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# Reaction $^3$He($\gamma$,2$p$)$n$ at intermediate photon energies

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Proton energy distributions and the differential cross section for the reaction $^3$He($\gamma$,2$p$)$n$ for incident photon energies between 80 and 120 MeV have been calculated. In order to compare with available experimental data, we consider the situation in which both protons emerge close to a direction perpendicular to the incident photon beam. It is shown that in that case the dominant contribution is given by the direct breakup of $^3$He into a free neutron and an interacting singlet $p$-$p$ pair. The nucleon momentum distribution in $^3$He used in the calculation is obtained from a phenomenological fit to momentum distributions observed in quasifree ($p$,2$p$) scattering experiments on $^3$He. If the Coulomb interaction between protons is ignored, the calculated energy distribution agrees, at lower photon energies ($\sim$ 50 MeV), with previous calculations in which the nuclear interaction was included fully in the final state, but only the dipole electromagnetic operator was included. The Coulomb interaction changes the shape of the proton energy distribution completely, bringing it and the differential cross section into agreement with the experimental data.

[NUCLEAR REACTIONS Three-nucleon photodisintegration of $^3$He; intermediate energy calculation; Coulomb effects.]

## I. INTRODUCTION

Proton-proton coincidence measurements in the three-nucleon photodisintegration of $^3$He have been carried out by Peridier *et al.*[1] In these experiments the protons are detected emerging at angles of $\theta_1 = 92°$ and $\theta_2 = 88°$ with the direction of the incident photon, all three directions being coplanar. Incident photon energies range from 80 to 160 MeV. The proton energy distribution $d^3\sigma/dE_1 d\Omega_1 d\Omega_2$ and differential cross section $d^2\sigma/d\Omega_1 d\Omega_2$ have been obtained by these authors (with $E_p \geq 9.5$ MeV).

Proton energy distributions for the proton angles of the experiment of Ref. 1 and for photon energies up to 50 MeV have been calculated by Gibson and Lehman.[2] In their calculation (1) the initial and final states are exact eigenstates of the three-nucleon Hamiltonian with Yamaguchi type nucleon-nucleon potentials; (2) in the electromagnetic interaction only electric dipole terms were included; and (3) the Coulomb interaction between protons was not included.

The authors of Ref. 1 have shown, however, that the shape of their proton energy distribution is reproduced very well by the Migdal formula[3] for $p$-$p$ final state interaction, which includes the effect of both the nuclear and Coulomb interactions. On the other hand, the authors of Ref. 2 have shown that already for photon energies as low as 50 MeV the effect of the $n$-$p$ interaction is small in the experimental configuration of Ref. 1. (This point is illustrated in Fig. 9 of Ref. 2.)

These points are taken into account in the calculation of cross sections presented in this note. In the next section the reaction amplitude and the approximations utilized in its calculation are discussed. The nucleon momentum distributions in the ground state of $^3$He that enter in the calculation of the reaction amplitude can be calculated from available theoretical $^3$He ground state wave functions, as in Ref. 2. However, since the main purpose of the present note is the determination of the dominant reaction mechanism in the experimental configuration of Ref. 1, it was felt that more reliable conclusions can be obtained if the relevant nucleon momentum distributions were taken directly from other experiments. This approach can be carried out by utilizing the results of quasifree scattering experiments on $^3$He. Phenomenological fits to these momentum distributions are derived in Sec. III.

In Sec. IV the energy distribution and differential cross section are calculated and compared with the results of Ref. 1.

## II. REACTION AMPLITUDE

The amplitude for the reaction

$$\gamma + {}^3\mathrm{He} \to p + p + n \tag{1}$$

 

is given by

$$M_{m_1 m_2 m_n, \epsilon m_3} = (\Psi^{(-)}_{\vec{p}_1 \vec{p}_2 \vec{p}_n, m_1 m_2 m_n}, \mathcal{H}_\gamma \Psi_{0 m_3}) , \qquad (2)$$

where the initial and final states are, respectively, the ground state of $^3$He and the eigenstate of the $ppn$ Hamiltonian characterized asymptotically by three free outgoing nucleons with momenta $\vec{p}_1, \vec{p}_2, \vec{p}_n$. The $m$'s are the $^3$He and spin components of the three nucleons, $\epsilon$ the photon polarization. $\mathcal{H}_\gamma$ is the photon-proton electric interaction operator[4]

$$\mathcal{H}_\gamma = -\frac{e}{M}\left(\frac{2\pi}{\omega}\right)^{1/2} [e^{i\vec{k}\cdot\vec{r}_1}(\hat{\epsilon}\cdot\vec{p}_1) + e^{i\vec{k}\cdot\vec{r}_2}(\hat{\epsilon}\cdot\vec{p}_2)] , \quad (3)$$

$\omega, \vec{k}$ are the photon energy and momentum, $\vec{r}_i, \vec{p}_i$ ($i = 1, 2$) the proton position and momentum operators, and $M$ the nucleon mass.

The experiments under consideration here explore the final state situation in which the two protons emerge with momenta almost parallel to each other, and perpendicular to the incident photon direction. The neutron will then emerge in the direction opposite to that of the protons. For the range of photon energies in these experiments it can then be expected that the effects of the interaction between the emerging neutron and the proton pair are not important. This point was checked by Gibson and Lehman in their calculations and we take that assumption to be proven by their results. The final state in the calculation of the reaction amplitude (2) can then be taken to be that of the motion of a free neutron and an interacting proton pair

$$\Psi_f = e^{i(\vec{p}_n\cdot\vec{r}_n)} e^{i[(\vec{p}_1+\vec{p}_2)\cdot(\vec{r}_1+\vec{r}_2)/2]} \times u^{(-)}_{\vec{q}_{12}}(\vec{r}_{12})\chi'(12, n) . \qquad (4)$$

$\vec{p}_1, \vec{p}_2, \vec{p}_n$ are the nucleon momenta, $\vec{r}_1, \vec{r}_2, \vec{r}_n$ position vectors, and $u^{(-)}_{\vec{q}_{12}}(\vec{r}_{12})$ the two-proton wave function [$\vec{q}_{12} = \frac{1}{2}(\vec{p}_1 - \vec{p}_2)$ the relative momentum and $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$ the relative position of the protons]. The spin eigenstate is $\chi'(12, n)$, with total spin $\frac{1}{2}$, and the proton spins coupled to singlet.

The Pauli principle requires that only the even $[u(\vec{r}_{21}) = u(\vec{r}_{12})]$ part of $u^{(-)}_{\vec{q}_{12}}(\vec{r}_{12})$ be considered in $\Psi_f$. This is automatically taken into account when calculating the reaction amplitude since the initial state, the ground state of $^3$He, has the form

$$\Psi_0 = \psi(12, n)\chi'(12, n) \qquad (5)$$

with the space part $\psi(12, n)$ symmetric in the 1, 2 variables.

