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Calculated spectra of antineutrinos from the fission products of ^{235}U, ^{238}U, and ^{239}Pu, and antineutrino-induced reactions

F. T. Avignone III and Z. D. Greenwood
Department of Physics, University of South Carolina, Columbia, South Carolina 29208
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The theoretical spectra of antineutrinos from the beta decays of the products of fissioning ^{235}U, ^{238}U, and ^{239}Pu were recalculated using recent compilations of the level structure, beta branching ratios, and fission yields. In addition, a very recent semiempirical mass formula, particularly designed to achieve local accuracy on the (N,Z) plane, was used to calculate the beta Q values of isotopes of unknown decay schemes. Recent decay systematics, far from the beta stability line, have encouraged us to assume that most of the levels of the isotopes of a given Z, differing by $\Delta N = 2$, are very similar in order and in energy. The beta spectrum from equilibrium, thermal fission products of ^{235}U was calculated and compared with existing experimental data. Theoretical predictions are presented for the inverse beta decay of the proton, the neutral current disintegration of the deuteron, and the excitation of ^7Li by antineutrinos. Existing data from the elastic scattering of electrons by antineutrinos are reanalyzed with the present antineutrino spectra. The result is that these data agree with the predictions of the Weinberg-Salam model with $(0.24 < \sin^2\theta_W < 0.31)$ and do not agree with the predictions of the Feynman-Gell-Mann theory.

[RADIOACTIVITY, FISSION ^{235}U, ^{238}U, ^{239}Pu; antineutrino and beta spectra calculated in secular equilibrium. $\bar{\sigma}$ for $\bar{\nu}_e(p,n)\beta^+$, $\bar{\nu}_e(d,pn)\bar{\nu}_e$, $\bar{\nu}_e(^7Li, ^7Li^*)\bar{\nu}_e$, and $\bar{\nu}_e(e^-,e^-)\bar{\nu}_e$.]

INTRODUCTION

There is a strong renewed interest in the accurate knowledge of the spectra of antineutrinos from nuclear reactors particularly for the interpretation of experiments involving reactor antineutrinos. Fundamental questions concerning the nature of the weak interaction, which might well be resolved by such experiments, include the measurement of the cross section of the neutral current disintegration of the deuteron by electron antineutrinos,[1] $\bar{\nu}_e(d,pn)\bar{\nu}_e$, the elastic scattering of electrons by antineutrinos,[2] the direct search for neutrino oscillations over distances of several meters,[3] and the antineutrino excitation of nuclei.[4-6] The nuclear excitation experiment would be a direct test of the hypothesis that the axial neutral current is simply related to the axial charged current.

Our earlier attempts to calculate reactor antineutrino spectra were based on the spectrum from the beta decay of the fission products of ^{235}U alone,[7-9] and in addition, even that most recent attempt was made just prior to the appearance of a complete new compilation of fission yields by Crouch[10] and also a new semi-empirical mass formula by Jänecke,[11] which should predict binding energies far more accurately over small regions of the (N, Z) plane. This feature should allow more reliable prediction of the beta decay Q values of the nuclides with unknown decay properties, which lie far from the line of beta stability. One very strong motivation of the present work was the 20% to 30% disagreement at high energies between our most recent attempt, which included the spectra from the products of ^{238}U and ^{239}Pu,[12] and that of Borovoĭ *et al.*[13] Investigating this disagreement, we discovered that while the main difference lies in the method of selecting average values for the beta branching ratios and nuclear level structure of the unknown nuclides, there were also differences in the spectra from nuclides of known decay properties. This has caused us to completely reexamine the assumptions made in approximating decay properties and level structures of unknown nuclides. In addition, we have separately calculated the portions of the antineutrino spectrum from the products of ^{235}U with known and unknown decay schemes. Finally, an independent calculation of antineutrino spectra by Davis *et al.*[14] has recently appeared in the literature in which the assumption of a constant reduced matrix element for Gamov-Teller decays was made. The branching ratios of beta decays with unknown level schemes were then estimated on this basis. The spectrum given in Ref. 14 has significantly fewer antineutrinos at energies above 6 MeV than any of our previous results and the results of Borovoĭ *et al.*[13] and also predicts a conjugate beta spectrum of ^{235}U fission products with 135% fewer beta particles at 8 MeV than the most recent of the three well known experiments.[15-17]

A recent analysis by Rudstam and Aleklett[18] has also emphasized the possibility that our earlier work may have underestimated the feeding to the higher energy excited states in nuclei far from stability. No numerical values for their spectra are given in Ref. 18; however, our present results appear to be in agreement with their plotted spectra. Also, recently theoretical predictions of antineutrino-induced reactions, based on the spectra given in Ref. 13, were made by Fayans *et al.*[19]

ANTINEUTRINO SPECTRUM CALCULATION

In principle, one could calculate the antineutrino spectrum from the decays of fission fragments exactly, if one had exact knowledge of the primary fission yield $Y(ZA)$, the beta branching ratios $b_j(ZA)$, the excited state energies of daughter nuclides E_{zj}, and E_{oj}, the beta end point energies. In this case the spectrum is expressed as

$$N(E) = \sum_{Z,A,j} K(ZA) b_j(ZA) P_j(Z,A,E_{oj},E_\beta)\,, \qquad (1)$$

where $P_j(Z,A,E_{oj},E_\beta)$ is the normalized, allowed, Coulomb corrected antineutrino spectrum with end point energy E_{oj}, and $b_j(ZA)$ is the beta branching ratio of the jth beta branch in the isotope (Z,A). The weighting factor $K(AZ)$ is the sum over Z of the independent fission yields of all of the isotopes in the chain of mass A, up to and including Z, and is given by

$$K(ZA) = \sum_{z_0}^{z} Y(Z'A)\,, \qquad (2)$$

where z_0 is the lowest value of Z which is directly produced by binary fission in the mass chain A. Equations (1) and (2) are of course applicable only in the case that all fission products are in secular equilibrium as they approximately are in a stable nuclear reactor during normal operating conditions.

The normalization of the spectra at each energy point from each branch requires a very large number of evaluations of the Fermi function. This can be easily seen from the general form of $P_j(ZAW)$ which is given by

$$P_j(ZAW) = \frac{F(ZAW)(W^2-1)^{1/2}(W_0-W)^2 W}{\int_1^{W_0} F(ZAW)(W^2-1)^{1/2}(W_0-W)^2 W\,dW}\,, \qquad (3)$$

where $W = (E_0 - E_{\bar{\nu}} + m_0c^2)/m_0c^2$ and W_0 is the maximum value of W. The Fermi function can be approximated by[9]

$$F(ZAW) = \frac{4(1+s/2)}{[\Gamma(3+2s)]^2}\left[\frac{2R(A)}{\hbar/m_0c}\right]^{2s} \frac{2\pi y}{1-e^{-2\pi y}} \times \{\tfrac{1}{4}[W^2(1+4(Z/137)^2)-1]\}^s\,, \qquad (4)$$

where $s = [1-(Z/137)^2]^{1/2} - 1$, $y = ZW/137\eta$, $R(A)$ is the radius of the nucleus which is approximated by $R(A) = (1.2\times10^{-13})A^{1/3}$ cm, and η is the momentum of the beta particle given by $\eta^2 = (W^2-1)$.

