PHYSICAL REVIEW C VOLUME 22, NUMBER 2 AUGUST 1980

Variation of ground-state α-particle strengths for sd- and fp-shell nuclei

N. Anantaraman, H. W. Fulbright, and P. M. Stwertka
Nuclear Structure Research Laboratory, University of Rochester, Rochester, New York 14627
(Received 26 February 1980)

It is shown that when (^{6}Li,d) experimental results are reduced via a distorted-wave Born approximation procedure taking into account the finite size of ^{6}Li, the systematic variation in α-particle transfer strengths in the $A = 20$ to $A = 68$ region agrees with recent theoretical strength predictions based on shell-model wave functions.

[NUCLEAR REACTIONS DWBA analyses of (^{6}Li, d) reactions on 19 sd- and 17 fp-shell targets. Derived S_α(g.s.). Compared with the shell-model predictions.]

I. INTRODUCTION

The success achieved in treating (^{6}Li, d) and (d, ^{6}Li) data via direct reaction theory based on the model of simple alpha cluster transfer is well known.[1] Both zero-range and finite-range distorted-wave Born approximation (DWBA) calculations have produced good fits to angular distributions; relative strengths extracted for transitions to various final states of the same nucleus via different analyses are generally found rather stable against substantial changes in the optical model and bound-state parameters used, provided that angular distributions are fitted reasonably well, and it has therefore been possible to apply this type of analysis in spectroscopy. However, such analyses have not yielded credible absolute α-particle spectroscopic strengths or even relative strengths of ground-state transitions ($S_{g.s.}$) as a function of the mass of the target nucleus. There are several reasons for this. Values of $S_{g.s.}$ extracted are sometimes very sensitive to the choice of parameters, e.g., the radius parameter of the bound-state potential well. The choice of the ^{6}Li optical well parameters is somewhat arbitrary, since the best guide in their choice, analysis of elastic scattering data, can typically yield several sets of parameters between which a choice can be made only if additional constraints are introduced. In principle, this difficulty can be circumvented through some procedure introducing information from other types of experiments. For example, α-particle decay rates can be related to the data from the α-particle transfer experiments, as can reaction data from (p,$p\alpha$) and (α, 2α) experiments. Or, should sufficiently reliable theoretical predictions of α-particle transfer strengths exist, a normalization and parametrization procedure to produce consistent results over a range of nuclei could be employed. Substantial progress in these directions has been made, but certain contradictions remain and the variation of $S_{g.s.}$ with $A_>$ (mass of residual nucleus) has not been established until now. The situation as of a year ago is reviewed in Ref. 1.

When early results of systematic (^{6}Li, d) reaction studies in the sd and lower fp shells were assembled,[2] the plot of $S_{g.s.}$ with $A_>$ for even values of $A_>$ from 20 to 66 showed first a fall, then a striking rise to a strong, broad peak at $A_> = 36$, then a fall to a minimum at about $A_> = 52$, and finally a rise to $A_> = 66$. Although detailed theoretical calculations covering this region had not been made, the strong variations seen were puzzling. Later data and analysis[1,3] have modified the picture somewhat, bringing out steps attributable to shell effects and changing the general shape a little, but the overall strong strength variations with $A_>$ have remained unexplained. [See Fig. 3(a).] Also shown in Fig. 3(a) is a plot of theoretical values of $S_{g.s.}$ calculated from the formalism of Ichimura *et al.*[4] as programmed by Bennett,[5] applied with shell-model wave functions: (1) throughout the sd shell in the full $(sd)^m$ space,[6] (2) for the single case ^{40}Ca$\rightarrow{}^{44}$Ti in the $(fp)^4$ space,[5] and (3) for ^{58,60,62,64}Ni$\rightarrow{}^{62,64,66,68}$Zn in the $(f_{5/2}, p_{3/2}, p_{1/2})^m$ space.[7] The predicted values are shown connected by straight lines. The peak seen in the experimental strength in the mass region 36–40 does not appear. Predicted values in the Zn region are much lower than the experimental values shown.

