Kaon-nucleus scattering

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Recent kaon-nucleon phase shift parametrizations are used to investigate the energy dependence of kaon-nucleus scattering. When nucleon motion is included the differential and total cross sections are found to vary smoothly with energy. The consequences for isobar dynamics are discussed.

[NUCLEAR REACTIONS ${}^{12}C(K^\pm, K^\pm){}^{12}C^{(0.0,4.4)}$, $E_K = 100-1000$ MeV; role of nucleon motion and kaon distortions; $\sigma(\theta)$ for various $\bar{K}N$ amplitudes.]

I. INTRODUCTION

The subject of kaon-nucleus scattering is a timely one in view of the interest in developing intense kaon beams,[1] and because of recent experiments on elastic and inelastic scattering of $K^\pm$ mesons from ${}^{12}C$ and ${}^{40}Ca$ nuclei.[2] In this paper, a distorted wave impulse approximation (DWIA) approach to kaon scattering is presented, which is kept simple to enable ready extraction of the main features of kaon probes, and to gauge the need for full treatments of the many-body dynamics. Several such DWIA studies have already been made.[3,4] Our work serves largely to confirm those contributions, but also differs from earlier works by using two recent analyses of the K^--nucleon basic amplitudes,[5,6] which are then used to indicate the sensitivity of K^--nucleon results to that input information. In addition we emphasize the influence of nucleon motion on the myriad of narrow, often overlapping K^--nucleon hyperon resonances. That aspect is studied by constructing optical potentials and examining their energy dependence as obtained from the basic kaon amplitudes. Finally these parameter free optical potentials are used to obtain K^+ and K^- elastic and inelastic cross sections for ${}^{12}C$ and ${}^{40}Ca$ which are then compared to the recent Carnegie-Mellon University data.[2]

II. HYPERON RESONANCES IN NUCLEI

We begin by discussing the basic K^--nucleon amplitudes, which display a rich resonance structure. These amplitudes will later be used to construct our optical potentials.

A. Hyperon resonances

For kinetic energies between 200 MeV and 1 GeV, the K^-N scattering amplitudes are as reliable as any two-body amplitudes within this energy range. The experimental data available to Gopal *et al.*[6] included elastic scattering, charge and strangeness exchange, and polarization information, while the more recent Alston-Garnjost *et al.*[5] amplitudes made use of improved charge exchange data. These two amplitude sets are quite similar in the important partial waves (S, P, and D) and lead to similar but not identical K^-N total cross sections, as shown in Fig. 1. The broad drop in cross section from zero to 700 MeV/c is a dominating feature of the interaction and is a result of subthreshold structure such as the S-wave $\Lambda(1450)$ which has long been known to dominate the kaonic atom dynamics.

The differences between the phase shift parametrizations of Refs. 5 and 6 are shown in the Argand diagrams of Fig. 2. The existence of resonances was ascertained by these authors from rapid counterclockwise variations of the amplitudes with energy, which are fit by resonance forms with proper threshold, penetration, and background behavior. That arduous task yields the spectrum of Y_I^* states displayed in Fig. 3. The major features are the subthreshold resonances, the strong D-wave $\Lambda(1520)$ with a width of 15 MeV,

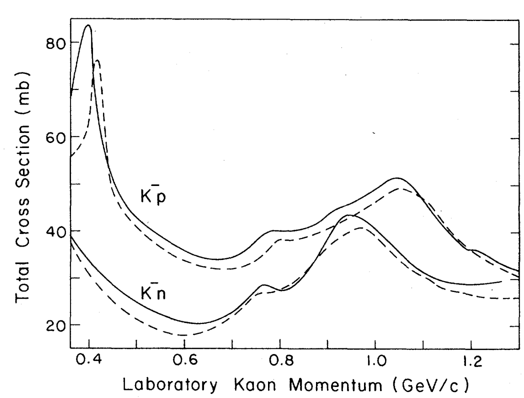


FIG. 1. Kaon-nucleon cross sections calculated from the amplitude parametrization of Ref. 5 (solid) and Ref. 6 (dashed).

