Baer-Kouri-Levin-Tobocman and Faddeev equations for the three-body problem with pairwise interactions

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We reanalyze the Baer-Kouri-Levin-Tobocman and Faddeev equations for the three-body problem with pairwise interactions. The approach to the Baer-Kouri-Levin-Tobocman equations is more in the spirit of the usual Faddeev equations. The detailed operators which appear as "effective interactions" are different from those appearing in earlier forms of the Faddeev equations. It is shown that the same effective interactions appear in both the Baer-Kouri-Levin-Tobocman and Faddeev-type equations. The resulting form of the Baer-Kouri-Levin-Tobocman equations may be more suitable for the approximate treatment of breakup effects than previous forms of these equations.

[NUCLEAR REACTIONS Scattering theory Baer-Kouri-Levin-Tobocman three-body scattering equations, Faddeev three-body scattering equations, three-body scattering with pairwise interactions.]

I. INTRODUCTION

Recently there has been considerable progress in the development of alternatives to the Faddeev equations[1] for treating N-body scattering.[2-19] One approach which has received attention is the channel coupling array method due to Baer and Kouri,[12] Kouri and Levin,[16] and Tobocman.[15] The Baer, Kouri, Levin, and Tobocman (BKLT) equations emphasize the various arrangements and their interactions so that the naturally occurring interactions are the ordinary "channel potentials" and in place of the totally free Green's function, one encounters partially interacting Green's functions which describe the various asymptotic arrangements. While this may have advantages for certain considerations (e.g., in the treatment of chemical rearrangement processes where the interactions are basically known and of electrostatic origin), it has disadvantages for applications to nuclear problems and to dissociative processes. For such problems, it is more convenient to utilize the totally free Green's function and introduce, by appropriate summing of graphs, effective interactions which contain the pole structure arising from collisions of subgroups of particles. Such a structure has not been introduced into the (BKLT) equations even for the simplest three-body problem with pairwise interactions. In the case of the Faddeev equations, this is in fact the form of the equations which is most commonly used and the corresponding effective interactions are t operators describing the collision of two of the particles while the third acts simply as a spectator (it can affect the two-body collision only in the sense that it can possess part of the energy and momentum present in the three-body system). In this paper, we address the problem of deriving an analogous form of the BKLT equations. In the next section, we show that it is possible to derive a similar form for these equations for the three-body problem but that the effective interactions which arise have a more complex (and perhaps interesting) structure than those appearing in the ordinary Faddeev equations. Again, we find that the effective interactions have a straightforward physical interpretation. We also consider the connectivity of the resulting form of the BKLT equations. In addition, we show that Faddeev-type equations can be derived which also involve these new effective interactions in place of the more usual "two-body t operators." Finally, in Sec. III, we briefly discuss our results.

II. DERIVATION OF ALTERNATE FORM OF THE EQUATIONS

We follow the standard notation for the three-body problem with pairwise interactions. Thus, we label the arrangement channels with the index of the body which is free (the breakup amplitude will be obtained in the standard fashion as a linear combination of the partially bound transition amplitudes). Then the transition operator for going from arrangement j (where particle j is free and i and k are bound) to arrangement i, where particle i is free and j and k are bound, is taken to be

$$T_{ij}=V_i+V_iG^+V_j, \tag{1}$$

where G^+ is

$$G^+ = 1/(E - H + i\epsilon)\,, \tag{2}$$

and

$$H = H_i + V_i = H_j + V_j = H_k + V_k\,. \tag{3}$$

For example, H_i is the Hamiltonian describing the free motion of particle i and bound cluster (jk), so that V_i is the interaction of particle i with the bound cluster. Since we assume pairwise interactions, we have that

$$V_i = V_{ik} + V_{ij}\,, \tag{4}$$

where the V_{lm} is the interaction between particles l and m. To derive a channel permuting form of the BKLT equations, we begin by noting that the three-body Schrödinger equation can be written as[1,2,8,16]

$$(E - H_i)\psi_i = V_{i+1}\psi_{i+1}\,,\quad i = 1 \text{ to } 3\,,$$

and

$$V_{i+1}\psi_{i+1} = V_1\psi_1\,,\quad \text{for } i = 3\,. \tag{5}$$

These equations sum up to yield the Schrödinger equation (in differential form), and their formal solution is

$$\vec{\Psi} = \vec{\Phi} + \underline{G_0^+}\,\underline{\mathfrak{V}}^{\mathrm{BKLT}}\vec{\Psi}\,. \tag{6}$$

