Theory of large angle p-nucleus scattering. I. pd elastic scattering and deuteron form factor at large q^2

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We derive a new approach to the proton-nucleus large angle scattering which accounts in a systematic way for Pauli, binding, and Fermi motion effects. We concentrate here on pd scattering. After antisymmetrization of incident and target protons, the pd amplitude can be separated into (1) the standard multiple scattering series with the last pp amplitude antisymmetrized and (2) a neutron exchange amplitude which includes rescattering terms. The optimal approximation designed to minimize corrections is derived for the antisymmetrized pd amplitude. The single scattering amplitude is factorized into an on-shell (antisymmetrized) pN amplitude and the deuteron form factor, and it is found to play a main role in the large angle pd scattering at sufficiently high energy. The results are applied to analysis of pd elastic scattering data ($T_p \gtrsim 300$ MeV) which are well reproduced by the calculations. This analysis also permits an extraction of the deuteron body form factor for values of q^2 which far exceed those measured in ed elastic data.

[NUCLEAR REACTIONS Proton-nucleus scattering. Pauli, binding, and Fermi motion effects. Antisymmetrization. Minimization of corrections. pd large angle scattering. Comparison with data. Deuteron form factor.]

I. INTRODUCTION

Our aim is the derivation of an approach to large angle proton-nucleus scattering which will account for Pauli, Fermi motion, and binding effects, and be simple enough for applications. In this paper we restrict ourselves to the case of pd large angle elastic scattering, although the results can be generalized to heavier nuclei. (This will be done in separate publications.)

We start with the derivation of the multiple scattering equations which include an antisymmetrization of the incident proton with the target one. This procedure is similar to that of Takeda and Watson.[1,2] However, we do not neglect the target exchange term[1,2]; this term is included in the properly written, exact final equations. These equations represent the total pd amplitude as a sum of two components. The first is a standard multiple scattering series. However, those multiple scattering terms which end with a pp amplitude contain the latter in antisymmetrized form. The second component is the target exchange amplitude which includes all projectile rescatterings.

In Sec. III we search for the optimal approximation for the formal exact solution of the problem. That is, we seek an approximation for the scattering amplitude (including binding and Fermi motion effects) such that the first order correction terms vanish.[3,4] The procedure differs from that derived in Refs. 3 and 4, since the exchange interaction complicates the problem. The expressions for neutron exchange obtained here have not been used before. However, the final result for multiple scattering series coincides with that of Refs. 3 and 4 where the last pp amplitude is taken to be antisymmetrized. In particular, the single scattering amplitude factors into deuteron body form factor and on-shell proton-nucleon amplitude for energy argument increasing with momentum transfer.[3]

Our result manifests the compensation of Fermi motion and binding effects. The difference from the usual treatments, which neglect binding potential effects [e.g., impulse approximation[5] (IA)] is pronounced in the large angle scattering region. The binding effects are very important there even for high energy scattering. It can be demonstrated in the following way.[6] Let us consider the single scattering amplitude for elastic pd scattering. It can be written as a ground state matrix element τ, the scattering operator of the projectile ($\tilde{p}$) on the target nucleon (p or n) which is bound by the other nucleon

$$F^{(1)}_{\tilde{p}d}(E,\vec{k}',\vec{k})=\langle\phi_d,\vec{k}'|\tau_{\tilde{p}p}(E)+\tau_{\tilde{p}n}(E)|\phi_d,\vec{k}\rangle\,. \qquad (1)$$

Here E is the total energy, $\vec{k}'$ and $\vec{k}$ are the momenta of projectile proton in initial and final states, and ϕ_d is deuterons wave function. The τ matrix satisfies

$$\tau_{\tilde{p}N}=V_{\tilde{p}N}+V_{\tilde{p}N}(E-K_p-K_n-V_{pn}-K_{\tilde{p}})^{-1}\tau_{\tilde{p}N}\,, \qquad (2)$$

where K is the kinetic energy of a nucleon and

V_{pn} is the pn interaction. The IA is correspondent to replacement of $\tau_{\bar{p}N}$ by $t^{(0)}_{\bar{p}N}$, a free scattering operator satisfying Eq. (2) with $V_{pn}=0$. The lowest order binding correction to the IA have been estimated in[2] to be

$$(\Delta F^{(1)}/F^{(1)})_{\rm IA} \propto \langle \phi_0, \vec{k}' | V_{pn}/E | \vec{k}, \phi_0 \rangle .$$

Invoking the Schrödinger equation one finds

$$(\Delta F^{(1)}/F^{(1)})_{\rm IA} \propto \langle \phi_0, \vec{k}' | (K_p + K_n)/E | \phi_0, \vec{k} \rangle .$$

The momentum due to Fermi motion is of order of magnitude of the momentum transfer $\vec{q}=\vec{k}-\vec{k}'$. Therefore in the backward scattering region $K_p+K_n \propto E$ and binding effects are very important.

Section IV is devoted to the application of our results to the analysis of large angle pd data. The peculiar feature of these data is the pronounced backward scattering peak. It has been studied in many papers, but the standard analysis (without the inclusion of additional degrees of freedom) did not go beyond the evaluation of the neutron pick-up reaction or of some Feynman diagrams. The binding effects have not been considered and the Pauli principle has been included only partially. Our results presented in Sec. IV are, we believe, the first systematic analysis of these data.

II. CONSEQUENCES OF THE PAULI PRINCIPLE FOR *pd* SCATTERING

Consider pd scattering in the frame where the total pd momentum equals $\vec{K}$, Fig. 1. Two protons, with total momentum $\vec{P}$, are denoted by indices "1" and "3," where the latter is the projectile. All derivations are done in the momentum representation.

The wave function ψ_3 of the system with projectile 3 to be asymptotically in a plane wave of momentum $\vec{k}$ is

$$\psi_3 = \chi_3\phi_{12} + G(V_{13}+V_{23})\chi_3\phi_{12} , \tag{3}$$

where $\chi_3=\delta(\vec{p}-\vec{k})$ and ϕ_{12} is the deuteron ground state wave function. Hence we get

$$\begin{aligned}\chi_3\phi_{12} &\equiv \delta(\vec{p}-\vec{k})\phi_d[\vec{P}-\tfrac{1}{2}(\vec{K}+\vec{p})]\\ &= \delta(\vec{p}-\vec{k})\phi_d[\vec{P}-\tfrac{1}{2}(\vec{K}+\vec{k})] .\end{aligned} \tag{4}$$

G is the total Green's function

$$G=\Big(E-\sum_{i=1}^{3}K_i-\sum_{i>j}V_{ij}\Big)^{-1} , \tag{5}$$

where K_i is the kinetic energy of nucleon i and V_{ij} is two nucleon potential. The total energy E is

$$E=\frac{k^2}{2m}+\frac{(\vec{K}-\vec{k})^2}{4m}+\epsilon_0 , \tag{6}$$

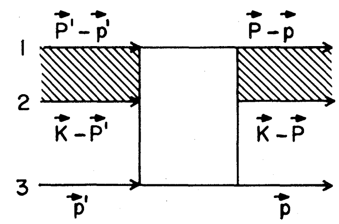

FIG. 1. Schematic representation of *pd* scattering. Target and projectile protons are denoted by indices "1" and "3," respectively.

where ϵ_0 is the deuteron binding energy:

$$(K_{12}^{\rm rel}+V_{12})\phi_d=\epsilon_0\phi_d . \tag{7}$$

The antisymmetrized wave function ψ can be written[1,2]

$$\begin{aligned}\psi=\frac{1}{\sqrt{2}}\{&(\chi_3\phi_{12}-\chi_1\phi_{23})\\ &+G[(V_{13}+V_{23})\chi_3\phi_{12}-(V_{12}+V_{13})\chi_1\phi_{23}]\} ,\end{aligned} \tag{8}$$

where $\chi_1=\delta(\vec{P}-\vec{p}-\vec{k})$ and

$$\begin{aligned}\chi_1\phi_{23} &\equiv \delta(\vec{P}-\vec{p}-\vec{k})\phi_d[\tfrac{1}{2}(\vec{P}-\vec{K}+\vec{p})]\\ &= \delta(\vec{P}-\vec{p}-\vec{k})\phi_d[\vec{P}-\tfrac{1}{2}(\vec{K}+\vec{k})] .\end{aligned} \tag{9}$$

In order to find the scattering amplitude into the final state $\chi_3'\phi_{12}$ we use the relation[1,2]

$$G=\tilde{G}+\tilde{G}(V_{13}+V_{23})G , \tag{10}$$

where

$$\tilde{G}=\Big(E-\sum_{i=1}^{3}K_i-V_{12}\Big)^{-1} . \tag{11}$$

With Eqs. (10) and (11) we rewrite (8) as

$$\begin{aligned}\psi=&\frac{1}{\sqrt{2}}(\chi_3\phi_{12}-\chi_1\phi_{23})+\frac{1}{\sqrt{2}}\tilde{G}(V_{23}-V_{12})\chi_1\phi_{23}\\ &+\tilde{G}(V_{13}+V_{23})\psi .\end{aligned} \tag{12}$$

The second term of Eq. (12) is the so-called target exchange term and it is neglected in the standard treatment.[1,2] We account for it by rewriting Eq. (12) accordingly.

