The equation for T_{3p} is solved by iteration and the solution is then substituted into the driving term $T_{4p}^{(0)}$, Eq. (3.13), for the 4p multiple scattering equation (3.15) for T_{4p}. The equation for T_{4p} is also solved by iteration and then the solution is used through Eq. (3.16) to give a Q box $\hat{Q}_{C4p2h}(j_p^4 JT)$ which fully includes a set of 4p2h intermediate state contributions to the two-body effective interaction.

The 3p amplitude T_{3p} converges very quickly. The strong 4p correlation shows up in the solution of T_{4p}; therefore the convergence of iteration highly depends on the 4p angular momentum and isospin $J_4 T_4$. As naturally expected, the T_{4p} for $T_4 = 0$ is most enhanced and gives dominant contribution to $\hat{Q}_{C4p2h}$, since this corresponds to the α-cluster-like correlation. Figures 5(a) and 5(b) show the contribution to $\hat{Q}_{C4p2h}(JT = 01)$ and $\hat{Q}_{C4p2h}(JT = 10)$, respectively, from different $(J_h T_h, J_4 T_4)$ components in the case of the energy denominator $D = 11$ MeV. All these components contribute attractively to $\hat{Q}$ in this case.

By varying the energy denominator D we can find out the position of the pole for each $J_4 T_4$ component, which somehow corresponds to the energy gain of the corresponding 4p "state" due to the internal correlation. Figure 6 displays the worst convergence behaviors of contributions to $\hat{Q}_{C4p2h}$ as a function of iterations for T_{4p}. We can see that the $T_4 = 0$, $J_4 = 0$, 2, and 4 contributions start to diverge at $D \simeq 9.5$, 9.0, and 8.5 MeV, respectively, implying that these 4p2h states come down from the unperturbed energy by those amounts of energies. The location of the pole is of physical interest so that divergent series can be of physical interest. A choice of $D = 11$ MeV used in Fig. 5 may thus correspond to the situation where a $J_4 T_4 = 0\ 0$ 4p2h state is 1.6 MeV above the 2p state. It is interesting that even in the $D = 10$ MeV case, which is only 0.5 MeV above the pole for $J_4 = 0$, the contribution to $\hat{Q}_{C4p2h}$ still re-

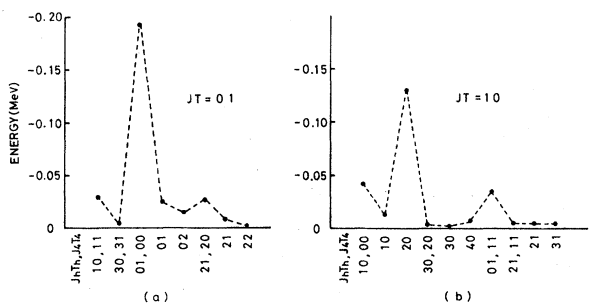

FIG. 5. Contributions to $\hat{Q}_{C4p2h}(JT)$ from 4p2h correlations T_{4p} with different $(J_h T_h, J_4 T_4)$. The energy denominator is taken as $D = 11$ MeV.

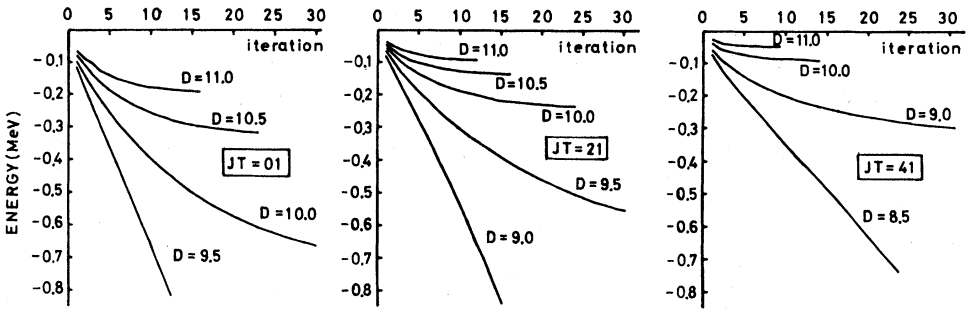

FIG. 6. Convergence behaviors of contributions of $T_{4p}(J_4T_4 = J\,0,\ J_hT_h = 0\,1)$ to $\hat{Q}_{C4p2h}(J, T = 1)$ as a function of iterations.

mains not too large. Anyway the extreme sensitivity to D as seen in Fig. 6 indicates the necessity of treating the energy denominator self-consistently with the G matrix in the realistic case. It is to be noted here that the present multiple scattering equation could be solved in a noniterative way, because it forms a simple inhomogeneous linear equation, although all the omponents are not independent, owing to antisymmetrization. Convergence of T_{4p} for other than $J_4 =$ even, $T_4 = 0$ is very quick.