For the evaluation of the reaction amplitude Eq. (2) with the final state Eq. (4), it is convenient to express the operator Eq. (3) in terms of c.m. coordinates and their canonically conjugate momenta

$$\vec{R} = \tfrac{1}{3}(\vec{r}_1 + \vec{r}_2 + \vec{r}_n), \quad \vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_n ,$$
$$\vec{\rho} = \vec{r}_n - \tfrac{1}{2}(\vec{r}_1 + \vec{r}_2), \quad \vec{p} = \tfrac{2}{3}\vec{p}_n - \tfrac{1}{3}(\vec{p}_1 + \vec{p}_2), \qquad (6)$$
$$\vec{r} = \vec{r}_1 - \vec{r}_2, \quad \vec{q} = \tfrac{1}{2}(\vec{p}_1 - \vec{p}_2) .$$

Then (the $\hat{\epsilon}\cdot\vec{P}$ term does not contribute to the reaction amplitude)

$$\mathcal{H}_\gamma = -\frac{e}{M}\left(\frac{2\pi}{\omega}\right)^{1/2} e^{i\vec{k}\cdot\vec{R} - i\vec{k}\cdot\vec{\rho}/3}\{(e^{i\vec{k}\cdot\vec{r}/2} + e^{-i\vec{k}\cdot\vec{r}/2})[-\tfrac{1}{2}(\hat{\epsilon}\cdot\vec{p})] + (e^{i\vec{k}\cdot\vec{r}/2} - e^{-i\vec{k}\cdot\vec{r}/2})(\hat{\epsilon}\cdot\vec{q})\} .$$

To the extent that terms of order $(\vec{k}\cdot\vec{r})^2$ and higher can be neglected $[(\vec{k}\cdot\vec{r})^2 \cong 0.03$ at $E_\gamma = 150$ MeV$]$,

$$\mathcal{H}_\gamma = +\frac{e}{M}\left(\frac{2\pi}{\omega}\right)^{1/2} e^{i\vec{k}\cdot\vec{R} - i\vec{k}\cdot\vec{\rho}/3}[\hat{\epsilon}\cdot\vec{p}) - i(\vec{k}\cdot\vec{r})(\hat{\epsilon}\cdot\mathrm{q})] . \quad (7)$$

The second term in Eq. (7) would produce $E2$ and $M1$ transitions between $pp$ pairs in the initial and final state. These contributions will be neglected for the present, since the $p$-$p$ pair is predominantly $s$ wave in the ground state and in this experiment we are restricted to relatively small $q$'s. The contribution to the reaction amplitude (2) is then given by

$$M_{m_1 m_2 m_n, \epsilon m_3} = \frac{e}{M}\left(\frac{2\pi}{\omega}\right)^{1/2} \int d^3\rho \int d^3 r\, e^{-i(\vec{p}+\vec{k}/3)\cdot\vec{\rho}}\, u^{(-)}_{\vec{q}}(\vec{r})^* \times(\hat{\epsilon}\cdot\vec{p})\psi_0(\vec{r},\vec{\rho}) \times\langle\tfrac{1}{2}m_1, \tfrac{1}{2}m_2, \tfrac{1}{2}m_n|\tfrac{1}{2}m_3\rangle , \qquad (8)$$

where $\psi_0(\vec{r},\vec{\rho})$ is the space part of the $^3$He ground state [Eq. (5)] and the spin part of the amplitude is

$$\langle\tfrac{1}{2}m_1, \tfrac{1}{2}m_2, \tfrac{1}{2}m_n|\tfrac{1}{2}m_3\rangle = \langle\tfrac{1}{2}m_1, \tfrac{1}{2}m_2|\tfrac{1}{2}\tfrac{1}{2}; 00\rangle \times\langle[\tfrac{1}{2}{}_1\tfrac{1}{2}{}_2]_{00}, \tfrac{1}{2}m_n|[\tfrac{1}{2}{}_1\tfrac{1}{2}{}_2]_0, \tfrac{1}{2}{}_n; \tfrac{1}{2}m_3\rangle$$
$$= \langle\tfrac{1}{2}m_1, \tfrac{1}{2}m_2|\tfrac{1}{2}\tfrac{1}{2}; 00\rangle\delta_{m_n m_3} . \qquad (9)$$

Since the plane wave $\exp[i(\vec{p} + \frac{1}{3}\vec{k})\cdot\vec{\rho}]$ is an eigenstate of the $n$-$(pp)$ relative momentum operator $\vec{p}$, the amplitude (8) is

$$M_{m_1 m_2 m_n, \epsilon m_3} = \frac{e}{M}\left(\frac{2\pi}{\omega}\right)^{1/2} (\hat{\epsilon}\cdot\vec{p}) \int d^3\rho \int d^3 r\, e^{-i(\vec{p}+\vec{k}/3)\cdot\vec{\rho}} \times u^{(-)}_{\vec{q}}(\vec{r})^*\psi_0(\vec{r},\vec{\rho}) \times\langle\tfrac{1}{2}m_1, \tfrac{1}{2}m_2, \tfrac{1}{2}m_n|\tfrac{1}{2}m_3\rangle ,$$

or, in a more concise notation,

$$M_{m_1 m_2 m_n, \epsilon m_3} = \frac{e}{M}\left(\frac{2\pi}{\omega}\right)^{1/2}(\hat{\epsilon}\cdot\vec{p}) \times \langle n, (pp)_0;\ \vec{p}+\tfrac{1}{3}\vec{k}, \vec{q}\,|{}^3\mathrm{He}\rangle\,. \quad (10)$$

That is, the reaction amplitude is proportional to the component of $n$-$(pp)$ relative momentum in the polarization direction times the overlap of the ground state of $^3$He with the neutron in free relative motion with respect to the interacting proton pair. This overlap is the probability amplitude for $n$-$(pp)$ in the ground state of $^3$He and can be calculated given the ground state wave function of $^3$He. The experimental configuration of Ref. 1 provides then a probe of the neutron-singlet proton pair relative momentum distribution in the ground state of $^3$He. However, before using that probe as a test of the theoretical $^3$He ground state wave function, it is desirable to check the validity of the assumptions leading to the expression (10) for the reaction amplitude. This can be accomplished if instead of calculating the amplitude $\langle n(pp)_0|{}^3\mathrm{He}\rangle$ from theoretical wave functions, we use experimentally measured amplitudes in Eq. (10).

## III. NUCLEON MOMENTUM DISTRIBUTION IN $^3$He

At the present time no direct measurement of the neutron momentum distribution in $^3$He, $\langle n(pp)_0|{}^3\mathrm{He}\rangle$, has been carried out. Instead, the momentum distributions $\langle pd|{}^3\mathrm{He}\rangle$ and $\langle p(pn)_0|{}^3\mathrm{He}\rangle$ have been observed separately by Frascaria *et al.*,[5] in the quasifree scattering of high energy protons on $^3$He.

Charge independence plus the predominance (92%) of the fully symmetric component[6] in the ground state of $^3$He yield the relation

$$\langle n(pp)_0|{}^3\mathrm{He}\rangle = -\sqrt{2}\,\langle p(pn)_0|{}^3\mathrm{He}\rangle\,. \quad (11)$$

To the extent that charge independence and full symmetry are valid in $^3$He, it is then possible to use the data of Ref. 5 in the calculation of the reaction amplitude. This is accomplished by means of a phenomenological fit to the data of Frascaria *et al.* The form chosen for the function used in this fit is based on the following considerations.