First we write down the matrix elements of potentials V_{ij}:

$$\langle \vec{p}', \vec{P}' | V_{13} | \vec{p}, \vec{P}\rangle = V_{pp}(\vec{p}'-\vec{p})\delta(\vec{P}'-\vec{P}) , \tag{13a}$$

$$\langle \vec{p}', \vec{P}' | V_{23} | \vec{p}, \vec{P}\rangle = V_{pn}(\vec{P}'-\vec{P})\delta(\vec{p}'-\vec{P}'-\vec{p}+\vec{P}) , \tag{13b}$$

$$\langle \vec{p}', \vec{P}' | V_{12} | \vec{p}, \vec{P}\rangle = V_{pn}(\vec{P}'-\vec{P})\delta(\vec{p}'-\vec{p}) . \tag{13c}$$

Then introducing the "exchange" potentials $\tilde{V}_{ij}\equiv V_{ij}P_{13}$, where P_{13} is the permutation operator on particle labels "1" and "3" (hence $\tilde{V}_{ij}$ are very long range potentials):

$$\langle \vec{p}', \vec{P}'|\tilde{V}_{13}|\vec{p}, \vec{P}\rangle = V_{pp}(\vec{p}+\vec{p}'-\vec{P})\delta(\vec{P}-\vec{P}')\,, \quad (14a)$$

$$\langle \vec{p}', \vec{P}'|\tilde{V}_{23}|\vec{p}, \vec{P}\rangle = V_{pn}(\vec{P}'-\vec{P})\delta(\vec{p}'+\vec{p}-\vec{P}')\,, \quad (14b)$$

$$\langle \vec{p}', \vec{P}'|\tilde{V}_{12}|\vec{p}, \vec{P}\rangle = V_{pn}(\vec{P}'-\vec{P})\delta(\vec{p}'+\vec{p}-\vec{P})\,, \quad (14c)$$

one obtains, using Eqs. (4) and (9)

$$\begin{aligned}(V_{23}-V_{12})\chi_1\phi_{23} &\equiv (\tilde{V}_{23}-\tilde{V}_{12})\chi_3\phi_{12}\,,\\ (V_{13}+V_{23})\chi_1\phi_{23} &\equiv (\tilde{V}_{13}+\tilde{V}_{23})\chi_3\phi_{12}\,.\end{aligned} \quad (15)$$

We note that only the potential $\tilde{V}_{13}$ does conserve the total momentum of the pp pair and corresponds to the standard pp exchange potential. The potentials $\tilde{V}_{23}$ and $\tilde{V}_{12}$ do not conserve the total pair momentum. (One sees in the following that $\tilde{V}_{23}$ and $\tilde{V}_{12}$ generate the neutron pick-up processes.) Now we introduce the T matrix:

$$\frac{1}{\sqrt{2}}(\tilde{V}_{23}-\tilde{V}_{12})\chi_3\phi_{12} + (V_{13}+V_{23})\psi = T\chi_3\phi_{12}\,, \quad (16)$$

so that the elastic scattering amplitude to the state $\chi_3'\phi_{12}$ is $\langle\chi_3'\phi_{12}|T|\chi_3\phi_{12}\rangle$. From Eqs. (12) and (15) we find

$$T = \frac{1}{\sqrt{2}}(V_{13}+V_{23}) - \frac{1}{\sqrt{2}}(\tilde{V}_{12}+\tilde{V}_{13}) + (V_{13}+V_{23})\tilde{G}T\,. \quad (17)$$

One obtains after some algebra that the operator T can be divided into direct ($T^{\rm d}$) and exchange ($T^{\rm ex}$) parts:

$$T = \frac{1}{\sqrt{2}}(T^{\rm d} - T^{\rm ex})\,, \quad (18a)$$

$$T^{\rm d} = (V_{13}+V_{23}) + (V_{13}+V_{23})\tilde{G}T^{\rm d}\,, \quad (18b)$$

$$T^{\rm ex} = (\tilde{V}_{12}+\tilde{V}_{13}) + T^{\rm d}\tilde{G}(\tilde{V}_{12}+\tilde{V}_{13})\,. \quad (18c)$$

One sees from Eq. (18b) that $T^{\rm d}$ corresponds to the scattering of nonidentical nucleons and can be expressed through the multiple scattering series[2]:

$$T^{\rm d} = \tau_{31}^{\rm d} + \tau_{32}^{\rm d} + \tau_{31}^{\rm d}\tilde{G}\tau_{32}^{\rm d} + \tau_{32}^{\rm d}\tilde{G}\tau_{31}^{\rm d} + \tau_{31}^{\rm d}\tilde{G}\tau_{32}^{\rm d}\tilde{G}\tau_{31}^{\rm d} + \cdots\,, \quad (19)$$

where $\tau_{3i}^{\rm d} \equiv \tau_{pN}^{\rm d}$ is the proton 3-target nucleon i direct scattering matrix

$$\tau_{3i}^{\rm d} = V_{3i} + V_{3i}\tilde{G}\tau_{3i}^{\rm d}\,. \quad (20)$$

Next we write $T^{\rm ex} = T_p^{\rm ex} + T_n^{\rm ex}$, so that

$$T_p^{\rm ex} = \tilde{V}_{13} + T^{\rm d}\tilde{G}\tilde{V}_{13}\,, \quad (21a)$$

$$T_n^{\rm ex} = \tilde{V}_{12} + T^{\rm d}\tilde{G}\tilde{V}_{12}\,. \quad (21b)$$

When we substitute Eq. (19) into (21a) we find

$$T_p^{\rm ex} = \tau_{31}^{\rm ex} + \tau_{32}^{\rm d}\tilde{G}\tau_{31}^{\rm ex} + \tau_{31}^{\rm d}\tilde{G}\tau_{32}^{\rm d}\tilde{G}\tau_{31}^{\rm ex} + \cdots\,, \quad (22)$$

where $\tau_{31}^{\rm ex} \equiv \tau_{pp}^{\rm ex}$ is

$$\tau_{31}^{\rm ex} = \tilde{V}_{31} + \tau_{31}^{\rm d}\tilde{G}\tilde{V}_{31}\,. \quad (23)$$

Combining Eq. (22) with Eq. (18a) one gets finally

$$T = \frac{1}{\sqrt{2}}(T^t - T_n^{\rm ex})\,, \quad (24a)$$

where

$$T^t = \tau_{31} + \tau_{32}^{\rm d}\tilde{G}\tau_{31} + \tau_{31}^{\rm d}\tilde{G}\tau_{32}^{\rm d} + \tau_{31}^{\rm d}\tilde{G}\tau_{32}^{\rm d}\tilde{G}\tau_{31} + \cdots\,, \quad (24b)$$

and τ_{31} is the antisymmetrized pp scattering matrix

$$\tau_{31} = \tau_{31}^{\rm d} - \tau_{31}^{\rm ex}\,. \quad (24c)$$

$T_n^{\rm ex}$ is defined by Eq. (21b). One sees that Eqs. (24b) and (24c) define the scattering matrix T^t in the form of the standard multiple scattering series where the last pp scattering matrix is antisymmetrized. Eqs. (24a)–(24c) are the formal exact solution for pd scattering.

III. OPTIMAL APPROXIMATION FOR pd ELASTIC SCATTERING AMPLITUDE

In Eqs. (24) we find operators τ, which are the solution of the many-body scattering equations (20) and (23). Equations (24) also contain Green's function $\tilde{G}$ (Eq. 11), which includes the full target Hamiltonian. Hence, for the practical treatment we need an approximation for τ and $\tilde{G}$. For the direct amplitude $T^{\rm d}$ (Eq. 19) an optimal approximation has been derived in Refs. 3 and 4. We considered there each term $T^{(n)}$ of expansion (19) and we showed how best to choose the approximation for $\tilde{G}$ and τ. This procedure defines the approximation $T_a^{(n)}$, so that the first correction to $T^{(n)} - T_a^{(n)}$ vanishes for the elastic scattering.

The approximate amplitudes $T_a^{(n)}$ have been found to be factorized into a multinucleon density in momentum space and projectile target nucleons rescattering amplitude with energy argument dependent on momentum transfer. In particular, for pd scattering the (direct) single scattering amplitude, written in the pd Breit frame, factors into a deuteron body form factor and on-shell pN amplitude for the kinematics of Fig. 2 (i.e., as if the struck nucleon takes all momentum of

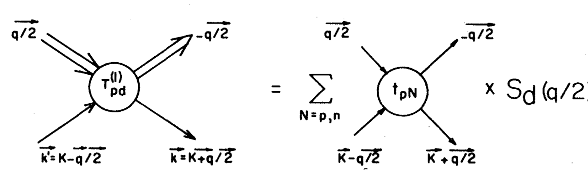

FIG. 2. The single scattering pd amplitude in optimal approximation. $\vec{k}'$ and $\vec{k}$ are projectile momenta in the pd Breit frame.

the deuteron[3]).

$$T^{(1)}_{pd}(E,\vec{k},\vec{k}')=[t_{pp}(E^{\rm eff},\vec{k},\vec{k}')+t_{pn}(E^{\rm eff},\vec{k},\vec{k}')]S_d(\tfrac{1}{2}q)\,, \qquad (25a)$$

where the total energies $(E, E^{\rm eff})$ for relativistic kinematics are

$$E=\left(m^2+K^2+\frac{q^2}{4}\right)^{1/2}+\left(m_d{}^2+\frac{q^2}{4}\right)^{1/2}, \qquad (25b)$$

$$E^{\rm eff}=\left(m^2+K^2+\frac{q^2}{4}\right)^{1/2}+\left(m^2+\frac{q^2}{4}\right)^{1/2}, \qquad (25c)$$

and $\vec{K}=\frac{1}{2}(\vec{k}+\vec{k}')$, $\vec{q}=\vec{k}-\vec{k}'$.

The first nonvanishing correction to the optimal approximation (OA) appear to be of order $(\Delta F^{(1)}/F^{(1)})_{\rm OA}\propto(p_0/k_{\tilde{p}})^2$ where p_0 and $k_{\tilde{p}}$ are, respectively, nucleon rms and projectile momenta and are thus much smaller than the correction to the impulse approximation $(\Delta F^{(1)}/F^{(1)})_{\rm IA}\propto V_{pn}/E$.

