Statistical analysis of the reaction $^{12}C(^{12}C,\alpha)^{20}Ne$ in the energy range 7–15 MeV

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Statistical analyses involving deviation functions, channel cross correlations, and the number of maxima method were applied to the $^{12}C(^{12}C,\alpha)$ reaction in the energy range 7.4–15 MeV. They show that the structures found in the angle integrated excitation functions are not consistent with a pure statistical reaction mechanism.

[NUCLEAR REACTIONS Statistical analysis of the reaction $^{12}C(^{12}C,\alpha)$, $E=7.4$–15 MeV; existence of nonstatistical features deduced.]

I. INTRODUCTION

Excitation functions of light heavy ion systems show typically narrow structures ($\Gamma \approx 100$–400 keV) if measured with sufficiently good energy resolution. In most cases these structures can be explained as statistical fluctuations. There are, however, a few systems the excitation functions of which contain besides statistical fluctuations narrow structures which cannot be explained in the framework of the statistical model. These structures are called nonstatistical structures in the following. It should be pointed out that we are not referring to a particular reaction mechanism by using this notation.

The most pronounced narrow structures of nonstatistical origin exist in the $^{12}C+{}^{12}C$ system. They were found for the first time at sub-Coulomb energies by Bromley *et al.*[1] two decades ago. A subsequent investigation of $^{12}C+{}^{12}C$ excitation functions above the barrier showed that narrow structures exist at these energies, too.[2,3] From statistical analyses of these data it was, however, concluded that these structures are in accord with statistical model predictions. (Similar conclusions were drawn later on also by other authors.[4,5]) Therefore it was believed that nonstatistical structures exist only at sub-Coulomb energies.

For a few years it has been known, however, that $^{12}C+{}^{12}C$ excitation functions above the barrier contain, besides statistical fluctuations, nonstatistical resonant structures[6-8] in contrast to the results of the statistical analyses mentioned above. In this paper a statistical analysis of new $^{12}C+{}^{12}C$ data measured above the barrier[8] is carried out. The aim of this investigation is to shed light upon the apparent discrepancy between new data and the analyses of Refs. 2–5.

II. STATISTICAL ANALYSIS

Before applying a statistical analysis to measured data it is necessary to determine if the assumptions of the statistical model are fulfilled in the particular reaction. Two of these assumptions which are frequently rather crucial in case of heavy ion reactions are the following: (i) The background part of the scattering matrix should be roughly energy independent yielding an average cross section which varies only weakly with energy. (ii) The mean width Γ of compound nuclear states should be large compared with the mean level spacing D $(\Gamma/D \gg 1)$. Condition (ii) is fulfilled in the energy range studied in this work ($E_{c.m.}=7.4$–15 MeV); condition (i), however, is not. Therefore we had to remove the effect of the energy dependent average cross section to get, nevertheless, meaningful results from this statistical analysis.

There are quite a few different types of statistical analyses known from the literature which can be applied to experimental data. The choice of a particular analysis should depend on the questions one is dealing with.

In the context of this paper we are interested in the following two questions: (i) Do the $^{12}C+{}^{12}C$ data contain nonstatistical effects? (ii) Which of the structures are of nonstatistical origin? In order to answer these questions we have performed a deviation function analysis of the data since we believe that this analysis is most suited in order to answer the above questions. In addition we have applied a channel cross correlation analysis and the number-of-maxima method recently proposed by Dennis *et al.*[9]

III. DEVIATION FUNCTION ANALYSIS

A deviation function $D(E)$ can be defined by

$$D(E)=\frac{\sigma(E)}{\langle\sigma\rangle}-1, \tag{1}$$

where $\langle\sigma\rangle$ is the average cross section which should be roughly energy independent according to the assumptions of the statistical model.

As mentioned above, $\langle\sigma\rangle$ is not energy independent in the present case. It turns out that the data averaged over a sufficiently large energy interval Δ ($\Delta \gtrsim 10\Gamma$) still contain broad structures. This can be seen from Figs. 1 and 2. These figures show the experimental data[8,10] (angle integrated excitation functions) which are subject of this analysis. The solid lines are Hauser-Feshbach calculations to be discussed later. The dashed lines are sliding averages obtained with $\Delta = 1.4$ MeV.