The momentum distributions $\langle p(pn)_s|{}^3\mathrm{He}\rangle$ $(s=0,1)$ have the general form

$$\langle p(pn)_s|{}^3\mathrm{He}\rangle = \left(B_{ppn} + \frac{3}{4}\frac{p^2}{M} + \frac{q^2}{M}\right)^{-1} h_s(p,q)\,, \quad (12)$$

with $\vec{p}$ the relative momentum of the proton-$(pn)$ pair, $\vec{q}$ the relative momentum of the nucleons in the interacting pair, $B_{ppn} = 7.72$ MeV the three-nucleon binding energy, and

$$h_s(p,q) = \langle p(pn)_s|V_{pp} + V_{pn}|{}^3\mathrm{He}\rangle \quad (13)$$

the $^3\mathrm{He} \to p + (pn)_s$ vertex amplitude.

The fact, well established both experimentally[7] and theoretically,[8] that the momentum distribution $\langle p(pn)_1|{}^3\mathrm{He}\rangle$ is practically zero, compared with $\langle pd|{}^3\mathrm{He}\rangle$ and $\langle p(pn)_0|{}^3\mathrm{He}\rangle$, leads us to assume for the vertex amplitude (13) the form

$$h_s(p,q) = g_s(p)\,(u_{\vec{q}}^{(s)}, u_0^{(1)})\,, \quad (14)$$

where $u_{\vec{q}}^{(s)}$ is the continuum $pn$ eigenstate of spin $s$ and $u_0^{(1)}$ the deuteron eigenstate.

In the approximation provided by the phenomenological form of Eq. (14) the $^3\mathrm{He} \to p + (pn)_1$ amplitude is identically zero, a consequence of the orthogonality of continuum and ground state two-nucleon triplet states, $(u_{\vec{q}}^{(1)}, u_0^{(1)}) = 0$. The function $g(p)$ is assumed to have the form

$$g(p) = g(0)(1 + p^2/\beta^2)^{-1}. \quad (15)$$

In order to determine the constants $g(0), \beta$, we introduce the momentum distribution given by Eqs. (12), (14), and (15) into the expression for the quasifree scattering cross section for the reaction[9] $p({}^3\mathrm{He}, 2p)d^*$ and adjust those constants to fit the experimental cross section. In Fig. 1 we show the result obtained with the value $g(0) = 95$ MeV fm$^{3/2}$, $\beta = 0.98$ fm$^{-1}$. The data are from Ref. 5. The same constants give a good fit to recent quasifree experiments on $^3$He by the Georgetown University group.[10]

The momentum distribution to be used in the calculation of the reaction amplitude of Eq. (10) is then obtained from that just described by use of Eq. (11). In addition, Coulomb effects are introduced naturally by the form of Eq. (14). Namely, the overlap between deuteron and singlet nucleon pair that appears in Eq. (14) will become, for the $\langle n(pp)_0|{}^3\mathrm{He}\rangle$ momentum distribution, the overlap between deuteron and singlet $p$-$p$ pair wave functions.

In the experimental situation of Ref. 1 the rela-

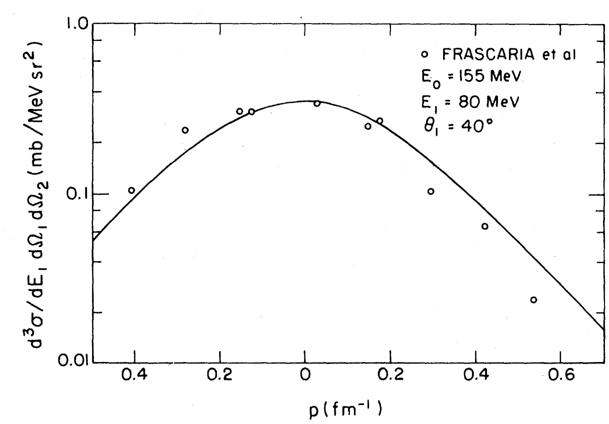


FIG. 1. $pd^*$ momentum distribution. Data from Frascaria *et al.* (Ref. 5). Curve is phenomenological fit described in text.

tive $p$-$p$ momentum is restricted to small values. (Typically, for $E_\gamma = 100$ MeV, $0.03 \leq q \leq 0.15$ fm$^{-1}$.) The overlap function $A(q) = (u_{\vec{q}}^{(0)}, u_0^{(1)})$ can, therefore, be obtained to sufficient accuracy by use of zero-range wave functions: If the Coulomb interaction is ignored, the singlet continuum zero-range function is given by (we use the *incoming* spherical wave boundary condition, as is proper for created particles)

$$u_{\vec{q}}^{(0)} = e^{i\vec{q}\cdot\vec{r}} + \frac{1}{\alpha_0 + iq}\,\frac{e^{-iqr}}{r}, \tag{16}$$

and the deuteron zero-range (triplet bound state) wave function by

$$u_0^{(1)} = \left(\frac{2\gamma}{4\pi}\right)^{1/2} \frac{e^{-\gamma r}}{r}, \tag{17}$$

where

$$\gamma = (MB_{np})^{1/2}, \tag{18}$$

$B_{np}$ being the deuteron binding energy, and

$$\alpha_0 = q\cot\delta_0. \tag{19}$$

We then find

$$A(q) = \frac{(8\pi\gamma)^{1/2}}{q^2+\gamma^2}\,\frac{\alpha_0+\gamma}{\alpha_0 - iq}. \tag{20}$$

When the Coulomb interaction is included we have

$$A_c(q) = (u_{c,q}^{(0)}, u_0^{(1)}), \tag{21}$$

where $u_0^{(1)}$ is given in Eq. (17) and

$$u_{c,q}^{(0)}(r) = e^{-i\delta_0}(F_0\cos\delta_0 + G_0\sin\delta_0) \tag{22}$$

is the $s$ wave singlet continuum zero-range wave function including the Coulomb interaction. Here $F_0$ and $G_0$ are, respectively, the regular and irregular Coulomb functions having the asymptotic form

$$F_0(r) \sim \frac{\sin(qr + \sigma_0 - \eta\ln 2qr)}{qr}, \tag{23a}$$

$$G_0(r) \sim \frac{\cos(qr + \sigma_0 - \eta\ln 2qr)}{qr}, \quad qr \to \infty. \tag{23b}$$

$\delta_0$ is the nuclear phase shift and $\sigma_0$ is the Coulomb phase shift, given by $\sigma_0 = \arg\Gamma(1+i\eta)$. Here $\eta$ is the Coulomb parameter

$$\eta = (2qR)^{-1} \tag{24}$$

in terms of the proton-proton relative momentum $q$, and the Coulomb radius $R = \hbar^2/Me^2 (= 28.9\text{ fm})$.

The functions $F_0$ and $G_0$ are defined in terms of confluent hypergeometric functions by[11]

$$\begin{aligned} F_0 &= e^{-\pi\eta/2}|\Gamma(1+i\eta)|e^{iqr}F(1+i\eta; 2; -2iqr) \\ &= Q + Q^* \\ G_0 &= i(Q - Q^*) \end{aligned} \tag{25}$$

with

$$Q \equiv -e^{\pi\eta/2}e^{i\sigma_0}e^{iqr}\Psi(1+i\eta; 2; -2iqr). \tag{26}$$

(We follow the notation of Bateman[12] in defining the irregular solution $\Psi$.)