We derive now the optimal approximation for the exchange amplitudes, where the procedure used in Refs. 3 and 4 cannot be applied straightforwardly. (From this point we work in pd Breit frame only.) First we deal with the single scattering term Eq. (23). We rewrite this equation in terms of matrix elements

$$\langle\vec{p}',\vec{P}'|\tau^{\rm ex}_{31}|\vec{p},\vec{P}\rangle=\langle\vec{p}',\vec{P}'|\tilde{V}_{31}|\vec{p},\vec{P}\rangle+\int\langle\vec{p}',\vec{P}'|\tau^{\rm d}_{31}|\vec{p}_1,\vec{P}_1\rangle\langle\vec{p}_1,\vec{P}_1|\tilde{G}|\vec{p}_2,\vec{P}_2\rangle\langle\vec{p}_2,\vec{P}_2|\tilde{V}_{31}|\vec{p},\vec{P}\rangle \times d^3P_1\,d^3P_2\,d^3p_1\,d^3p_2\,. \qquad (26)$$

Consider the Born amplitude of Eq. (26) for elastic pd scattering. Using Eq. (14a) one obtains

$$\langle\phi_d,\vec{k}'|\tilde{V}_{31}|\phi_d,\vec{k}\rangle=\int\phi_d(\vec{P}-\vec{k}')V_{pp}(\vec{P}-\vec{k}-\vec{k}')\phi_d(\vec{P}-\vec{k})d^3P\,. \qquad (27)$$

We expand the potential V_{pp} around an optimal point which minimizes first order corrections. This point is $\vec{P}=\frac{1}{2}(\vec{k}+\vec{k}')=\vec{K}$. Indeed,

$$\langle\phi_d,\vec{k}'|\tilde{V}_{31}|\phi_d,\vec{k}\rangle=\int\phi_d\left(\vec{Q}+\frac{\vec{q}}{4}\right)[V_{pp}(\vec{K})+(\vec{Q}\,\vec{\nabla}_x)V_{pp}(\vec{x})|_{\vec{x}=\vec{K}}+\cdots]\phi_d\left(\vec{Q}-\frac{\vec{q}}{4}\right)d^3Q\,, \qquad (28)$$

where $\vec{P}=\vec{K}+\vec{Q}$, $\vec{q}=\vec{k}-\vec{k}'$.

Using the definite parity of ϕ_d, which implies

$$\phi_d(\vec{X}_1)\phi_d(\vec{X}_2)=\phi_d(-\vec{X}_1)\phi_d(-\vec{X}_2) \qquad (29)$$

we find that the second term of expansion (28) is zero after integration over d^3Q and

$$\langle\phi_d,\vec{k}'|\tilde{V}_{31}|\phi_d,\vec{k}\rangle\cong V_{pp}(\vec{K})S_d(\tfrac{1}{2}\vec{q})\,. \qquad (30)$$

The first nonvanishing correction from the third term of expansion (28) equals

$$\langle\phi_d,\vec{k}'|\delta^{(2)}\tilde{V}_{13}|\phi_d,\vec{k}\rangle=\tfrac{1}{6}[\nabla^2V(\vec{x})]|_{\vec{x}=\vec{K}}\int Q^2\phi_d(\vec{Q}+\tfrac{1}{4}\vec{q})\phi_d(\vec{Q}-\tfrac{1}{4}\vec{q})d^3Q\,. \qquad (31)$$

In the backward scattering region $\vec{K}\approx 0$. If the pp potential $V_{pp}(\tilde{Q})$ is rather smooth for small $\tilde{Q}$, this correction is a small one.

Now we consider the second term of Eq. (26). Similar to Refs. 3 and 4 we are looking for the approximation $\tilde{G}_a\cong\tilde{G}$ in the form

$$\langle\vec{p}_1,\vec{P}_1|\tilde{G}_a|\vec{p}_2,\vec{P}_2\rangle=\frac{\delta(\vec{p}_1-\vec{p}_2)\delta(\vec{P}_1-\vec{P}_2)}{E-p_1{}^2/2m-(\vec{K}-\vec{p}_1)^2/4m-\bar{\epsilon}(\vec{p}_1,\vec{K},q)}, \qquad (32)$$

where the quantity $\bar{\epsilon}$ is a function of the projectile momentum $\vec{p}_1$ and external parameters $\vec{K}$ and q and is independent of the total projectile-nucleon momenta $\vec{P}_1$ $(\vec{P}_2)$.

The approximation for the direct scattering matrix $t^{\rm d}_{31}\cong\tau^{\rm d}_{31}$ is defined by Eq. (20) with $\tilde{G}\to\tilde{G}'_a$:

$$t^{\rm d}_{31}=V_{31}+V_{31}\tilde{G}'_at^{\rm d}_{31}\,, \qquad (33)$$

where G'_a is taken in the form (32) with $\bar{\epsilon}=\bar{\epsilon}'$. The approximation $\tilde{V}_a\cong\tilde{V}_{13}$ is the zero order term of expansion V_{pp} $(\vec{P}-\vec{p}_2-\vec{p})$ around some optimal point $\vec{P}=\vec{K}_0$. The quantities $\bar{\epsilon}$, $\bar{\epsilon}'$, and $\vec{K}_0$ should be found from the condition that the first order correction to the approximation

$$\langle\phi_d,\vec{k}'|t^{\rm d}_{31}\tilde{G}_a\tilde{V}_a|\phi_d,\vec{k}\rangle\cong\langle\phi_d,\vec{k}'|\tau^{\rm d}_{31}\tilde{G}\tilde{V}_{31}|\vec{k},\phi_d\rangle$$

is zero. It can be written as

$$\Delta_1=\langle\phi_d,\vec{k}'|\delta_1\tau^{\rm d}_{13}\tilde{G}_a\tilde{V}_a+t^{\rm d}_{13}\delta_1\tilde{G}\tilde{V}_a+t^{\rm d}_{13}\tilde{G}_a\delta_1\tilde{V}_{13}|\vec{k},\phi_d\rangle=0\,, \qquad (34a)$$

where $\delta_1\tau^{\rm d}_{13}$, $\delta_1\tilde{G}$ are the first order terms of expansion $\tau^{\rm d}_{13}$ and $\tilde{G}$ in terms of $t^{\rm d}_{13}$ and $\tilde{G}_a$. As we found in[3,4]

$$\delta_1\tau^{\rm d}_{13}=t^{\rm d}_{13}\tilde{G}'_a(\tilde{G}'^{-1}_a-\tilde{G}^{-1})\tilde{G}'_at^{\rm d}_{13} \qquad (34b)$$

and

$$\delta_1\tilde{G}=\tilde{G}_a(\tilde{G}_a{}^{-1}-\tilde{G}^{-1})\tilde{G}_a\,. \tag{34c}$$

For $\delta_1\tilde{V}_{13}$ one gets

$$\delta_1\tilde{V}_{13}=(\vec{P}-\vec{K}_0)\vec{\nabla}_x V_{pp}(\vec{X})|_{\vec{X}=\vec{K}_0}\,. \tag{34d}$$

With[3,4]

$$\langle\vec{p}',\vec{P}'|t^{d}_{13}|\vec{p},\vec{P}\rangle=\langle\vec{p}'|\hat{t}^{\,d}_{13}|\vec{p}\rangle\delta(\vec{P}-\vec{P}')\,, \tag{35}$$

and the diagonality of $\tilde{G}$ in projectile momentum space the first term of Eq. (34a) reads

$$\Delta_1'=\int\phi_d\left(\vec{P}'-\vec{K}+\frac{\vec{q}}{4}\right)\langle\vec{k}'|\hat{t}_{31}|\vec{p}_2\rangle\frac{\langle\vec{P}',\vec{p}_2|\tilde{G}_a'^{-1}-\tilde{G}^{-1}|\vec{p}_2,\vec{P}\rangle}{\left(E-\frac{p_2{}^2}{2m}-\frac{(\vec{K}-\vec{p}_2)^2}{4m}-\bar{\epsilon}'(\vec{p}_2,\vec{K},q)\right)^2}\langle\vec{p}_2|\hat{t}_{31}|\vec{p}_1\rangle$$
$$\times\frac{1}{E-\frac{p_1{}^2}{2m}-\frac{(\vec{K}-\vec{p}_1)^2}{4m}-\bar{\epsilon}(\vec{p}_1,\vec{K},q)}\tilde{V}(\vec{K}_0-\vec{p}_1-\vec{k})\phi_d\left(\vec{P}-\vec{K}-\frac{\vec{q}}{4}\right)d^3P\,d^3P'\,d^3p_1\,d^3p_2\,, \tag{36a}$$

Here

$$\langle\vec{P}',\vec{p}_2|\tilde{G}_a'^{-1}-\tilde{G}^{-1}|\vec{p}_2,\vec{P}\rangle=\left[\left(\vec{P}-\frac{\vec{K}+\vec{p}_2}{2}\right)^2\frac{1}{m}-\bar{\epsilon}'(\vec{p}_2,\vec{K},q)\right]\delta(\vec{P}'-\vec{P})+V_{pn}(\vec{P}'-\vec{P}) \tag{36b}$$

and $\hat{t}_{31}$ does not depend on $\vec{P}(\vec{P}')$.[3]

Using the Schrödinger equation (8) for elimination of potential V_{pn} in integral (36) we obtain

$$\int V_{pn}(\vec{P}'-\vec{P})\phi_d(\vec{P}'-\vec{K}+\vec{q}/4)d^3P'$$
$$=[\epsilon_0-(\vec{P}-\vec{K}+\vec{q}/4)^2/m]\phi_d(\vec{P}-\vec{K}+\vec{q}/4)\,, \tag{37}$$

and with the definite parity of ϕ_d [Eq. (29)] we find that $\Delta_1'=0$, if

$$\bar{\epsilon}'(\vec{p}_2,\vec{K},q)=\epsilon_0+\frac{(\vec{K}-\vec{p}_2)^2}{4m}-\frac{q^2}{16m}\,. \tag{38}$$

This quantity defines the Green's function of Eq. (33). From an analysis of Eq. (33) one can show[3,4] that $\langle\vec{p}'|t^{d}_{13}|\vec{p}\rangle$ is the half-off-shell (direct) pp scattering matrix $t^{d}_{pp}(E^{\rm eff},\vec{K},\vec{k},\vec{p}_1)$ corresponding to kinematics $[\vec{K}-\vec{k}']+[\vec{k}']\to[\vec{K}-\vec{p}_1]+[\vec{p}_1]$, where $\vec{K}=(\vec{k}+\vec{k}')/2$ is total pp momentum and total energy $E^{\rm eff}$ equals

$$E^{\rm eff}=\frac{k'^2}{2m}+\frac{(\vec{K}-\vec{k}')^2}{2m}\,. \tag{39}$$

Applying similar algebra one can find the quantity $\bar{\epsilon}$ (Eq. 32) from the vanishing of the second term of Eq. (34a). It appears to be of the same form as $\bar{\epsilon}'$ (Eq. 38) with $\vec{p}_2\to\vec{p}_1$. The vanishing of the last term of Eq. (35) gives the parameter $\vec{K}_0=\vec{K}$.

