Unified description of inelastic pion nucleus reactions in the A^* model

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The role of nuclear A^* excitations for a unified description of pion induced reactions on nuclei in the medium energy range is discussed. As specific examples we treat the excitation of the $1^+, T=1$ (15.11 MeV) state in ^{12}C by inelastic pion scattering and compare it with the single charge exchange reaction and the radiative capture of pions in flight into the corresponding ground states of the residual nuclei. It is shown that those reactions provide further useful tools to investigate the multipole distribution of the A^* excitation strength and thus complement elastic pion scattering data.

[NUCLEAR REACTIONS $^{12}C(\pi,\pi')^{12}C(1^+, 15.11\ \mathrm{MeV})$, $^{12}C(\pi^+,\pi^0)^{12}N(\mathrm{g.s.})$, $^{12}C(\pi^+,\gamma)^{12}N(\mathrm{g.s.})$, $\Delta(3,3)$ energy region, A^* reaction mechanism.]

I. INTRODUCTION

With the operation of the new meson factories, pion nucleus scattering in the energy range of the $\Delta(33)$ resonance has received a great deal of theoretical interest. In addition to the conventional quasifree excitation models a new kind of excitation mechanism has been introduced to treat the interaction of a resonating pion with a complex nucleus.[1-5] The basic idea, common to all those models, is the creation of $(\Delta\bar{N})$ particle hole configurations, acting as doorway states for the πA reaction.

One of the most important results of such a many body approach to the πA interaction is that each nucleus is characterized by a specific pionic excitation strength distribution, i.e., a typical multipole spectrum which dominates the pion nucleus interaction in the corresponding energy range. This excitation pattern has been calculated and applied to elastic π scattering on several light nuclei [^4He,[3-5] ^{12}C,[1-3] ^{16}O (Ref. 2–4)] and also consistently to inelastic π-^{12}C scattering.[2,6] In contrast to the well known N^* resonances we prefer to denote the corresponding nuclear analogs as A^* resonances.[2]

In this paper we will use the A^* model[2] as the general framework of our discussion. The main purpose is to point out the unifying role of the A^* multipole resonances for a description of various pion induced reactions on nuclei in the medium energy range. To some extent this has already been demonstrated in Refs. 2 and 6, where we have shown that a realistic description of elastic and inelastic π-^{12}C scattering can be achieved simultaneously. In this paper we want to discuss specifically the excitation of unnatural parity states, which carry the pionic signature and which are connected also to other pionic reactions on nuclei. As an example we treat the excitation of the 1^+ state at 15.11 MeV in inelastic π-^{12}C scattering and compare it with the single charge exchange reactions (SCE) and radiative π capture in flight (RCF) on ^{12}C into the ground states of the corresponding final nuclei (^{12}N, ^{12}B), respectively. We observe a close connection between the SCE and the corresponding (π,π') excitation, which should exhibit very similar cross sections. On the other hand, we find marked differences between the 1^+ excitation and the (π,γ) reaction, which are due to the different nature of the outgoing particle. Therefore, we have a means to investigate those A^* excitations under quite different conditions simply by changing the final nuclear state and (or) the outgoing particle in the final channel. In this sense the selection of different reaction channels acts as a kind of multipole filter, suppressing or favoring certain A^* multipole excitations.

The paper is organized as follows: In Sec. II we briefly review the A^* formalism and some technical details concerning the calculation of the A^* resonances. In Sec. III we shall discuss the 1^+ excitation together with the corresponding SCE reaction. The (π,γ) reaction is studied in Sec. IV, and finally we compare the different reactions and analyze the corresponding role of the various A^* multipoles in Sec. V. The Appendix contains some formal discussion about the relation of the coupled channels method [and the distorted-wave impulse approximation (DWIA)] to the present A^*-resonance mechanism.

II. THE T MATRIX

As discussed in the Appendix the resonant part of the T matrix for π-nucleus scattering in the A^* model is given by (see Ref. 2)

$$T_{fi}^{\mathrm{res}}(\vec{k}',\vec{k}) = \sum_{J^\pi,\mu} \langle \vec{k}',\phi_f|\mathcal{L}_\pi^+|A_\mu^{*J^\pi}\rangle \times \frac{1}{\omega - \mathcal{E}_\mu^{J^\pi}} \langle \tilde{A}_\mu^{*J^\pi}|\mathcal{L}_\pi|\phi_i,\vec{k}\rangle . \quad (1)$$

Here ϕ_i (ϕ_f) denotes the initial (final) state of the target nucleus and $\vec{k}$ ($\vec{k}'$) the initial (final) momentum of the projectile. J^π and μ are the quantum numbers of the A^* resonances, $|A_\mu^{*J^\pi}\rangle$; those A^* resonances are considered as the eigenmodes of a generalized A-baryon system (here $[1\Delta, (A-1)N]$). $\mathcal{E}_\mu^{J^\pi}$ is the corresponding (complex) resonance energy and ω is the π energy in the πA c.m. system. $\mathcal{L}_\pi$ is the usual $\pi N\Delta$ transition operator[7]

$$\mathcal{L}_\pi = \frac{f_\pi^*}{\omega_m} \vec{S}^\dagger \cdot \vec{k}\, \vec{T}^\dagger \hat{\phi}_\pi . \quad (1a)$$

The differential cross section inthe πA c.m. system is then given by

$$\frac{d\sigma}{d\Omega} = \left(\frac{M_T}{4\pi E_c}\right)^2 \frac{k'}{k} \overline{\sum_i} \sum_f |T_{fi}|^2 , \quad (2)$$

where M_T denotes the target mass and $E_{\mathrm{c.m.}}$ the πA c.m. energy.