At the outset of the present work, it was recognized that the disagreement might have at least three contributors. First, there were uncertainties in the values of the experimental cross sections. Systematic examination of the data raised a few questions which were answered by making several new measurements, after which all the Rochester sd-shell data were internally consistent[8] and the Rochester[9] and Los Alamos[3] fp-

shell data appeared consistent to within 30% or better, except possibly for $^{48}Ca \rightarrow {}^{52}Ti$, which has not been remeasured. See the comparison made in Fig. 12 of Ref. 1. It was concluded that the experimental trends of the cross sections were sufficiently well established.

Second, deficiencies in the theoretical results could be significant. There are deficiencies in the wave functions used, the most serious of which may be the neglect of core polarization effects at the ends of the shells,[10,11] and in the Ni → Zn region. The *sd*-shell wave functions used (the Chung-Wildenthal wave functions[12]) have been checked via the (t,p) reaction.[13] This is a sensitive test, since the two-nucleon transfer strengths depend strongly on nucleon-nucleon correlations. Satisfactory results were found, except at the ends of the shell, where increases of 20% and 100% were seen in the $^{16}O(t,p)$ and $^{36}Ar(t,p)$ cross sections, respectively, compared to *sd*-shell predictions. These deviations can be explained in terms of the neglected polarization effects.[13] Moreover, use of these wave functions in general successfully explains the relative strengths of excited levels found in *sd*-shell $(^6Li, d)$ reactions. The wave functions used for the Ni → Zn transitions[7,14] have been tested similarly against the observed strengths of (p,t) reactions, where good agreement is found.[15] The normalization factor required differs by less than 50% from that found for the (p,t) reaction in the *sd* shell. Some reassurance is also derived from the fact that Bennett *et al.*[7] used three different α-particle operators in calculating $S_{g.s.}$ for various Ni → Zn transitions, all of which gave for each case internal agreement within 20%, except for $^{64}Ni \rightarrow {}^{68}Zn$, where the greatest difference was 33%. For these reasons the existing values of $S_{g.s.}$ calculated from shell-model wave functions appear to give a sufficiently reliable prediction of the general systematic pattern of strengths with $A_>$, even though uncertainties exist at the ends of shells.

A third probable contributor to the disagreement is the lack of a proper prescription for reducing experimental cross sections to strengths over a range of target nuclei. In particular, the proper variation of optical model parameters associated with ^6Li is in question, as is the variation of the bound-state radius parameter. (Good deuteron parameters have long been known.[16]) The straightforward procedure of obtaining the ^6Li parameters from analysis of elastic scattering data gives ambiguous results, as mentioned earlier. Part of the ambiguity can be removed by requiring the radius parameter to vary as $A^{1/3}$, as is often done. Parameters found this way typically lead to relative α-particle transfer strength values like those plotted in Fig. 3(a).

The Strohbusch-Bauer-Fulbright (SBF) parameter set,[9,17] found early in the study of the $(^6Li, d)$ reaction in the *fp* shell by a trial and error process requiring good fits to the reaction angular distributions, includes an entrance channel radius parameter variation making allowance for the finite size of the ^6Li projectile, $R \propto (A^{1/3} + 1.9)$. When the SBF set is used to reduce the *fp*-shell data, the strength variation found is in better agreement with theoretical predictions.[1,7,18] The improvement in trend is due primarily to the different variation of entrance channel radius. Calculated cross sections are quite sensitive to that parameter. Although providing allowance for the size of the projectile is reasonable and seems to improve the variation of $S_{g.s.}$ with $A_>$, the SBF parameter set is unsuitable for present purposes because it fits neither the *fp*-shell elastic scattering data nor the *sd*-shell $(^6Li, d)$ reaction data.