Let us now define

$$BK(Z,A) = \frac{4(1+s/2)}{[\Gamma(3+2s)]^2}\left[\frac{R(A)}{\hbar/m_0c}\right]^{2s} \frac{2\pi Z}{137} \qquad (5)$$

and

$$g(W) = \frac{W}{1-e^{-2\pi y}}[W^2(1+4(Z/137)^2)-1]^s\,. \qquad (6)$$

The normalized probability that a given beta decay of nuclide (ZA) will result in an antineutrino of energy W can then be written by substituting these quantities into Eq. (3) and by noting that the same energy independent factor $BK(Z,A)$ appears in both the numerator and denominator, hence cancel. Using this approximate form of the Fermi function, we find that no gamma functions need be evaluated which is a considerable savings of computational time. Finally, then

$$P_j(ZAW) = \frac{g(W)(W_0-W)^2 W}{\int_1^{W_0} g(W)(W_0-W)^2 W\,dW}\,. \qquad (7)$$

The accuracy in such a calculation of the antineutrino spectrum is a direct reflection of the accuracy of the three main sets of input data to Eq. (1), namely, the primary fission yields $Y(ZA)$ used to calculate the partial chain yields $K(ZA)$, the beta branching ratios $b_j(ZA)$, and the end point energies of the beta branches E_{oj}. In these calculations, the primary fission yields were taken from the recent compilation by Crouch,[10] while a completely new set of beta branching ratio data and end point energy data, for nuclides with known decay schemes, was provided for us by the Oak Ridge Nuclear Data Project.[20] The beta end point energies used in approximating the decays of nuclides far from the line of beta stability whose decay schemes are unknown were calculated using the semi-empirical mass formula of Jänecke.[11] This formulas has the form

$$\Delta M(N,Z) = N\Delta M_n + Z\Delta M_H + \beta Z^2 + \eta(N-Z)^2 + g_1(N) + g_2(Z) + g_3(A)\,. \qquad (8)$$

The functions $g_1(N)$, $g_2(Z)$, and $g_3(A)$ give rise to a high degree of local accuracy on the N-Z plane; hence, we concluded that this prescription would be the most accurate one for determining the mass excesses of nuclides far from the line of stability. A conclusion of an earlier paper by one of us (FTA[8]) was that one of the most serious sources of error would be the calculation of those mass excesses. It now appears that with more accurate mass formulas, much more spectroscopic data and more accurate fission yields, the most serious source of uncertainty by far is probably in the method of selection of the average excitation energies and beta branching ratios for nuclides with unknown decay schemes.

There are several methods which have been used to approximate the beta spectra from the decay of these nuclides. The simplest assumption is that each of these nuclides can be replaced with a fictitious nuclide whose ground state mass is calculated using Eq. (8) with the parameters given by Jänecke,[11] or by use of some other mass formula, and whose excited states are replaced by a single excited state. The energy of this excited state is then assumed to be the average excitation energy of all of the nuclides with known decay schemes. These known nuclides can be separated into classes, for example, (even Z-even N), (odd Z-even N), (even Z-odd N), and (odd Z-odd N). In addition, each of these four classes can be separated into two classes depending on which of the two fission mass peaks ($A \simeq 100$ or $A \simeq 140$) the nuclide of interest falls under. This was the approach taken in Ref. 12. We find that the predicted beta spectrum is very sensitive to the method in which these average parameters are calculated; hence, we have reinvestigated this problem. We found, for example, that the recent spectrum of beta particles from the fission fragments of ^{235}U in equilibrium which we calculated separating the averages into the eight categories discussed above, was in disagreement with that calculated by Borovoĭ, Dobrynin, and Kopeĭkin.[13] In their work, they used an average over selected nuclides in only three classes: (even Z-even N), (odd Z-odd N), and (all odd A). Their predicted beta spectrum was in better agreement with the experimental spectrum of Ref. 17 than our more detailed analysis which motivated the present investigation. Using our computer codes and input data and using the average excitation energies and branching ratios for the unknown nuclides given in Ref. 13, we did not, however, find very close agreement with the beta spectrum of Ref. 13; hence, we conclude that there must have been other differences in the input data. However, the spectra will depend very sensitively on the particular method of selecting the average properties of the nuclides with unknown decay schemes. We also noted that the mean square deviation of excitation energies and branching ratios becomes smaller when the number of classifications becomes larger. In that case, then our results using the eight classifications should be more accurate than those given in Ref. 13, but it is in poorer agreement with experiment which is somewhat confusing.

The disagreement with the beta spectrum of Ref. 13 can possibly be explained by noting that in the averaging process discussed in Ref. 13, only certain nuclides were chosen; hence, a fortuitous choice might have been made which produced good agreement with experiment. We would also have to conclude that the averaging process we used in our earlier work, while far more complete than that used in Ref. 13, is still inadequate. The replacement of many beta branches by only two could also be a serious oversimplification. In the present analysis, we have chosen to avoid broad averages and to attribute to a given nucleus (ZA) of unknown decay scheme all of the excited levels and beta branches found in all of the nuclides with known decay schemes of the same Z, but differing by even numbers of neutrons so that odd A and even A nuclides are separate. This entire collection of branching ratios is then renormalized to unity and the beta end point energies were calculated for the jth branch as $[Q(ZA)-E_j]$, where $Q(ZA)$ is the beta Q value computed with Jänecke's mass excess formula and E_j is one of the experimental excited state energies from the known nuclides averaged as discussed above. This procedure can be more strongly justified by a careful examination of nuclear level systematics than by

TABLE I. Comparison of several recently calculated antineutrino spectra using various methods for approximating the properties of nuclides with unknown decay schemes.

Energy (MeV)	Present results	Ref. 13	Ref. 14	Ref. 12
1.5	1.62 (0)	1.60 (0)	1.65 (0)	1.66 (0)
2.0	1.35 (0)	1.26 (0)	1.21 (0)	1.34 (0)
2.5	1.04 (0)	8.82 (−1)	8.42 (−1)	9.90 (−1)
3.0	7.69 (−1)	6.61 (−1)	5.95 (−1)	7.28 (−1)
3.5	5.26 (−1)	4.65 (−1)		5.10 (−1)
4.0	3.49 (−1)	3.22 (−1)	2.73 (−1)	3.39 (−1)
4.5	2.12 (−1)	2.04 (−1)		2.10 (−1)
5.0	1.39 (−1)	1.30 (−1)	1.03 (−1)	1.33 (−1)
5.5	8.57 (−2)	7.93 (−2)		8.66 (−2)
6.0	4.93 (−2)	4.89 (−2)	3.50 (−2)	5.40 (−2)
6.5	2.87 (−2)	3.03 (−2)		3.24 (−2)
7.0	1.50 (−2)	1.74 (−2)	1.01 (−2)	1.80 (−2)
7.5	6.93 (−3)	9.51 (−3)		9.96 (−3)
8.0	3.10 (−3)	4.35 (−3)	1.87 (−3)	5.37 (−3)
8.5	9.30 (−4)	1.65 (−3)		2.48 (−3)
9.0	5.52 (−4)	8.23 (−4)		1.53 (−3)
9.5	2.86 (−4)	4.35 (−4)		8.36 (−4)
10.0	1.71 (−4)	1.61 (−4)		3.79 (−4)

taking broader averages, while also it avoids the undesirable replacement of many beta branches by a single branch. We have chosen to pursue the calculation of the antineutrino spectra from fission products using the method outlined above rather than that used earlier, and we based this decision on the experimental evidence discussed below.