The technical procedure for the calculation of those A^* resonances was similar to that discussed in Refs. 2 and 6. Therefore we will not go into detail, but mention only the main points. We used a 10 $\hbar\omega$ basis of $(\Delta\bar{N})$ states and we explicitly calculated the coupling of these doorway states to the elastic channel, formally described by the exchange matrix element of the one-pion exchange (OPE) ΔN interaction. (In addition we have included correlated ρ exchange and the direct ΔN interactions, with the same parameters as in Ref. 2.)

The influence of the other channels was taken into account by a damping width, adjusted to reproduce the integrated elastic cross sections.[2,6] For the $\pi N\Delta$ transition operator $\mathcal{L}_\pi$ we included nonstatic corrections as described in Ref. 6. Therefore, for the present calculation, the input was fixed by the calculations of elastic scattering. Clearly, for inelastic reactions we need as additional information the wave function of the final nuclear state $|\phi_f\rangle$. This was taken from a (1p1h) random-phase approximation (RPA) calculation of Ref. 8, yielding a similar wave function, as quoted by Gillet *et al.*[9]

Finally in the present treatment we have neglected the nonresonant background interactions, which seems to be justified in the resonance region from the satisfactory description of elastic as well as inelastic πA scattering.[2,6] In fact, using standard optical model amplitudes for the nonresonant part of the T matrix, their effect on the cross sections is of the same order as present uncertainties in the calculation of the resonant amplitude. They are however, more important far outside the proper resonance region.

III. INELASTIC PION SCATTERING AND SINGLE CHARGE EXCHANGE INTO ISOBARIC ANALOG STATES

The connection between these two types of reactions on ^{12}C is schematically shown in Fig. 1. According to the A^* assumption, those three processes

$$^{12}\mathrm{C}(\pi,\pi)^{12}\mathrm{C}(1^+, 15.11) , \quad (3a)$$

$$^{12}\mathrm{C}(\pi^+,\pi^0)^{12}\mathrm{N}\ (1^+,\mathrm{g.s.}) , \quad (3b)$$

$$^{12}\mathrm{C}(\pi^-,\pi^0)^{12}\mathrm{B}\ (1^+,\mathrm{g.s.}) , \quad (3c)$$

are dominated by the same mechanism, the excitation of the intermediate A^* resonances and their subsequent decay into the specific final channel, as indicated in Fig. 1. Assuming isospin invariance, the wave function for the different nuclear final states are the same (up to Coulomb corrections). Therefore, we expect the cross section to be very similar—if not the same—for the three processes. Of course, there are slight differences, due to Coulomb energy differences in the nuclear states involved and the mass difference of the charged and neutral pions, respectively. However, those purely kinematical corrections are found to be small. Therefore, we give in Fig. 2 only the differential cross sections for the 1^+ excitation in inelastic π-^{12}C scattering for several energies in the resonance region.

From those results we can draw the following conclusions:

(i) The 1^+ state is much more weakly excited than the well known excitations of the low lying 2^+ or 3^- states,[6] the integrated cross sections being of the order of 0.3 mb for the 1^+ excitation, compared to about 5–15 mb for 2^+ and 3^- excitations, respectively. Experimentally, only very scarce information is available about this 1^+ excitation. In Fig. 2 we show experimental data points from Binon *et al.*[10] However, in their measurement they could not resolve this 1^+ excitation uniquely, so that those numbers are probably too large. There is a single measurement at SIN[11] of the 1^+ cross section at a pion energy of $T_\pi = 148$ MeV and at an angle of $\theta_{\mathrm{Lab}} = 58°$. A value of $d\sigma$ of about 40 μb was observed which roughly agrees with our calculated value of 34 μb.

