Single nucleon knockout in heavy-ion collisions at high energy

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A distorted-wave impulse approximation formalism is developed for the knockout contribution in high energy (> 100 MeV/A) heavy-ion collisions. Computed inclusive proton spectra based on this model, which require only the nuclear densities, nucleon-nucleon scattering amplitude, and proton optical potential as input quantities, agree well in magnitude and shape with the available measured spectra in the forward directions.

[NUCLEAR REACTIONS DWIA formalism for the single nucleon knockout in high energy heavy-ion collisions.]

I. INTRODUCTION

By now sufficient experimental data exist on the inclusive proton and pion reactions induced by high energy heavy-ion collisions.[1] These reactions are described by the double differential cross section, with respect to energy and angle of proton (pion), and summed over all other particles. Theoretically, these spectra are found to be consistent with the "fireball" description of the heavy-ion collision.[2] However, since this model assumes the thermalization of the participating nucleons in a time interval of the order of 10^{-22} sec for the fast moving ions, the literal interpretation of the model is open to question. Microscopic calculations are also done.[3] Specifically, the recent one by Hüfner and Knoll[4] is of interest. Here, in the spirit of high energy approximation, the ion-ion collision is described as a one-dimensional cascade of a row of nucleons in one ion on a row of nucleons in the other ion. This model reproduces the data very well. The authors of this model also attempt to answer the question regarding the extent of thermalization achieved in these reactions. General findings seem to suggest that beyond 400 MeV/A there is no thermalization, while around 250 MeV/A the answer is not clear cut. On the other extreme of thermalization is the possibility of a complete nonequilibrium process of single step knockout of nucleons. This process, in fact, is found to be dominant in proton-nucleus scattering beyond 100 MeV (Ref. 5) and reasonable in accounting for the inclusive proton and pion spectra in the scattering of light ions (p, d, α, etc.) on nuclei.[6] In the heavy-ion interactions the contribution of the knockout process has been calculated by Koonin[7] and is found to agree well with the experiments. However, in this estimate, either an undetermined parameter (N_{BT}) is introduced which is determined by normalizing the calculated cross section to the measured one, or a nuclear density with extraordinary rich high momentum components is employed. This work also assumes, implicitly, that the contribution to knockout comes from the whole nuclear volume. This seems a little unphysical as the direct reactions are expected to be peripheral. In the present paper we develop the distorted wave impulse approximation (DWIA) formalism for the proton inclusive spectra for heavy-ion collisions. This formalism takes care of the attenuation of the incoming and outgoing particles and also predicts the absolute cross sections. The formalism is presented in Sec. II and the results, in Sec. III.

II. FORMALSIM

Consider first that the proton is knocked out from the projectile. (See Fig. 1). The matrix element for this in the distorted wave Born approximation (DWBA) may be written as

$$T_{fi} = A_P^{1/2} \left\langle \chi^-_{\vec{k}_f}(\vec{R}_C) \Phi^T_\beta(\xi_T) \Phi^C_\gamma(\vec{\xi}_c) \chi^-_{\vec{k}_p}(\vec{r}_1) \middle| \sum_{1=1}^{A_T} v_{1i}(\vec{r}) \middle| \Phi^T_0(\xi_T) \Phi^P_0(\vec{\xi}_C, \vec{r}_1) \chi^+_{\vec{k}_i}(\vec{R}) \right\rangle, \tag{1}$$

where the notations P, T, and C stand, respectively, for projectile, target, and core of $(A_P - 1)$ nucleons, and ξ stands collectively for internal coordinates. The Φ_α are the intrinsic nuclear wave functions in the state α and 0 denotes the ground state. $\chi^+_{\vec{k}_i}$ is the relative wave function of the target and projectile in the incident channel, $\chi^-_{\vec{k}_f}$ that of target and the core nucleus in the exit channel, and $\chi^-_{\vec{k}_p}$ that of outgoing proton with respect to the core. In the distorted wave approximation, these wave functions can be described by the appropriate optical potential. Since in the inclusive proton spectra nothing but the momentum $\vec{k}_p$ of the proton is observed, the inclusive cross section is obtained by summing the squared T matrix (1) over $\vec{k}_f$, β, and γ. Integration over $\vec{k}_f$ gives