Here we use an obvious matrix notation and we note that $\vec{\Phi}$ has zeros in every row except for that corresponding to the initial arrangement. The matrix $\underline{G_0^+}$ is diagonal in arrangement channel space with elements $\delta_{ij}G_i^+$, and

$$G_i^+ = 1/(E - H_i + i\epsilon)\,. \tag{7}$$

Then the matrix of arrangement channel transition operators is defined by

$$\underline{T}\,\vec{\Phi} = \underline{\mathfrak{V}}^{\mathrm{BKLT}}\vec{\Psi}\,, \tag{8}$$

or

$$\underline{T} = \underline{\mathfrak{V}}^{\mathrm{BKLT}}(\underline{1} + \underline{G_0^+}\,\underline{T})\,, \tag{9}$$

with

$$\underline{\mathfrak{V}}^{\mathrm{BKLT}} = \begin{pmatrix} 0 & V_2 & 0 \\ 0 & 0 & V_3 \\ V_1 & 0 & 0 \end{pmatrix}. \tag{10}$$

Clearly, taking the initial arrangement to be labeled one, we have for one column of T the equations

$$T_{11} = V_2G_2^+T_{21}\,, \tag{11}$$

$$T_{21} = V_3G_3^+T_{31}\,, \tag{12}$$

$$T_{31} = V_1 + V_1G_1^+T_{11}\,. \tag{13}$$

The structure of the iterated kernel becomes connected after two iterations and the iterated kernel is well known to be[15,16]

$$\mathfrak{K} = V_2G_2^+V_3G_3^+V_1G_1^+\,. \tag{14}$$

The connectivity is determined solely by the structure of the product $V_1V_2V_3$ since G_i^+, $i = 1, 2, 3$ is always disconnected. Explicitly, one has

$$\begin{aligned} V_1V_2V_3 = {} & (V_{12}{}^2 + V_{12}V_{23})(V_{13} + V_{23}) \\ & + (V_{13}V_{12} + V_{13}V_{23})(V_{13} + V_{23})\,, \end{aligned} \tag{15}$$

which is obviously connected. In order to write Eqs. (11)–(13) in a form more analogous to the Faddeev equations, we first examine $V_2G_2^+$. We note

$$V_2G_2^+ = (V_{12} + V_{23})(E - H_0 - V_{13} + i\epsilon)^{-1}\,. \tag{16}$$

Consider the factor $V_{12}(E - H_0 - V_{13} + i\epsilon)^{-1}$. We write

$$\begin{aligned} & V_{12}(E - H_0 - V_{13} + i\epsilon)^{-1} \\ & \quad = [V_{12} + V_{12}(E - H_0 - V_{13} + i\epsilon)^{-1}V_{13}](E - H_0 + i\epsilon)^{-1}\,, \end{aligned} \tag{17}$$

and then we identify τ_{213} by

$$\tau_{213} = V_{12} + V_{12}(E - H_0 - V_{13} + i\epsilon)^{-1}V_{13}\,. \tag{18}$$

We note that by Eqs. (17)–(18), we also have that

$$V_{12}(E - H_0 - V_{13} + i\epsilon)^{-1} = \tau_{213}(E - H_0 + i\epsilon)^{-1}\,. \tag{19}$$