(ii) The most conspicuous feature of the differential cross section, however, is the fact that it vanishes for $\theta = 0°$ and $\theta = 180°$. This is not accidental, but of purely geometric origin. Without going into a detailed derivation of the T matrix

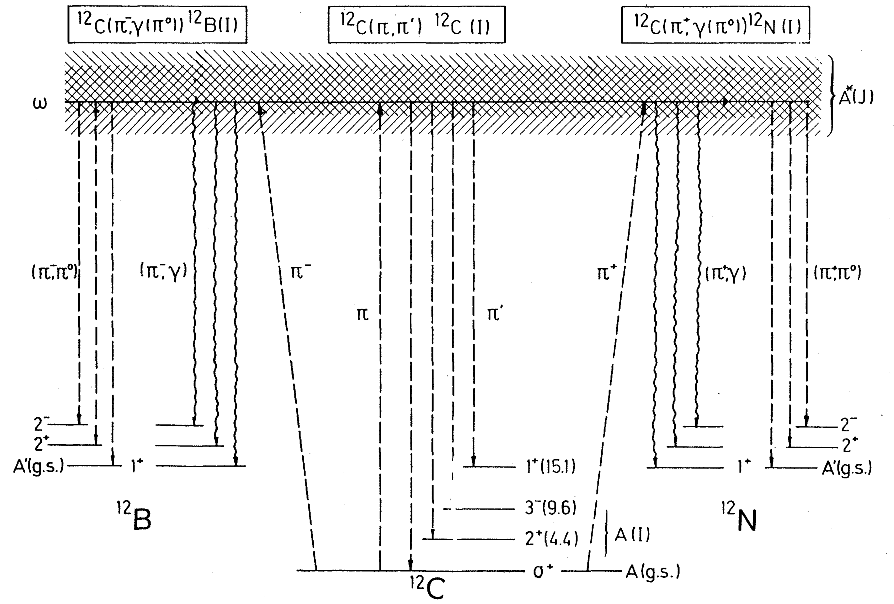

FIG. 1. Schematic picture for the A^* reaction mechanism of the 1^+ excitation in inelastic π-^{12}C scattering [Eq. (3a)] in the single charge exchange reaction [Eq. (3b)] and in radiative pion capture in flight [Eq. (3c)].

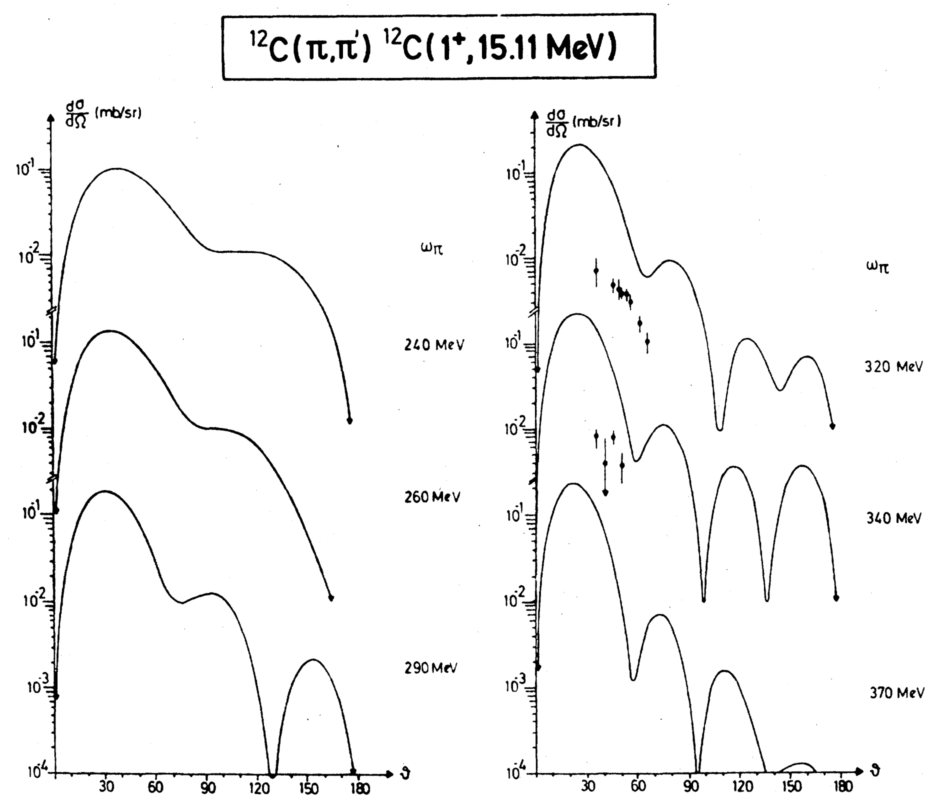

FIG. 2. Differential cross sections for the excitation of the 1^+, $T=1$ (15.11 MeV) state in inelasitc π-^{12}C scattering. Experimental data for $\omega=240$–370 MeV from Ref. 10 (see text).

we may give a simple explanation. Let I_f denote the angular momentum of the final nuclear state and $\lambda_i(\lambda_f)$ the multipolarity of the incoming (outgoing) pion; if we start from a 0^+ target nucleus, λ_i has to be equal to the angular momentum J^π of the A^* resonance, and parity conservation for the (π,π') reaction requires $(\lambda_f+\lambda_i+I_f)$ to be odd for a final nuclear state of unnatural parity. However, from angular momentum conservation the T matrix contains a vector coupling coefficient

$$\begin{bmatrix} \lambda_i & \lambda_f & I_f \\ -m_i & m_f & m_f \end{bmatrix} .$$

For forward (backward) scattering we then have $m_i = m_f = 0$. This implies $(\lambda_i+\lambda_f+I_f)$ to be even in contradiction to parity conservation.