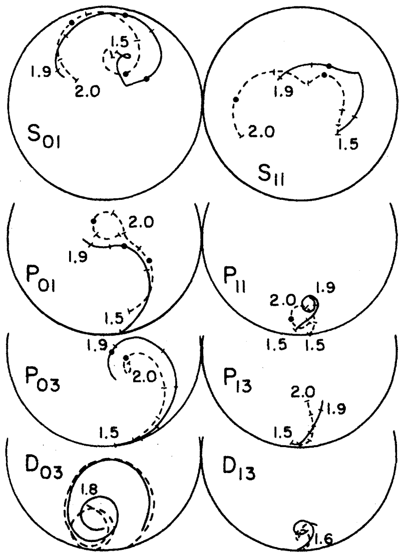


FIG. 2. Kaon-nucleon amplitudes of Ref. 5 (solid) and Ref. 6 (dashed).

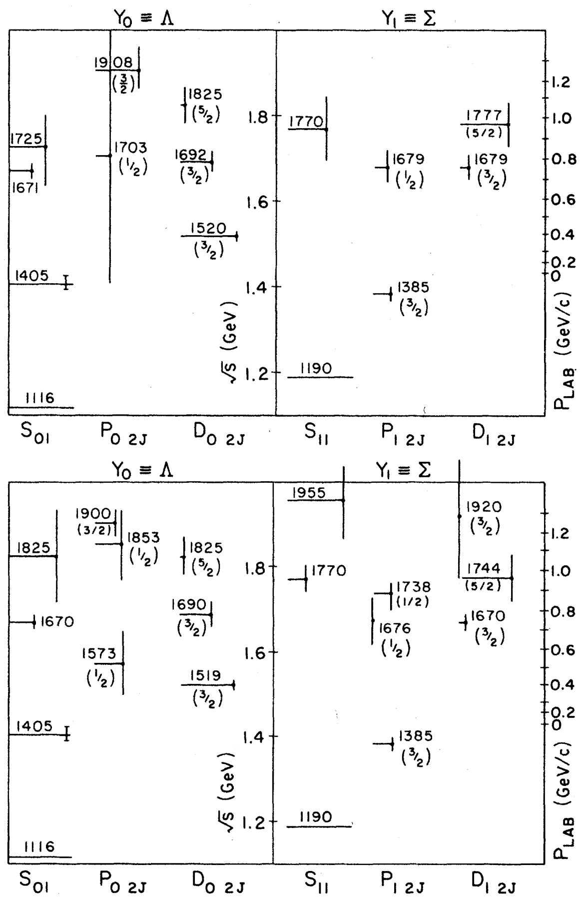


FIG. 3. Deduced Y^* resonances of Ref. 5 (top) and Ref. 6 (bottom). Vertical lines represent resonance widths.

a group of overlapping Λ and Σ resonances near 850 MeV/c with an average energy width of 90 MeV, and a region of broad resonances beyond 1200 MeV/c. From Fig. 3, the major differences between the two parametrizations is apparently that Gopal *et al.* see two resonances in the P01 channel at 1573 and 1853 MeV, while Alston-Garnjost *et al.* see only a single weak, extremely broad resonance centered at 1703 MeV. Nevertheless the extracted phase shifts do not differ significantly at any of these three energies and although the P-wave amplitudes have an erratic energy dependence, probably due to a P01 resonance, those resonance parameters are too sensitive to the details of analysis to be considered well determined.

Starting with the amplitudes of Ref. 5 or 6, the standard methods of multiple scattering theory may be used to infer the properties of the kaon-nuclear interaction, which then forms the starting point of an investigation of hyperon resonance-nuclear dynamics. An interesting question is whether the differences in these amplitudes affect the kaon-nucleus results.