$$\int d\vec{k}_f |T_{fi}|^2 = A_P \int d\vec{R}_C \left| \left\langle \Phi_\beta^T(\vec{\xi}_T)\Phi_\gamma^C(\vec{\xi}_C)\chi_{\vec{k}_p}^-(\vec{r}_1) \right| \sum_{i=1}^{A_T} v_{1i}(\vec{r}) \left| \Phi_0^T(\vec{\xi}_T)\Phi_0^P(\vec{\xi}_C,\vec{r}_1)\chi_{\vec{k}_i}^+(\vec{R}) \right\rangle \right|^2 .$$

Since we are interested in high energy collisions, we will use a high energy approximation for the distorted waves χ. Ignoring the corrections of order a $(=1/A_P)$ and its powers, the $\vec{k}_f$ integrated $|T|^2$ may be written as

$$\int d\vec{k}_f |T_{fi}|^2 \simeq A_P \int d\vec{R}_C e^{-\sigma_{NN}^T(E)T(\vec{b})/2} |F_{\beta\gamma}(\vec{k}_p,\vec{R}_C)|^2 , \tag{2}$$

where

$$F_{\beta\gamma}(\vec{k}_p,\vec{R}_C) = \left\langle \Phi_\beta^T(\vec{\xi}_T)\Phi_\gamma^C(\vec{\xi}_C)\chi_{\vec{k}_p}^-(\vec{r}_1) \right| \sum_{i=1}^{A_T} v_{1i}(\vec{r}) \left| \Phi_0^T(\xi_T)\Phi_0^P(\xi_C,\vec{r}_1)e^{ia\vec{k}_i\cdot\vec{r}_1} \right\rangle . \tag{3}$$

In writing Eqs. (2) and (3), considering the smallness of a, it is assumed that

$$\chi_{\vec{k}_i}^+(\vec{R}) = \chi_{\vec{k}_i}^+(\vec{R}_C + a\vec{r}_1) \approx e^{ia\vec{k}_i\cdot\vec{r}_1}\chi_{\vec{k}_i}^+(\vec{R}_C)$$

and

$$\chi_{\vec{k}_i}^+(\vec{R}_C) = e^{i\vec{k}_i\cdot\vec{R}_C} \exp\left(-\frac{ik_i}{2E_i}\int_{-\infty}^{z} V(\vec{R}_C')dz'\right) ,$$

where V is the average optical potential. Using again the impulse approximation and the limit of large A, the optical potential is written as

$$V(R) = \frac{2E_i}{k_i}\tfrac{1}{2}\sigma_{NN}^T(i+\nu)A_P A_T \int d\vec{R}'\rho_P(\vec{R}-\vec{R}')\rho_T(\vec{R}') , \tag{4}$$

where ν is the ratio of the real to imaginary part of the nucleon-nucleon forward scattering amplitude. $T(b)$ is the thickness function defined as

$$T(\vec{b}) = A_P A_T \int_{-\infty}^{+\infty} dz \int d\vec{R}'\rho_P(\vec{R}-\vec{R}')\rho_T(\vec{R}') . \tag{5}$$

The description of distorted waves by (4) assumes the dominance of forward scattering in N-N collision, which should be reasonable at high energy. Now summing over β and γ,

$$\sum_{\beta\gamma} |F_{\beta\gamma}(\vec{k}_p,\vec{R}_C)|^2 = \int d\vec{\xi}_T d\vec{\xi}_C \left| \left\langle \chi_{\vec{k}_p}^-(\vec{r}_1) \right| \sum_{i=1}^{A_T} v_{1i}(r) \left| \Phi_0^t(\vec{\xi}_T)\Phi_0^P(\vec{\xi}_C,\vec{r}_1)e^{ia\vec{k}_i\cdot\vec{r}_1} \right\rangle \right|^2 . \tag{6}$$