II. DISTORTED-WAVE BORN APPROXIMATION PROCEDURE

An attempt was made to find a DWBA procedure which would reduce the data to agreement with theoretical predictions. The plan was to demand that (1) the entrance channel radius parameter should include an allowance for the radius of ^6Li, as suggested by the results from use of the SBF parameters; (2) the $(^6Li, d)$ ground-state angular distributions should be fitted throughout the $A_> = 20$ to 68 region; (3) ^6Li elastic scattering should also be fitted. Fortunately there already existed a successful ^6Li parameter set derived from elastic scattering of 50-MeV ^6Li on ^{58}Ni by Chua *et al.*[19] which gave good fits to reaction data in both the *sd* and *fp* shells.

The depth and diffuseness of the real and imaginary parts of the Chua potential for ^{58}Ni were used for all the *sd*- and *fp*-shell nuclei, while the radii for the two parts of the potential were varied according to the expressions (in fm) $R_R = 0.836 (A^{1/3} + 1.9)$ and $R_I = 1.093 (A^{1/3} + 1.9)$. These expressions give the same values for the entrance channel radii, $R_R = 1.3\, A^{1/3}$ and $R_I = 1.7\, A^{1/3}$, as does the Chua potential at $A = 40$, but not for other values of A. By a rough search procedure the constant 1.9 was found to yield the best agreement of $S_{g.s.}$ with theoretical predictions. (In these searches, the value of the additive constant in R_R and R_I was kept the same.) The deuteron potential used was that of Ref. 16. The bound-state radius was varied as $1.029 (A^{1/3} + 1.0)$, corresponding to $R = 1.33\, A^{1/3}$ fm at $A = 40$, the additive constant again being chosen by a rough search. With these potential parameters used in the zero-range

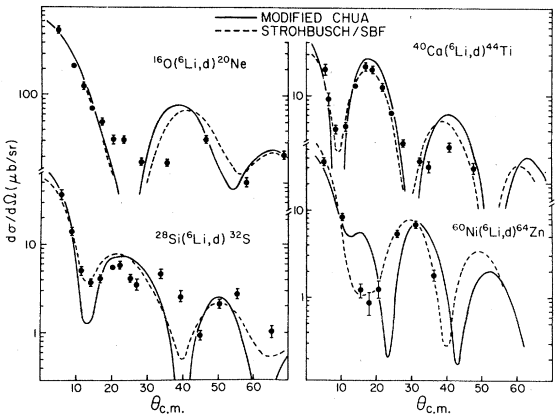

FIG. 1. Angular distributions for four (^{6}Li, d) ground-state transitions. The ^{6}Li energy is 32 MeV for all except the ^{60}Ni(^{6}Li, d) reaction, for which it is 28 MeV. The solid and dashed curves are results of zero- or finite-range DWBA calculations (see text).

DWBA code DWUCK,[20] (^{6}Li, d) angular distributions were calculated and fitted to the data.

Figure 1 compares the ground-state (^{6}Li, d) angular distributions obtained with the modified Chua ^{6}Li potential used here and with the potentials used earlier, for four cases. For the reactions on ^{16}O and ^{28}Si, the dashed curves are those obtained using the potential of Strohbusch *et al.*[21] in a finite-range DWBA calculation with the code LOLA.[22] For the reactions on ^{40}Ca and ^{60}Ni, the dashed curves are those obtained using the SBF potential in a DWUCK analysis. It is seen that the quality of fits to the angular distributions continues to be as good with the modified Chua potential as with the earlier potentials, with the exception of the ^{60}Ni(^{6}Li, d) reaction. The modified Chua potential gives a much improved fit in the case of the ^{62}Ni(^{6}Li, d) reaction.

We have measured ^{6}Li elastic scattering on targets of ^{16}O, ^{28}Si, ^{40}Ca, ^{56}Fe, ^{60}Ni, and ^{90}Zr at 28 or 32 MeV. Figure 2 shows the data for the same four targets as in Fig. 1. In the *fp* shell the fits are considerably better with the modified Chua potential than with the SBF potential, while in the *sd* shell they are comparable to those obtained with the Strohbusch *et al.* potential[21] (acceptable for ^{28}Si, poor for ^{16}O). The four dash-dotted curves are the results of using the ^{6}Li optical potentials determined by Chua *et al.*[19] from elastic scattering of 50-MeV ^{6}Li on ^{16}O, ^{16}O, ^{40}Ca, and ^{58}Ni, respectively. Note that the fits obtained with these Chua potentials for our 28- or 32-MeV data are on the whole worse than with the modified Chua potential.