B. Nucleon motion effects

It is clear that nucleon motion must appreciably modulate the effects of a resonance as narrow as the $\Lambda(1520)$. For purposes of estimating that modulation we begin by using simple multiple scattering theory, which requires us to consider the K^--nucleus t matrix in the appropriate reference frame.

Consider the kaon to have laboratory four-momentum $(\vec{k},\omega)$ and an individual nucleon in the nucleus to have a momentum four-vector $(\vec{p},\epsilon)$. We assume ϵ is on shell: $\epsilon^2 = p^2 + M_N{}^2$. A boost velocity $\vec{\beta} = (\vec{k}+\vec{p})/(\epsilon+\omega)$ is used to transform to the kaon-nucleon center-of-momentum (c.m.) frame, in which the particle four-momenta are $(\vec{\kappa},\bar{\omega})$ and $(-\vec{\kappa},\bar{\epsilon})$. Within this c.m. frame, the free scattering matrix is

$$t(q,\bar{\omega}+\bar{\epsilon}) = \sum_{l=0}^{\infty} t_l(\bar{\omega}+\bar{\epsilon}) P_l(\hat{\kappa}\cdot\hat{\kappa}')\,, \tag{1}$$

where $\vec{q} = \vec{\kappa} - \vec{\kappa}'$. For a bound nucleon the "in-the-medium" scattering matrix is required, and we approximate it by averaging over the energy dependent factors in (1):

$$\tau(q,\omega) = \sum_{l=0}^{\infty} \langle t_l(\bar{\omega}+\bar{\epsilon})\rangle P_l(\hat{\kappa}\cdot\hat{\kappa}')\,. \tag{2}$$

Here we average over the possible values of the nucleon momentum vector $\vec{p}$ as determined by the nuclear momentum density $\rho(\vec{p})$:

$$\langle t_l(\tilde{\omega}+\tilde{\epsilon})\rangle \equiv \int d^3p\, \rho(\vec{p}) t_l(E)$$

$$= 2\pi \int_0^\infty dp\, p^2 \rho(p)$$

$$\times \int_{-1}^{+1} d\mu\, t_l\{E = [\epsilon(p)+\omega][1-\beta^2(p,\mu)]^{1/2}\}.$$

The $\rho(p)$ was taken from simple harmonic oscillator distributions $\sum_\alpha |\phi_\alpha(p)|^2$. Using these τ matrices an approximate transformation to the $(A+1)$-body center of momentum is obtained from the equation

$$\tau^{(A+1)} = \left(\frac{\epsilon\epsilon'\omega\omega'}{\bar{\epsilon}\bar{\epsilon}'\bar{\omega}\bar{\omega}'}\right)^{1/2} \tau ,$$

where the barred quantities refer to the $(A+1)$ c.m. and the unbarred quantities to a two-body system defined by nucleon momentum $\vec{p} = -\vec{k}/A$, with the above energies on shell. At this stage, for simplicity, we ignore the l-mixing effects of the "angle transformation" and use the forward scattering formula

$$\tau_l^{(A+1)} = \frac{\epsilon\omega}{\bar{\epsilon}\bar{\omega}} \tau_l . \qquad (3)$$

Finally, the optical parameters are defined by

$$b_l = \frac{16\pi^3}{\bar{K}^2}\left(\frac{A-1}{A}\right)\omega \tau_l^{(A+1)} = \frac{16\pi^3}{\bar{K}^2}\left(\frac{A-1}{A}\right)\frac{\epsilon\omega}{\bar{\epsilon}} \tau_l . \qquad (4)$$

We do not claim rigor for this procedure, only easy computability, our aim being to identify the general effects of nucleon motion. Impulse approximation optical parameters are defined by substituting t_l for τ_l in (4).