We have found thus the optimal approximation for the exchange amplitude $t^{\rm ex}_{31}\cong\tau^{\rm ex}_{31}$ of Eq. (26). Using Eqs. (30), (32), (38), and (39) this amplitude reads for the elastic pd scattering

$$\langle\phi_d,\vec{k}'|t^{\rm ex}_{31}|\phi_d,\vec{k}\rangle=\left[V_{pp}(\vec{K})+\int t^{d}_{pp}(E^{\rm eff},\vec{K},\vec{k}',\vec{p}_1)\frac{d^3p_1}{E^{\rm eff}-\frac{p_1{}^2}{2m}-\frac{(\vec{K}-\vec{p}_1)^2}{2m}}V_{pp}(\vec{K}-\vec{p}_1-\vec{k})\right]S_d\left(\frac{\vec{q}}{2}\right)\,. \tag{40}$$

One can easily see that this expression equals $t^{\rm ex}_{pp}(E^{\rm eff},\vec{K},\vec{k},\vec{k}')S_d(\vec{q}/2)$, where $t^{\rm ex}_{pp}$ is on-shell exchange pp scattering matrix for Breit kinematics.

Combining Eq. (40) with Eq. (25a) for the direct single scattering amplitude one finds that the inclusion of the Pauli principle results only in the on-shell pp scattering matrix in Eq. (25a) being replaced by an antisymmetrized one.

In the same way we derive the optimal approximation for the multiple scattering amplitudes including proton exchange [Eqs. (22) (24b)]. The final expressions coincide with those derived in Ref. 4 with the last pp t matrix in the corresponding multiple scattering terms antisymmetrized. The expressions for the first nonvanishing correction terms are similar to those in Refs. 3 and 4, but they also include the terms of type Eq. (31).

Consider now the part of the pd amplitude corresponding to neutron exchange, Eq. (21b). The

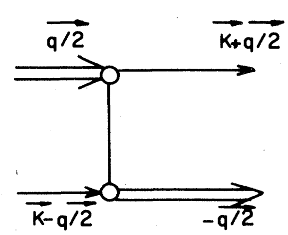

FIG. 3. Neutron exchange amplitude in Born approximation.

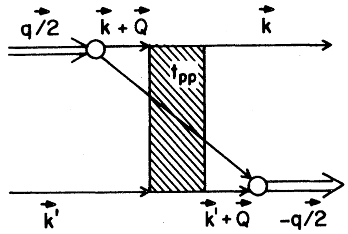

FIG. 4. Schematic representation of neutron exchange process with rescattering on proton, calculated in the optimal approximation.

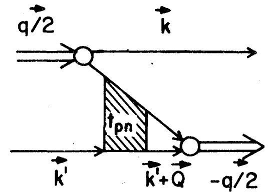

FIG. 5. The same as Fig. 4 with rescattering on the neutron.

first term for the elastic scattering reads

$$\langle \phi_d, \vec{k}' | \tilde{V}_{12} | \phi_d, \vec{k}\rangle = \int d^3P\, d^3P'\, \phi_d(\vec{P}' - \vec{K} + \vec{q}/4) \times \langle \vec{P}', \vec{k}' | \tilde{V}_{12} | \vec{P}, \vec{k}\rangle \phi_d(\vec{P} - \vec{K} - \vec{q}/4)\,, \tag{41}$$

where $\tilde{V}_{12}$ is defined by Eq. (14c). Using Eq. (37) to eliminate potential V_{pn} one easily obtains that Eq. (41) gives the standard neutron pick-up amplitude in Born approximation (Fig. 3):

$$\langle \phi_d, \vec{k}' | \tilde{V}_{12} | \phi_d, \vec{k}\rangle = \left(\epsilon_0 - (K^2 + q^2/16)\frac{1}{m}\right) \times \phi_d(\vec{K} - \vec{q}/4)\phi_d(\vec{K} + \vec{q}/4)\,. \tag{42}$$

For the remaining rescattering terms of Eq. (21b) we develop again an optimal approximation. (The derivation may be found in the Appendix.) It gives for the first rescattering term

$$T_n^{(1)\mathrm{ex}}(E, \vec{k}, \vec{k}') = \langle \phi_d, \vec{k}' | \tau_{31}^d G \tilde{V}_{12} | \vec{k}, \phi_d\rangle$$

the following result:

$$T_n^{(1)\mathrm{ex}} \cong \langle \phi_d, \vec{k}' | t_{31}^d \tilde{G}_a \tilde{V}_{12} | \vec{k}, \phi_d\rangle = \int \phi_d(\vec{K} + \vec{q}/4 + \vec{Q}) t_{pp}(E_{pp}, \vec{k}', \vec{k}' + \vec{Q}) \frac{1}{\dfrac{(\vec{k}+\vec{Q})^2}{2m} - \dfrac{(\vec{k}'+\vec{Q})^2}{2m}} \times \left(\epsilon_0 - \frac{(\vec{K} + \vec{q}/4 + \vec{Q})^2}{m}\right)\phi_d(\vec{K} - \vec{q}/4 + \vec{Q}) d^3Q\,, \tag{43}$$

where the amplitude t_{pp} corresponds to kinematics $[\vec{k}'] + [\vec{k} + \vec{Q}] \to [\vec{k}' + \vec{Q}] + [\vec{k}]$ for the total energy argument

$$E_{pp} = \frac{k'^2}{2m} + \frac{(\vec{k} + \vec{Q})^2}{2m}\,. \tag{44}$$

Expression (43) can be represented schematically by Fig. 4, where the effective energy of the pp amplitude and propagator are calculated as if the target proton were on mass shell (cf. Ref. 4).

In the same way one can show that the third term of Eqs. (19) and (21b), which corresponds to the rescattering on the neutron, can be represented in the optimal approximation by Fig. 5, which gives

$$\langle \phi_d, \vec{k}' | \tau_{32} \tilde{G} \tilde{V}_{12} | \phi_d, \vec{k}\rangle \cong \langle \phi_d, \vec{k}' | t_{32} \tilde{G}_a \tilde{V}_{12} | \phi_d, \vec{k}\rangle$$

$$= \phi_d(\vec{K} + \vec{q}/4)\left(\epsilon_0 - \frac{K^2 + \dfrac{q^2}{16}}{m}\right) \int \frac{t_{pn}\left(E_{pn} = \dfrac{K^2}{2m} + \dfrac{k^2}{2m}, \vec{k}', \vec{k}' + \vec{Q}\right)}{\dfrac{k'^2}{2m} + \dfrac{K^2}{2m} - \dfrac{(\vec{k}'+\vec{Q})^2}{2m} - \dfrac{(\vec{K}+\vec{Q})^2}{2m}} \phi_0(\vec{K} - \vec{q}/4 + \vec{Q}) d^3Q\,. \tag{45}$$

We mention that there is no overcounting in the case of Fig. 5, since this is only a schematic representation for the approximation of amplitude $\tau_{32}\tilde{G}\tilde{V}_{12}$. The latter is clearly free of overcounting.

The evaluation of higher order terms of expansions (19) and (21b) in the optimal approximation can be shown to be reduced to the calculation of diagrams where the struck target nucleon is taken to be on-shell (cf. Ref. 4). For example, consider the next two terms of expansion (19) and (21b) [Fig. 6(a), (b)]. We find that the amplitude corresponding to Fig. 6(a) equals

$$\langle \phi_d, \vec{k}' | t_{31}\tilde{G}_a t_{32}\tilde{G}_a \tilde{V}_{12} | \phi_d, \vec{k}\rangle = \int \phi_d(\vec{K}+\vec{q}/4+\vec{Q}_1) t_{pp}(E_{pp}^{(1)}, \vec{k}', \vec{k}'+\vec{Q}_1) \times \frac{t_{pn}(E_{pn}^{(2)}, \vec{k}'+\vec{Q}_1, \vec{k}'+\vec{Q}_2)\left(\epsilon_0 - (\vec{K}+\vec{q}/4+\vec{Q}_1)^2 \frac{1}{m}\right)}{\left(E_{pp}^{(1)} - \frac{(\vec{k}'+\vec{Q}_1)^2}{2m} - \frac{k^2}{2m}\right)\left(E_{pn}^{(2)} - \frac{(\vec{k}'+\vec{Q}_2)^2}{2m} - \frac{(\vec{K}+\vec{Q}_2)^2}{2m}\right)} \phi_d(\vec{K}-\vec{q}/4+\vec{Q}_2) d^3Q_1 d^3Q_2 \tag{46}$$

and the one corresponding to Fig. 6(b) is

$$\langle \phi_d, \vec{k}' | t_{32}\tilde{G}_a t_{31}\tilde{G}_a \tilde{V}_{12} | \phi_d, \vec{k}\rangle = \int \phi_d(\vec{K}+\vec{q}/4+\vec{Q}_1) t_{pn}(E_{pn}^{(1)}\vec{k}', \vec{k}'+\vec{Q}_1) \frac{t_{pp}(E_{pp}^{(2)}, \vec{k}'+\vec{Q}_1, \vec{k}'+\vec{Q}_2)}{\left(E_{pn}^{(1)} - \frac{(\vec{k}'+\vec{Q}_1)^2}{2m} - \frac{(\vec{K}+\vec{Q}_2)^2}{2m}\right)} \times \frac{\epsilon_0 - \frac{\left(K+\frac{q}{4}+Q_2-Q_1\right)^2}{m}}{\left(E_{pp}^{(2)} - \frac{(\vec{k}'+\vec{Q}_2)^2}{2m} - \frac{(\vec{K}+\vec{Q}_2)^2}{2m}\right)} \phi_d(\vec{K}-\vec{q}/4+\vec{Q}_2) d^3Q_1 d^3Q_2 , \tag{47}$$

where $E_{pN}^{(1),(2)}$ are total pN energies after the first and second rescatterings:

$$E_{pp}^{(1)} = \frac{(\vec{k}+\vec{Q}_1)^2}{2m} + \frac{k'^2}{2m} ,$$
$$E_{pn}^{(2)} = E_{pp}^{(1)} - \frac{k^2}{2m} + \frac{(\vec{K}+\vec{Q}_1)^2}{2m} ,$$
$$E_{pn}^{(1)} = \frac{(\vec{K}+\vec{Q}_1+\vec{Q}_2)^2}{2m} + \frac{k'^2}{2m} , \tag{48}$$
$$E_{pp}^{(2)} = E_{pn}^{(1)} - \frac{(\vec{K}+\vec{Q}_2)^2}{2m} + \frac{(\vec{k}+\vec{Q}_2-\vec{Q}_1)^2}{2m} .$$

Our results may be extended to the relativistic case if one uses the relativistic propagators and reduces them to the form similar to the non-relativistic one (cf. Ref. 7).