Calculated values of $\hat{Q}_{C4p2h}(j_p^4 JT)$ with $D = 11$ MeV are -0.241, -0.133, -0.115, -0.134 MeV for $J = 1,3,5,7$, $T = 0$, and -0.300, -0.169, -0.104, -0.085 MeV for $J = 0,2,4,6$, $T = 1$. These are sizable enough to give a physical effect on the 2p system in view of the fact that realistic bare G-matrix elements are smaller than $G(j_p^4 JT)$ used here, since the latter simulates full $(0f1p)^2$ effects.

V. SUMMARY

We have developed a general multiple scattering formalsim for the effective interaction between valence nucleons, which fully takes into account low-lying 3p1h and 4p2h intermediate state contributions. Five kinds of correlation amplitudes, 2p2h, 2p1h, 3p1h, 3p2h, and 4p2h, are introduced to evaluate only the connected series of diagrams and they constitute a hierarchy of mutaully dependent multiple scattering equations. This is a generalization of the previously used 2p1h and 3p1h multiple scattering formalisms. As already proved, this approach is flexible in the sense that the energy dependence of G-matrix vertices, center-of-mass treatment, and rearrangement effects can be easily incorporated, and at the same time maintains a transparent connection with the linked-valence connected diagram perturbation theory. Combination with the Q-box approach for evaluating folded diagrams is naturally achieved. Explicit angular momentum coupled expressions for the general equations diagrammatically shown in Sec. II can be given straightforwardly, although we have presented them only for a particular path of the hierarchy that we used in our numerical investigation. The multiple scattering equations have a form to be easily solved by a simple linear iteration. For strongly correlated amplitudes it will also be possible to solve them as a set of coupled algebraic linear equations. The noniterative way of solving is the only way when the iterative series is diverging. Divergent iterative series does not mean that the solution does not exist.

Numerical calculation has been carried out for a simple two-level model which simulates mass-42 systems. A series of equations, $T_{2p2h} \to T_{3p2h} \to T_{4p2h} \to \hat{Q}$, have been solved by iteration, with the neglect of the p-h and h-h interactions. The 4p correlation with isospin 0 has dominant contributions to the two-body effective interaction, as expected. The net contribution of the connected 4p-2h correlation is sizably attractive for a reasonable choice of the unperturbed energy denominator. Of course this schematic model calculation is not meant to draw any definite conclusion for the realistic 4p2h effect; however, this calculation gives us a prospect for applying the present approach to a more realistic situation. In mass-42 systems, for example, we need to take into consideration three hole orbits and four particle orbits, or moreover to incorporate the possible α clustering, and the sensitive dependence of the 2p2h matrix element $G(p_1p_2h_1h_2JT)$ on the relevant orbits. Such aspects can only be treated in realistic calculations.

The formalsim proposed in this paper is more general than those used to include 2p1h (Refs. 8, 9) and 3p2h (Refs. 10, 11) correlations and also maintains the advantages of these approaches. A *more* realsitic calculation of 2p1h, 3p1h, and 4p2h correlations should involve the solution of the coupled set of equations presented in Sec. II and not the neglect of the coupling as in previous works. The number of coupled equations depend on the definition (2.3) of the truncated Q^t space. The contribu-

tion (relative importance) of the various terms of the Q' space to the effective interaction is not obvious. We can include more terms in this Q' space, in principle, and this will increase the number of coupled equations. There have been several calculations which use the shell model[22] or cluster model[25] to couple 2p and 4p2h configurations. These calculations give valuable information on the coupling structure, especially through close comparison with various experimental data; on the other hand, their treatments are not within the rigorous effective interaction theory in that, for example, disconnected diagrams are included, while folded diagram contributions are not included. The present approach could hopefully contribute to putting the problem on a firmer theoretical basis.

ACKNOWLEDGMENTS

We are greatly indebted to Dr. E. M. Krenciglowa for helpful discussions and suggestions. This work was partially supported by the CNPq of Brasil and the FINEP of Brasil. One of us (H.B.) would like to thank the exchange program between ABC of Brasil and JSPS of Japan for supporting his visit to Brasil. We are thankful to Professor H. T. Coelho and the nuclear theory group of UFPE for encouragement.

---

*Present address.

[1]B. R. Barrett and M. W. Kirson, *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum, New York, 1973), Vol. 6, p. 219; T. T. S. Kuo, Annu. Rev. Nucl. Sci. 24, 101 (1974); *Proceedings of the Tucson International Topical Conference on Nuclear Physics, Tucson, 1975*, Lecture Notes in Physics, Vol. 40, edited by B. R. Barrett (Springer, New York, 1975); P. J. Ellis and E. Osnes, Rev. Mod. Phys. 49, 777 (1977).

[2]G. Bertsch, Nucl. Phys. 24, 234 (1965); T. T. S. Kuo and G. E. Brown, *ibid.* 85, 40 (1966).