In order to eliminate the effect of these gross structures which can invalidate the statistical

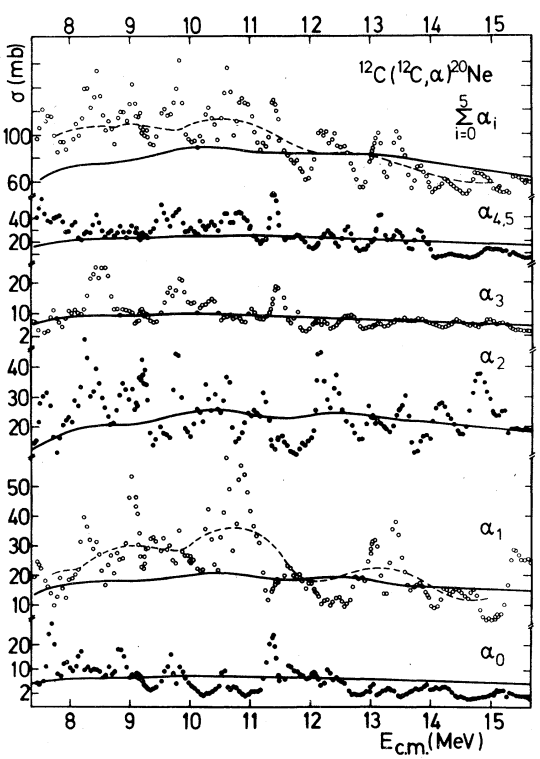

FIG. 1. Angle integrated cross sections of the reaction $^{12}C(^{12}C,\alpha)$ leading to several states in ^{20}Ne. The data are from Refs. 8 and 10. Solid lines are HF calculations, dashed lines are sliding averages obtained with $\Delta = 1.4$ MeV.

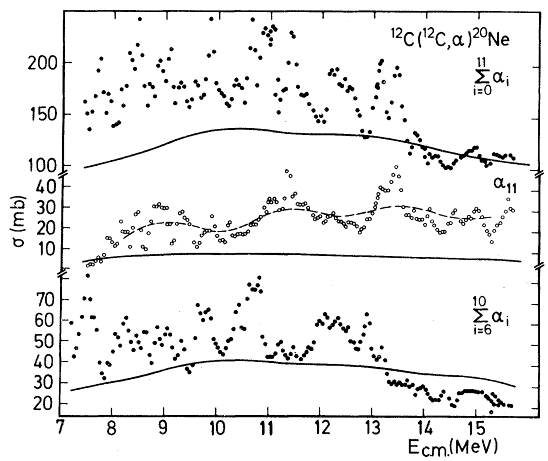

FIG. 2. Angle integrated cross sections of the reaction $^{12}C(^{12}C,\alpha)$ leading to several states in ^{20}Ne. The data are from Refs. 8 and 10. Solid lines are HF calculations, the dashed line is a sliding average ($\Delta = 1.4$ MeV).

analysis we used instead of a constant $\langle\sigma\rangle$ a running average $\langle\sigma(E)\rangle_\Delta$ which was obtained by averaging the data over the energy interval Δ. This interval has to be chosen in such a way that $\langle\sigma(E)\rangle_\Delta$ contains only very little of the fluctuating structures and most of the gross structures. In order to find a suitable Δ we studied the dependence of the normalized variance $C(\epsilon=0)$ on the averaging interval Δ where $C(\epsilon)$ is the energy correlation function defined as

$$C(\epsilon)=\left\langle\frac{\sigma(E)\sigma(E+\epsilon)}{\langle\sigma(E)\rangle_\Delta\langle\sigma(E+\epsilon)\rangle_\Delta}\right\rangle_I-1\,. \tag{2}$$

The brackets $\langle\ \rangle_I$ define an average over the entire energy range I.