This purely geometric consideration shows that $d\sigma/d\Omega$ for the 1^+ excitation has to be zero for forward and backward scattering. Obviously this conclusion generally holds for the excitation of unnatural parity states in inelastic π scattering from a 0^+ ground state. Moreover it is impossible to excite a 0^- state in a (π,π') reaction. It should be stressed that those conclusions do not depend on the reaction mechanism; they are completely general since they are based uniquely on angular momentum and parity conservation. Finally it should be noted that for the 1^+ excitation one can easily verify from the above discussion that $\lambda_i = \lambda_f$ holds. This is similar to elastic scattering $(\lambda_i = \lambda_f)$ except that the 0^- partial wave cannot contribute to the 1^+ excitation. This restricted interference pattern is responsible for the strongly diffractive structure of the cross sections of Fig. 2, a phenomenon which is also observed in the case of elastic scattering[2,6] [but is by far not as pronounced for the well known 2^+ (4.4 MeV) and 3^- (9.6 MeV) excitations of ^{12}C(Refs. 2 and 6)]. Nevertheless the multipole excitation strength distribution, shown in the upper part of Fig. 4, is distinctly different from that of elastic scattering (see Ref. 2) [and of the 2^+ and 3^- excitations of ^{12}C (see Ref. 12)].

Summarizing, we can conclude that in a (π,π') reaction the different structure of various excited nuclear states leads to a kind of multipole filter, which weights the A^* multipole resonances in a distinct and characteristic manner. Therefore a consistent comparison of several reaction channels may finally help to identify present uncertainties in the detailed description of the A^* multipole strength distribution.

IV. RADIATIVE π CAPTURE IN FLIGHT (RCF)

The connection of the (π,γ) reaction to the previously discussed processes is also indicated in Fig. 1. The corresponding T matrix is easily obtained from Eq. (1) by changing the matrix element for the decay of the A^* resonances. That means the final channel state consists now of a photon and the ground state of ^{12}N or ^{12}B, respectively. The $\gamma N\Delta$ transition operator has been chosen as

$$\mathcal{L}_\gamma = f_\gamma^* \vec{S}^\dagger (\vec{k}_\gamma \times \vec{\epsilon}) T_0^\dagger . \tag{4}$$

This form is the leading term of a nonrelativistic reduction of the $\gamma N\Delta$ vertex.[13,14] $S^\dagger$ $(T^\dagger)$ denote the spin (isospin) transition operators and are the same as those contained in $\mathcal{L}_\pi$. $\vec{k}_\gamma$ denotes the photon momentum and $\vec{\epsilon}$ the photon polarization. The $\gamma N\Delta$ coupling constant has been chosen so as to fit the dominant M_{1+} multipole in photo-pion production[13,15,16]: $f_\gamma^* = 0.163$ fm.

The differential cross sections for some energies in the Δ energy range are shown in Fig. 3. Clearly, the cross section ranges now in the μb

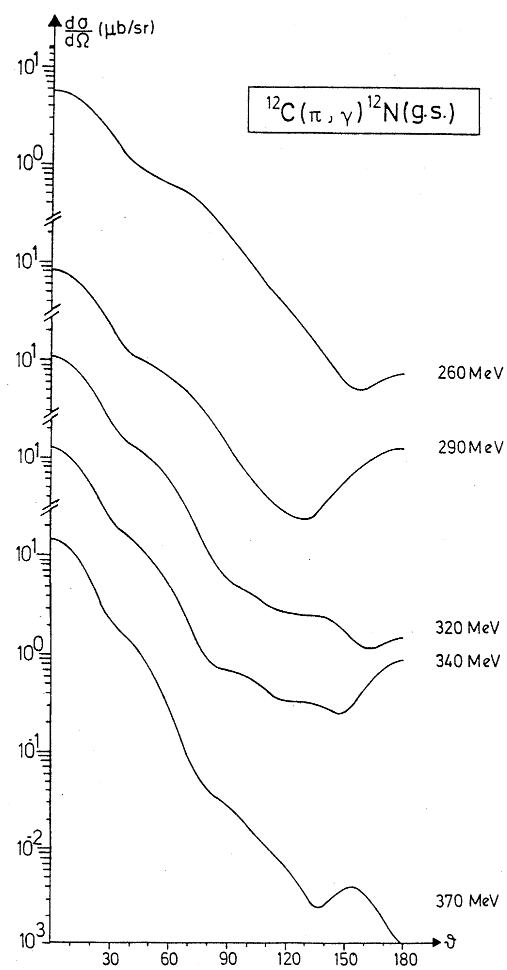

FIG. 3. Differential cross sections for radiative π capture on ^{12}C into the ground state of ^{12}N.

region (compared to inelastic πA scattering). This magnitude, however, is essentially determined by the ratio of the $\gamma N\Delta$- and $\pi N\Delta$-coupling constants. The most interesting feature, however, is seen in the angular distribution of the outgoing photon; the differential cross section remains rather flat if we compare it to the corresponding 1^+ excitation discussed in the previous section (see Fig. 2). In our calculation we assumed isospin invariance; thus from the nuclear point of view, the final states $^{12}C(1^+, 15.11$ MeV) and ^{12}N(g.s.) [^{12}B(g.s.)] are identical. Therefore the striking difference observed in the present calculation is exclusively due to the different nature of the outgoing particles, i.e., the pion and the photon, respectively. This will be discussed in more detail in the next section.