The values of $S_{g.s.}$ obtained with the modified Chua potential for even-A nuclei from ^{20}Ne to ^{68}Zn are shown in Fig. 3(b), along with the theo-

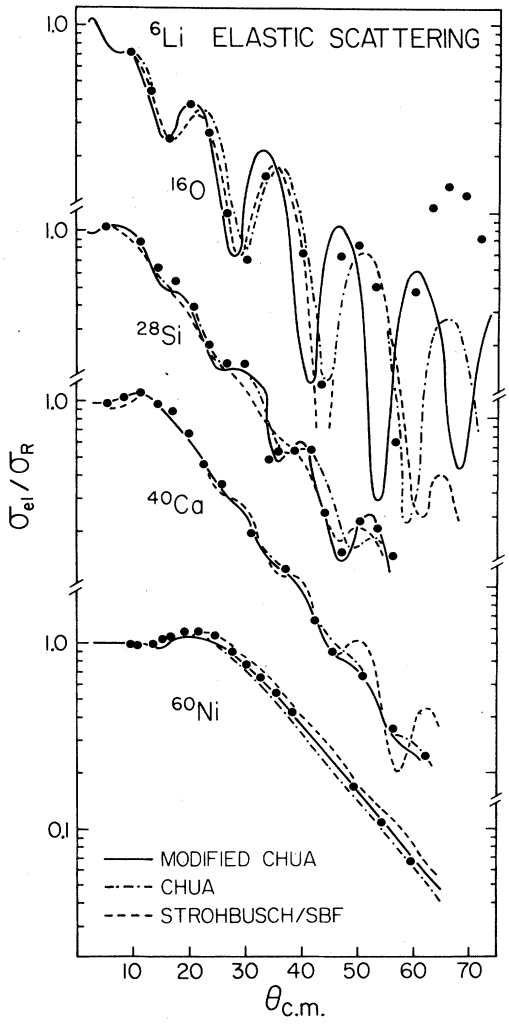

FIG. 2. Elastic scattering angular distributions for 32-MeV ^{6}Li on ^{16}O, ^{28}Si, and ^{40}Ca, and 28-MeV ^{6}Li on ^{60}Ni, together with the results of optical model calculations.

retical predictions. The experimental and theoretical strengths have both been normalized to unity at ^{20}Ne. For comparison, Fig. 3(a) shows the systematics obtained using the unmodified ^{6}Li Chua potential for ^{58}Ni (with R_R and R_I varying as $A^{1/3}$) and a bound-state radius of 1.33 $A^{1/3}$ fm; this is similar to the data reduction scheme employed in Ref. 2. The solid circles in Fig. 3 are obtained from Rochester data; of the two circles at $A_> = 54$ and 58, the upper ones are for ^{54}Cr and ^{58}Fe, the lower for ^{54}Fe and ^{58}Ni. The crosses are from Los Alamos data[3] for nuclei not studied at Rochester: ^{50}Cr, ^{52}Cr, and ^{62}Ni.

III. DISCUSSION

A dramatic improvement in agreement between the extracted and predicted trends is evident,