IV. ANALYSIS OF pd LARGE ANGLE SCATTERING DATA

The two-body scattering amplitude $F_{1,2\to 1,2}$ is connected with the corresponding scattering matrix $T_{1,2\to 1,2}$ through the relation

$$F = -4\pi^2 \frac{m_1 m_2}{m_1+m_2} T , \tag{49}$$

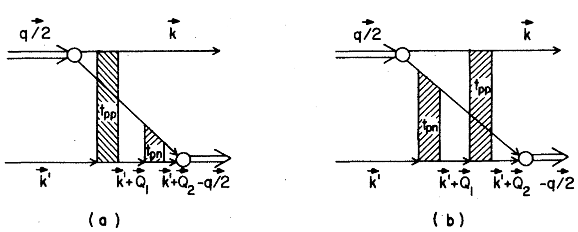

FIG. 6(a), (b). The same as Fig. 4 with rescattering on two nucleons.

where m_1 and m_2 are the scattering masses and F is normalized as

$$\frac{d\sigma}{d\Omega_{\text{c.m.}}} = |F|^2 . \tag{50}$$

Consider the single scattering amplitude Eq. (25a). At forward and therefore also backward angles for sufficiently large energies [$k_L^{\text{eff}} \gtrsim 2.6$ GeV/c in Eqs. (25a)–(25c), which correspond to $k_L \gtrsim 1.7$ GeV/c, or $T_p \gtrsim 1$ GeV] the scalar part of the pp amplitude is dominant, although at lower energies the influence of spin dependent parts in the amplitude is rather important.[8] Assuming also the dominance of the scalar part in the pn amplitude for backward scattering and using Eqs. (25a), (49), and (50) one obtains the following expression for the unpolarized pd cross section (given by single scattering amplitude[9])

$$\frac{d\sigma_{pd}^{(1)}(k_L, q^2)}{dq^2} = \left[\frac{i+\rho_{pp}}{(1+\rho_{pp}^2)^{1/2}}\left(\frac{d\sigma_{pp}(k_L^{\text{eff}}, q^2)}{dq^2}\right)^{1/2} + \frac{i+\rho_{pn}}{(1+\rho_{pn}^2)^{1/2}}\left(\frac{d\sigma_{pn}(k_L^{\text{eff}}, q^2)}{dq^2}\right)^{1/2}\right]^2 \times \left(\frac{k_L^{\text{eff}}}{k_L}\right)^2 \left[S_s^2\left(\frac{q^2}{4}\right) + S_Q^2\left(\frac{q^2}{4}\right)\right], \tag{51}$$

where k_L, k_L^{eff} are lab momenta corresponding to total energies of Eqs. (25b) and (25c), $\rho_{pN} = \text{Re} f_{pN}/\text{Im} f_{pN}$. $S_s \equiv S_s(\frac{1}{4}q^2)$ and $S_Q \equiv S_Q(\frac{1}{4}q^2)$ are scalar and quadrupole deuteron body form factors. One can easily check that Eq. (51) holds when one uses relativistic normalization of amplitudes.[7] From (25b) and (25c) one sees that effective energy E^{eff} increases with momentum transfer q provided the maximum $q_{\max}^2$ for pd backward scattering is equal to $q_{\max}^2$ for pN backward scattering. Since the pN on-shell amplitude

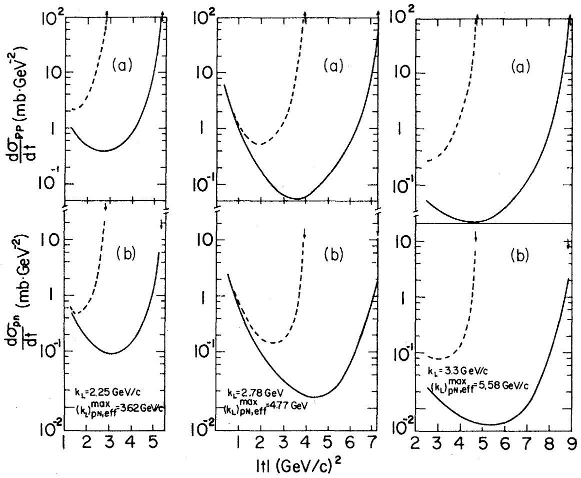

FIG. 7. $d\sigma_{pN}(k_L^{eff}, q^2)/dq^2$ (solid line) and $d\sigma_{pN}(k_L, q^2)/dq^2$ (dashed line) for $T_p = 1.5$, 2.08, 2.5 GeV. Arrows indicate point $\theta_{c.m.} = 180°$.

increases rapidly with q^2 towards $\theta_{c.m.} = 180°$, we expect that in spite of the falling form factor S_d the amplitude (25a) increases, thus giving the important contribution to the backward scattering cross section. In order to provide the information required for application of Eq. (51) we have used an interpolation procedure[9] of existing pN data.[10,11] As an example we show on Fig. 7 the differential cross sections $d\sigma_{pN}(k_L^{eff}, q^2)/dq^2$ (solid line) compare to $d\sigma_{pN}(k_L, q^2)/dq^2$ (dashed line) for three different energies ($T_p = 1.5$, 2.08, 2.5 GeV).

The cross section $d\sigma_{pN}(k_L^{eff}, q^2)/dq^2$ can be found analytically using Gaussian parametrization for pN amplitudes:

$$f_{pN}(k_L, q^2) = \frac{k_{c.m.}\sigma_{pN}^{tot}}{4\pi}(i + \rho_{pN})\exp(-\tfrac{1}{2}\beta_{pN}{}^2 q^2)\,. \quad (52)$$

One thus obtains

$$\frac{d\sigma_{pN}(k_L^{eff}, q^2)}{dq^2} = \left.\frac{d\sigma_{pN}}{dq^2}\right|_{\theta_{c.m.}=180°} \exp[-\beta^2(q_{max}{}^2 - q^2)]\,, \quad (53a)$$

where

$$q_{max}{}^2 - q^2 = 2\left(m^2 + \frac{q^2}{4}\right)^{1/2} \times\left[\left(K^2 + m^2 + \frac{q^2}{4}\right)^{1/2} - \left(m^2 + \frac{q^2}{4}\right)^{1/2}\right] \cong K^2\,, \quad (53b)$$

and $q = q_{max}$ corresponds to $\theta_{c.m.} = 180°$. The backward pp cross section equals

$$\left.\frac{d\sigma_{pp}}{dq^2}\right|_{\theta_{c.m.}=180°} = \left.\frac{d\sigma}{dq^2}\right|_{\theta_{c.m.}=0°} = \frac{\sigma_{pp}^2(1 + \rho_{pp}{}^2)}{16\pi}\,.$$

Data[11] show that $d\sigma_{pn}/dq^2|_{\theta_{c.m.}=180°}$ fall rapidly as a function of energy and $d\sigma_{pn}(k_L^{eff}, q^2)/dq^2$ hardly influences Eq. (51) for $k_L \gtrsim 1.7$ GeV/c ($k_L^{eff} \gtrsim 2.6$ GeV/c).

Now consider the multiple scattering terms of amplitude T^t [Eq. (24b)]. We mentioned above that the optimal approximation (including Pauli principle) leads to the same expressions, which have been derived in Ref. 4. From analysis of these expressions one finds that the eikonal approximation[12] can be applied for calculation of these terms (in the same way as has been done for fixed scatterers case[13-15]). It results in that only the second and the third order multiple scattering amplitudes (with eikonalized propagators) are important in the pd large angle cross sections,[14] Fig. 8.

Our eikonalized expressions for these amplitudes are similar to those of the fixed scatterers case. However, they contain on-shell pp, pn amplitudes f_{pN} ($E^{eff}, \vec{p}_1, \vec{p}_2$) which correspond to the kinematics[4]

$$[\vec{p}_1] + \left[\frac{\vec{p}_2 - \vec{p}_1}{2}\right] \to [\vec{p}_2] + \left[\frac{\vec{p}_1 - \vec{p}_2}{2}\right] \quad (54)$$

and the last pp amplitude in Figs. 8(a), 8(d), and 8(f) is antisymmetrized. $\vec{k}'$ ($\vec{k}$) is the proton's initial (final) momentum in pd Breit frame. Since pp, pn amplitudes are strongly peaked in the forward (backward) direction and since the backward pn amplitude is small (for sufficiently high energies), the large angle pd scattering occurs essentially when the proton scatters backward on the target proton and forward on the neutron. Since only the antisymmetrized pp amplitude is large for backward scattering, the process shown in Fig. 8(a) contributes mainly in the cross section.

Thus using Eqs. (49) and (52)–(64) and the expression of Ref. 4 for the double scattering term one obtains in the eikonal approximation

$$F_{S,Q}^{(2)}(E, \vec{k}, \vec{k}') = \frac{k_L^{eff} k_{c.m.}^{(pd)}}{k_{Br}} \frac{\sigma_{pp}^{tot}\sigma_{pn}^{tot}}{(4\pi)^3}(i + \rho_{pp})(i + \rho_{pn}) \times \int e^{-(\beta^2/2)Q_1{}^2} e^{-(\beta^2/2)(\vec{K} + \vec{Q}_1/2)^2} \times S_{S,Q}[k_{Br}{}^2 + (\vec{Q}_1 + \vec{K})^2] d^2Q_1\,, \quad (55)$$

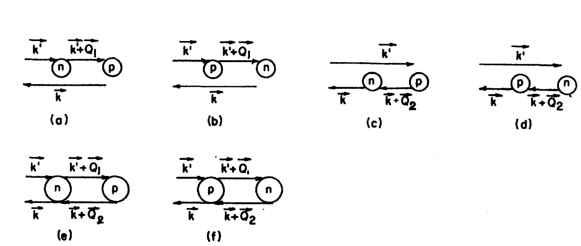

FIG. 8. Schematic representation of second and third order scattering processes for pd backward scattering.

where $F^{(2)}_{S,Q}$ is the double scattering amplitude calculated with the scalar or quadrupole component of the deuteron body form factor. This expression also holds if one takes the relativistic normalization of pN amplitude and uses the reduced relativistic propagator (as in Ref. 7).