Any treatment of nucleon motion significantly more accurate than the above approach is likely to involve the integration of nucleon momentum wave functions over (off-shell) kaonic amplitudes, a procedure which has become common in pion physics. This type of treatment usually leads to a momentum-space optical potential. However, the associated wave equation is more difficult to solve for kaons than for the lighter mass pions. Our experience has been that existing momentum space computer programs, because of the way they extract double-Bessel transforms of nuclear densities, are not yet suitable for kaon-nucleus reaction studies. For this reason, only coordinate space results are presented in this paper.

Figure 4 shows the energy variation of the b_l's calculated with the amplitudes of Ref. 5. Note that the scales for b_0, b_1, and b_2 differ. It is b_0 which dominates (due to the subthreshold Λ and Σ resonances), even over the isolated D03(1520). The nucleon motion averaged curves are drastically smoother. The 1520 is practically wiped out and replaced by a much smoother energy dependence. The other significant features are broad bumps in b_{1R} and b_2 caused mainly by the 850 MeV/c group of resonances and the elimination of resonance absorption in b_{1I} near 800 MeV/c.

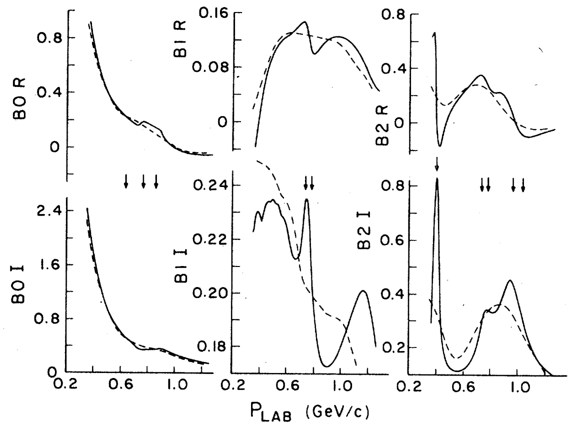


FIG. 4. Optical parameters in the impulse approximation (solid) and after Fermi averaging (dashed). The basic amplitudes are those of Ref. 5. Arrows indicate resonance positions.

Kaon-nucleon form factor effects are expected to be small and we study the role of such effects by adopting two different potential models. The S-wave model $U_{\text{opt}}^{(\text{SW})}$ is determined by ignoring the momentum dependence of the form factor, so that the optical potential is of the form

$$2\bar{\omega} U_{\text{opt}}^{(\text{SW})} = \bar{b}\bar{K}^2 \rho(r) ,$$

where

$$\bar{b} = \Sigma_l b_l . \qquad (5)$$

The second potential model has been described by Kisslinger[7] and is based on a strongly momentum dependent form factor including up to $l=2$ kaon-nucleon effects

$$2\bar{\omega} U_{\text{opt}}^{(\text{MD})} = \bar{K}^2 b_0' \rho(r) + b_1' \nabla\cdot\rho\nabla + b_2' \nabla^2\rho , \qquad (6)$$

where the (b_l')'s are linear combinations of the b_l's as discussed in Ref. 7. All partial waves higher than the second have been incorporated in b_0'. The above models permit us to use the coordinate space computer code PIRK.[14] However the actual off-shell behavior of the KN form factor is not well known and stronger nonlocalities than those considered here might be necessary for higher accuracy.

Figure 5 shows the K^--^{12}C elastic and inelastic cross sections deduced by Fermi averaging the parameters of Ref. 5 for the two types of optical potential. The results are in good quantitative agreement with the preliminary data of Ref. 2. Although