This expression, as we see, includes the effect of v in first order only. Its effect in all orders can be easily included by introducing proton-nucleon scattering matrix in place of v. In addition, using the linear energy approximation, we get

$$\begin{aligned} \sum_{\beta\gamma} |F_{\beta\gamma}(\vec{k}_p,\vec{R}_C)|^2 &\approx \int d\vec{\xi}_T d\vec{\xi}_C \left| \left\langle \chi_{\vec{k}_p}^-(\vec{r}_1) \right| \sum_{i=1}^{A_T} t_i^P(\vec{q}^P)\delta(\vec{x}_i - \vec{R}_C - \vec{r}_1) \left| \Phi_0^T(\vec{\xi}_T)\Phi_0^P(\vec{\xi}_C,\vec{r}_1)e^{ia\vec{k}_i\cdot\vec{r}_1} \right\rangle \right|^2 \\ &\approx \int d\vec{\xi}_T d\vec{\xi}_C |\bar{t}^P(\vec{q}^P)|^2 \left| \left\langle \chi_{\vec{k}_p}^-(\vec{r}_1) \right| \sum_{i=1}^{A_T} \delta(\vec{x}_i - \vec{R}_C - \vec{r}_1) \left| \Phi_0^T(\vec{\xi}_T)\Phi_0^P(\vec{\xi}_C,\vec{r}_1)e^{ia\vec{k}_i\cdot\vec{r}_1} \right\rangle \right|^2 . \end{aligned} \tag{7}$$

Here $\bar{t}^P(\vec{q}^P)$ is the averaged proton-nucleon t matrix, defined as

$$\bar{t}^P(\vec{q}^P) = \frac{N_T}{A_T} t_{pn}(\vec{q}^P) + \frac{Z_T}{A_T} t_{pp}(\vec{q}^P) , \tag{8}$$

with (N,Z,A) as the neutron, proton, and mass number of the target. $t_{NN}(\vec{q}^P)$ is the nucleon-nucleon scattering amplitude at the momentum transfer q^P. This momentum transfer is approximated as

$$q^P(\vec{r}_1) = \bar{k}_N(r_1)\hat{k}_i - \bar{k}_p\hat{k}_p , \tag{9}$$

where

$$\bar{k}_N(r_1) = \{(ak_i)^2 + [k^P(r_1)]^2\}^{1/2} ,$$

$$\bar{k}_p(r_1) = \left[k_p^{\,2} + \frac{2m}{\hbar^2}U(k_p, r_1)\right]^{1/2} . \tag{10}$$

Here $k^P(r_1)$ is the average local momentum of the proton in the projectile at the point $\vec{r}_1$ and may be written in terms of density. U is the real part of the optical potential of outgoing proton at energy E_p due to the core. In writing (9), we have modified only the magnitude of the momenta $\vec{k}_i$ and $\vec{k}_p$.

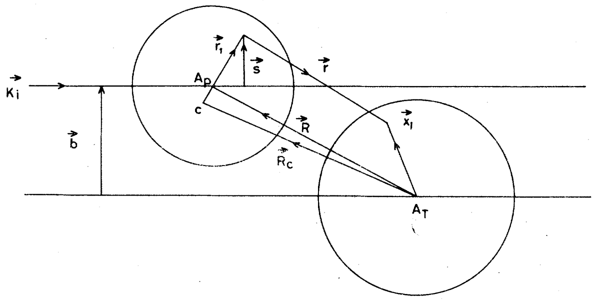

FIG. 1. Vector diagram.

Their directions are assumed to remain unchanged. This might not be justified at the low energy end of the proton spectrum. Regarding the energy at which t_{NN} is to be evaluated, it may be mentioned that, in principle, this scattering matrix is off shell. In any case, the choice of this energy need not be critical due to little variation in nucleon-nucleon cross section above 200 MeV. This energy may be taken as the incident energy itself.