The justification of the assumption that most of the energy levels in a nuclide of a given Z remains stable against the addition or subtraction of pairs of neutrons comes mainly from results recently obtained in the neutron deficient nuclei far from the line of beta stability. The technique using an isotope separator on-line to a heavy ion accelerator has been used successfully at UNISOR (Universities Isotope Separator at Oak Ridge) for several years for the investigation of the level schemes of the short-lived isotopes far from the line of stability. Earlier observations of this phenomenon were made by Diamond and Stevens[21] and by Uyttenhove *et al.*[22] in the thallium isotopes. Later investigations in Au, Tl, and Hg isotopes[23-26] showed a similar strong trend, which stimulated a general search for this behavior in all known regions of the nuclide chart.

It was observed that while one or more single-particle levels might change relative position as a function of neutron number, much of the low lying structure was very stable with the addition or subtraction of neutron pairs. In some of the lighter nuclei, it was seen that the spectra would oscillate slightly, but an extensive calculation of the average excitation energy generally did not show a systematic increase or decrease with increasing or decreasing neutron pairs. It seems that on the average, and in many individual cases, the beta end point energies systematically increase as one moves further from the stability line in accordance with the increased mass difference between adjacent nuclei. We then adopted the view that the nuclear structure and decay systematics were stable against the addition of neutron pairs, and the beta Q values change in accordance with Jänecke's mass formula. All other assumptions remained the same as in our earlier work discussed above, with the exception of the new input data.

The new data set obtained from the Oak Ridge Nuclear Data Project contained experimental values for beta end point energies and branching ratios of 2161 known beta branches, which accounts for $4.472\bar{\nu}_e$/fission. The method described above resulted in 872 fictitious beta branches which are intended to produce a realistic contribution to the antineutrino spectrum due to the nuclei with yet unmeasured decay scheme properties. The fission yields and branching ratios were used to determine that $1.613\bar{\nu}_e$/fission were due to these nuclei. The antineutrino spectrum due to only nuclides with known decay schemes was calculated separately and is shown in Fig. 1. The importance of the contribution due to beta decay to nuclei with unknown level structures is immediately obvious. For comparison to earlier spectra see Table I.

ERROR ANALYSIS

A complete error analysis of the spectrum would be a monumental task. For simplification, we assumed the decay schemes of the known nuclides to be absolutely correct and calculated an uncertainty in the spectrum due to the uncertainties in each of the yields, the Q values used for the unknown nuclides, and the decay schemes for the unknowns. In Crouch's tables of mass chain yields,[10] the standard deviation for each chain yield is listed as a percent (coefficient of variation) of the chain yield. Jänecke gives a standard deviation of 118 keV for the differences between his calculated mass excesses and the experimental mass excesses. By changing each mass yield, end point energy, and branching ratio individually

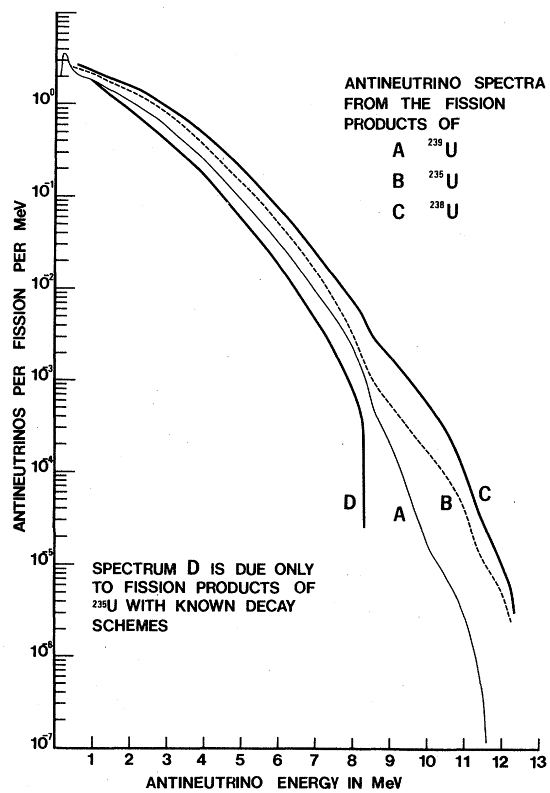

FIG. 1. Spectra of antineutrinos from the fission products of ^{235}U, ^{238}U, and ^{239}Pu in secular equilibrium. Curve D shows the antineutrino spectrum of the fission products of ^{235}U which have known decay schemes.

by its standard deviation, and by recalculating the spectrum each time, a total uncertainty was computed from the deviations in the spectrum. The error analysis then required many hundreds of calculations of the spectrum, and hence required orders of magnitude more computational time than the calculation of the spectrum alone. The quoted errors at each energy were computed from the square root of the sum of the squares of all of the fluctuations of the spectrum observed when each piece of input data was varied by an amount equal to its uncertainty. The spectra and computed errors are given in Table II.

PREDICTED BETA SPECTRUM FROM THERMAL FISSION OF ^{235}U

There are two types of experiments which can be used to check the methodology used in theoretical calculations of antineutrino spectra. One is the direct comparison of the calculated antineutrino spectrum with that derived from the positron spectrum observed in the inverse beta decay reaction $\bar{\nu}_e(p,n)\beta^+$. The other is the comparison of the theoretically predicted beta spectrum, from the thermal fission products of ^{235}U, with that observed experimentally. Three well known measurements exist in the literature; however, we shall compare our present results with the later work of Tsoulfanidis *et al.*[17] In 1979, we calculated such a spectrum[27] and gave a detailed analysis of the comparison of our theoretical results with the experimental results of Ref. 17. That calculation was based on almost the same analysis given here; however, the present results are slightly different due to the use of the complete new data set supplied to us by the Oak Ridge Nuclear Data Project.[20] In particular, the present beta spectrum is below the experimental spectrum at higher energies, as can be seen in Table III. This change was totally caused by the new data set which includes only isotopes with known decay schemes. The effect was mainly due to changes in the branching ratios from new more precise measurements and not due to the inclusion of isotopes which were not known before. The agreement of the present spectrum with experiment is comparable to that obtained by Borovoĭ *et al.*,[13] but much better than that obtained by Davis *et al.*[14] For example, in Table III of Ref. 14, the predictions of Davis *et al.* are 50% and 135% below the experimental spectrum at 7 and 8 MeV, respectively, whereas our predictions are 48% and 39% below the experimental spectrum at 7.5 and 8.5 MeV, respectively.