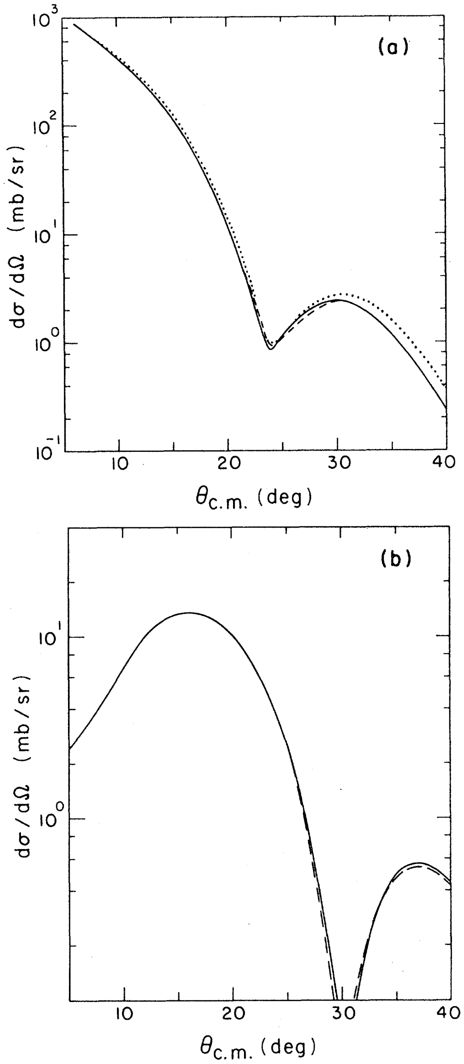


FIG. 5. (a) 800 MeV/c K^- ^{12}C elastic cross sections in the SW model using Fermi averaged parameters of Ref. 5 (solid) and Ref. 6 (dashed) and in the MD model (dotted). The differences between Ref. 5 and Ref. 6 parameters in the MD model are not visible on the scale shown. (b) 800 MeV/c K^- ^{12}C* (4.44) cross sections calculated with $\beta_2 = 0.56$. SW calculation is solid; MD dashed. The differences between the two amplitude sets are not visible on the scale shown.

the projectile energy is in a region of overlapping KN resonances, the simple optical models outlined above reproduce most details of the observed (preliminary) angular distributions. Similar remarks apply to the ^{40}Ca cross sections, not shown.

Figure 6 shows $\sigma_{tot}(K^- \, ^{12}\text{C})$ deduced from the S-wave model, in the impulse approximation and compared to the data of Bugg *et al.*[8] The increase of the total cross section at low energies arises from the imaginary part of the optical potential. The open channels at zero energy result in a $1/v$

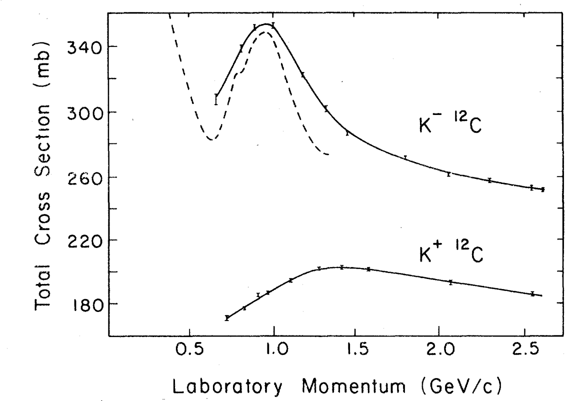


FIG. 6. $K^{\pm}$ ^{12}C total cross sections of Bugg *et al.* (Ref. 8) (solid) and the predictions of an S-wave impulse approximation calculation using parameters of Ref. 5. The parameters of Ref. 6 give results which differ by typically 5%.

behavior of the reaction cross section, a behavior also exhibited by pions.

The overall variation of the elastic differential cross section for K^- ^{12}C is given in Fig. 7 for laboratory energies from 118 to 718 MeV. The variation of $\sigma(\theta)$ with T_{LAB} is unremarkable; the simple diffractive behavior indicates a smooth vari-

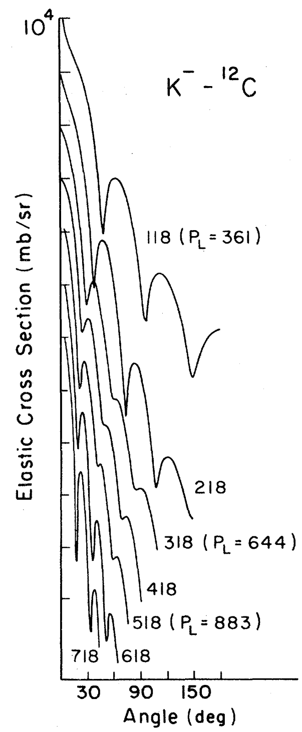


FIG. 7. K^- ^{12}C differential cross sections for the S-wave, impulse approximation model, using parameters of Ref. 5.