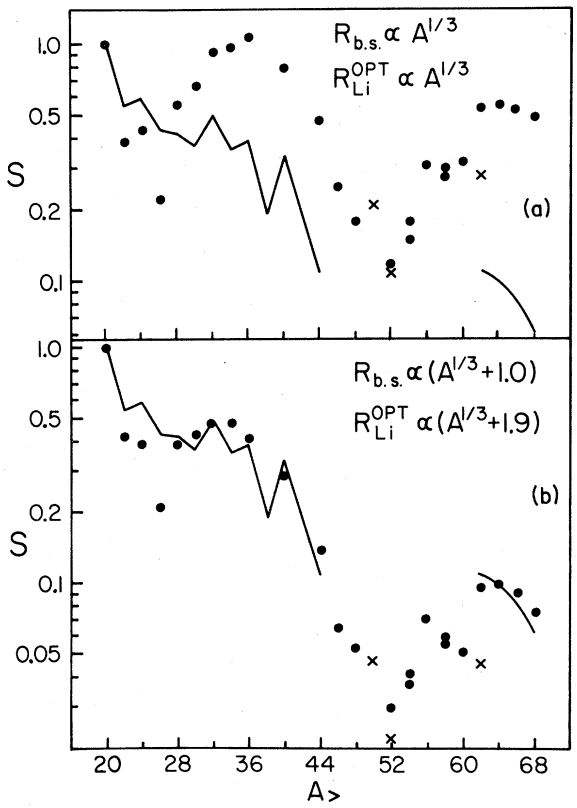

FIG. 3. Systematics of $S_{g.s.}$, obtained (a) using the unmodified Chua potential for ^{58}Ni and with $A^{1/3}$ variation in the bound-state radius, and (b) using additive constants in the ^{6}Li potential and bound-state potential radii, as indicated, for doubly even nuclei, $A_>$ being the mass of the final nucleus populated in the (^{6}Li, d) reaction. The solid circles are from Rochester data, the crosses from Los Alamos. The solid line represents the shell-model predictions.

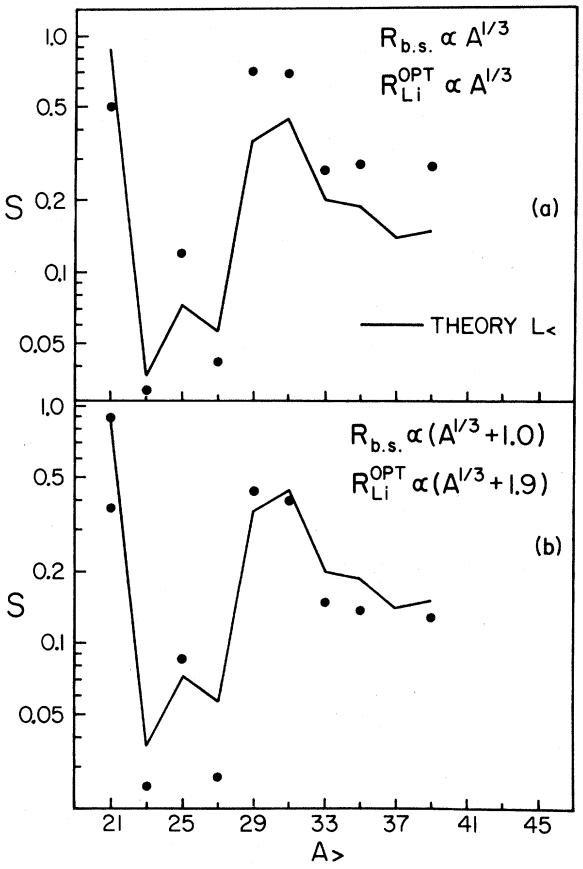

FIG. 4. (a) and (b): As in Figs. 3(a) and 3(b), for odd-A sd-shell nuclei.

brought about by the use of additive constants in the optical and bound-state potential radii. The small differences between experiment and theory at $A_> = 40$ and 44 are due partly to uncertainties in the experimental points (estimated to be about 25%) and partly to the assumption made in the theory of a closed shell for ^{40}Ca. There is also a marked disagreement with the shell-model prediction at $A_> = 26$ and small deviations are present at $A_> = 20$ and 24, evidently not due to errors in the cross section measurements. It is interesting to note that the extracted values for these cases [in both Fig. 3(a) and Fig. 3(b)] are in much better agreement with SU(3) predictions.[2] On the other hand, the strengths extracted for $A_> = 26$ from two separate (d, ^{6}Li) measurements[23,24] are in agreement with each other and with the shell-model predictions.