After carrying out the integration over $r$ in Eq. (21) (see Appendix) we find

$$A_c(q) = \frac{(8\pi\gamma)^{1/2}}{q^2+\gamma^2}\,\frac{[F(\eta)]^{1/2}\{e^{2\eta y}\alpha_c(q) + \gamma[1+\epsilon(q)]\}}{\alpha_c(q) - h(\eta)/R - iqF(\eta)} \tag{27}$$

with

$$\alpha_c(q) = F(\eta)q\cot\delta_0 + h(\eta)/R \tag{27a}$$

and

$$h(\eta) = \mathrm{Re}[\psi(1+i\eta)] - \ln\eta, \tag{27b}$$

$$F(\eta) = 2\pi\eta/(e^{2\pi\eta} - 1) \tag{27c}$$

is the Coulomb penetration factor and

$$\epsilon(q) = \frac{1}{\gamma R}\,e^{2\eta y}\Big[C + \ln\frac{1}{\gamma R} - \tfrac{1}{2}\ln(1+q^2/\gamma^2) - 2\eta y + \eta^2 y^2 + \tfrac{2}{9}\eta y^3 + \text{terms which are negligible for all values of } q\Big] \tag{27d}$$

with $y = \tan^{-1}(q/\gamma)$. ($C$ = Euler's constant $= -\psi(1) = 0.5772\ldots$). [Details of the derivation of Eq. (27) are given in the Appendix.] Note that if we let $e^2 \to 0$ then the Coulomb radius $R \to \infty$, $\eta \to 0$, and $A_c(q)$ of Eq. (27) goes to $A(q)$ of Eq. (20).

We also note that for $q/\gamma \ll 1$ (in which case our amplitudes are independent of the model for the $^3$He ground state) our expressions for $A(q)$ and $A_c(q)$ [Eqs. (20) and (27)] are of the form

$$A(q) = \frac{\text{const}}{\alpha_0 - iq}, \tag{28}$$

$$A_c(q) = \frac{\text{const}[F(\eta)]^{1/2}}{\alpha_c(q) - h(\eta)/R - iqF(\eta)}, \tag{29}$$

which lead to cross sections of the form predicted some time ago by Migdal.[3]

In the calculation of cross sections, the effective range expansion[13] $\alpha(q) = -1/a + \frac{1}{2}r_0q^2 + \ldots$, was used for the functions $\alpha_0(q), \alpha_c(q)$.

## IV. CROSS SECTIONS

The cross section for the reaction $^3$He$(\gamma, 2p)n$ is given by

$$d^3\sigma/dE_1 d\Omega_1 d\Omega_2 = (2\pi)^{-5}\sum_{m_1 m_2 m_n}\tfrac{1}{2}\sum_{\epsilon}\tfrac{1}{2}\sum_{m_3}|M_{m_1m_2m_n,\epsilon m_3}|^2\frac{M}{D}p_1p_2^{\,2}, \tag{30}$$

where $D = |2p_2 - (\vec{k} - \vec{p}_1)\cdot\hat{p}_2|/M$. With the reaction amplitude in the approximation given by Eq. (10) and application of Eqs. (11), (13), and (15), one obtains, after performing sums and averages over spins and polarization,

$$d^3\sigma/E_1 d\Omega_1 d\Omega_2 = (2\pi)^{-4}\frac{e^2}{\hbar c}$$
$$\times\frac{(p_1\sin\theta_1+p_2\sin\theta_2)^2}{\omega^3\left[1+\frac{1}{2M}\left(\frac{k}{2}-p_1\cos\theta_1-p_2\cos\theta_2\right)\right]^2}$$
$$\times|g(|\vec{p}+\tfrac{1}{3}\vec{k}|)A(q)|^2\frac{p_1 p_2{}^2}{D}. \qquad (31)$$

A test of the approximations used in obtaining Eq. (31) can be made by calculating the cross section for the kinematical conditions used by Gibson and Lehman[2] and comparing the cross section given by Eq. (31) with their results. For this purpose we use the form of $A(q)$ given by Eq. (20) but employ in it the $n$-$n$ effective range parameters used in Ref. 2, viz., $a_{nn}=-17.0$ fm, $r_0=2.84$ fm. The results of Gibson and Lehman for $E_\gamma=50$ MeV and the results obtained from Eq. (31) are shown in Fig. 2. The difference between the magnitudes of these results is a consequence of the different amounts of higher momentum components in Tabakin's ground state (used in Ref. 2) and in the fit used here. Otherwise, both results are quite similar.

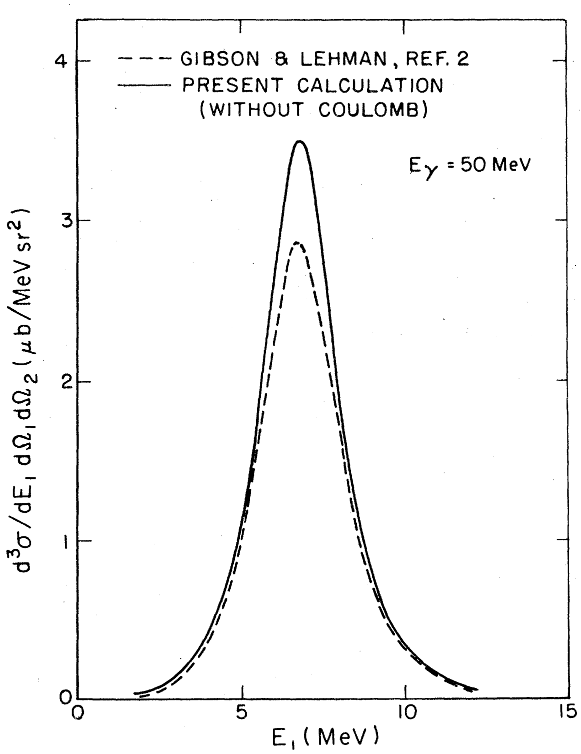


FIG. 2. $d^3\sigma/dE_1 d\Omega_1 d\Omega_2$ as a function of proton energy $E_1$. Results using Gibson and Lehman (dashed line) and using Eq. (20) in Eq. (31) of the present paper (solid line) ($\theta_1=92°$, $\theta_2=88°$).

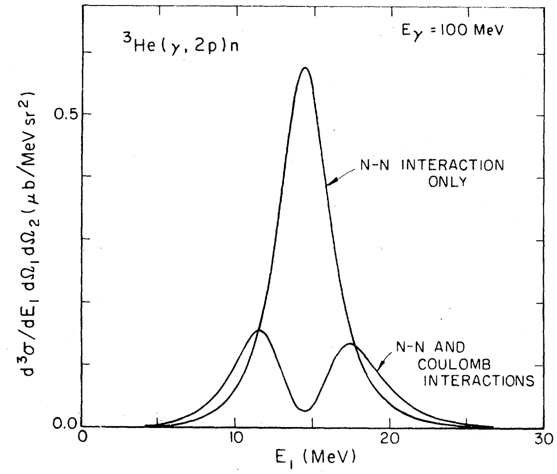


FIG. 3. $d^3\sigma/dE_1 d\Omega_1 d\Omega_2$ as a function of proton energy $E_1$. Results with and without Coulomb effects ($\theta_1=92°$, $\theta_2=88°$).