[3]B. H. Brandow, Rev. Mod. Phys. 39, 171 (1967).

[4]G. Oberlechner, F. Owono-N'Guema, and J. Richert, Nuovo Cimento 68B, 23 (1970).

[5]M. B. Johnson and M. Baranger, Ann. Phys. (N.Y.) 62, 172 (1971).

[6]T. T. S. Kuo, S. Y. Lee, and K. F. Ratcliff, Nucl. Phys. A176, 65 (1971); K. F. Ratcliff, S. Y. Lee, and T. T. S. Kuo (unpublished).

[7]H. Bandō, T. Murota, and S. Nagata, Prog. Theor. Phys. 29, 221 (1963); B. H. Brandow, *Lectures in Theoretical Physics*, edited by K. Y. Mahanthappa and W. E. Brittin (Gordon and Breach, New York, 1969) Vol. XIB, p. 55; G. E. Brown, Rev. Mod. Phys. 43, 1 (1971); B. R. Barrett, Nucl. Phys. A221, 299 (1974); E. M. Krenciglowa, C. L. Kung, T. T. S. Kuo, and E. Osnes, Phys. Lett. 63B, 141 (1976).

[8]K. Andō and H. Bandō, Prog. Theor. Phys. 53, 1711 (1975); 51, 954 (1974); K. Andō, H. Bandō, and S. Nagata, *ibid.* 57, 1303 (1977); 57, 1584 (1977).

[9](a) H. Bandō and E. M. Krenciglowa, Nucl. Phys. A273, 95 (1976); (b) E. M. Krenciglowa and H. Bandō, *ibid.* A294, 191 (1978).

[10](a) K. Andō, H. Bandō, and E. M. Krenciglowa, Nucl. Phys. A309, 59 (1978); (b) H. Bandō, E. M. Krenciglowa, and K. Andō, *ibid.* A329, 109 (1979).

[11]K. Andō, H. Bandō, and E. M. Krenciglowa, Prog. Theor. Phys. Suppl., No. 65, 38 (1979).

[12]M. W. Kirson, Ann. Phys. (N.Y.) 82, 345 (1974); 66, 624 (1971).

[13]N. Loludice, D. J. Rowe, and S. S. M. Wong, Nucl. Phys. A245, 479 (1975); A219, 171 (1974); H. M. Hofmann, Y. Starkand, and M. W. Kirson, *ibid.* A266, 138 (1976).

[14]J. Shurpin, H. Müther, T. T. S. Kuo, and A. Faessler, Nucl. Phys. A293, 61 (1977).

[15]T. H. Schucan and H. A. Weidenmüller, Ann. Phys. (N.Y.) 76, 483 (1973); 73, 108 (1972); H. M. Hofmann, S. Y. Lee, J. Richert, and H. A. Weidenmüller, *ibid.* 85, 410 (1974).

[16]E. M. Krenciglowa and T. T. S. Kuo, Nucl. Phys. A235, 171 (1974)

[17]S. Pittel, Nucl. Phys. A271, 62 (1976).

[18]H. M. Hofmann, Y. Starkand, and M. W. Kirson, Nucl. Phys. A266, 138 (1976).

[19]S. F. Tsai and T. T. S. Kuo, Phys. Lett. 39B, 427 (1972); E. M. Krenciglowa, C. L. Kung, T. T. S. Kuo, and E. Osnes, Ann. Phys. (N.Y.) 101, 154 (1976).

[20]S. Nagata, H. Bandō, and Y. Akaishi, Prog. Theor. Phys. Suppl., No. 65, 10 (1979).

[21]E. M. Krenciglowa and H. Bandō, Nucl. Phys. A322, 145 (1979).

[22]H. G. Benson and B. H. Flowers, Nucl. Phys. A216, 305 (1969); A216, 332 (1969); P. J. Ellis and T. Engeland, *ibid.* A181, 368 (1972); A144, 161 (1970); J. B. McGrory and B. H. Wildenthal, Phys. Rev. C 7, 974 (1973); A. Watt, B. J. Cole, and R. R. Whitehead, Phys. Lett. 51B, 435 (1974); T. Motoba and K. Itonaga, Prog. Theor. Phys. Suppl., No. 65, 136 (1979).

[23]E. M. Krenciglowa, Ph.D. thesis, State University of New York at Stony Brook, 1975 (unpublished).

[24]E. M. Krenciglowa, H. Bandō, and K. Andō, Nucl. Phys. A301, 29 (1978).

[25]T. Sakuda, S. Nagata, and F. Nemoto, Prog. Theor. Phys. 56, 1126 (1976); 53, 1841 (1975); T. Sakuda, *ibid.* 57, 855 (1977); B. Buck, H. Friedrich, and A. A. Pilt, Nucl. Phys. A290, 205 (1977).