The pd cross section given by single and double scattering terms reads

$$\frac{d\sigma_{pd}}{d\Omega_{\text{c.m.}}} = 2\left|F'^{(1)} + F^{(2)}_S - \frac{1}{4\sqrt{2}}F^{(2)}_Q\right|^2 + \left|F''^{(1)} + F^{(2)}_S - \frac{1}{2\sqrt{2}}F_Q\right|^2 + \tfrac{9}{16}|F^{(2)}_Q|^2, \qquad (56a)$$

where

$$F'^{(1)} = k_L^{\text{eff}}\frac{k^{(pd)}_{\text{c.m.}}}{k_L}[f_{pp}(E^{\text{eff}},q^2) + f_{pn}(E^{\text{eff}},q^2)] \times\left[S_S(q^2/4) - \frac{1}{\sqrt{2}}S_Q(q^2/4)\right] \qquad (56b)$$

and $F''^{(1)}$ equals the same expression with $(-1/\sqrt{2}S_Q \to \sqrt{2}S_Q)$.

Consider the neutron exchange amplitude Eq. (21b). The Born term has been calculated with the same normalization factors as in Ref. 16. We evaluated rescattering terms again in the eikonal approximation. In that case only single and double scattering amplitudes (Figs. 4–6) contribute effectively. The calculations have been done using parametrization (52) for pN amplitude. We obtained that the rescatterings reduce the Born exchange amplitude approximately by the same factor as has been found earlier in the fixed scatterers case.[17]

The results of our calculations are shown in Figs. 9–11. The dashed line corresponds to the contribution from the single scattering [Eq. (51)]. The dashed-dotted line shows the contribution from the single and double scattering [Eq. (56)] and the solid one corresponds to full result when the neutron exchange is added. The data are taken from Refs. 18–24.

For the calculation of the single scattering cross section we need information about $S_S{}^2+S_Q{}^2$ for high momentum transfer. Up to $q^2 \lesssim 4$ GeV/c^2 we may extract this quantity from ed elastic scattering measurements,[25] taking the neutron electric form factor to be zero and the proton electric form factor from Ref. 26. For larger values of q^2 we postulate a form of $S_S{}^2+S_Q{}^2$ in order to fit the data. The square of the deuteron body form factors $S_S{}^2+S_Q{}^2$ is presented in Fig. 12 (solid line),

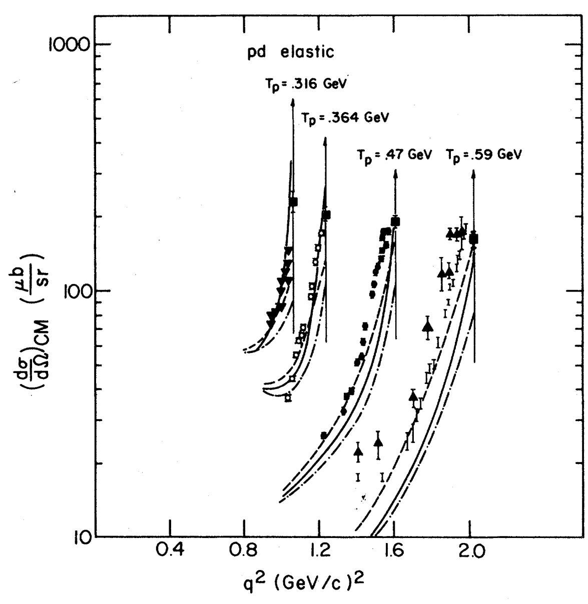

FIG. 9. pd large angle scattering data and our prediction for T_p = 0.316, 0.364, 0.47, 0.59 GeV scattering. Data ▪ (Ref. 18), ▲ (Ref. 20), I for T_p = 0.59 GeV, ● for T_p = 0.47 GeV, ☐ for T_p = 0.364 GeV, ▼ for T_p = 0.316 GeV (Ref. 19). Legends: (---) single scattering (Eq. 51); (– • – • –) single + double scattering (Eq. 56); (——) single + double + n exchange with rescattering corrections.

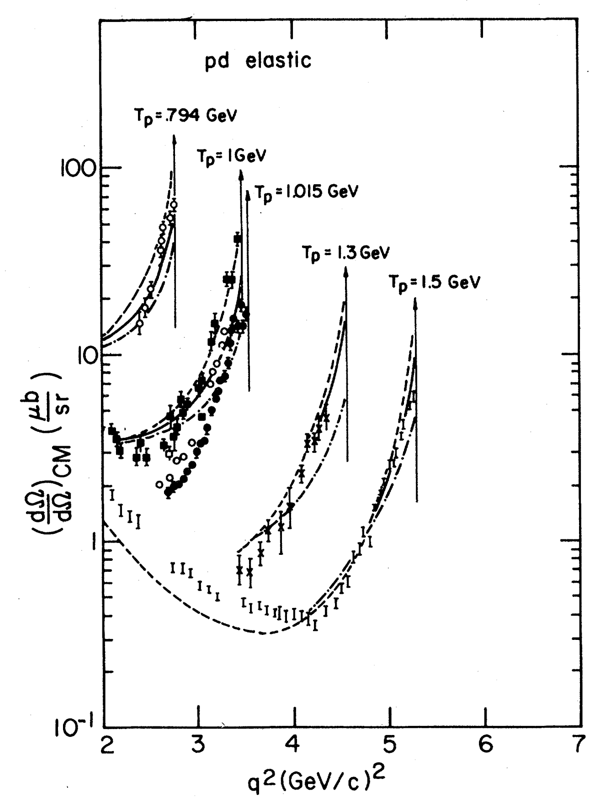

FIG. 10. $d\sigma_{pd}/d\Omega_{c.m.}$ for T_p = 0.794, 1, 1.015, 1.3, 1.5 GeV. Data ϕ (Ref. 21); ▪ (Ref. 22); ○ for T_p = 1 GeV, ✗ for T_p = 1.3 GeV (Ref. 23); ● for T_p = 1.015 GeV, I for T = 1.5 GeV (Ref. 24). Legends: as in Fig. 9.

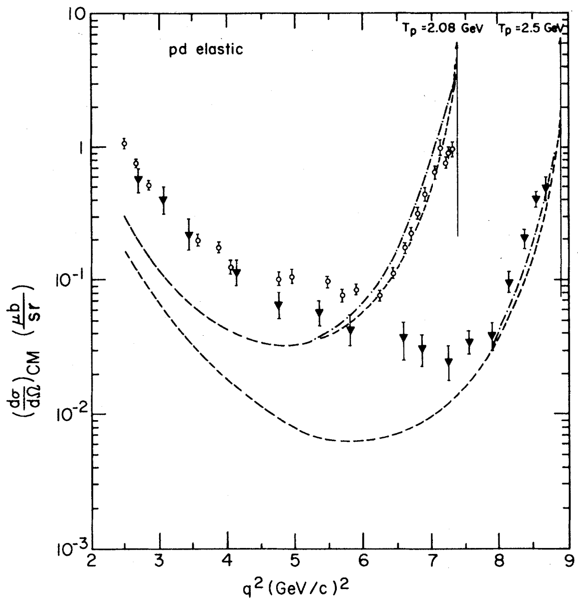

FIG. 11. $d\sigma_{pd}/d\Omega_{c.m.}$ for $T_p = 2.08$, 2.5 GeV. Data ♁ for $T_p = 2.08$ GeV, ▼ for $T_p = 2.5$ GeV (Ref. 24). Legends: as in Fig. 9.

where we show also, for the comparison, the result of formal calculation of $S_S{}^2 + S_Q{}^2$ using Humberston-Wallace deuteron wave functions[27] (dashed line). In order to calculate the contribution of the double scattering amplitude in pd cross section [Eqs. (55), (56)], we need information on S_S and S_Q separately. We show in Fig. 13 one possible extrapolation of these form factors (solid

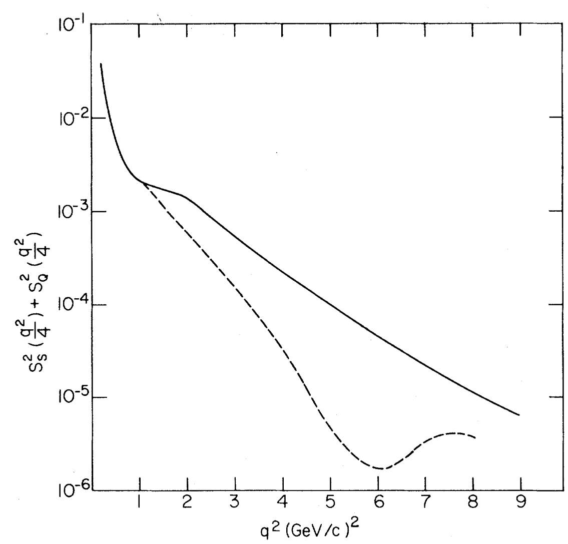

FIG. 12. Square of deuteron body form factors $S_S^2 + S_Q^2$. Solid curve corresponds to the measurement[25] [up to $q^2 \leq 4$ (GeV/c)^2] and to our predictions for $q^2 > 4$ (GeV/c)^2. Dashed curves give the calculation with Humberston-Wallace wave functions.