Consideration of Coulomb effects in Eq. (31) changes the shape of the energy distribution drastically, as illustrated in Fig. 3, where the cross section at $E_\gamma=100$ MeV obtained without Coulomb effects [$A(q)$ given by Eq. (20)] and with Coulomb effects [$A_c(q)$ given by Eq. (27)] are exhibited. It should be noted that the result obtained here reproduces the shape given by application of Migdal's theory.[3] This was noted by Peridier *et al.* That theory, however, considered the final state distribution for a general reaction; it specified the shape of the distribution, but not the overall normalization. In the present calculation the absolute value of the cross section is determined.

In Fig. 4 we show the result obtained for the differential cross section

$$d^2\sigma/d\Omega_1 d\Omega_2 = \int_{(E_p\geq 9.5\ \mathrm{MeV})} dE_1 d^3\sigma/dE_1 d\Omega_1 d\Omega_2 . \qquad (32)$$

The data are from Peridier *et al.* As shown in Fig. 4, the result obtained for the differential cross section [Eq. (32)], based on the nucleon momentum distribution in $^3He$ as measured by Frascaria *et al.*,[5] agrees reasonably well with the observed cross section. As is to be expected, given that the experimental conditions restrict us to small values of $q$, there is little difference between the cross section values obtained in effective range and zero-range theory. The calculated values of the differential cross section are also given in Table I. The results obtained when Coulomb effects are ignored are also shown in Table I. In that case, in the range of photon energies consid-

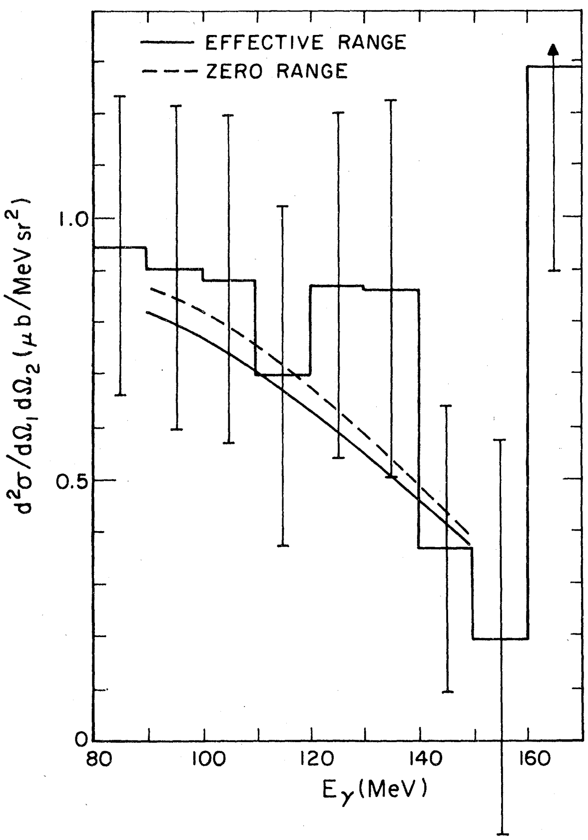


FIG. 4. $d^2\sigma/d\Omega_1 d\Omega_2$ as a function of $E_\gamma$. Experimental results from Peridier *et al.* (Ref. 1). Curves are calculated from momentum distribution shown in Fig. 1. Full curve includes effective range. Dashed curve is in zero range ($\theta_1 = 92°$, $\theta_2 = 88°$). If the Coulomb interaction is neglected, the theoretical values for the cross section are larger by a factor of 3, well outside the experimental measurements (see Table I).

ered here, the calculated cross section is about three times the experimental value.

It should be noted that when we calculate the cross section using, in the final state, the terms where the proton-neutron interaction and the photo breakup of virtual deuterons are included, we obtain cross sections that are three to four orders of magnitude below the results presented here. From the present calculation we can then conclude that the combination of high incident photon energy and proton coincidence detector arrangement permits the selection of that piece of the three-nucleon final state in which a neutron is moving freely relative to an interacting proton pair. As Eq. (31) shows, this type of experiment probes the momentum distribution $|\langle n(pp)_0|^3\mathrm{He}\rangle|^2$. On the other hand, the same type of experiment, but rearranging the proton detectors to be on opposite sides of the incident photon beam, would probe essentially the momentum distribution $|\langle p(pn)_0|^3\mathrm{He}\rangle|^2$. [As we noted before, the overlap $\langle p(pn)_1|^3\mathrm{He}\rangle$ is practically zero.] Experimental observation of the $p(pn)_0$ distribution is of relevance in connection with the question of the ($^3$He, $pd^*$) asymptotic normalization parameter.[7]

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## APPENDIX

In this appendix we derive the expression for the overlap function including the Coulomb interaction $A_c(q)$, defined in Eq. (21), and given explicitly in Eq. (27). The regular and irregular Coulomb functions, $F_0$ and $G_0$, which are needed are defined in Eqs. (25) and (26) in terms of confluent hypergeometric functions. We choose instead to write them in terms of the closely related Whittaker functions, viz.,

TABLE I. Calculated differential cross section.

| | No Coulomb[a] | | Coulomb[b] | | | | | |
|---|---|---|---|---|---|---|---|---|
| $E_\gamma$ | Zero | Effective | Zero range | | | Effective range | | |
| MeV | range | range | $M_1$ | $M_2$ | $M_3$ | $M_1$ | $M_2$ | $M_3$ |
| 100 | 2.648 | 2.570 | 0.8262 | 0.8272 | 0.8277 | 0.7561 | 0.7571 | 0.7575 |
| 120 | 1.778 | 1.735 | 0.6632 | 0.6640 | 0.6645 | 0.6169 | 0.6177 | 0.6181 |
| 140 | 1.212 | 1.187 | 0.4859 | 0.4865 | 0.4869 | 0.4579 | 0.4585 | 0.4588 |

[a] Effective range parameters from Ref. 2: $a_{nn} = -17.0$ fm, $r_0 = 2.84$ fm.