To simplify Eq. (7) further, we define the nucleon density distribution (or one particle density function) and the two particle density function in the usual way:

$$\rho(\vec{x}) = \frac{1}{A}\left\langle 0 \left| \sum_i (\vec{x}-\vec{x}_i) \right| 0 \right\rangle ,$$

$$C(\vec{x},\vec{x}') = \frac{1}{A(A-1)}\left\langle 0 \left| \sum_{i \neq j} \delta(\vec{x}-\vec{x}_i)\delta(\vec{x}'-\vec{x}_j) \right| 0 \right\rangle . \tag{11}$$

Then

$$\sum_{\beta\gamma} |F_{\beta\gamma}(\vec{k}_p, \vec{R}_C)|^2 = A_T \int d\vec{r}_1 \rho_T(\vec{r}_1 + \vec{R}_C)\rho_P(\vec{r}_1)\chi^{-*}_{\vec{k}_p}(\vec{r}_1)\chi^{+}_{\vec{k}_p}(\vec{r}_1)\, |\tilde{t}^{\,P}(E_N, q^P(r_1))|^2$$
$$+ A_T(A_T-1)\int\int d\vec{r}_1 d\vec{r}_1{}' \{C_T(\vec{r}_1+\vec{R}_C, \vec{r}_1{}'+\vec{R}_C) C_P(\vec{r}_1,\vec{r}_1{}')\chi^{-*}_{\vec{k}_p}(\vec{r}_1)\chi^{-*}_{\vec{k}_p}(\vec{r}_1{}')\chi^{+}_{\vec{k}_p}(\vec{r}_1)\chi_{\vec{k}_p}(\vec{r}_1{}')$$
$$\times e^{i\alpha\vec{k}_i\cdot(\vec{r}_1-\vec{r}_1{}')}\, |\tilde{t}^{\,P}(E_N, q^P)\tilde{t}^{\,P+}(E_N, q^P)| . \tag{12}$$

If we retain only the one body term,

$$\sum_{\beta\gamma} |F_{\beta\gamma}(\vec{k}_p, \vec{R}_C)|^2 \approx A_T \int d\vec{r}_1 \rho_T(\vec{r}_1 + \vec{R}_C)\rho_P(\vec{r}_1)\chi^{-*}_{\vec{k}_p}(\vec{r}_1)\chi^{+}_{\vec{k}_p}(\vec{r}_1)\, |\tilde{t}^{\,P}(E_N, q^P(r_1))|^2 . \tag{13}$$

This expression is quite compact and simple to evaluate numerically. Distorted waves χ can be calculated with the standard techniques of distorted wave theory. Alternatively, one can also use the approximation Eq. (4) used for incident wave. With this approximation,

$$\sum_{\beta\gamma} |F_{\beta\gamma}(\vec{k}_p, \vec{R}_C)|^2 = A_T \int d\vec{r}_1 \rho_T(\vec{r}_1 + \vec{R}_C)\rho_P(\vec{r}_1) e^{-\sigma^T_{NN}(E_p)T_P(\vec{s})/2}\, |\tilde{t}^{\,P}(E_N, q^P(r_1))|^2 , \tag{14}$$

where $T_P(\vec{s})$ is the thickness function analogous to Eq. (5) for outgoing proton,

$$T_P(\vec{s}) = (A_P - 1)\int_{-\infty}^{+\infty} dz\, \rho_C(\vec{s}, z) . \tag{15}$$

Finally, then, in one body approximation,

$$\sum_{\beta\gamma}\int d\vec{k}_f |T_{fi}|^2 = A_P A_T \int d\vec{R}_C\, e^{-\sigma^T_{NN}(E_N)T(b)/2}\int d\vec{r}_1\, e^{-\sigma^T_{pn}(E_p)T_P(s)/2}\rho_T(\vec{r}_1 + \vec{R}_C)\rho_P(\vec{r}_1)\, |\tilde{t}^{\,P}(E_N, q^P)|^2 . \tag{16}$$