The apparent disagreement between the present results and the experimental results of Ref. 17

TABLE II. Theoretical spectra of antineutrinos from the fission products of ^{235}U, ^{238}U, and ^{239}Pu in secular equilibrium. $N(w)$ is given in antineutrinos per MeV per fission.

w (MeV)	$N(w, {}^{235}\mathrm{U})$	$N(w, {}^{238}\mathrm{U})$	$N(w, {}^{239}\mathrm{Pu})$
0.1	(2.02 ± 0.03)	(1.94 ± 0.03)	(2.17 ± 0.04)
0.5	(2.76 ± 0.05)	(2.72 ± 0.04)	(2.48 ± 0.04)
1.0	(2.12 ± 0.03)	(2.26 ± 0.04)	(1.89 ± 0.03)
1.5	(1.62 ± 0.03)	(1.77 ± 0.03)	(1.31 ± 0.02)
2.0	(1.35 ± 0.02)	(1.53 ± 0.03)	(1.05 ± 0.02)
2.5	(1.04 ± 0.02)	(1.19 ± 0.02)	$(7.47\pm0.12)\times10^{-1}$
3.0	$(7.69\pm0.13)\times10^{-1}$	$(9.21\pm0.15)\times10^{-1}$	$(5.43\pm0.09)\times10^{-1}$
3.5	$(5.26\pm0.12)\times10^{-1}$	$(6.73\pm0.16)\times10^{-1}$	$(3.70\pm0.09)\times10^{-1}$
4.0	$(3.49\pm0.11)\times10^{-1}$	$(4.61\pm0.15)\times10^{-1}$	$(2.36\pm0.07)\times10^{-1}$
4.5	$(2.12\pm0.08)\times10^{-1}$	$(3.01\pm0.12)\times10^{-1}$	$(1.40\pm0.06)\times10^{-1}$
5.0	$(1.39\pm0.07)\times10^{-1}$	$(2.00\pm0.09)\times10^{-1}$	$(8.39\pm0.39)\times10^{-2}$
5.5	$(8.57\pm0.46)\times10^{-2}$	$(1.29\pm0.07)\times10^{-1}$	$(5.14\pm0.28)\times10^{-2}$
6.0	$(4.93\pm0.30)\times10^{-2}$	$(7.69\pm0.47)\times10^{-2}$	$(3.00\pm0.18)\times10^{-2}$
6.5	$(2.87\pm0.20)\times10^{-2}$	$(4.47\pm0.31)\times10^{-2}$	$(1.84\pm0.13)\times10^{-2}$
7.0	$(1.50\pm0.11)\times10^{-2}$	$(2.51\pm0.19)\times10^{-2}$	$(9.96\pm0.75)\times10^{-3}$
7.5	$(6.93\pm0.57)\times10^{-3}$	$(1.37\pm0.11)\times10^{-2}$	$(4.83\pm0.39)\times10^{-3}$
8.0	$(3.10\pm0.32)\times10^{-3}$	$(7.30\pm0.74)\times10^{-3}$	$(2.26\pm0.23)\times10^{-3}$
8.5	$(9.30\pm0.82)\times10^{-4}$	$(3.03\pm0.27)\times10^{-3}$	$(4.84\pm0.42)\times10^{-4}$
9.0	$(5.52\pm0.48)\times10^{-4}$	$(1.95\pm0.17)\times10^{-3}$	$(2.22\pm0.20)\times10^{-4}$
9.5	$(2.86\pm0.29)\times10^{-4}$	$(1.07\pm0.11)\times10^{-3}$	$(5.73\pm0.58)\times10^{-5}$
10.0	$(1.71\pm0.22)\times10^{-4}$	$(5.83\pm0.76)\times10^{-4}$	$(1.46\pm0.19)\times10^{-5}$
10.5	$(9.34\pm1.62)\times10^{-5}$	$(2.90\pm0.50)\times10^{-4}$	$(7.57\pm1.31)\times10^{-6}$
11.0	$(3.95\pm0.91)\times10^{-5}$	$(9.66\pm2.22)\times10^{-5}$	$(2.64\pm0.61)\times10^{-6}$
11.5	$(1.14\pm0.34)\times10^{-5}$	$(2.66\pm0.80)\times10^{-5}$	$(2.79\pm0.84)\times10^{-7}$
12.0	$(4.52\pm1.75)\times10^{-6}$	$(1.05\pm0.41)\times10^{-5}$	$(1.10\pm0.43)\times10^{-7}$

TABLE III. Comparison between theoretical and experimental beta spectra in beta particles per MeV per fission.

E_β (MeV)	$N(E_\beta)$ theoretical	$N(E_\beta)$ experimental	$\Delta\%$ [a]
0.1	5.78 ± 0.05	3.40	+41
0.3	3.29 ± 0.04	3.17	+4
0.5	2.54 ± 0.04	2.91	−15
0.7	2.21 ± 0.03	2.63	−19
0.9	1.90 ± 0.02	2.34	−23
1.1	1.66 ± 0.02	2.01	−21
1.3	1.47 ± 0.02	1.69	−15
1.5	1.31 ± 0.02	1.40	−7
1.7	1.17 ± 0.02	1.18	−1
1.9	1.03 ± 0.01	1.00	+3
2.5	6.89 ± 0.09 (−1) [b]	6.17 (−1)	+10
3.5	3.03 ± 0.08 (−1)	2.75 (−1)	+9
4.5	1.19 ± 0.05 (−1)	1.06 (−1)	+11
5.5	4.24 ± 0.22 (−2)	3.95 (−2)	+7
6.5	1.21 ± 0.07 (−2)	1.35 (−2)	−12
7.5	2.70 ± 0.23 (−3)	4.00 (−3)	−48
8.5	4.82 ± 0.53 (−4)	6.70 (−4)	−39

[a] $\Delta\% \equiv [N(\text{theor}) - N(\text{exp})]/N(\text{theor}) \times 100$.
[b] Read as $6.89 \pm 0.09 \times 10^{-1}$.

at very low energy are probably due in part to electrons scattering out of the detector which is very difficult to correct. In addition, the experiment was run for 8 hours of irradiation time, while the present calculation assumes the fission fragments to be in secular equilibrium; hence, we should expect the calculation to yield a larger number of beta particles at low energies but better agreement at high energies. The reason for the low energy discrepancy is mainly based on the fact that the fission fragments which have low β-Q values are generally long lived and have not reached their equilibrium value in 8 hours. Those with large β-Q values have half-lives on the order of minutes and easily reach secular equilibrium in that time. Experimental errors are not quoted point by point in Ref. 17 which makes it difficult to determine how serious the observed differences in the spectra are, since the theoretical and experimental spectra oscillate about one another. Finally, it was noted, in our distribution of end point energies, that there are several groupings that lead to bumps and valleys in the spectrum. Such oscillations could easily have been washed out of the experimental spectrum due to the intrinsic resolution of the scintillator and the complication resulting from partial energy deposition of a significant fraction of the electrons.

The prescription used in calculations of this type should certainly yield results for the spectrum of beta particles from the thermal fission products which are in agreement with experiment because no special assumptions about the physical conditions under which the experiments are done need be made. In the case of the antineutrino spectra from operating reactors, severe assumptions about the reactor dynamics are made, namely, that the fission product densities are not affected by reactions other than thermal fission.

We conclude that our prescription for predicting the end point energies and beta branching ratios of isotopes far from beta stability lead to a predicted fission beta spectrum which is in better agreement with experiment at energies above 6 MeV than the prescriptions of Davis *et al.*[14]

INVERSE BETA DECAY OF THE PROTON

The only direct experimental test of the methodology used to calculate the fission spectra of antineutrinos from a given nuclear reactor will certainly be the accurate measurement of the spectrum of positrons from the reaction $\bar{\nu}_e(p,n)\beta^+$ for which the nuclear matrix element is well known from neutron β^- decay.[28] Unfortunately, the interpretation of such data will be somewhat complicated by the possibility of neutrino oscillations.[3] Accurate interpretation of other antineutrino-induced reactions will eventually depend on precision measurements of this β^+ spectrum for the given reactor used. The most recent data[29] published for this spectrum is in serious disagreement with any of the predictions, especially at energies above 7 MeV.[7,15] There are presently several measurements underway[30,31,32] and there will also be a spectrum forthcoming which is a spin-off of the recent measurement of the electron-antineutrino elastic scattering experiment of Reines and his co-workers,[33] which is discussed later. Whenever a theoretical fission spectrum of antineutrinos is reported, it is also important to report the conjugate β^- spectrum from ^{235}U thermal fission as well as the β^+ spectrum from inverse beta decay. Agreement with the β^- spectrum and not with the β^+ spectrum would imply an interesting and possibly fundamental paradox.