One expects according to Pappalardo[11] that $C(0)$ increases with increasing Δ until Δ is large enough to remove all rapid fluctuations from the measured excitation function. A further increase in Δ should leave $C(0)$ approximately constant yielding a plateau in the $C(0)$ versus Δ plot. The occurrence of the plateau indicates that Δ is sufficiently large to meet the requirements on $\langle\sigma\rangle_\Delta$ mentioned above. Inspection of Fig. 3 shows that such a plateau occurs between 1 MeV$<\Delta<$2 MeV for almost all excitation functions. Therefore we chose $\Delta = 1.4$ MeV as the appropriate averaging interval for the $^{12}C + {}^{12}C$ data under study. This Δ value is approximately ten times the average width of normal states in ^{24}Mg having an excitation energy which corresponds to $E_{c.m.} = 7-15$ MeV.[4]

Figures 4 and 5 show the deviation functions obtained. In order to draw any conclusions concerning questions (i) and (ii) one has to compare these

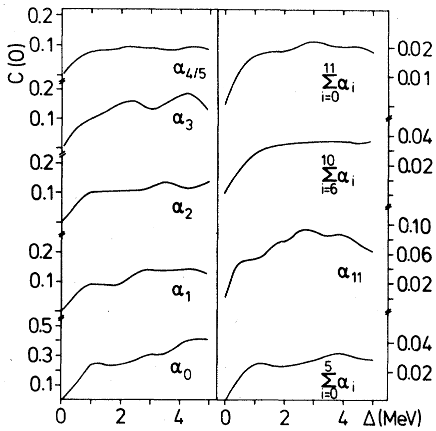

FIG. 3. Normalized variances $C(0)$ as a function of the averaging interval Δ.

deviation functions with predictions of the statistical model. A suitable prediction to compare with is the statistical probability distribution $P(\sigma/\langle\sigma\rangle)$ given by the expression

$$P(\sigma/\langle\sigma\rangle)=\frac{N^N}{(N-1)!}\left(\frac{\sigma}{\langle\sigma\rangle}\right)^N \exp\left(-N\frac{\sigma}{\langle\sigma\rangle}\right), \qquad (3)$$

where N is the number of effective channels which contribute to a particular transition.[12] Equation

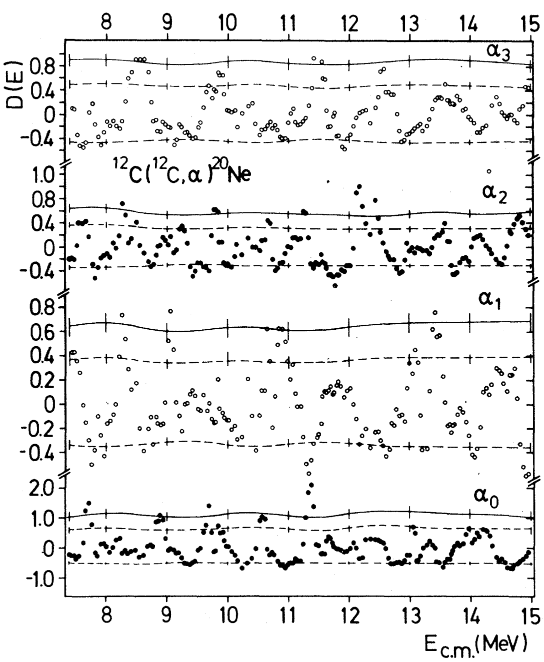

FIG. 4. Deviation functions of the transitions α_0–α_3. For the solid and dashed lines see text.