Our notation is designed to indicate that for τ_{ijk}, the inhomogeneity will be V_{ij} and the final interaction will be V_{jk}. Thus, in general,

$$\tau_{ijk} = V_{ij} + \tau_{ijk}(E - H_0 + i\epsilon)^{-1}V_{jk}\,. \tag{20}$$

If we consider the portion of Eq. (16) given by $V_{23}(E - H_0 - V_{13} + i\epsilon)^{-1}$, we see analogously that

$$V_{23}(E - H_0 - V_{13} + i\epsilon)^{-1} = \tau_{231}(E - H_0 + i\epsilon)^{-1}\,, \tag{21}$$

where

$$\tau_{231} = V_{23} + \tau_{231}(E - H_0 + i\epsilon)^{-1}V_{13}\,. \tag{22}$$

In general, we will require V_iG_i in order to write our Eqs. (11)–(13) in terms of the effective interactions τ_{ijk} and the totally free Green's function $(E - H_0 + i\epsilon)^{-1}$. We note by Eqs. (3)–(8) and (16)–(20), that

$$V_iG_i^+ = V_{ij}G_i^+ + V_{ik}G_i^+\,, \tag{23}$$

and therefore

$$V_iG_i^+ = \tau_{ijk}G_0^+ + \tau_{ikj}G_0^+\,, \tag{24}$$

where i, j, k take on values 1, 2, or 3 (but $i \neq j$, $i \neq k$, $j \neq k$). Then the BKLT equations become

$$T_{11} = (\tau_{213} + \tau_{231})G_0^+T_{21}\,, \tag{25}$$

$$T_{21} = (\tau_{312} + \tau_{321})G_0^+T_{31}\,, \tag{26}$$

and

$$T_{31}=(V_{12}+V_{13})+(\tau_{123}+\tau_{132})G_0^+T_{11}\,. \tag{27}$$

Now we consider the Faddeev-type equations. They can be derived in several forms and we shall consider two possible ones. The Lovelace form is given by[2,16]

$$T_{11}=t_{13}G_0^+T_{21}+t_{12}G_0^+T_{31}\,, \tag{28}$$

$$T_{21}=V_{23}+t_{23}G_0^+T_{11}+t_{12}G_0^+T_{31}\,, \tag{29}$$

and

$$T_{31}=V_{23}+t_{23}G_0^+T_{11}+t_{13}G_0^+T_{21}\,. \tag{30}$$

However, we can obtain an alternate form in a fashion similar to that used in obtaining Eqs. (11)–(13). The ordinary three-body Schrödinger equation can be written as[1-2,8,16]

$$(E-H_0-V^{(1)})\psi_1=V^{(1)}\psi_2+V^{(1)}\psi_3\,, \tag{31}$$

$$(E-H_0-V^{(2)})\psi_2=V^{(2)}\psi_1+V^{(2)}\psi_3\,, \tag{32}$$

$$(E-H_0-V^{(3)})\psi_3=V^{(3)}\psi_1+V^{(3)}\psi_2\,. \tag{33}$$

It is again readily verified that these equations sum up to yield the ordinary Schrödinger equation. The formal solution of Eqs. (31)–(33) is

$$\vec{\psi}=\vec{\phi}+\underline{G}_0^+\begin{pmatrix}0 & V^{(1)} & V^{(1)}\\ V^{(2)} & 0 & V^{(2)}\\ V^{(3)} & V^{(3)} & 0\end{pmatrix}\vec{\psi} \tag{34}$$

$$\equiv\vec{\phi}+\underline{G}_0^+\underline{\mathfrak{V}}\,\vec{\psi}\,,$$

where $\vec{\phi}$ and $\underline{G}_0^+$ are the same as in Eq. (6). Then $\underline{T}$ is again defined by

$$\underline{T}\,\vec{\phi}=\underline{\mathfrak{V}}\,\vec{\psi}\,, \tag{35}$$

so that

$$\underline{T}=\underline{\mathfrak{V}}+\underline{\mathfrak{V}}\,\underline{G}_0^+\underline{T}\,. \tag{36}$$