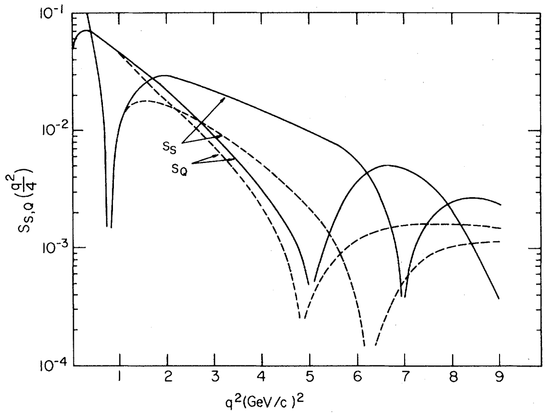

FIG. 13. Scalar and quadrupole form factors S_S, S_Q with legend as in Fig. 12.

line) in agreement with $S_S{}^2 + S_Q{}^2$ of Fig. 12. The dashed line shows the calculations with Humberston-Wallace wave functions. We examined the other possible extrapolations and found that the calculated pd cross section is rather stable.

The parameters of the pN amplitudes [Eq. (52)] used in the calculations were taken from Ref. 10. The magnitude of the pd cross section is insensitive to the uncertainties in these parameters.

Consider first the results shown in Fig. 9. The magnitude of the cross section for $T_p = 470$, 590 MeV seems not to be reproduced by the calculations. However, new accurate measurements of Bonner *et al.*[18] of nd scattering at 180° for neutron energies from 200 to 800 MeV show a smaller magnitude of the backward cross section than the measurements of Adler *et al.*[19] and Boschitz *et al.*[20] Therefore the latter should probably be reduced by an overall normalization factor (0.6–0.7) and then our calculations of angular distribution will reproduce the data.

Concerning the calculation for $T_p = 316$, 364 MeV one should be aware that for backward scattering the effective energy in pN amplitudes $T_p^{\text{eff}} \cong 500$ MeV. For this energy region the application of eikonal approximation can be hardly justified. In addition the backward scattering amplitudes f_{pn} and f_{pp} are of the same magnitude for $T_p^{\text{eff}} \cong 500$ MeV. Thus all the processes in Fig. 8 should be taken into account (which will decrease the calculated pd cross section). Therefore, our results for $T_p = 316$, 364 MeV should be considered only as an estimate.

The results shown in Fig. 10 demonstrate agreement with the data. However, the evaluation of the neutron exchange requires information of the deuteron wave function for large relative momenta Q (even for $T_p \gtrsim 1$ GeV, $Q \gtrsim 400$ MeV/c). Unfortunately, the standard deuteron wave functions

(Hamada-Johnson, Reid, etc.) do not reproduce the deuteron form factor for $q^2 \gtrsim 2$ (GeV/c)^2.[25] Since it is the high momentum part of the wave function which matters in that q^2 region, our evaluation of the neutron pick-up amplitude with the Humberston-Wallace[27] wave function should be considered only as an estimate for $T_p > 1$ GeV. For $T_p = 2.08$, 2.5 GeV the neutron exchange has not been included at all, since we have no reliable information for its estimate.

Figure 11 shows the large angle data for $k_L = 2.87$, 3.3 GeV/c and predictions. The backward scattering region is reproduced, but for smaller q^2 we disagree with experiment. It may be the influence of the neutron exchange, which has not been included. It may indicate also that an additional mechanism which is outside of our description, plays an important role in this energy region.

One sees from Figs. 9–11 that one form factor reproduces the different sets of data with the same q^2 but different energies. That means that the agreement of our calculation with experiment is not trivial. Since large angle pd scattering is governed in general by the single scattering mechanism, one obtains from Eqs. (53a) and (53b) that the ratio of pd cross sections for the same q^2 but for different energies is independent of the deuteron form factor and equals

$$\exp[-2\beta^2(m^2+q^2/4)^{1/2}(E_p^{(1)}-E_p^{(2)})]\,,$$

where E_p is the proton energy in the pd Breit

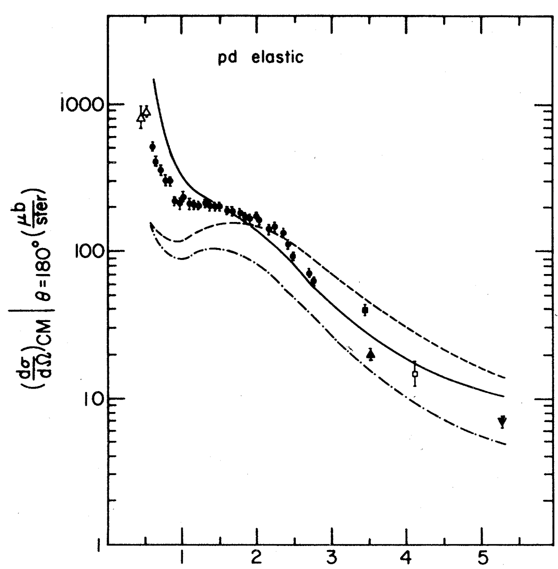

FIG. 14. $d\sigma_{pd}/d\Omega_{\mathrm{c.m.}}|_{\theta_{\mathrm{c.m.}}=180°}$ as a function of q^2. Data (Ref. 18, 21); (Ref. 22); (Ref. 24); (Ref. 28); (Ref. 29). Legends: as in Fig. 9.

frame. The measurements of backward pd scattering for $T_p > 2.5$ GeV will be very interesting in verifying this prediction. If confirmed one can extract the deuteron body form factors at $q^2 > 9$(GeV/c)^2 using proton beams.

In Fig. 14 we show the data for the pd cross section at $\theta_{\mathrm{c.m.}} = 180°$ for different T_p (Refs. 18, 21, 22, 24, 28, 29) plotted as a function of q^2. Comparing this data with the deuteron form factor in Fig. 12, one sees that the shoulder in pd data from 1 (GeV/c)$^2 \leq q^2 \leq 2$ (GeV/c)^2 corresponds to the shoulder in $S_S^2 + S_Q^2$ at the same q^2 region. It is naturally explained in our approach, since the pd cross section given by single scattering $d\sigma_{pd}^{(1)}/d\Omega|_{\theta_{\mathrm{c.m.}}=180°}$ is proportional to $S_S^2+S_Q^2$ [Eq. (51)]. We have already mentioned that in the region $q^2 \lesssim 1.2$ (GeV/c)^2 ($T_p \lesssim 360$ MeV) the rescattering terms are underestimated. Thus the evaluated cross section in this region should probably be reduced. The results of our partial analysis of analyzing power data can be found in Ref. 6.

V. CONCLUSION

In spite of many years of extensive study of hadron-nucleus scattering, the processes with large momentum transfer still challenge the theory. In particular, the observed pd backward scattering peak stimulated the appearance of various models for its explanation. Since the neutron exchange could not reproduce the data, these models introduced explicitly an additional degree of freedom in the deuteron wave function, such as the exchange of N^*.[30] The other explanation invoked pion exchange effects.[31] All these models were outside of the usual multiple scattering description. The latter has not been investigated seriously since the binding effects were always neglected.

In this paper we have studied the multiple scattering mechanism in pd backward scattering. The binding effects *have been included* in a consistent way using the method derived in Refs. 3 and 4. Owing to compensation of Fermi motion and binding effects, the multiple scattering amplitudes were found to be dependent on the on-shell pN amplitudes and deuteron form factors. The important step in that development was the inclusion of the Pauli principle. It led to the pp amplitude being antisymmetrical (i.e., physical). Since the pp amplitude is strongly peaked in backward direction the pp multiple scattering amplitudes are also peaked towards $\theta_{\mathrm{c.m.}} = 180°$. Therefore the dominant mechanisms of pd high energy scattering are: (a) the backward on-shell pp (single) scattering, and (b) the double scattering term consisting of backward pp scattering followed by forward pn rescattering.

The second process which contributes in the pd backward scattering is the neutron pick-up distorted by the proton's rescattering. In the calculation of the distortion the binding effects have been taken into account in the same way as in Refs. 3 and 4. Finally, the distortion has been found to be essentially due to forward rescattering of the proton from target nucleons.

These results have been applied for the analysis of the existing pd large angle scattering data. Since the multiple scattering amplitudes were found in factorized form we needed only the information on the deuteron body form factor at large q^2. We have shown that all pd large angle data for 0.3 GeV $\lesssim T_p \lesssim$ 1.3 GeV could be reproduced with a deuteron form factor taken from *direct measurements in ed scattering.*[25] These measurements provided the information only up to $q^2 \leq 4$ (GeV/c)^2. For the analysis of pd large angle data for $T_p \gtrsim$ 1.3 GeV we needed information on the deuteron form factor for $q^2 > 4$ (GeV/c)^2. We have shown that these pd data can be fitted with one extrapolation of the deuteron form factor up to $q^2 \lesssim 9$ (GeV/c)^2. Therefore, using our approach, the large angle pd data could be used as a source of information on the deuteron form factor at large q^2. It is interesting to compare the extracted form factor with the prediction of the dimensional-scaling quark model.[32] One sees from Fig. 12 that our squared form factor decreases more slowly [from $q^2=3$ (GeV/c)^2 up to $q^2=9$ (GeV/c)^2] than one predicted by this model [$S_S{}^2+S_Q{}^2$ at $q^2=9$ (GeV/c)^2 is an order of magnitude larger than the predictions].

Finally we note that the analysis of pd large angle scattering using well-defined multiple scattering theory should be considered as a necessary step which should be performed before considering exotic models. Only a disagreement of the prediction with experimental data may be a reason for inclusion of different effects which are external to the multiple scattering description.