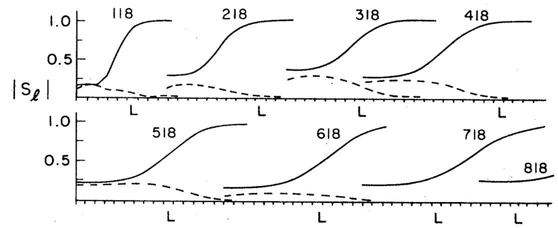


FIG. 8. Absorption parameters associated with the calculations of Fig. 7. Dashed lines show $\text{Im}(S_l)$.

ation of absorption with energy. No dramatic resonance effects associated with sudden increases in absorption are seen. The corresponding absorption coefficients $\eta_l = |S_l|$ are displayed in Fig. 8, which demonstrate the smooth evolution of absorption with energy.

To detect the effects of underlying K^-N resonances, one needs to scrutinize closely the energy dependence of $\sigma(\theta)$ and $|S_l|$. Absorption parameters for $K^-{}^{12}C$ scattering in the vicinity of the $\Lambda(1520)$ are shown in Fig. 9 for the two potential types, with and without Fermi averaging. A resonance effect is seen in $|S_2|$ for the MD form, Eq. (6), and in all low l for the SW, Eq. (5), but these effects are considerably modified by nucleon motion. It is this level of detail that will be required to learn about the D03 resonance in nuclei and it is clear that both nucleon motion and form factor effects need to be included carefully if resonance-nuclear dynamical information is to be extracted.

III. K^+-NUCLEAR INTERACTION

A. Status of K^+N phase shifts

There are several recent parametrizations of K^+-nucleon scattering[9,10] and the existence of newer data will hopefully lead to others in the near future. For a review, see Kelly.[11] In the energy region likely to be available to medium energy nuclear physics, these amplitudes do not yet have the degree of agreement which exists between different K^-N phase shift sets.

Recent work in this area has concentrated on the possibility of resonant structure. Any K^+N resonance would be "exotic" in the sense that it could not exist in standard three-quark models of hadrons. Such resonances have not been unambiguously identified, a typical candidate being the Arndt *et al.*[12] P13 structure near 1 GeV/c with a width of 100 MeV/c and a large inelasticity. Direct effects on nuclear scattering and reaction cross sections of any such weak, broad, high energy resonance are unlikely to be observed.

B. K^+-nucleus scattering

This topic has been covered extensively[3,4] elsewhere and we limit ourselves to the following observations. Lowest order multiple scattering theory gives a qualitatively reliable model. In Fig. 10 we show the 800 MeV/c $K^+{}^{12}C$ angular distributions calculated with this model. The Martin amplitudes in particular give an acceptable description of the data of Ref. 2, and we hope that the agreement improves with the next generation of K^+N phase shifts. With better phase shifts it will be possible to explore corrections to the model at lower energies. In particular, the fact that for K^+ mesons there is no annihilation channel would make any evidence for significant deviations from the model calculations at low energies intriguing. Such effects are expected in some theories.[13]

IV. CONCLUSION

Although the K^-N system is characterized by a rich resonance structure, the K^-A interaction has only a slow energy dependence. The modulation in the energy dependence is due to the resonances being either extremely narrow and therefore strongly affected by nucleon motion, or else being broad but overlapping so that even in the impulse approximation the optical potential shows no rapid energy variation. The extraction of resonance-nuclear dynamics is therefore quite complicated. To accomplish such an extraction with any reliability requires a high degree of accuracy in the treatment of both nucleon motion and K^-N form factor effects. This is a very difficult task and even if it were done the dynamical information of interest would reside in fine details of angular distributions.