[b] Effective range parameters from Ref. 14: $a_{pp} = -7.80$ fm, $r_0 = 2.75$ fm. Values given under the headings $M_1$, $M_2$, and $M_3$ refer to expression (27d) in which we use, respectively, the first four, five, and six terms in the square brackets there.

$$F_0 = e^{-\pi\eta/2}\,|\Gamma(1+i\eta)|\,\frac{M_{-i\eta,1/2}(-2iqr)}{-2iqr}$$
$$= \frac{1}{i}e^{\pi\eta/2}\left(e^{i\sigma_0}\frac{W_{-i\eta,1/2}(-2iqr)}{2qr} - e^{-i\sigma_0}\frac{W_{i\eta,1/2}(2iqr)}{2qr}\right)$$
$$= e^{\pi\eta/2}\mathrm{Im}\left(e^{i\sigma_0}\frac{W_{-i\eta,1/2}(-2iqr)}{qr}\right), \quad \text{(A1)}$$
$$G_0 = e^{\pi\eta/2}\left(e^{i\sigma_0}\frac{W_{-i\eta,1/2}(-2iqr)}{2qr} + e^{-i\sigma_0}\frac{W_{i\eta,1/2}(2iqr)}{2qr}\right)$$
$$= e^{\pi\eta/2}\mathrm{Re}\left(e^{i\sigma_0}\frac{W_{-i\eta,1/2}(-2iqr)}{qr}\right) \quad \text{(A2)}$$

[Ref. 12, p. 264 (1)–(4)]. Thus, instead of considering the integrals over $F_0$ and $G_0$ separately, obtained when Eq. (22) is substituted in Eq. (21), we need consider only the one function

$$\psi_0 = e^{i\sigma_0}e^{\pi\eta/2}\frac{W_{-i\eta,1/2}(-2iqr)}{qr} \quad \text{(A3)}$$

and then take the real and imaginary parts of the final result since $u_0^{(1)}$ is real.

From the asymptotic form of the Whittaker function $W_{\kappa,\mu}$ [Ref. 12, p. 264 (4) and p. 278 (1)], we have

$$\psi_0 \sim \frac{e^{i(qr+\sigma_0-\eta\ln 2qr)}}{qr}, \quad qr\to\infty. \quad \text{(A4)}$$

The asymptotic forms for $F_0$ and $G_0$, given in (23a) and (23b), follow directly from (A4) on taking the real and imaginary parts of $\psi_0$, as they should.

From (A1)–(A3), (17), and (22), we see that the integral required in (21) is

$$I = \int_0^\infty e^{-\gamma r}W_{-i\eta,1/2}(-2iqr)dr. \quad \text{(A5)}$$

To transform this integral to a form suitable for our calculations we use the integral representation for the Whittaker function in (A5) [Ref. 12, p. 264 (4) and p. 256 (3)],

$$W_{-i\eta,1/2}(-2iqr) = \frac{-2iqr\,e^{iqr}}{\Gamma(1+i\eta)}\times\int_0^{\infty e^{i\phi}} e^{2iqrt}t^{i\eta}(1+t)^{-i\eta}dt, \quad \text{(A6)}$$

where $0<\phi<\pi$.

Substituting (A6) in (A5) and interchanging orders of integration we have, after integration over $r$,

$$I = \frac{-2iq}{\Gamma(1+i\eta)}\int_0^{\infty e^{i\phi}}\left(\frac{t}{1+t}\right)^{i\eta}\frac{dt}{[\gamma-iq(1+2t)]^2}.$$

Making the substitution of variable

$$z = \frac{t}{1+t}$$

we obtain

$$I = \frac{-2iq}{\Gamma(1+i\eta)}\int_0^1\frac{z^{i\eta}dz}{[(\gamma-iq)-(\gamma+iq)z]^2}.$$

Writing

$$\frac{1}{[(\gamma-iq)-(\gamma+iq)z]^2} = \frac{1}{(\gamma+iq)}\frac{d}{dz}\left(\frac{1}{(\gamma-iq)-(\gamma+iq)z}-\frac{1}{(\gamma-iq)}\right) = \frac{1}{(\gamma-iq)}\frac{d}{dz}\left(\frac{z}{(\gamma-iq)-(\gamma+iq)z}\right),$$

we integrate by parts, giving

$$I = \frac{1}{\Gamma(1+i\eta)(\gamma-iq)}\times\left(1-2\eta q\int_0^1\frac{z^{i\eta}dz}{(\gamma-iq)-(\gamma+iq)z}\right). \quad \text{(A7)}$$

Making the further change of variable,

$$z = \left(\frac{\gamma-iq}{\gamma+iq}\right)s = e^{-2iy}s,$$

where

$$y = \tan^{-1}\frac{q}{\gamma}, \quad \text{(A8)}$$

we have

$$\int_0^1\frac{z^{i\eta}dz}{(\gamma-iq)-(\gamma+iq)z} = \frac{1}{(\gamma+iq)}e^{2\eta y}\times\int_0^{z_0}\frac{s^{i\eta}ds}{1-s}, \quad \text{(A9)}$$

where

$$z_0 = e^{2iy}.$$

For the integral on the right hand side of (A9) we separate the pole in the integrand at $s=1$ from the more complicated part of the integrand by writing

$$\int_0^{z_0}\frac{s^{i\eta}}{1-s}ds = \int_0^{z_0}\frac{ds}{1-s}+\int_0^{z_0}\frac{s^{i\eta}-1}{1-s}ds = \int_0^{z_0}\frac{ds}{1-s}+\int_0^1\frac{s^{i\eta}-1}{1-s}ds - \int_{z_0}^1\frac{s^{i\eta}-1}{1-s}ds. \quad \text{(A10)}$$

Of the three integrals here, the first may be evaluated directly:

$$\int_0^{z_0}\frac{ds}{1-s} = -\ln(1-z_0), \quad |\arg(1-z_0)|<\pi.$$

Furthermore,

$$1-z_0=1-e^{2iy}=-2i\,e^{i\eta}\sin\eta$$
$$=\frac{-2i\,e^{i\eta}\left(\frac{q}{\gamma}\right)}{\left[1+\left(\frac{q}{\gamma}\right)^2\right]^{1/2}},$$

from (A8). Thus

$$\int_0^{z_0}\frac{ds}{1-s}=\frac{i\pi}{2}-iy-\ln\left(\frac{2q}{\gamma}\right)+\tfrac{1}{2}\ln\left[1+\left(\frac{q}{\gamma}\right)^2\right]. \tag{A11}$$

The second integral in (A10) is most simply obtained as a limit:

$$\int_0^1\frac{s^{i\eta}-1}{1-s}ds=\lim_{\epsilon\to 0^+}\left(\int_0^1\frac{s^{i\eta}ds}{(1-s)^{1-\epsilon}}-\int_0^1\frac{ds}{(1-s)^{1-\epsilon}}\right)$$
$$=\lim_{\epsilon\to 0^+}\left(\frac{\Gamma(1+i\eta)\Gamma(\epsilon)}{\Gamma(1+i\eta+\epsilon)}-\frac{1}{\epsilon}\right)$$
$$=\lim_{\epsilon\to 0^+}\left[\frac{\frac{\Gamma(1+i\eta)\Gamma(1+\epsilon)}{\Gamma(1+i\eta+\epsilon)}-1}{\epsilon}\right]$$
$$=\psi(1)-\psi(1+i\eta) \tag{A12}$$

using l'Hôpital's rule and noting that $\psi(z)=\Gamma'(z)/\Gamma(z)$. In the last integral in (A10) we note that both limits of integration are on the unit circle. We therefore make the change of variable

$$s=e^{2i\mu}$$

and obtain

$$\int_{z_0}^1\frac{s^{i\eta}-1}{1-s}ds=\int_0^y\frac{(1-e^{-2\eta\mu})e^{i\mu}}{\sin\mu}d\mu\,. \tag{A13}$$