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APPENDIX

We derive the optimal approximation for the rescattering terms of neutron exchange amplitude, Eq. (19b). As an example we consider the first rescattering term

$$T_n^{(1)\mathrm{ex}}(E,\vec{k},\vec{k}')=\langle\phi_d,\vec{k}'|\tau_{31}^d\tilde{G}\tilde{V}_{12}|\vec{k},\phi_d\rangle=\int\phi_d(\vec{P}'-\vec{K}+\vec{q}/4)\langle\vec{k}',\vec{P}'|\tau_{31}^d|\vec{P}_1,\vec{p}_1\rangle\langle\vec{P}_1,\vec{p}_1|\tilde{G}|\vec{P}_2,\vec{p}_1\rangle V_{pn}(\vec{P}_2-\vec{P})$$
$$\times\delta(\vec{p}_1+\vec{k}-\vec{P})\phi_d(\vec{P}-\vec{K}-\vec{q}/4)d^3P_1\,d^3P_2\,d^3P\,d^3P'\,d^3p_1\,. \qquad \text{(A1)}$$

We choose the approximations $\tilde{G}_a\cong\tilde{G}$ and $t_{31}^d\cong\tau_{31}^d$ in the forms (32), (33), but with the quantities $\bar{\epsilon}$, $\bar{\epsilon}'$ different from those of Eq. (38). ϵ and ϵ' should be determined from the vanishing of the first order correction to the approximation $t_{31}^d\tilde{G}_a\tilde{V}_{12}\cong\tau_{31}^d\tilde{G}\tilde{V}_{12}$ which reads

$$\Delta_1=\langle\phi_d,\vec{k}'|\delta_1\tau_{13}^d\tilde{G}_a\tilde{V}_{12}+t_{13}^d\delta_1\tilde{G}\tilde{V}_{12}|\vec{k},\phi_d\rangle\,, \qquad \text{(A2)}$$

where $\delta_1\tau_{13}^d$ and $\delta_1\tilde{G}$ are defined by Eqs. (34a), (34b).

Consider the first term of Eq. (A2):

$$\Delta_1'=\int\phi_d(\vec{P}'-\vec{K}+\vec{q}/4)\langle\vec{k}'|\hat{t}_{31}^{\,d}|\vec{p}_2\rangle\frac{\langle\vec{P}',\vec{p}_2|\tilde{G}_a'^{-1}-\tilde{G}^{-1}|\vec{p}_2,\vec{P}''\rangle}{\left(E-\dfrac{p_2^2}{2m}-\dfrac{(\vec{K}-\vec{p}_2)^2}{4m}-\bar{\epsilon}'\right)^2}\langle\vec{p}_2|\hat{t}_{31}^{\,d}|\vec{p}_1\rangle\tilde{G}_a(\vec{p}_1,\bar{\epsilon})V_{pn}(\vec{P}''-\vec{P})$$
$$\times\delta(\vec{P}-\vec{p}_1-\vec{k})\phi_d(\vec{P}-\vec{K}-\vec{q}/4)d^3P\,d^3P'\,d^3P''\,dp_1\,d^3p_2\,. \qquad \text{(A3)}$$

Using Eqs. (36a), (37) one performs the $d^3P\,d^3P'$ integration to obtain

$$\Delta_1'=\int\phi_d(\vec{P}''-\vec{K}+\vec{q}/4)\langle\vec{k}'|\hat{t}_{31}^{\,d}|\vec{p}_2\rangle\tilde{G}_a'^2(\vec{p}_2,\bar{\epsilon}')\left[(\vec{P}''-(\vec{K}+\vec{p}_2)/2)^2\frac{1}{m}+\epsilon_0-(\vec{P}''-\vec{K}+\vec{q}/4)^2\frac{1}{m}-\bar{\epsilon}'\right]$$
$$\times\langle\vec{p}_2|\hat{t}_{31}^{\,d}|\vec{p}_1\rangle\tilde{G}_a(\vec{p}_1,\bar{\epsilon})V_{pn}(\vec{P}''-\vec{p}_1-\vec{k})\phi_d(\vec{p}_1+\vec{q}/4)d^3p_1\,d^3p_2\,d^3P''. \qquad \text{(A4)}$$

Neglecting the terms $\propto(\vec{P}''-\vec{p}_1-\vec{k})V_{pn}(\vec{P}''-\vec{p}_1-\vec{k})$ in the integrand (A4) [since in the coordinate space they are proportional to $\partial/\partial\vec{r}V_{pn}(r)$] one can perform the integration over d^3P'' to eliminate the potential V_{pn}

in Eq. (A4). Then we find that $\Delta_1'=0$ if

$$\bar{\epsilon}' = \left(\vec{p}_1 + \frac{\vec{K}+\vec{q}-\vec{p}_2}{2}\right)^2 \frac{1}{m} + \epsilon_0 - \left(\vec{p}_1 + \frac{3\vec{q}}{4}\right)^2 \frac{1}{m}\,. \tag{A5}$$

Substituting Eq. (A5) into (32) and Eq. (32) into (33) we obtain after some algebra that the equation for $\langle\vec{k}'|\hat{t}^{\,d}_{31}|\vec{p}_1\rangle$ reads

$$\langle\vec{k}'|\hat{t}^{\,d}_{31}|\vec{p}_1\rangle = V_{pp}(\vec{k}'-\vec{p}_1) + \int V_{pp}(\vec{k}'-\vec{p}_2)\,\frac{\langle\vec{p}_2|\hat{t}^{\,d}_{31}|\vec{p}_1\rangle\, d^3p_2}{\dfrac{k'^2}{2m} - \dfrac{(\vec{q}+\vec{p}_1)^2}{2m} - \dfrac{p_2^{\,2}}{2m} - \dfrac{(\vec{k}+\vec{p}_1-\vec{p}_2)^2}{2m}}\,. \tag{A6}$$

One can see easily from Eq. (A6) that $\langle\vec{k}'|\hat{t}^{\,d}_{31}|\vec{p}_1\rangle$ is a (direct) pp scattering matrix corresponding to kinematics $[\vec{k}'] + [\vec{p}_1+\vec{q}] \to [\vec{p}_2] + [\vec{k}+\vec{p}_1-\vec{p}_2]$ for the total energy argument

$$E_{pp} = \frac{k'^2}{2m} + \frac{(\vec{p}_1+\vec{q})^2}{2m}\,. \tag{A7}$$

The vanishing of the second term of Eq. (A2) defines the quantity $\bar{\epsilon}$, which determines the approximate Green's function $\tilde{G}_a(\vec{p}_1,\bar{\epsilon}) \cong \tilde{G}$ of Eq. (A1).

Performing the same derivation as above one finds

$$\bar{\epsilon} = \frac{(\vec{K}+\vec{q}+\vec{p}_1)^2}{4m} + \epsilon_0 - \left(\vec{p}_1 + \frac{3\vec{q}}{4}\right)^2 \frac{1}{m}\,. \tag{A8}$$

With these now defined $\hat{t}^{\,d}_{31}$ and $\tilde{G}_a$ one obtains Eq. (43) for $T_n^{(1)\mathrm{ex}}$. The optional approximation for second and higher order rescattering terms of expansions (19), (21b) is derived in the same way.

[1]G. Takeda and K. M. Watson, Phys. Rev. 97, 1336 (1955).
[2]M. L. Goldberger and K. M. Watson, *Collision Theory* (Wiley, New York, 1964).
[3]S. A. Gurvitz, J.-P. Dedonder, and R. D. Amado, Phys. Rev. C 19, 142 (1979).
[4]S. A. Gurvitz, Phys. Rev. C 20, 1256 (1979).
[5]C. F. Chew, Phys. Rev. 80, 196 (1950); C. F. Chew and M. L. Goldberger, *ibid.* 87, 778 (1952).
[6]S. A. Gurvitz, Phys. Lett. 25B, 5 (1979).
[7]S. A. Gurvitz, Y. Alexander, and A. S. Rinat, Ann. Phys. (N.Y.) 93, 152 (1975).
[8]W. de Boer *et al.*, Phys. Rev. Lett. 34, 558 (1975); I. P. Auer *et al.*, Phys. Lett. 67B, 113 (1977).
[9]This formula was proposed in S. A. Gurvitz and A. S. Rinat, Phys. Lett. 60B, 405 (1976); S. A. Gurvitz, Y. Alexander, and A. S. Rinat, Ann. Phys. (N.Y.) 98, 346 (1976) as phenomenological description of pd backward scattering data. However, the factor $(k_L^{\mathrm{eff}}/k_L)^2$ has been erroneously omitted.
[10]Particle Data Group Report No. UCRL 20 000 NN, 1970.
[11]E. L. Miller *et al.*, Phys. Rev. Lett. 26, 984 (1971).
[12]R. J. Glauber, in *Lecture in Theoretical Physics*, edited by W. Britten *et al.* (Interscience, New York, 1959), Vol. I.
[13]T. W. Chen and D. W. Hoock, Phys. Rev. D 12, 1765 (1975).
[14]T. W. Chen, Phys. Rev. C 13, 1974 (1976).
[15]T. Ishihara, Ann. Phys. (N.Y.) 110, 180 (1978).
[16]J. V. Noble and H. J. Weber, Phys. Lett. 50B, 233 (1974).
[17]M. Levitas and J. V. Noble, Nucl. Phys. A251, 385 (1975).
[18]B. E. Bonner *et al.*, Phys. Rev. Lett. 39, 1253 (1977).
[19]J. C. Adler *et al.*, Phys. Rev. C 6, 2010 (1972).
[20]E. T. Boschitz *et al.*, Phys. Rev. C 6, 457 (1972).
[21]B. E. Bonner *et al.*, Phys. Rev. C 17, 671 (1978).
[22]G. W. Bennet *et al.*, Phys. Rev. Lett. 19, 387 (1967).
[23]E. Coleman, R. M. Heinz, O. E. Overseth, and D. E. Pellett, Phys. Rev. 164, 1655 (1967).
[24]L. Dubal *et al.*, Phys. Rev. D 9, 597 (1974).
[25]R. Arnold *et al.*, Phys. Rev. Lett. 35, 776 (1975).
[26]Ch. Berger *et al.*, Phys. Lett. 35B, 87 (1971); W. Bartel *et al.*, Phys. Lett. 33B, 245 (1970).
[27]J. W. Humberston and J. B. G. Wallace, Nucl. Phys. A141, 362 (1970).
[28]G. Igo *et al.*, Nucl. Phys. A195, 33 (1972).
[29]Y. Banaigs *et al.*, Nucl. Phys. B23, 596 (1970).
[30]A. K. Kerman and L. S. Kisslinger, Phys. Rev. 180, 1483 (1969).
[31]N. S. Craigie and C. Wilkin, Nucl. Phys. B14, 477 (1969).
[32]S. J. Brodsky and B. T. Chertok, Phys. Rev. D 14, 3003 (1976).