The theoretical cross section for inverse beta decay, averaged over the antineutrino spectrum, can be simply derived from the expression for first order decay of the neutron with the following result (given in $cm^2/\bar{\nu}_e$):

$$\bar{\sigma} = \frac{2\pi^2\hbar^3}{m^5c^8}\,\frac{\ln 2}{ft} \times \int_{w_1}^{w_2} P(w)(w-\delta)[(w-\delta)^2 - m_e{}^2c^4]^{1/2}dw, \tag{9}$$

where w is the incident antineutrino energy, $\delta = (M_n - M_p)c^2 = 1.3$ MeV, $P(w)$ is the normalized antineutrino spectrum, and w_2 is the end point energy of the antineutrino spectrum. Using the recent ft value from Raman *et al.*,[28] the constant before the integral becomes 9.28×10^{-44} cm^2/MeV^2.

The resulting total cross sections are given in Table IV and the predicted β^+ spectrum of inverse beta decay is shown in Fig. 2. Numerical values are presented in Table V for comparison to forthcoming experimental positron spectra from this reaction. The integrated value of Eq. (9), using the ^{235}U, ^{238}U, and ^{239}Pu antineutrino spectra individually, results in the following values (in units of 10^{-43} cm^2/fission): $\bar{\sigma}(^{235}\mathrm{U}) = 7.99$, $\bar{\sigma}(^{238}\mathrm{U}) = 11.15$, and $\bar{\sigma}(^{239}\mathrm{Pu}) = 5.34$. These are to be compared with those given in Ref. 19 which are $\bar{\sigma}(^{235}\mathrm{U}) = 7.66$ and $\bar{\sigma}(^{239}\mathrm{Pu}) = 5.63$. The experimental value given in Ref. 29 is 5.64 ± 0.78 in the same units. The value given in Ref. 14 is 6.0.

An accurate measurement of the positron spectrum from inverse beta decay, for several types of reactors, will settle many questions of importance. It would be surprising if a method which underestimates the higher energy portion of the β^- spectrum from ^{235}U fission fragments, conversely overestimated the high energy portion of the conjugate antineutrino spectrum from a reactor.

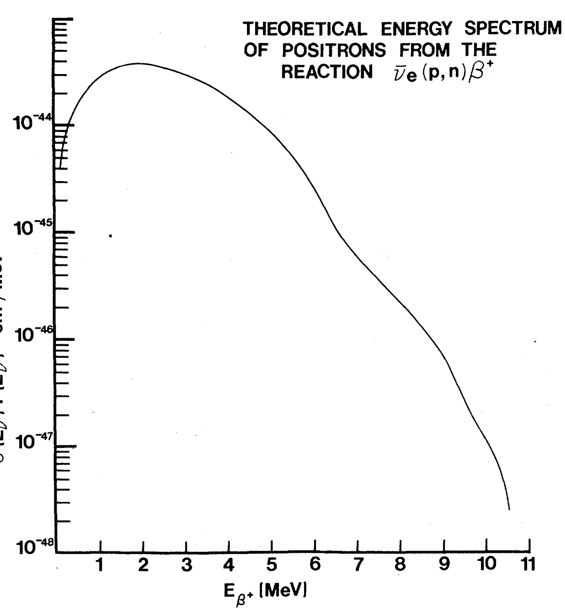

FIG. 2. Predicted spectrum of positrons from the reaction $\bar{\nu}_e(p,n)\beta^+$ as a function of positron kinetic energy. The units are cm^2 per MeV per antineutrino. These values must be multiplied by 6.06 antineutrinos per fission to convert to units of cm^2 per MeV per fission.

NEUTRAL DISINTEGRATION OF THE DEUTERON

The theoretical investigation of the weak neutral disintegration of the deuteron $\bar{\nu}_e(d,pn)\bar{\nu}_e$ has had a long history (see Ref. 1). The main value of a measurement of the cross section for this reaction, with low energy antineutrinos, is that the allowed approximation can be made, hence, a single coupling constant can be independently determined. The quantity of interest is the axial-vector coupling constant associated with the semileptonic weak interaction of the neutral components of the isovector currents. High energy neutrino experiments are not nearly as reliable for this determination because induced tensor interactions, momentum-transfer-dependent form factors, and the presence of both vector and axial-vector interactions complicate the interpretation.

A recent measurement of the total cross section for this reaction has been reported by Pasierb *et al.*[34] as $\bar{\sigma} = (3.8 \pm 0.9)\times 10^{-45}$ cm^2. This measurement was based on the detection of single, double, and triple neutron events observed in a collection of ^{3}He proportional counters immersed in a bath of pure (99.85%) heavy water. A more recent update of the analysis of this data[35] gives a value of $\bar{\sigma} = (5.0 \pm 0.8)\times 10^{-45}$ cm^2. It has recently been pointed out, however, that the analysis of these data do not include the effects of the observed reactor-associated triple neutron signal.[36] A more sophisticated experiment involving a deuterated scintillator coaxially located inside a neutron detecting scintillator is in its final stages of development.[37] Accuracies of 10% in the cross section or better will be required to adequately determine the coupling constant. The uncertainties in the antineutrino spectrum must also be reduced to properly interpret the experimental data.

The average theoretical cross section can be written in the low energy limit as follows[9]:

$$\bar{\sigma} = \frac{2G^2}{\pi^2}\int_w \int_{E_k} \frac{m^{3/2}\gamma(\gamma a_s - 1)^2 E_k^{\,1/2}(w - E_d - E_k)^2}{(mE_k a_s^{\,2} + 1)(\gamma^2 + mE_k)^2} \times P(w)\, dE_k\, dw\,, \qquad (10)$$

where (m = reduced nucleon rest mass energy) $\gamma = (mE_d)^{1/2} = 45.71$ MeV, a_s is the singlet scattering length (-0.1201 MeV^{-1}), E_d is the binding energy of the deuteron, E_k = nucleon energy, w is the incident antineutrino energy, and $P(w)$ is the normalized antineutrino spectrum. A complete theoretical treatment of this problem was given recently by Ahrens and Gallaher.[38]

The cross sections for all three fission spectra expected from a nuclear reactor are given in Table IV. Using the appropriate weighting[35] for the

TABLE IV. Integrated cross sections $\int N(w)\sigma(w)dw/\int N(w)dw$ for inverse beta decay, disintegration of the deuteron and excitation of ^{7}Li.