Finally we would like to point out that our results on RCF have been obtained by a strict resonance assumption ignoring nonresonant background contributions.[13] In the case of (π,γ) [or (γ,π)] reactions the importance of the nonresonant mechanism is less well established. However, from present experience, the relatively unimportant E_{1+} electric quadrupole part of $\mathcal{L}_{\gamma N\Delta}$ can be safely neglected. Therefore, this calculation should be regarded as an exploratory one, giving some reasonable idea of what can be expected. Furthermore, it is thought to demonstrate the universality of the A^* mechanism to describe various π-induced reactions.

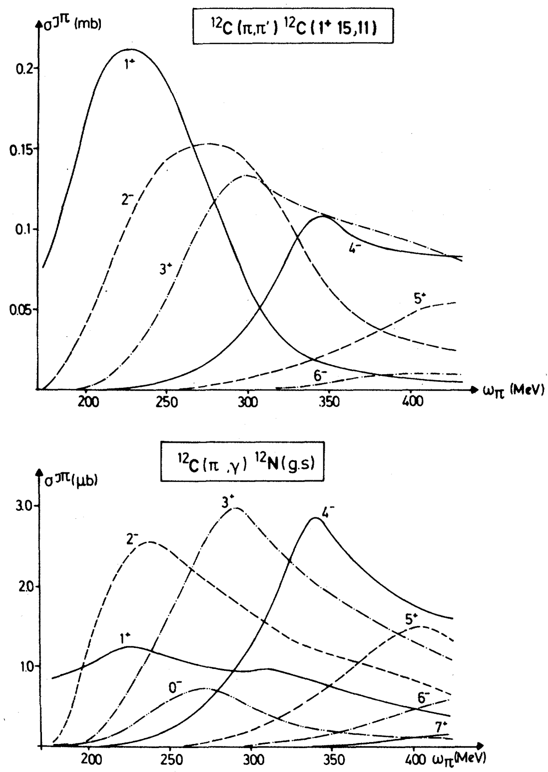

FIG. 4. Partial contributions σ^{J^π} of an A^* (J^π) resonance to the total cross section for the 1^+ excitation of ^{12}C in (π,π') (SCE) and RCF.

V. COMPARISON

In the last two sections we have presented our results for the 1^+ excitation (SCE) and the coherent RCF on ^{12}C. Although both processes are considered to be dominated by the excitation of the same A^* resonances and although the same nuclear states are involved, we observed a quite different behavior of the corresponding differential cross sections. The reason for this phenomenon is the different nature of the outgoing particles, i.e., of the pion (0^-) and the photon (1^-), respectively.

To understand this difference we refer to Table I. The first row contains the various multipolarities λ_i of the incoming pion which are (for a 0^+-target) identical to the angular momentum of the intermediate A^* resonance. In the second and third rows the possible multipolarities λ_f are shown for the outgoing pion (λ_f^π) and photon (λ_f^γ), respectively, with the residual nucleus left in a 1^+ state.

Obviously, in RCF each multipole resonance can decay by emitting an outgoing photon of, generally, several multipolarities λ_f^γ, whereas for the pionic 1^+ excitation the selection rules are by far more restrictive. In detail, for a given value of λ_i, the pion has to leave the nucleus as an $M(\lambda_i)$ pion, whereas the photon can be of the $E(\lambda_i - 1)$ type.

TABLE I. Multipolarities for the excitation λ_i and decay λ_f of an A^* resonance of a given multipolarity $J^\pi = \lambda_i$; in the (π,π') [Eq. (3a)] or SCE [Eq. (3b)] and RCF [Eq. (3c)], respectively (see text).

		$J^\pi = \lambda_i$					
		0^-	1^-	2^-	3^+	4^-	···
λ_f	λ_f^π		1^+	2^-	3^+	4^-	···
	λ_f^γ	1^-	0^+, 1^+, 2^+	1^-, 2^-, 3^-	2^+, 3^+, 4^+	3^-, 4^-, 5^-	···

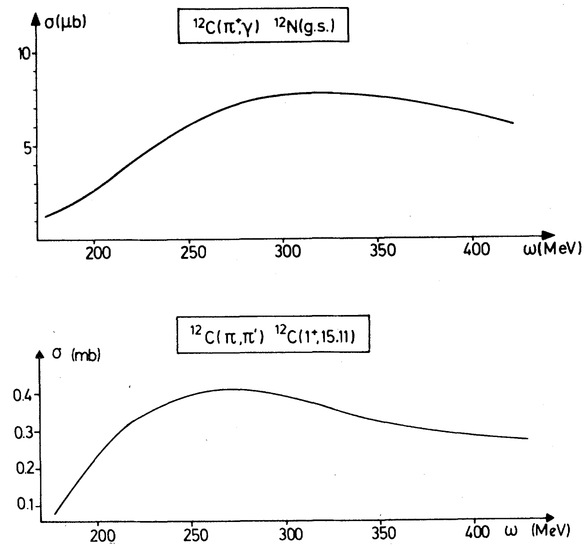

FIG. 5. Total cross section for the 1^+ excitation in (π,π') and in the RCF, respectively.