The integral on the right hand side of this equation has the great advantage that real and imaginary parts may be separated straightforwardly, and the imaginary part evaluated in closed form:

$$\int_0^y\frac{(1-e^{-2\eta\mu})e^{i\mu}}{\sin\mu}d\mu$$
$$=i\int_0^y(1-e^{-2\eta\mu})d\mu+\int_0^y(1-e^{-2\eta\mu})\cot\mu\,d\mu$$
$$=\frac{i}{2\eta}(2\eta y-1+e^{-2\eta y})+\int_0^y(1-e^{2\eta\mu})\cot\mu\,d\mu\,. \tag{A14}$$

From (A5), (A7), and (A9)–(A14) we then have, after observing that a number of terms cancel,

$$\int_0^\infty e^{-\gamma r}W_{-i\eta,1/2}(-2iqr)dr=\frac{1}{\Gamma(1+i\eta)(\gamma^2+q^2)}\left(\gamma+2\eta q\,e^{2\eta y}\left\{-\frac{i\pi}{2}+\frac{i}{2\eta}+\psi(1+i\eta)-\psi(1)+\ln\left(\frac{2q}{\gamma}\right)-\tfrac{1}{2}\ln\left[1+\left(\frac{q}{\gamma}\right)^2\right]\right.\right.$$
$$\left.\left.-\int_0^y(1-e^{-2\eta\mu})\cot\mu\,d\mu\right\}\right). \tag{A15}$$

Anticipating the desired separation of real and imaginary parts, we note that [Ref. 15, p. 259 (6.3.13)]

$$\mathrm{Im}\,\psi(1+i\eta)=-\frac{1}{2\eta}+\frac{\pi}{2}\frac{\cosh\pi\eta}{\sinh\pi\eta}.$$

Thus, collecting the imaginary terms in the square brackets in (A15) we have

$$\frac{-i\pi}{2}+\frac{i}{2\eta}+i\,\mathrm{Im}\,\psi(1+i\eta)=\frac{i}{2\eta}\left(\frac{2\pi\eta}{e^{2\pi\eta}-1}\right). \tag{A16}$$

The factor appearing here, $2\pi\eta/(e^{2\pi\eta}-1)$, defined as $F(\eta)$ in (27c), also arises in the overall normalization: From (A3) and (A5) we have

$$\int_0^\infty\frac{e^{-\gamma r}}{r}\psi_0 r^2dr=\frac{1}{q}e^{i\sigma_0}e^{\pi\eta/2}I=\frac{\Gamma(1+i\eta)}{q}I\cdot\frac{e^{\pi\eta/2}}{|\Gamma(1+i\eta)|}, \tag{A17}$$

and here we have

$$\frac{e^{\pi\eta/2}}{|\Gamma(1+i\eta)|}=\left(\frac{2\pi\eta}{e^{2\pi\eta}-1}\right)^{-1/2}=\frac{1}{[F(\eta)]^{1/2}}. \tag{A18}$$

From (A5) and (A15)–(A18) we thus have

$$\int_0^\infty\frac{e^{-\gamma r}}{r}\psi_0 r^2dr=i\frac{[F(\eta)]^{1/2}}{(\gamma^2+q^2)}e^{2\eta y}$$
$$+\frac{1}{[F(\eta)]^{1/2}(\gamma^2+q^2)}\left(\frac{\gamma}{q}+2\eta\,e^{2\eta y}\left\{\ln\left(\frac{2q}{\gamma}\right)-\tfrac{1}{2}\ln\left[1+\left(\frac{q}{\gamma}\right)^2\right]+\mathrm{Re}[\psi(1+i\eta)-\psi(1)]\right.\right.$$
$$\left.\left.-\int_0^y(1-e^{-2\eta\mu})\cot\mu\,d\mu\right\}\right). \tag{A19}$$

The separation into real and imaginary parts is now complete. As a check we note that the imaginary term on the right hand side of (A19) is indeed the known integral over the regular Coulomb function [Ref. 12, p. 270 (6) and p. 271 (13)].

Of significance in the expression on the right hand side of (A19) is the appearance of the factor $(\gamma^2+q^2)^{-1}$ common to both the regular and irregular parts of the integral. This factor, familiar from the amplitudes without Coulomb interaction, is thus not changed by the presence of the Coulomb interaction. In the amplitudes given in Ref. 3 this factor is simply replaced by $1/\gamma^2$, since effective range theory is supposed to be good for small $q$. However, this theory is in fact valid for $q$ up to 10 or 20 MeV, in which case $q/\gamma$ is not small. Thus we actually do not want to restrict ourselves to an expansion in $q/\gamma$. Although we use an effective range formalism, we could in principle use the phase shift calculation of $q\cot\delta_0$. With this perspective, we retain the factor $(\gamma^2+q^2)^{-1}$ and leave $y=\tan^{-1}(q/\gamma)$ as is, in the evaluation of the integral on the right hand side of (A19).

Next, in the real part of the expression on the right hand side of (A19) we have the term $\mathrm{Re}[\psi(1+i\eta)-\psi(1)]$. This term has been noted in connection with other problems involving Coulomb wave functions.[16] In the notation of Ref. 3 we have

$$\begin{aligned}\mathrm{Re}[\psi(1+i\eta)-\psi(1)] &= [\mathrm{Re}\psi(-i\eta)-\ln\eta] \\ &\quad +\ln\eta-\psi(1) \\ &= h(\eta)+\ln\eta-\psi(1). \qquad \text{(A20)}\end{aligned}$$

A number of expansions for this term may be found in the literature (Ref. 15, Sec. 6.3, pp. 258–259; Ref. 12, Sec. 1.17; and Ref. 16). Of these, the series

$$\mathrm{Re}[\psi(1+i\eta)-\psi(1)]=\eta^2\sum_{n=1}^{\infty}\frac{1}{n(n^2+\eta^2)} \qquad \text{(A21)}$$

is indeed convergent for all $\eta$, but because of the slowness of the convergence it is not very satisfactory for numerical evaluation. A series which is more useful for this purpose may be derived from it (Ref. 12, p. 45) by expanding the summand in a power series in $\eta$ and interchanging the order of the summation; giving [Ref. 15, p. 259 (6.3.17)]

$$\mathrm{Re}[\psi(1+i\eta)-\psi(1)] =\frac{\eta^2}{1+\eta^2}+\sum_{j=1}^{\infty}(-1)^{j+1}[\zeta(2j+1)-1]\eta^{2j}, \qquad \text{(A22)}$$

where

$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}, \quad \mathrm{Re}\, s>1 \qquad \text{(A23)}$$

is the Riemann zeta function. The series (A22) converges for $|\eta|<2$, as may be seen from (A23), from which we have, for large $s$,

$$\zeta(s)-1\sim 1/2^s.$$

Thus for large $j$, the terms in the sum in (A22) are approximately $\frac{1}{2}(-1)^{j+1}(\eta/2)^{2j}$. The numerical values of $\zeta(n)$ are given with great accuracy in Ref. 15, Table 23.2, p. 811. The first few terms of the expansion are

$$\mathrm{Re}[\psi(1+i\eta)-\psi(1)]=\frac{\eta^2}{1+\eta^2}+0.202\,06\eta^2-0.0369\eta^4 +0.0083\eta^6-0.002\eta^8+\cdots. \qquad \text{(A24)}$$