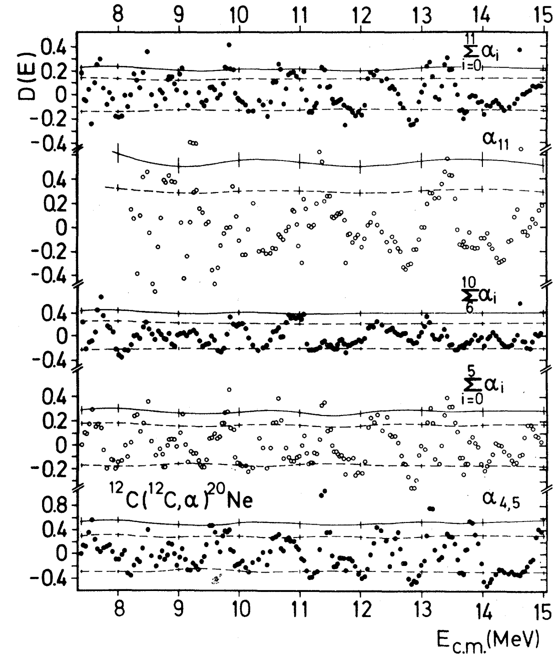

FIG. 5. Deviation functions for several transitions to ^{20}Ne. For the solid and dashed lines see text.

(3) does not contain the effect of direct contributions. They are assumed to be small in the present case as will be shown later.

For angle integrated cross sections N can be expressed (in the absence of direct contributions) by[13]

$$N=\frac{(\sum \sigma^J_{\alpha s l,\alpha' s' l'})^2}{\sum(\sigma^J_{\alpha s l,\alpha' s' l'})^2}, \qquad (4)$$

where $\sum \sigma^J_{\alpha s l,\alpha' s' l'}$ is the energy averaged angle integrated cross section for the transition from the initial state α with total spin J, angular momentum l, and channel spin s to the final state α' with angular momentum l' and channel spin s'. The summation is over J, l, s, l', and s'. The energy averaged cross section $\sum \sigma^J_{\alpha s l,\alpha' s' l'}$ is identical with the angle integrated Hauser Feshbach (HF) cross section σ^{HF}. Therefore N can be obtained from HF calculations.

We have performed HF calculations[14] in order to determine N. The parameters chosen for these calculations were identical with those used by Greenwood *et al.*[15] with the exception of the depth of the real potential V for the $^{12}C+{}^{12}C$ channel which was chosen to be 6.0 MeV over the entire energy range. The reason for this change was that better fits were obtained with the revised potential for the elastic angular distributions recently measured in the energy range 7–14 MeV by Treu *et al.*[16]

Figures 1 and 2 show the calculated HF cross sections (solid lines). They represent approximately the smooth background part of the measured cross sections. Only the calculations for the sum of the first 12 transitions and for the transition to the 7.83 MeV 2^+ state in ^{20}Ne result in cross sections which are too small. Since the 7.83 MeV state is assumed to have a 8p-4h configuration,[17] it is most likely that direct contributions exist in the (^{12}C, α) reaction which are not included in the HF calculations.

The damping coefficients N_{HF} deduced from these HF calculations are slightly energy dependent. Energy averaged values $\langle N_{HF}\rangle$ (averaged over the entire energy range I) are given in Table I. In cases where m transitions are summed up N_{HF} was calculated according to

$$N_{HF}=\frac{\left(\sum_{i=0}^{m-1}\sigma_i^{HF}\right)^2}{\sum_{i=0}^{m-1}\left(\frac{\sigma_i^{HF}}{N_i}\right)^2}. \tag{5}$$

Table I also contains the damping coefficient $N(C)$ which are deduced from the normalized variance $C(0)$ according to $N(C)=C(0)^{-1}$. This relation holds in cases where significant direct contributions are absent. The $N(C)$ values are corrected for the fact that a sliding average $\langle\sigma\rangle_\Delta$ has been used in the autocorrelation analysis. The correction factor f depends on the averaging interval Δ and the ratio Γ/D. It was deduced[18] from a study of synthetic excitation functions similar to that in Ref. 4. It turns out that f is roughly constant for Γ/D values larger than approximately 8. We obtain $f=0.58\pm0.13$ for $\Delta\approx10\Gamma$. The errors given in Table I are due to the uncertainty in the value of f and due to the finite range of the data.

TABLE I. Damping coefficients $\langle N_{HF}\rangle$ and $N(C)$ as deduced from HF calculations (averaged over I) and from an autocorrelation analysis, respectively. The numbers in brackets are the lower and upper limits on $N(C)$ due to the errors involved.