Consequently we generally expect a stronger excitation of higher A^* multipoles in the RCF as for the 1^+ excitation. Furthermore, this different importance of a given A^* resonance for the two reactions will also be reflected in the structure of the corresponding differential cross sections.

This is particularly exemplified in Fig. 4 where the partial contributions σ^{J^π} of the various multipole resonances to the total cross section are shown. We note that the 0^- resonance, which did not contribute to the 1^+ excitation, gives some contribution in the RCF; however, it plays only a minor role. The $J^\pi = 1^+$ resonance, which dominates inelastic pion scattering into the 1^+ state at low energies, is less important for RCF (in the same energy range), whereas the 2^- and even the 3^+ resonances play a more important role in RCF. As a general feature, we note that the higher multipolarities tend to be more important for the RCF. As a consequence the differential cross sections are relatively flattened out for the RCF (see Figs. 2 and 3).

Moreover, we find that the total RCF excitation strength distribution (see Fig. 4) is shifted towards higher excitation energies. This is also clearly seen in Fig. 5 where the integrated cross sections are shown for the two reactions. It should be added that this striking difference between reactions with pions and photons, respectively, appears to be quite general. Similar effects are also found by a comparison of elastic pion scattering with the photoproduction of neutral pions into the target ground state (see Ref. 17).

VI. SUMMARY AND CONCLUSIONS

In Refs. 2 and 6 it has been shown that the A^*-resonance spectrum dominates elastic and inelastic pion nucleus scattering in the region of the $\Delta(3,3)$ resonances. In this paper we apply the same reaction mechanism to different pion induced reactions; as particular examples, we compare inelastic pion scattering, single charge exchange reactions, and radiative pion capture in flight leading to the equivalent final states of the residual nucleus. We found that the various A^* resonances behave quite differently for the SCE and the equivalent RCF reaction. This leads us to the observation that, by varying the nature of the outgoing boson, different components of the A^* spectrum contribute differently to the reaction cross section. A similar sensitivity of the reaction cross section is also observed for purely pionic reactions leading into different states of the residual nucleus. In this sense the nucleus provides us with a whole laboratory for the study of those A^* excitations. The selection of different final channels acts here as a sort of multipole filter, favoring and suppressing certain multipole resonances, respectively.

In view of the complexity of the A^* spectrum this sensitivity of the various reaction cross sections on rather subtle details of the multipole excitation strength distribution offers an important tool for reliably disentangling the structure of the multipole resonances. It is obvious that systematic and consistent data are required for such an analysis and, furthermore, that a clean separation of the various final states is indispensible for such an analysis.

Obviously, the A^* spectrum is much more complex than the elementary N^* spectrum simply because the nuclear many body degrees of freedom do couple to the internal excitations of an individual nucleon. Unfortunately, however, the various A^* resonances are not sufficiently displaced from each other so that we are faced with a rather complicated excitation spectrum of broad overlapping resonances of different multipolarity. In order to disentangle this complexity and to identify reliably the excitation strength distribution, we can make use of the variety of low lying nuclear excitations and/or of the different decay channels as indicated in the present note.

As a byproduct we observe that the excitation of states of unnatural parity (such as $1^+, 2^-, 3^+$) in inelastic pion scattering leads to an interesting structure: The corresponding cross sections have to vanish for forward and backward angles—irrespective of the reaction mechanism and of possible distortion effects. This feature is of purely geometrical origin and depends only on the scalar nature of the pion and the unnatural parity of the final nuclear state.

Finally our results call for the use of electromagnetic probes (photons, inelastic electron scattering) for the investigation of the nuclear many baryon system. With photons one can investigate certain properties of the A^* system which are of minor importance for a pionic reaction. Moreover, real (or virtual) photons are able to excite the A^* states of electric multipolarity whereas with pions only the magnetic multipolarities are seen. Therefore the study of electromagnetic processes is essential for a detailed investigation of the A-baryon system as it leads to complementary and new information, compared to purely pionic reactions.

APPENDIX: COUPLED CHANNELS APPROACH FOR THE T MATRIX

There has been some discussion in the literature about the description of the SCE reaction in the Isobar Doorway Model (IDM) (Refs. 18 and 19) as an explicit coupled channels problem,[19] which also applies to inelastic scattering. Therefore we will show in the following that the coupled channel T matrix leads to precisely the form of Eq. (1). Of course, consistent with the assumption of the A^* model, we will neglect nonresonant interacts ("Δ dominance assumption").