The same analysis applied to the ground-state transitions for odd-A sd-shell nuclei[25] gives $S_{g.s.}$ values which are also in improved agreement with the predictions (Fig. 4).

The additive constant in the radius of the bound-state potential of the transferred α-particle was introduced for the sake of consistency and it improves the agreement with the predictions, but it is not essential.

It is well established that the strengths of excited states in a given nucleus relative to $S_{g.s.}$ are insensitive to changes in the optical and bound-state potential parameters. Thus the generally good agreement with theory found in previous analyses of sd-shell excited state strengths is not expected to be affected in the present analysis. This has been explicitly verified in a number of cases.

In Fig. 3(b), two interesting breaks can be seen among the fp-shell strengths, one between ^{54}Fe and ^{56}Fe, the other between ^{62}Ni and ^{62}Zn. These occur where according to the simplest shell-model picture the $p_{3/2}$ subshell is beginning to be filled with neutrons and with neutrons and protons, respectively. The breaks have been observed also by Hanson *et al.*[3] The first of the two breaks

was predicted by Kurath and Towner.[26] Following each break, there is a falloff in the strength with increasing $A_>$, which might indicate a blocking effect due to the Pauli principle.

It would, of course, be useful to have proper shell-model predictions for more *fp*-shell nuclei. Using the Rochester–Oak Ridge shell-model program,[27] we have calculated $S_{g.s.}$ for the ^{90}Zr(^{6}Li, *d*) ^{94}Mo reaction, with the target treated as a closed core and with the valence nucleons in ^{94}Mo having the configuration $(p_{1/2}, g_{9/2})_\pi^2 \times (d_{5/2}, s_{1/2})_\nu^2$. The strength found is an order of magnitude smaller than the experimental strength of 0.009 [using a normalization identical with that of Fig. 3(b)], probably because a sufficiently large configuration space for ^{94}Mo could not be used.

In the present work, the zero-range DWBA code DWUCK was used instead of a finite-range code such as LOLA. This was to reduce the time spent in performing the calculations. In the analysis of Ref. 2, the *sd*-shell strengths were extracted using LOLA and they are very nearly the same in relative values as those found with the DWUCK analysis shown in Fig. 3(a). Thus we believe that the use of LOLA would not alter the results in any significant way. It must be pointed out, however, that the use of the same optical and bound-state parameters in DWUCK and LOLA calculations results in differing angular distributions. Thus the parameters used here will have to be changed if the measured angular distributions are to be fitted with LOLA calculations.

In Refs. 1 and 6 are shown variations of $S_{g.s.}$ with $A_>$ in the *sd* shell, extracted from various transfer and knockout reactions by analyses involving optical potential radii varying as $A^{1/3}$. We have not attempted to determine how the trends shown for the (d, ^{6}Li) and (^{3}He, ^{7}Be) reactions will be modified if the optical potentials for the outgoing particles have a radius variation similar to that used for ^{6}Li in the present work. It is disturbing that although the $S_{g.s.}$ trend obtained from the (α, 2α) reaction agrees with the shell-model predictions, the trend from (p, $p\alpha$) does not.

Possibly the trend of the predicted $S_{g.s.}$ values may have to be revised from that used in the present work, for example if a larger self-model calculation in the Zn region predicts somewhat different $S_{g.s.}$ values relative to the *sd*-shell values. In that event it is likely that a change in the additive constant in the radius could reproduce the changed trend; the present procedure is capable of accommodating moderate changes in the pattern of the predicted $S_{g.s.}$ values.

Finally, our aim has been to show that at least one physically reasonable scheme exists for obtaining agreement between experimental and theoretical α-particle strengths. Our rough parameter search could of course be refined, e.g., by using the additive radius constant as an adjustable parameter in optical model analyses of elastic scattering data or in fitting transfer data.

This work was supported by the National Science Foundation under Grant No. PHY 77-27943 Amend. No. 1.

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