Transition	$\langle N_{HF}\rangle$	$N(C)$
α_0	2.8	2.4 (1.5, 4.0)
α_1	7.3	6.7 (4.2, 11)
α_2	9.0	5.9 (3.7, 9.7)
α_3	4.2	4.6 (2.9, 7.6)
$\alpha_{4,5}$	10.6	7.5 (4.6, 12)
$\sum_{i=0}^{5}\alpha_i$	33.5	23 (14, 38)
$\sum_{i=6}^{10}\alpha_i$	19.1	19 (12, 31)
α_{11}	4.4	9.5 (6.0, 16)
$\sum_{i=0}^{11}\alpha_i$	55.9	28 (17, 45)

Comparison between the two damping coefficients $\langle N_{HF}\rangle$ and $N(C)$ shows good agreement in most cases within the calculated errors. This justifies the assumption that significant direct contributions are absent [within the uncertainties for $N(C)$]. The transition to the 2^+ state at 7.83 MeV in ^{20}Ne is an exception, since $\langle N_{HF}\rangle$ is noticeably smaller than $N(C)$. This is an additional hint for the existence of a direct contribution. The magnitude of this contribution expressed as the ratio y_d of direct to background contribution was calculated from $y_d^2=1-\langle N_{HF}\rangle/N(C)$. We find the rather large value of $y_d=0.73\,(+0.12,-0.22)$.

The calculated damping coefficients N_{HF} have been used in order to determine probability distributions $P(\sigma/\langle\sigma\rangle,N)$ according to Eq. (3). For the 7.83 MeV state a different formula had to be used due to the existence of direct contributions. We used the expression for $P(\sigma/\langle\sigma\rangle,N,y_d)$ given in Ref. 19. Since all probabilities are to be compared with data which contain the effect of averaging over Δ it is necessary to correct N_{HF} with the factor f.

The percentage n of data points in a measured excitation function which are allowed according to the statistical model to have a deviation $\sigma/\langle\sigma\rangle=x\geq M$, where M is an arbitrary value, can be obtained from

$$n=\left|\int_M^{\pm\infty}P(x,N)\,dx\right|. \tag{6}$$

Instead of calculating n we have determined M for given values of $n=0.02$ and 0.10. Figure 6 shows these M values as a function of the damping coefficient N [the notation -10% means that the integral $\int_{-\infty}^{M}P(x)\,dx$ was evaluated]. Finally we have inserted these M values into Figs. 4 and 5 and connected by solid ($n=0.02$) and dashed ($n=0.10$) lines. This offers a convenient way to compare experimental deviation functions with predictions of the statistical model (for the first time proposed by Dayras *et al.*[20]).

Since our excitation functions contain approximately 160 data points one expects roughly 16 points to fall outside the dashed lines ($n=10\%$). It is, however, obvious from these figures that in most cases many more points are above (below) these lines. This is also true for $n=2\%$, where roughly three points should lie above the solid

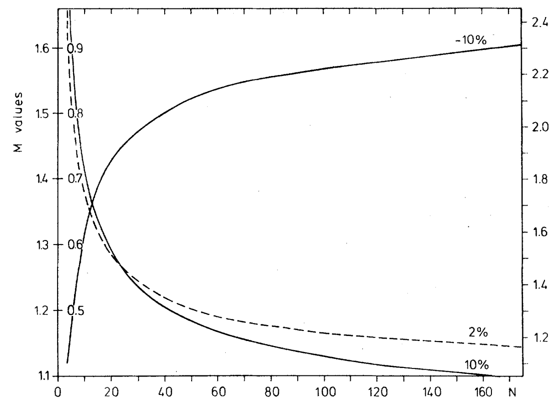

FIG. 6. M values as a function of the damping coefficient N. The values are obtained from $n=|\int_M^{\pm\infty}P(x)dx|$ for $n=2\%$ and 10%. The notation -10% means that the integral $n=\int_{-\infty}^{M}P(x)dx$ was evaluated. Notice that different scales belong to the different curves. The scale to the right belongs to the 2% curve, the scales to the left to the ± 10% curves (the very left scale belongs to the 10% curve).

line. Table II gives the number of points above (below) the 2% and 10% lines with the corresponding upper and lower limits. Inspection of this table shows that significant deviations from the statistical prediction exist. That means that nonstatistical effects play a considerable role in the $^{12}C(^{12}C,\alpha)$ reaction studied, i.e., some of the narrow structures observed must be of nonstatistical origin.