To make this connection we will also use the general projection operator formalism of the IDM.[1] That means we divide the whole Hilbert space for πA scattering into the following subspaces:

(i) P *space*. It contains the elastic channel $|\phi_0\rangle$ and a specific inelastic channel $|\phi_1\rangle$. Accordingly we have

$$P=P_0+P_1\,. \tag{A1}$$

(ii) D *space*. It contains all $[\Delta,(A-1)N]$ doorway states.

(iii) R space. It contains all the remaining unspecified channels.

We denote the corresponding orthogonal projection operators by P_0, P_1, D, and R, respectively; XHY is abbreviated by H_{XY}.

Before going into detail let us briefly discuss the resonant part of the T matrix of the IDM, which has been derived in Ref. 1 using the doorway hypothesis:

$$T_{fi}=\left\langle\phi\left|H_{PD}\frac{1}{\omega-\mathcal{H}_{DD}}H_{DP}\right|\phi_i\right\rangle\,, \tag{A2}$$

where $\mathcal{H}_{DD}$ is the Hamiltonian for the $[\Delta,(A-1)N]$ system including the coupling to all the various decay channels in P and R space

$$\mathcal{H}_{DD}=H_{DD}+W_{DD}^{(\sigma)}+W_{DD}^{(1)}+\tilde{W}_{DD} \tag{A3}$$

with

$$W_{DD}^{(\nu)}=H_{DP_\nu}\frac{1}{\omega^+-H_{P_\nu P_\nu}}H_{P_\nu D}\,, \tag{A4a}$$

$$\tilde{W}_{DD}=H_{DR}\frac{1}{\omega^+-H_{RR}}H_{RD}\,. \tag{A4b}$$

If we denote the eigenmodes of $\mathcal{H}_{DD}$ by $|A_\mu^*\rangle$, the corresponding eigenenergies by $\mathcal{E}_\mu$, and identify H_{PD} with $\mathcal{L}_{\pi N\Delta}$, we immediately recognize that this formal expression of the IDM T matrix is equivalent to that of Eq. (1).

Let us now discuss the coupled channel approach and its relation to Eq. (A2). It is easily shown[19] that with the above projection operators the Schrödinger equation for πA scattering can be transformed into the following set of coupled equations:

$$(\omega-\mathcal{H}_{00})P_0\psi=\mathcal{H}_{01}P_1\psi\,, \tag{A5a}$$

$$(\omega-\mathcal{H}_{11})P_1\psi=\mathcal{H}_{10}P_0\psi\,, \tag{A5b}$$

with

$$\mathcal{H}_{ij}=H_{ij}+H_{iD}\frac{1}{\omega^+-H_{DD}-\tilde{W}_{DD}}H_{Dj}\quad(i,j=0,1)\,. \tag{A5c}$$

In Eq. (A5c) we have further introduced $H_{P_iP_j}=H_{ij}$. Neglecting nonresonant interactions we have $H_{ij}=0$ for $i\neq j$. From Eqs. (A5a) and (A5b) the T matrix is given by[20]

$$T_{10}=\langle\chi_1|\mathcal{H}_{10}|P_0\psi\rangle\,. \tag{A6}$$

$P_0\psi$ is the exact solution for elastic scattering; it is a solution of

$$\left(\omega-H_{00}-H_{0D}\frac{1}{e_D-W_{DD}^{(1)}}H_{D0}\right)P_0\psi=0 \tag{A7}$$

with $e_D=\omega^+-H_{DD}-\tilde{W}_{DD}$.

χ_1 describes the "elastic" scattering on the final nuclear state (without coupling to channel $|0\rangle$); it is a solution of

$$[\omega-H_{11}-H_{1D}(1/e_D)H_{D1}]\chi_1=0\,. \tag{A8}$$

Inserting $P_0\psi$ and χ_1 from Eqs. (A7) and (A8) we obtain for the T matrix

$$T_{10}=\left\langle\phi_1\right|\left(1+H_{1D}\frac{1}{e_D}H_{D1}\frac{1}{\omega^+-H_{11}-H_{1D}\frac{1}{e_D}H_{D1}}\right)H_{1D}\cdot\frac{1}{e_D}H_{D0}$$
$$\times\left(1+\frac{1}{\omega^+-H_{00}-H_{0D}\frac{1}{e_D-W_{DD}^{(1)}}H_{D0}}H_{0D}\frac{1}{e_D-W_{DD}^{(1)}}H_{D0}\right)\left|\phi_0\right\rangle\,, \tag{A9}$$

where ϕ_0 and ϕ_1 are the homogeneous solutions of Eqs. (A7) and (A8), respectively. To proceed further we need the identity

$$D\frac{1}{\bar{e}_D}D+D\frac{1}{\bar{e}_D}DHP_i\,\frac{1}{\omega-H_{ii}-H_{iD}\dfrac{1}{\bar{e}_D}H_{Di}}\,P_iHD\frac{1}{\bar{e}_D}D=D\frac{1}{\bar{e}_D-W_{DD}^{(i)}}D\,, \tag{A10}$$

where $W_{DD}^{(i)}$ has been defined in Eq. (A4a). We now apply this identity to Eq. (A8) with $\bar{e}_D=e_D$; the resulting equation can again be rewritten with the same identity of Eq. (A10) if we identify $\bar{e}_D=e_D-W_{DD}^{(1)}$. We then finally obtain

$$T_{10}=\left\langle\phi_1\middle|H_{1D}\,\frac{1}{\omega-H_{DD}-W_{DD}^{(0)}-W_{DD}^{(1)}-W_{DD}}\,H_{D0}\middle|\phi_0\right\rangle \tag{A11}$$

which exactly agrees with the expression of Eq. (A3) and which is equivalent to the amplitude of the A^* model, as discussed below Eq. (A4).