Spectrum $N(w)$	$\sigma[\bar{\nu}_e(p,n)\beta^+]$	$\sigma[\bar{\nu}_e(d,pn)\bar{\nu}_e']$	$\sigma[\bar{\nu}_e({}^7\mathrm{Li},{}^7\mathrm{Li}^*)\bar{\nu}_e']$
^{235}U	$(1.31\pm0.06)\times10^{-43}$ [a]	$(6.56\pm0.33)\times10^{-45}$	$(2.54\pm0.03)\times10^{-44}$
^{238}U	$(1.67\pm0.08)\times10^{-43}$	$(9.12\pm0.50)\times10^{-45}$	$(3.01\pm0.05)\times10^{-44}$
^{239}Pu	$(1.05\pm0.05)\times10^{-43}$	$(5.03\pm0.25)\times10^{-45}$	$(2.06\pm0.03)\times10^{-44}$

[a] All units are $\mathrm{cm}^2/\bar{\nu}_e$.

reactor used by Pasierb *et al.*,[34] the present predicted theoretical value is $\bar{\sigma}=(6.5\pm0.3)\times10^{-45}$ cm^2. This is in exact agreement with that of Ahrens and Gallaher[38] within a factor completely accounted for by a difference in the coupling constant used. This value is somewhat smaller than earlier predictions due to the fact that the present spectrum has fewer antineutrinos at energies above 5 MeV than the earlier spectra.

ANTINEUTRINO EXCITATION OF ^{7}Li

There is significant theoretical interest in the excitation of nuclei by the inelastic scattering of neutrinos (antineutrinos). Early motivation for the ideas described here came from the work of Donnelly *et al.*[4] and the later work of Lee[5]; a comprehensive review is given in a recent article by Donnelly and Peccei[6] in which both the underlying gauge theory models and their predictions for nuclear physics studies are discussed. It has recently been shown that this experiment is very probably feasible at an actual reactor site; hence, it is of great interest to evaluate the average cross section.[39] All of the popular gauge theory models are constructed so as to reproduce the ordinary charge-changing weak interactions; hence, a selection between the various models, as well as the measurement of the values of the appropriate gauge coupling constants, depends on accurate experimental data on weak neutral current reactions. As pointed out in Ref. 6, the experimental study of the cross sections for weak neutral current excitation of nuclei by antineutrinos can clearly eliminate certain gauge theory models from contention. Presently, the SU(2) × U(1) model proposed independently by Weinberg and by Salam, as modified by Glashow, Illiopoulos, and Mainani (the WS-GIM model), appears to be favored by most of the data on weak neutral currents. Even if the experimental results were conclusive, there would still be great value in having various direct experimental observations of antineutrino excitations of ^{7}Li. The ability to select specific parts of the weak neutral current with low energy neutrino excitation of nuclei using the available nuclear quantum numbers is of prime importance in this regard. Here we shall discuss the experiment involving the excitation of ^{7}Li from the ground state with $(J^\pi)T=(\frac{3}{2}^-)\frac{1}{2}$ to the $(\frac{1}{2}^-)\frac{1}{2}$ first excited state at 0.478 MeV in which the axial-vector coupling constants for the weak neutral current would be directly determined. We note, for example, that in the WS-GIM model the isoscalar coupling constant is identically zero so that this experiment would directly test this aspect of the model.

TABLE V. Differential cross section for the reaction $\bar{\nu}_e(p,n)\beta^+$, weighted by the normalized antineutrino spectrum $[\sigma(E_{\beta^+})P(w)]$.

E_{β^+} MeV	$d\sigma/dE_{\beta^+}$ cm^2/MeV	E_{β^+} MeV	$d\sigma/dE_{\beta^+}$ cm^2/MeV
1	2.82×10^{-44}	6	2.90×10^{-45}
2	3.86×10^{-44}	7	6.09×10^{-46}
3	3.01×10^{-44}	8	2.23×10^{-46}
4	1.91×10^{-44}	9	7.49×10^{-47}
5	8.85×10^{-45}	10	1.12×10^{-47}

[a] E_{β^+} is the positron kinetic energy.

The general expression for the cross section for this reactions has a very simple form after substitution of the appropriate values for the quantum numbers and matrix elements. The cross section as a function of antineutrino energy w is given by

$$\sigma(w)=(6.954\times10^{-38}\ \mathrm{cm}^2)\left(\frac{w-0.478}{M_n}\right)^2\kappa^2 \times|0.32\beta_A^{(0)}-0.327\phi\beta_A^{(1)}|^2, \tag{11}$$

where ($\phi=\pm1$) is the remaining relative phase between the isoscalar and isovector pieces. Using a single-particle description to get a rough idea of the magnitude of these matrix elements, the phase was determined[6] to be $\phi=+1$. The quantity κ^2 as well as the gauge coupling constants $\beta_A^{(0)}$ and $\beta_A^{(1)}$ are dependent on the particular gauge theory used. We have used the values $\beta_A^{(0)}=0$, $\beta_A^{(1)}=1$, and $\kappa^2=1$ predicted by the (WS-GIM) model.

The average cross sections were calculated in the present work by folding the cross section

$\sigma(w)$ into the spectrum of antineutrinos from reactor fission products of ^{235}U, ^{238}U, and ^{239}Pu. These are presented in Table IV.

ANTINEUTRINO-ELECTRON ELASTIC SCATTERING

The importance of the elastic scattering reaction $\bar{\nu}_e + e^- \to \bar{\nu}_e + e^-$ in the testing of current theories of the weak interaction has been discussed by Chen and Lee[40] and by Abers and Lee.[41] A clear understanding of this reaction is especially important in that it involves purely leptonic currents. In 1976, Reines, Gurr, and Sobel[33] reported experimental cross sections for this reaction in two energy ranges. To date these are still the most recent experimental data; however, there is a great deal of new data which has been taken but is not yet analyzed.[35] These cross sections were reported as factors multiplied by the theoretical predictions reported in 1970 by one of us (F.T.A.) and which were based on the Feynman-Gell–Mann (FG) theory.[8] Their results can be expressed in the usual way as follows:

$$\bar{\sigma}_{\exp}(1.5\text{–}3.0\ \text{MeV}) = (7.6 \pm 2.2)\times 10^{-46}\ \text{cm}^2$$

and

$$\bar{\sigma}_{\exp}(3.0\text{–}4.5\ \text{MeV}) = (1.86 \pm 0.48)\times 10^{-46}\ \text{cm}^2 ,$$

where the bar indicates a weighted average over the antineutrino spectrum and where the energy ranges are of the observed electron recoil energy. The main conclusions drawn from the analysis given in Ref. 33 were that the higher energy range results were in clear disagreement with (FG) theory and in agreement with the predictions of the Weinberg-Salam (WS-GIM) model with $\sin^2\theta_W = 0.32 \pm 0.05$. The results of the lower energy range were in fair agreement with (FG) theory and also with the (WS-GIM) model with $\sin^2\theta_W = 0.26 \pm ^{0.05}_{0.06}$. We later repeated the analysis with a later spectrum with similar results.[42] We have reanalyzed these results using a mixture of the antineutrino spectra from the fission products of ^{235}U, ^{238}U, and ^{239}Pu with slightly different results for the Weinberg angle. The crucial point, however, is that significantly differing antineutrino spectra do not change any of the major conclusions of either Ref. 33 or Ref. 42. An analysis of the 1976 data using the present spectra follows.