On the other hand, this slow energy dependence

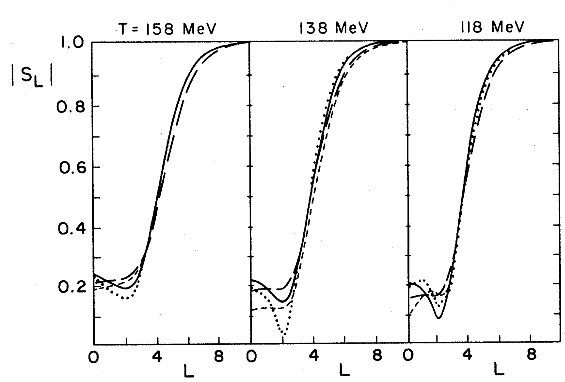
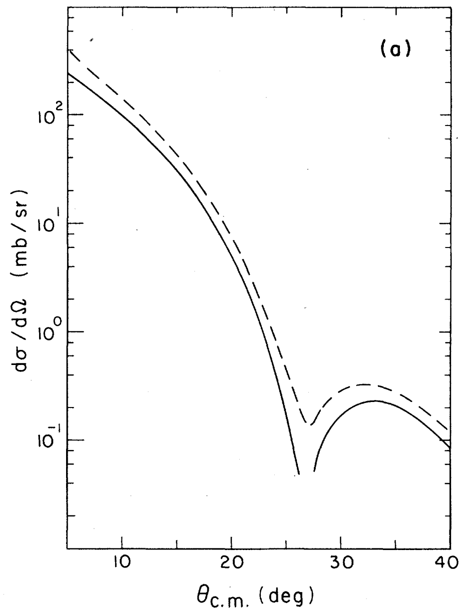


FIG. 9. Absorption parameters for $K^-{}^{12}C$ scattering near the $\Lambda(1520)$ calculated from the Fermi averaged MD potential (solid), impulse approximation MD (dotted), impulse approximation SW (short dashed), and Fermi averaged SW (long dashed).



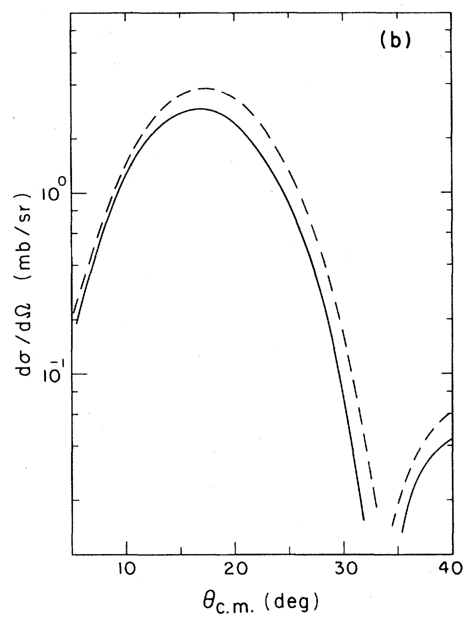


FIG. 10. (a) 800 MeV/c K^{+12}C elastic cross sections calculated from the parameters of Ref. 14 (solid) and Ref. 15 (dashed). (b) Same as (a) for $K^{+\,12}C^*$ (4.44).

of the optical potential has welcome consequences for nuclear structure investigations. The kaon distortions needed for hypernuclear physics are easily obtained with good accuracy. The inelastic and charge exchange reactions discussed in Ref. 4 are expected to have an energy variation which genuinely reflects the properties of nuclear states. Other applications, such as use of the strong low energy isovector character of the K^-N amplitudes to examine neutron distributions, are conceivable without the complexities of a multi-channel analysis.

ACKNOWLEDGMENTS

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