The first exponential in this expression describes the attenuation of the incoming ion and the second exponential of the outgoing nucleon. Owing to these absorption terms, the knockout reaction takes place only on the peripheries of the two nuclei. In Eq. (16) we also notice that the nucleon-nucleon scattering amplitude which is responsible for the knockout event appears inside the integral. It can be factored out from the integral if we can employ some kind of average over the nucleon momenta. However, due to the peripheral nature of the reaction, this average should be taken over the momenta of only those nucleons which are on the surface of nuclei. Due to this peripheral nature, the averaged scattering matrix might not even turn out very different from that obtained by ignoring the Fermi momenta altogether, except around the low energy end of the proton spectrum. The factorized expression for the proton inclusive cross section from the projectile is

$$\left(\frac{d^3\sigma}{dE_p d\Omega}\right)^P = \frac{Z_P}{A_P}\overline{\sigma_{pN}(E_N, q^P)}A_T N^P_{\text{eff}}, \tag{17}$$

where

$$N^P_{\text{eff}} = A_P \int d\vec{R}_C\, e^{-\sigma^T_{NN}(E_N)T_P(\vec{b})/2} \times \int d\vec{r}_1 e^{-\sigma^T_{pN}(E_p)T_P(\vec{s})/2}\rho_T(\vec{r}_1+\vec{R}_C)\rho_P(\vec{r}_1)\,. \tag{18}$$

The factor (Z/A), which represents charge to mass ratio, is introduced in Eq. (17) to account for the fact that only protons are observed. $\overline{\sigma}_{pN}$ is the averaged proton-nucleon cross section in their c.m. corresponding to momentum transfer q^P and laboratory energy E_N. N^P_{eff} is interpreted as the effective number of nucleons in the projectile. The actual number of nucleons A_P is reduced to N^P_{eff} because of attenuation and incomplete overlap of target and projectile densities.

For the contribution to knockout from the target, we can write down expressions similar to Eqs. (16) and (17),

$$\left(\sum_{\beta\gamma}\int d\vec{k}_f |T_{fi}|^2\right)^T = A_P A_T \int d\vec{R}_C\, e^{-\sigma^T_{NN}(E_N)T(b)/2}\int d\vec{r}_1\, e^{-\sigma^T_{pN}(E_p)T_T(s)/2}\rho_P(\vec{r}_1+\vec{R}_C)\rho_T(\vec{r}_1) \times |\bar{t}^T(E_N, q^T(\vec{r}_1))|^2, \tag{19}$$

$$\left(\frac{d^3\sigma}{dE_p d\Omega}\right)^T = \frac{Z_T}{A_T}\overline{\sigma_{pN}(E_N,q^T)}A_P N^T_{\text{eff}}, \tag{20}$$

with expressions for T_T and N^T_{eff} similar to Eqs. (15) and (18) with P replaced by T. The momentum transfer q^T to be used may be approximated as

$$\vec{q}^T(r_1) = \vec{k}^T(r_1) - \overline{k}_p(r_1)\hat{k}_p, \tag{21}$$

where $\vec{k}^T(r_1)$ is the internal momentum of the nucleon in the target. $\overline{k}_p(r_1)$ is defined analogous to Eq. (10) except that U refers to core of (A_T-1) nucleons. Since initially the knocked out nucleon from the target has only the internal momentum of the peripheral target nucleons, we may without much error, use an angle averaged value of q^T defined as

$$\langle q^T(r_1)\rangle = [k^{T^2}(r_1) + \overline{k}_p^{\,2}(r_1)]^{1/2}\,. \tag{22}$$

The total contribution to knockout protons is obtained by summing Eqs. (16) and (19), or (17) and (20), for projectile and target contributions. In the factorized approximation, for example, the summed cross section will be

$$\frac{d^3\sigma}{dE_p d\Omega} = \frac{Z_P}{A_P}\overline{\sigma_{pN}(E_N, q^P)}A_T N^P_{\text{eff}} + \frac{Z_T}{A_T}\overline{\sigma_{pN}(E_N,q^T)}A_P N^T_{\text{eff}}\,. \tag{23}$$