The only case where the statistical predictions seem to be fulfilled is the α_{11} transition. In this case, however, we have included a direct contribution ($y_d = 0.73$) in the calculation of $P(x)$. Thus the agreement merely gives additional support for the existence of large direct contributions in the transition to the 7.83 MeV state.

We have shown that the answer to the first question raised in the beginning is: The $^{12}C(^{12}C,\alpha)$ reaction is not purely statistical in the energy range in question. The answer to the second question is considerably more difficult. The problem is that one has to choose a value for the probability $P(x)$ which allows a separation between statistical and nonstatistical events. In any case, this is a subjective guess which makes this procedure ambiguous.

We have chosen $P(x)=0.02$ as this limit, i.e., all events which fall outside the 2% lines of Figs. 4 and 5 are defined to be of nonstatistical origin.

In Table III we have listed the energies $E(x)$ between 7.7 and 13.5 MeV at which deviations do exist having a statistical probability $P(x) \leq 0.02$. These energies are compared with resonance energies $E(\alpha)$ of the $^{12}C+^{12}C$ resonances which were determined according to the criteria given in Ref. 8. It turns out that deviations with $P(x) \leq 0.02$ exist at all resonance energies (with the exception of the 7.91 MeV resonance which is a rather weak resonance in the α-particle channel). Thus, this

TABLE II. Number of cross sections $S(n\%)$ above (below) the 2% and ±10% lines of Figs. 4 and 5. The numbers in brackets are the lower and upper limits of S due to the errors involved.

Transition	$S(2\%)$	$S(+10\%)$	$S(-10\%)$
α_0	8 (6, 11)	20 (18, 26)	27 (21, 36)
α_1	6 (3, 8)	20 (17, 23)	23 (17, 26)
α_2	11 (9, 14)	27 (24, 32)	32 (25, 41)
α_3	7 (3, 8)	21 (17, 23)	11 (7, 16)
$\alpha_{4,5}$	9 (5, 10)	33 (26, 36)	35 (28, 38)
$\sum_0^5 \alpha_i$	13 (9, 15)	33 (29, 37)	25 (21, 34)
$\sum_6^{10} \alpha_i$	5 (2, 8)	21 (17, 25)	23 (19, 24)
α_{11}	4 (2, 5)	17 (16, 20)	
$\sum_0^{11} \alpha_i$	20 (16, 25)	40 (36, 42)	32 (26, 34)

TABLE III. Energies $E(x)$ of presumably nonstatistical structures, resonance energies $E(\alpha)$, and number of maxima found at $E(\alpha)$. The energy range considered is 7.7–13.5 MeV.

$E(x)$	$E(\alpha)$	Number of maxima
7.71	7.71	3
	(7.91)	2
8.26	8.26	4
8.5	8.46	6
8.89	8.86	5
9.06	9.06	5
9.69		
9.83	9.84	7
10.65	10.63	5
10.88	10.90	1
11.38	11.38	4
11.48		
12.18		
12.44		
13.14	13.12	5
13.43	13.43	3

procedure seems to be indeed suited for the detection of nonstatistical structures. We find, however, deviations with $P(x) \leq 0.02$ at three energies which have not been assigned to be resonance energies. This reflects part of the problems mentioned above and the general difficulty of statistical analyses in giving clear-cut assignments of that particular type.

IV. CHANNEL CROSS CORRELATION ANALYSIS

Normalized cross correlation coefficients $C(\alpha,\alpha')$ have been calculated for those exit channels α and α' which are not intrinsically correlated, using the expression for $C(\alpha,\alpha')$ given in Ref. 15. The coefficients are plotted as a histogram in Fig. 7. This histogram should be symmetric around $C(\alpha,\alpha')=0$ if the structures in the excitation functions are uncorrelated, i.e., if the reaction mechanism is pure statistical. This is obviously not the case. In fact positive $C(\alpha,\alpha')$ values are enhanced indicating that correlations between maxima in the excitation function do exist (which is of course already obvious from the excitation functions). This is further evidence for the existence of nonstatistical effects in the $^{12}C(^{12}C,\alpha)$ reaction.