The theoretical cross section can be expressed in the convention used by Abers and Lee[41] as

$$\bar{\sigma} = \frac{G^2 Me}{2\pi}[(C_V - C_A)^2 G_1(T_1, T_2) + (C_V + C_A)^2 G_2(T_1, T_2) - (C_V{}^2 - C_A{}^2) G_3(T_1, T_2)] , \quad (12)$$

where in the (WS-GIM) model $C_A = +\frac{1}{2}$ and $C_V = \frac{1}{2} + 2\sin^2\theta_W$. It should be pointed out that in our earlier analysis,[42] C_A appears with the wrong sign in the text but this error was typographical and did not propagate into the analysis. The factors G_1, G_2, and G_3 are given by

$$G_1(T_1, T_2) = \int_{T_1}^{T_2}\int_{w_1}^{w_2} P(w)dw\, dT , \quad (13)$$

$$G_2(T_1, T_2) = \int_{T_1}^{T_2}\int_{w_1}^{w_2} P(w)(1 - T/w)^2 dw\, dT , \quad (14)$$

$$G_3(T_1, T_2) = \int_{T_1}^{T_2}\int_{w_1}^{w_2} P(w)\frac{mc^2 T}{w^2} dw\, dT , \quad (15)$$

where T is the electron recoil kinetic energy, w is the antineutrino energy, and the limits (T_1, T_2) and (w_1, w_2) represent their limiting values. The quantity $P(w)$ is the normalized antineutrino spectrum. These integrals have been numerically evaluated for the antineutrino spectra given here and are presented in Table VI. An appropriate mixture of the spectra from ^{235}U, ^{238}U, and ^{239}Pu fission products has also been used in our present analysis. In each case w_2 was chosen to be 12.4 MeV where all three spectra essentially vanish and T_2 is the largest kinetic energy of recoil possible with an incident antineutrino of energy w_2.

Figure 3 shows two solid and two dashed curves. The dashed curves correspond to the limits of the experiment corresponding to recoil electron kinetic energies between 1.5 and 3.0 MeV and include the uncertainties in the antineutrino spectrum as well as those in the experimental measurement of $\bar{\sigma}$. The region of agreement of these experimental data with theory is contained between the dashed curves. Similarly, the region of agreement between the higher energy range experiment (3.0–4.5 MeV) and theory lies between the solid lines. The values of the Weinberg angle corresponding to these data are: for (1.5–3.0 MeV), $\sin^2\theta_W = 0.24 \pm ^{0.07}_{0.06}$ and for (3.0–4.5 MeV), $\sin^2\theta_W = 0.29 \pm ^{0.07}_{0.05}$.

It is interesting to note that the higher energy range data are not in agreement with (FG) theory while the lower energy range data are. If one adopts the point of view that these two energy range data sets correspond to independent experiments, then the data rule out (FG) theory and result in a more accurate determination of the Weinberg angle, namely, $0.24 < \sin^2\theta_W < 0.31$. This point of view seems justifiable when one considers that the background contamination of this experiment is entirely different for these two ranges of energy. In that case, then the common regions of

TABLE VI. Numerical integrals G_1, G_2, and G_3 for $(\bar{\nu}_e - e^-)$ elastic scattering cross section.

T_1 (MeV)	$G_1(^{235}U)$	$G_2(^{235}U)$	$G_3(^{235}U)$
0.5	(1.09 ± 0.03)	$(2.63 \pm 0.07) \times 10^{-1}$	$(1.32 \pm 0.03) \times 10^{-1}$
1.0	$(7.63 \pm 0.21) \times 10^{-1}$	$(1.29 \pm 0.04) \times 10^{-1}$	$(8.74 \pm 0.19) \times 10^{-2}$
1.5	$(5.21 \pm 0.16) \times 10^{-1}$	$(6.43 \pm 0.23) \times 10^{-2}$	$(5.62 \pm 0.14) \times 10^{-2}$
2.0	$(3.43 \pm 0.12) \times 10^{-1}$	$(3.19 \pm 0.14) \times 10^{-2}$	$(3.47 \pm 0.10) \times 10^{-2}$
2.5	$(2.19 \pm 0.09) \times 10^{-1}$	$(1.57 \pm 0.08) \times 10^{-2}$	$(2.06 \pm 0.07) \times 10^{-2}$
3.0	$(1.36 \pm 0.06) \times 10^{-1}$	$(7.62 \pm 0.42) \times 10^{-3}$	$(1.19 \pm 0.05) \times 10^{-2}$
3.5	$(8.15 \pm 0.42) \times 10^{-2}$	$(3.65 \pm 0.22) \times 10^{-3}$	$(6.71 \pm 0.32) \times 10^{-3}$
4.0	$(4.78 \pm 0.28) \times 10^{-2}$	$(1.71 \pm 0.11) \times 10^{-3}$	$(3.71 \pm 0.20) \times 10^{-3}$
4.5	$(2.73 \pm 0.17) \times 10^{-2}$	$(7.72 \pm 0.56) \times 10^{-4}$	$(2.01 \pm 0.12) \times 10^{-3}$
5.0	$(1.49 \pm 0.10) \times 10^{-2}$	$(3.37 \pm 0.27) \times 10^{-4}$	$(1.04 \pm 0.07) \times 10^{-3}$
5.5	$(7.73 \pm 0.58) \times 10^{-3}$	$(1.42 \pm 0.12) \times 10^{-4}$	$(5.14 \pm 0.37) \times 10^{-4}$
6.0	$(3.82 \pm 0.31) \times 10^{-3}$	$(5.81 \pm 0.55) \times 10^{-5}$	$(2.41 \pm 0.19) \times 10^{-4}$
T_1	$G_1(^{238}U)$	$G_2(^{238}U)$	$G_3(^{238}U)$
0.5	(1.23 ± 0.03)	$(3.03 \pm 0.09) \times 10^{-1}$	$(1.40 \pm 0.03) \times 10^{-1}$
1.0	$(8.85 \pm 0.26) \times 10^{-1}$	$(1.55 \pm 0.05) \times 10^{-1}$	$(9.64 \pm 0.22) \times 10^{-2}$
1.5	$(6.22 \pm 0.20) \times 10^{-1}$	$(8.05 \pm 0.31) \times 10^{-2}$	$(6.41 \pm 0.17) \times 10^{-2}$
2.0	$(4.24 \pm 0.16) \times 10^{-1}$	$(4.17 \pm 0.19) \times 10^{-2}$	$(4.11 \pm 0.12) \times 10^{-2}$
2.5	$(2.81 \pm 0.12) \times 10^{-1}$	$(2.14 \pm 0.11) \times 10^{-2}$	$(2.55 \pm 0.09) \times 10^{-2}$
3.0	$(1.80 \pm 0.09) \times 10^{-1}$	$(1.09 \pm 0.06) \times 10^{-2}$	$(1.54 \pm 0.07) \times 10^{-2}$
3.5	$(1.13 \pm 0.06) \times 10^{-1}$	$(5.45 \pm 0.35) \times 10^{-3}$	$(9.02 \pm 0.44) \times 10^{-3}$
4.0	$(6.88 \pm 0.41) \times 10^{-2}$	$(2.69 \pm 0.19) \times 10^{-3}$	$(5.19 \pm 0.29) \times 10^{-3}$
4.5	$(4.09 \pm 0.27) \times 10^{-2}$	$(1.30 \pm 0.10) \times 10^{-3}$	$(2.92 \pm 0.18) \times 10^{-3}$
5.0	$(2.35 \pm 0.17) \times 10^{-2}$	$(6.20 \pm 0.52) \times 10^{-4}$	$(1.59 \pm 0.11) \times 10^{-3}$
5.5	$(1.31 \pm 0.10) \times 10^{-2}$	$(2.90 \pm 0.26) \times 10^{-4}$	$(8.36 \pm 0.63) \times 10^{-4}$
6.0	$(7.09 \pm 0.60) \times 10^{-3}$	$(1.34 \pm 0.13) \times 10^{-4}$	$(4.30 \pm 0.35) \times 10^{-4}$
T_1	$G_1(^{239}U)$	$G_2(^{239}U)$	$G_3(^{239}U)$
0.5	$(9.43 \pm 0.23) \times 10^{-1}$	$(2.25 \pm 0.06) \times 10^{-1}$	$(1.21 \pm 0.02) \times 10^{-1}$
1.0	$(6.42 \pm 0.17) \times 10^{-1}$	$(1.06 \pm 0.03) \times 10^{-1}$	$(7.64 \pm 0.16) \times 10^{-2}$
1.5	$(4.28 \pm 0.13) \times 10^{-1}$	$(5.15 \pm 0.18) \times 10^{-2}$	$(4.75 \pm 0.11) \times 10^{-2}$
2.0	$(2.76 \pm 0.09) \times 10^{-1}$	$(2.49 \pm 0.10) \times 10^{-2}$	$(2.84 \pm 0.08) \times 10^{-2}$
2.5	$(1.72 \pm 0.07) \times 10^{-1}$	$(1.20 \pm 0.06) \times 10^{-2}$	$(1.65 \pm 0.06) \times 10^{-2}$
3.0	$(1.04 \pm 0.05) \times 10^{-1}$	$(5.74 \pm 0.31) \times 10^{-3}$	$(9.29 \pm 0.37) \times 10^{-3}$
3.5	$(6.16 \pm 0.32) \times 10^{-2}$	$(2.72 \pm 0.17) \times 10^{-3}$	$(5.10 \pm 0.24) \times 10^{-3}$
4.0	$(3.56 \pm 0.20) \times 10^{-2}$	$(1.27 \pm 0.08) \times 10^{-3}$	$(2.76 \pm 0.15) \times 10^{-3}$
4.5	$(2.02 \pm 0.13) \times 10^{-2}$	$(5.75 \pm 0.41) \times 10^{-4}$	$(1.48 \pm 0.09) \times 10^{-3}$
5.0	$(1.12 \pm 0.08) \times 10^{-2}$	$(2.49 \pm 0.19) \times 10^{-4}$	$(7.81 \pm 0.52) \times 10^{-4}$
5.5	$(5.88 \pm 0.43) \times 10^{-3}$	$(1.02 \pm 0.08) \times 10^{-4}$	$(3.93 \pm 0.28) \times 10^{-4}$
6.0	$(2.90 \pm 0.23) \times 10^{-3}$	$(3.84 \pm 0.33) \times 10^{-5}$	$(1.86 \pm 0.14) \times 10^{-4}$