This series was used previously in Ref. 16. For $\eta\gg 1$ $(q\to 0)$ we have the following asymptotic expression [Ref. 15, p. 259 (6.3.19)]:

$$\mathrm{Re}[\psi(1+i\eta)-\psi(1)]\sim\ln\eta-\psi(1)+\frac{1}{12\eta^2} +\frac{1}{120\eta^4}+\cdots. \qquad \text{(A25)}$$

Thus, from (A20),

$$h(\eta)\to 0 \quad \text{for } q\to 0. \qquad \text{(A26)}$$

Finally, we consider the integral on the right hand side of (A19):

$$\begin{aligned}g(\eta,y) &\equiv \int_0^y (1-e^{-2\eta\mu})\cot\mu\, d\mu \\ &= y\int_0^1 (1-e^{-2\eta ys})\cot(ys)\,ds.\end{aligned}$$

Expanding both the exponential and the cotangent in power series, using [Ref. 15, p. 75 (4.3.70)]

$$\begin{aligned}\cot z &= \sum_{n=0}^{\infty}(-1)^n\frac{2^{2n}B_{2n}z^{2n-1}}{(2n)!} \\ &= \frac{1}{z}-\frac{z}{3}-\frac{z^3}{45}-\frac{2z^5}{945}-\frac{z^7}{4725}-\cdots \quad (|z|<\pi)\end{aligned}$$

(in which the $B_{2n}$ are the Bernoulli numbers, given in Ref. 12, Sec. 1.13 and Ref. 15, Chap. 23; $B_0=1$, $B_2=\frac{1}{6}$, $B_4=-\frac{1}{30}$, $B_6=\frac{1}{42}$, ...), we have, after integration,

$$\begin{aligned}g(\eta,y) &= -\sum_{p=1}^{\infty}\frac{(-2\eta y)^p}{p!}\sum_{n=0}^{\infty}\frac{(-1)^n 2^{2n}B_{2n}y^{2n}}{(2n)!}\frac{1}{p+2n} \\ &= (2\eta y)(1-\tfrac{1}{9}y^2-\tfrac{1}{225}y^4-\cdots) \\ &\quad -\tfrac{1}{2}(2\eta y)^2(\tfrac{1}{2}-\tfrac{1}{12}y^2-\tfrac{1}{270}y^4-\cdots) \\ &\quad +\tfrac{1}{6}(2\eta y)^3(\tfrac{1}{3}-\tfrac{1}{15}y^2-\tfrac{1}{315}y^4-\cdots) \\ &\quad +\cdots. \qquad \text{(A27)}\end{aligned}$$

It should be noted that in fact very few terms of this expansion are needed, since

$$0 \leq y < \frac{\pi}{2},$$
$$0 < 2\eta y \leq \frac{1}{\gamma R} \approx 0.15 . \quad \text{(A28)}$$

Substituting (17) and (22) in (21) we have, from (A19) and (A20) [and noting from (24) that $2\eta q = 1/R$],

$$\begin{aligned} A_c(q) &= \int u_{c,q}^{(0)*} u_0^{(1)} d^3 r \\ &= \frac{4\pi}{(q \cot\delta_0 - iq)} \int_0^\infty (q \cot\delta_0 \operatorname{Im} \psi_0 + q \operatorname{Re}\psi_0) u_0^{(1)} r^2 dr \\ &= \frac{(8\pi\gamma)^{1/2}}{(\gamma^2 + q^2)} \frac{1}{[F(\eta)]^{1/2}} \frac{1}{(q \cot\delta_0 - iq)} \\ &\times \left\{ e^{2\eta y}\left[F(\eta) q \cot\delta_0 + \frac{h(\eta)}{R}\right] + \gamma[1 + \epsilon(q)]\right\}, \end{aligned} \quad \text{(A29)}$$

where

$$\epsilon(q) = \frac{1}{\gamma R} e^{2\eta y}\left\{-\psi(1) + \ln\left(\frac{1}{\gamma R}\right) - \tfrac{1}{2}\ln\left[1 + \left(\frac{q}{\gamma}\right)^2\right] - g(\eta, y)\right\}. \quad \text{(A30)}$$

The expression for $g(\eta, y)$ is given in (A27). However, in view of (A28) we may write

$$g(\eta, y) = 2\eta y - \eta^2 y^2 - \tfrac{2}{9}\eta y^3 + \text{terms which are negligible for all } q . \quad \text{(A31)}$$

Substituting (A30) and (A31) in (A29) we then have the result given in (27)–(27d).

[1]C. A. Peridier, M. J. Leitch, J. L. Matthews, H. Jeremie, M. Irshad, and B. L. Roberts, Bull. Am. Phys. Soc. 23, 518 (1978).
[2]B. F. Gibson and D. R. Lehman, Phys. Rev. C 15, 545 (1977).
[3]A. B. Migdal, Zh. Eksp. Teor. Fiz. 28, 3 (1955) [Sov. Phys.—JETP 1, 2 (1955)].
[4]S. de Benedetti, *Nuclear Interactions* (Wiley, New York, 1964), p. 251.
[5]R. Frascaria, V. Comparat, N. Marty, M. Morlet, A. Willis, and N. Willis, Nucl. Phys. A178, 307 (1971).
[6]B. F. Gibson, Phys. Rev. B 139, 1153 (1965).
[7]G. R. Plattner, M. Bornand, and R. D. Viollier, Phys. Rev. Lett. 39, 127 (1977).
[8]D. R. Lehman, Phys. Rev. C 6, 2023 (1972).
[9]Expressions for this cross section can be found in Refs. 5 and 8.
[10]L. T. Myers, Ph.D. thesis, Georgetown University, 1978 (unpublished). We are indebted to Dr. Myers for providing us with a copy of his thesis.
[11]N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions*, 3rd ed. (Oxford University Press, London, 1965). Note Chap. III, Sec. 4, pp. 60, 61. Our functions $F_0$ and $G_0$ are denoted by $L_0$ and $K_0$ in this reference. See also, M. H. Hull, Jr. and G. Breit, in *Handbuch der Physik* (Springer, Berlin, 1959), Vol. XLI/1, pp. 408–465; M. Abramowitz, *Tables of Coulomb Wave Functions* (National Bureau of Standards, AMS 17, Washington, D.C., 1952).
[12]H. Bateman, *Higher Transcendental Functions* (McGraw-Hill, New York, 1953), Vol. I, Chap. VI.
[13]J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (Wiley, New York, 1952), Chap. II, Sec. 4, and R. G. Sachs, *Nuclear Theory* (Addison-Wesley, Cambridge, Mass., 1953), Chap. 4, Sec. 4–6; Chap. 6, Sec. 6-2.
[14]L. Heller, Rev. Mod. Phys. 39, 584 (1967). Values are those given in Table II, p. 589 for linear fit.
[15]*Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun (National Bureau of Standards, AMS 55, Washington, D.C., 1964).
[16]H. Davies, H. A. Bethe, and L. C. Maximon, Phys. Rev. 93, 788 (1954).