From this derivation it is clear that those effects, which are occasionally referred to in the literature as distortions, pion rescattering in the final channel, or final state interactions, are naturally included in the A^* Hamiltonian. Therefore, if we calculate the properties of the A^* resonance, those final state interactions are included (of course in practical calculations only to the extent that the corresponding effects are incorporated either explicitly in the configuration space or implicitly by appropriate effective interaction operators). Finally let us briefly comment on the technical procedure used to calculate the A^* spectrum in the present paper and its connection with the present derivation. In Sec. II we made two approximations: Firstly, we explicitly took into account the coupling to the elastic channel, only; secondly, we represented the $[\Delta, (A-1)N]$ system by (1p1h) configurations $(\Delta\bar{N})$ neglecting more complicated $(n\mathrm{p}, n\mathrm{h})$ structures.

In the notation used in this appendix the first approximation corresponds to an evaluation of all the matrix elements of $W_{DD}^{(0)}$ in a given restricted configuration space. The influence of the remaining channels has been taken into account only insofar as the matrix elements of $\tilde{W}_{DD}+W_{DD}^{(1)}$ are replaced by a state independent damping width. For the calculations of elastic scattering[2,6] this procedure turned out to be an effective way to sum up the influence of those complicated channels. Since the inelastic cross sections are only a small fraction (about 1% or 2%) of the elastic cross section at resonance,[10] the inelastic width is expected to be small compared to the elastic width—the latter being of the order of several hundred MeV. Therefore it appears justified to also incorporate the effects of $W_{DD}^{(1)}$ into this damping width.

Concerning the choice of the configuration space it seems technically impossible to enlarge it for the explicit inclusion of (npnh) configurations with $n>1$. Already the $(\Delta N\bar{N}\bar{N})$ states (with $n=2$) would blow up the configuration space to an unmanageably large size. However, for specific inelastic channels, those (2p2h) configurations might be quite important via the transitions $(\Delta N\bar{N}\bar{N})\to(N\bar{N})$ or $(\Delta N\bar{N}\bar{N})\to(NN\bar{N}\bar{N})$. As mentioned above, there is certainly no way to include rigorously those complicated configurations in an explicit calculation. To approximately include their effects, one could introduce distorted waves to write the T matrix of Eq. (A1) as

$$T_{10}=\left\langle\chi_1\middle|H_{1D}\,\frac{1}{\omega^+-H_{DD}-W_{DD}^{(0)}-W_{DD}}\,H_{D0}\middle|\phi_0\right\rangle. \tag{A12}$$

Here, we may obtain the distorted wave χ_1 from a potential model. This, however, might lead to serious double counting problems. An appropriate approximation, however, can be derived from Eqs. (A6), (A7), and (A8). If we also use in Eq. (A6) the homogeneous solution (distorted wave) χ_0 of Eq. (A5a), instead of $P_0\psi$ (this amounts to neglecting the presumably weak coupling of the elastic channel to the inelastic one), we obtain, after applying Eq. (A10),

$$T_{10}=\left\langle\phi_1\middle|\left(1+H_{1D}\frac{1}{e_D}H_{D1}\frac{1}{\omega^+-\mathcal{H}_{11}}\right)\times H_{1D}\,\frac{1}{\omega^+-H_{DD}-W_{DD}^{(0)}-\tilde{W}_{DD}}\,H_{DD}\middle|\phi_0\right\rangle. \tag{A13}$$

Obviously, Eq. (A13) is very similar to Eq. (A11); however, we have eliminated the coupling to the inelastic channel from the A^* Hamiltonian $\mathcal{H}_{DD}$ [see Eq. (A4)] and instead treat this coupling as a final pion rescattering, as we can see using the expansion

$$1+H_{1D}\frac{1}{e_D}H_{D1}\frac{1}{\omega^+-\mathcal{H}_{11}}=\sum_{n=0}^{\infty}\left(H_{1D}\frac{1}{e_D}H_{D1}\frac{1}{\omega^+-H_{11}}\right)^n \tag{A14}$$

in Eq. (A13). In a microscopic calculation of this rescattering mechanism we can have a perturbation scheme to take into account systematically the effects of the isobaric (2p2h) configurations without including them explicitly in the calculation of the doorway propagator. The convergence of this expansion is presently under study. Of course, for an outgoing photon (as in the RCF of Sec. IV) this final rescattering can be safely neglected, facilitating the evaluation of the amplitude from this point of view.

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