overlap of the two experiments and theory do not include the point $C_A = C_V = 1$, which are the values of the Feynman-Gell-Mann theory. In addition, we find that this general conclusion is not changed by the use of the spectrum of Ref. 8, Ref. 9, or the present spectrum. All three differ significantly at higher energies.

SUMMARY AND CONCLUSIONS

Antineutrino spectra from equilibrium fission products of ^{235}U, ^{238}U, and ^{239}Pu, as well as the beta spectrum from fission products of ^{235}U, have been calculated. The portion of the spectra due to beta decays to nuclides with known decay schemes was calculated using a completely new set of nuclear spectroscopic data supplied by the Oak Ridge Nuclear Data Project.[20] A recently published compilation of fission yields[10] was used as well as a recently published mass formula designed for local accuracy on the (NZ) plane.[11] The assumption was made that the general energy level structure of nuclei of a given Z does not change appreciably with changes in pairs of neutrons. This resulted in a new scheme for approximately constructing beta decay schemes for nuclei far from the stability line for which decay schemes have not been experimentally deter-

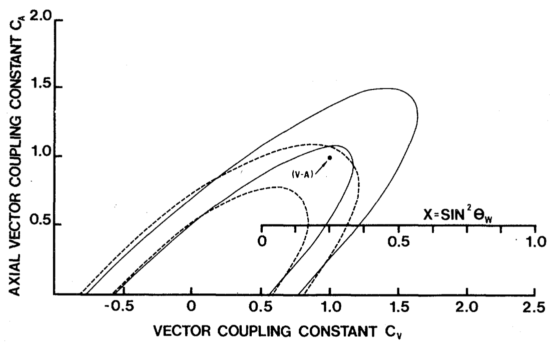

FIG. 3. Regions of the C_A, C_V plane in agreement with the experimental data of Reines *et al.* (Ref. 33). The region between the solid curves represent values of C_A and C_V in agreement with the data for electron kinetic energies in the range 3.0–4.5 MeV. The region between the dashed curves represent values of C_A and C_V in agreement with the data for electron kinetic energies in the range 1.5–3.0 MeV.

mined. This assumption was motivated by recently published energy level trends found in nuclei far from the stability line but on the proton rich side.[21–26]

The beta spectrum from the thermal fission fragments of ^{235}U in secular equilibrium were calculated using the prescription outlined above and was in general agreement with the most recent experimental results.[17] In fact, the present prescription leads to a calculated beta spectrum which is somewhat below the experimental spectrum above 7 MeV. This fact implies that the antineutrino spectra calculated using these prescriptions should not predict an artificially high number of antineutrinos at the higher energies, although our ^{235}U spectrum is still quite a bit above that of Ref. 14 in the high energy region.

The calculated spectra were used to compute the average cross sections for the reactions $\bar{\nu}_e(p,n)\beta^+$, $\bar{\nu}_e(^7\mathrm{Li},{}^7\mathrm{Li}^*)\bar{\nu}_e$, $\bar{\nu}_e(d,pn)\bar{\nu}_e$ and were also used to reanalyze existing data from $(\bar{\nu}_e - e^-)$ elastic scattering.[33] The calculated cross sections for $\bar{\nu}_e(p,n)\beta^+$ and $\bar{\nu}_e(d,pn)\bar{\nu}_e$ are larger than reported experimental results[29,34,35] which is somewhat perplexing in the light of the arguments presented above. If in the final analysis, future experimental data do in fact show that our prescription correctly predicts fission beta spectra but continues to predict higher average charge current cross sections than the experiments yield, there will exist a paradox. Such a paradox would imply that either our understanding of the processes occurring in the reactors in question is not clearly understood, or that there exist fundamental properties of the antineutrino itself which are not yet understood.

It is clear, from a consideration of all of the recent conflicting results,[9,12,13,14,18] that two experiments would be extremely valuable. First, an accurate remeasurement of the beta spectrum from ^{235}U fission fragments should be made, using a magnetic spectrometer, to energies above 10 MeV. Second, a precise measurement of the β^+ spectrum from the reaction $\bar{\nu}_e(p,n)\beta^+$ must be made so that it can be determined if there is indeed a paradox.

Finally, the existing data from $(e^- - \bar{\nu}_e)$ elastic scattering[33] were analyzed using the antineutrino spectrum of the present investigation, and the major conclusions reached earlier[33,42] are unchanged. If the data from the two different energy ranges are considered to be independent experiments, the data are not in agreement with the predictions of Feynman-Gell–Mann theory but are in agreement with those of the Weinberg-Salam model with $0.24 < \sin^2\theta_W < 0.31$.

Note added in proof. Since the acceptance of this paper, Reines and his co-workers have announced definite evidence for neutrino oscillations based on their observation of the reaction $\bar{\nu}_e(d,pn)\bar{\nu}_e$. The general conclusion is the same using either the spectrum presented here or that given in Ref. 14 to analyze their data.

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