V. NUMBER-OF-MAXIMA ANALYSIS

This analysis has been recently applied to the $^{12}C(^{12}C,\alpha)$ reaction (at somewhat higher energies) by Dennis *et al.*[9] The subject of this analysis is the number of maxima which occur simultaneously at one energy in a group of excitation functions. These maxima should obey a binomial distribution provided that several assumptions are fulfilled. Therefore the probability $P(y,z)$ of observing y maxima in a group of z excitation functions at one particular energy is given by the yth member of the binomial series

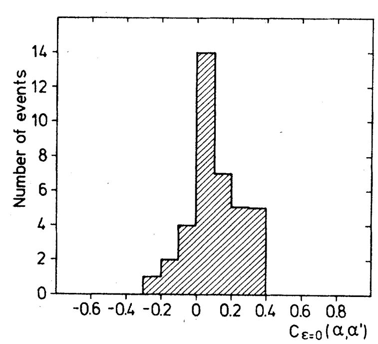

FIG. 7. Normalized channel cross correlation coefficients for independent excitation functions plotted as histogram.

$$P(y,z)=\frac{z!}{y!(z-y)!}p^y(1-p)^{z-y}, \quad (7)$$

where p is the probability parameter determined by the ratio of all maxima to the number of events. If y maxima are observed at one energy and if $P(y,z)$ is smaller than an arbitrarily chosen value $P_0(y,z)$ then the structures are believed to be of nonstatistical origin. The arbitrariness in choosing $P_0(y,z)$ weakens, as in the case of the deviation function analysis, the conclusiveness of this investigation.

We have included in our analysis only excitation functions which are independent from each other. As maxima we defined structures with deviations $\sigma/\langle\sigma\rangle \geq 1.2$. For $P_0(y,z)$ we chose the value 0.02 as in the deviation function analysis. From this choice it follows that a structure is likely to be of nonstatistical origin if it appears simultaneously in three or more of the seven excitation functions under investigation.

Column 3 of Table III shows how often those structures which were assigned to be real resonances in Refs. 8 and 10 show up simultaneously in all excitation functions. It is obvious that all resonant structures with the exception of the 10.9 MeV resonance and the weak resonance at 7.91 MeV meet the requirements for nonstatistical structures given above. Only at one energy ($E = 12.3$ MeV) which was not assigned to be a resonance energy are three maxima observed simultaneously indicating that these structures might be nonstatistical in origin.

VI. CONCLUSION

Several statistical methods have been applied to the $^{12}C(^{12}C,\alpha)$ reaction in the energy range 7.4–15 MeV. From the study of deviation functions and channel cross correlation coefficients it follows immediately that the reaction mechanism is not purely statistical and that part of the structures observed in the excitation functions is most likely of nonstatistical origin. Comparison between damping coefficients deduced from HF calculations and from the normalized variance $C(0)$ shows that direct contributions are absent in most transitions. Only the transition to the 7.83 MeV state with an assumed 8p-4h configuration takes place mainly via a direct mechanism.

With the deviation function analysis and the number-of-maxima method we were able to determine those energies at which most likely nonstatistical structure exists assuming that events occurring with a statistical probability smaller than 0.02 are of nonstatistical origin. This is, of course, an arbitrary choice. Nevertheless we find satisfactory agreement with known resonance energies determined according to the criteria of Ref. 8.

Our investigations show that statistical methods, properly chosen, are suited in order to detect nonstatistical structures among statistical fluctuations and, moreover, that there is a good chance to determine energies at which nonstatistical structures exist. Moreover, they eliminate a discrepancy for the $^{12}C+^{12}C$ system which originated from the fact that the existence of resonances between $E_{c.m.}=7$ and 15 MeV is well established, whereas statistical analyses gave no conclusive evidence for the existence of nonstatistical effects.

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