III. RESULTS AND DISCUSSION

Since the derivation of expressions for the knockout cross section in Sec. II incorporates the multiple scattering effects through forward scattering approximation, it is expected that the results will be more reliable for the emission of protons in the forward directions. We have, therefore, calculated the ^{4}He, ^{20}Ne, ^{40}Ar induced proton spectra at 30° emission on ^{238}U nucleus at 250 and 400 MeV/A. For the nuclear densities, we employ the three parameter Fermi distribution

$$\rho(r) = \rho_0(1 + wr^2/R_0^2)/[1+\exp(r-R_0)/a]\,.$$

The parameters for this distribution are taken from an analysis of electron scattering,[8] assuming that the charge and mass distributions are proportional. These parameters are listed in Table I. For the nucleon-nucleon scattering amplitude we have used the parametrized form

$$f_{NN}(E,q) = \frac{k}{4\pi}\sigma^T_{NN}(E)[\alpha(E)+i]e^{-\eta q^2}\,.$$

Various parameters are taken from literature which fit the nucleon-nucleon scattering data,[9,11] The optical potential required for protons at various energies are taken from the parametrization of Brieve and Rook and others.[10] With these parameters supplied, Eqs. (16) and (19) or (23) do not require any other parameter. The results corresponding to the unfactorized forms in Eqs. (16) and (19) are presented (solid lines) in Fig. 2. The experimental data of Gosset *et al.*[1] on ^{4}He

TABLE I. Nuclear density parameters.

Nucleus	w	a (fm)	R_0 (fm)	rms radius (fm)
^{4}He	0.445	0.327	1.01	1.71
^{20}Ne	0	0.571	2.80	3.03
^{23}Na	0	0.587	3.13	3.22
^{40}Ar	0	0.61	3.39	3.47
^{208}Pb	0.32	0.54	6.40	6.49
^{238}U	0	0.605	6.80	5.73

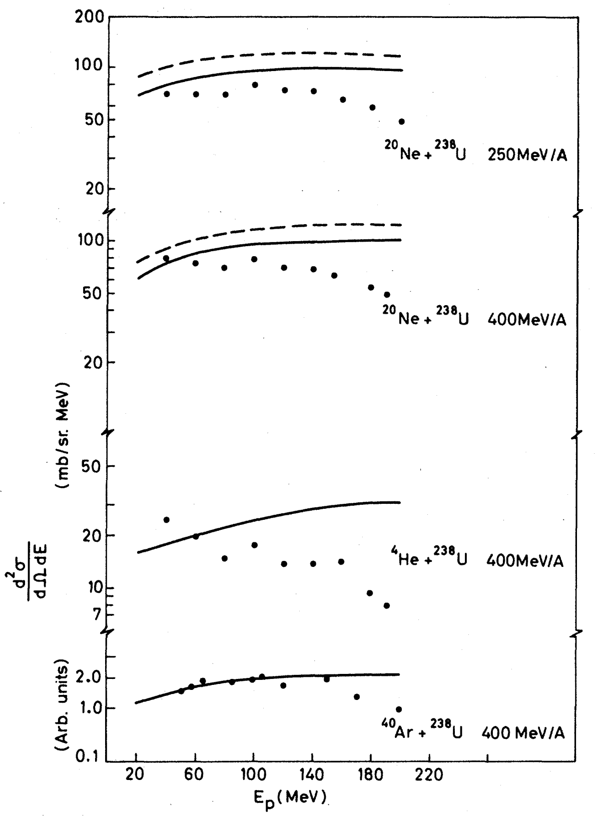

FIG. 2. Inclusive proton spectra for various reactions at 250 and 400 MeV/A. The solid line corresponds to the unfactorized expression and the broken line to the factorized expression. Experimental points are from Ref. 1.

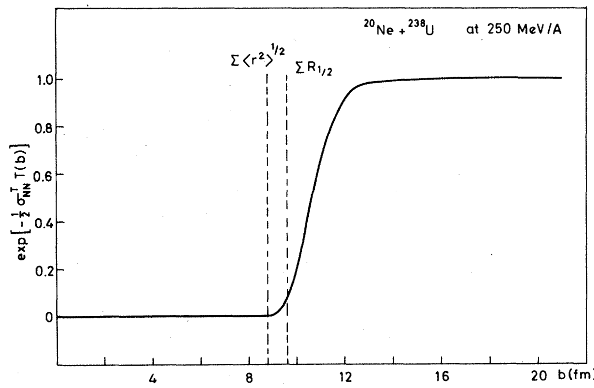

FIG. 3. Attenuation factor for ^{20}Ne + ^{238}U at 250 MeV/A incident energy (E_N).

+ ^{238}U and ^{20}Ne + ^{238}U at 250 and 400 MeV, and that of Poskanzer[1] on Ar + U at 400 MeV/A are also shown in Fig. 2. It is remarkable that the theoretical results reproduce the experimental data in both magnitude and shape. This suggests that the proton inclusive spectra in high energy heavy-ion collisions in the forward hemisphere are mainly due to single nucleon knockout.

In Fig. 2 we have also shown results for Ne + U for the factorized form, Eq. (23). In computing this we have neglected the Fermi momenta and computed the $\overline{\sigma}_{pN}$ corresponding to asymptotic momenta. These results are shown by broken lines. The shape of these curves are similar to those of the unfactorized curves, magnitudes are of course more by about 20%. This means that the effect of the Fermi momentum of nucleons is not very significant. This is expected as the reaction is very much localized on the surface of two nuclei. To demonstrate this we have plotted in Fig. 3 the attenuation factor $\exp[-\frac{1}{2}\sigma^T_{NN}T(b)]$ for Ne + U at 250 MeV. Similar curves result for other combinations of target and projectile. In this figure we have also indicated the points corresponding to a summed half value and r.m.s. radii. This shows that the contribution to the cross section comes only from densities beyond the half value radii.

Incidentally, if we neglect the multiple scattering effects (introduced through the distortion of the incoming and outgoing particle in the present formalism) completely, we will get the genuine single scattering approximation. Naively, in this approximation

$$\frac{d^3\sigma}{dE_p\, d\Omega} = \left[\frac{Z_P}{A_P}\overline{\sigma_{pN}(E_N, q^P)} + \frac{Z_T}{A_T}\overline{\sigma_{pN}(E_N, q^T)}\right] A_P A_T \,. \tag{24}$$

Comparison of this expression with Eq. (23) suggests that the multiple scattering essentially introduces two modifications to the "single scattering approximation" of Eq. (24).

(i) The number of nucleons in the projectile or target are changed to an effective number of nucleons $N_{\rm eff}$. This number ($N_{\rm eff}$) depends upon the incident energy and the energy of the outgoing proton. Since the nucleon-nucleon total cross section decreases with energy becoming approximately constant beyond around 200 MeV, $N_{\rm eff}$, apart from renormalizing the cross section, also influences the shape of the energy spectrum by lowering it at lower energies.

(ii) The proton-nucleon cross section $\overline{\sigma}_{pN}$ has to be averaged only over the momenta of peripheral nucleons, as against the averaging over the whole nuclear volume in Eq. (24). This effectively reduces the modification due to the Fermi momentum of nucleons and the Pauli blocking.

In order to get an idea about the value of $N_{\rm eff}$ and its variation with the incident energy, we have listed them in Table II for ^{20}Ne projectile on ^{23}Na, ^{208}Pb, and ^{238}U targets at 250, 400, and 800 MeV/A incident energies. Since $N_{\rm eff}$ is a func-

TABLE II. Effective number of nucleons.

Reaction	E_n (MeV/A)	250 N^P	250 N^T	400 N^P	400 N^T	800 N^P	800 N^T
^{20}Ne + ^{23}Na		0.65	0.75	0.75	0.86	0.71	0.82
^{20}Ne + ^{208}Pb		0.08	0.85	0.09	0.97	0.08	0.87
^{20}Ne + ^{238}U		0.08	0.94	0.09	1.07	0.08	0.96

tion of E_p, the listed numbers correspond to the high energy (~200 MeV) end of the proton spectrum. The smallness of these numbers is again the reflection of strong attenuation.

In conclusion we may state that the contribution to single nucleon knockout in high energy heavy-ion collisions comes from the periphery of the nuclei, more precisely, beyond the half value radii. Calculated results in DWIA agree well with the experiments in forward direction. It would be interesting to know how many scatterings (double, triple, etc.) would suffice to fit the data at all angles. If the findings of this paper are any indication, not many scatterings should be required. The work of Hufner and Knoll,[4] for example, takes care of all scatterings but it does not tell the contributions of single, double, etc., scatterings individually.

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