Then the resulting coupled equations for the T_{i1} are

$$T_{11}=V^{(1)}G_2^+T_{21}+V^{(1)}G_3^+T_{31}\,, \tag{37}$$

$$T_{21}=V_{13}+V^{(2)}G_1^+T_{11}+V^{(2)}G_3^+T_{31}\,, \tag{38}$$

and

$$T_{31}=V_{12}+V^{(3)}G_1^+T_{11}+V^{(3)}G_2^+T_{21}\,. \tag{39}$$

However, by our previous analysis, these can be written in terms of the $\tau_{ijk}G_0^+$ as

$$T_{11}=\tau_{231}G_0^+T_{21}+\tau_{321}G_0^+T_{31}\,, \tag{40}$$

$$T_{21}=V_{13}+\tau_{132}G_0^+T_{11}+\tau_{321}G_0^+T_{31}\,, \tag{41}$$

and

$$T_{31}=V_{12}+\tau_{123}G_0^+T_{11}+\tau_{213}G_0^+T_{21}\,. \tag{42}$$

These equations may then be compared with the BKLT equations written in the form Eqs. (25)–(27). It is also of interest to note that Eqs. (31)–(33) can be used to derive Faddeev-type equations. To do so, we note that

$$\underline{G}_0^+\underline{\mathfrak{V}}=\underline{g}_0^+\underline{t}\,, \tag{43}$$

where

$$\underline{g}_0^+=(E-H_0+i\epsilon)^{-1}\,\underline{1}\,, \tag{44}$$

$$\underline{1}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}, \tag{45}$$

and

$$\underline{t}=\begin{pmatrix}0 & t_{23} & t_{23}\\ t_{13} & 0 & t_{13}\\ t_{12} & t_{12} & 0\end{pmatrix}. \tag{46}$$

Then Eq. (35) can be written as

$$\vec{\psi}=\vec{\phi}+\underline{g}_0^+\underline{t}\,\vec{\psi}\,, \tag{47}$$

and we define the full three-body T matrices by the relation

$$\underline{t}\,\vec{\psi}=\underline{T}\,\vec{\phi}\,. \tag{48}$$

Use of Eqs. (47) and (48) then leads to

$$\underline{T}\vec{\phi}=\underline{t}\,\vec{\phi}+\underline{t}\,\underline{g}_0^+\underline{T}\vec{\phi}\,, \tag{49}$$

or

$$\underline{T}=\underline{t}+\underline{t}\,\underline{g}_0^+\underline{T}\,, \tag{50}$$

which is an alternate form of the Faddeev equations (see e.g., the discussion in Ref. 20). The T matrices in Eqs. (50) and (40)–(42) are essentially related in a similar fashion as are the "post" and "prior" definitions of rearrangement T operators. Thus, as is well known, there is an infinity of definitions of rearrangement T operators, all of which agree only in their physical (on-shell) matrix elements. One may also apply a similar analysis to the BKLT version of Eq. (34). In that case, however, not only does one encounter the t_{ij} [which constitute the elements of the t matrix in Eq. (46)] but also the τ_{ijk} still appear. This is then an interesting distinction between the Faddeev-type sequence of arrangement channel coupling and that used by BKLT. The Faddeev sequence of coupling is such that one either encounters equations involving *only* the t_{ij} or the τ_{ijk} as the effective interactions. The BKLT sequence leads either to equations involving only the τ_{ijk} effective interactions or a mixture of the τ_{ijk} and t_{ij} effective interactions. The two types of BKLT equations have been discussed previously[21] but only in the form which involves ordinary potentials V_{ij}. In that case, the potential matrix occurring in one form is the transpose of that occurring in the other. We now turn to a brief consideration of connectivity. In order to do so, it is convenient to examine the structure of the effective interactions τ_{ijk}. By Eq. (20), upon iteration,

$$\tau_{ijk} = V_{ij} + V_{ij}G_0^+V_{jk} + V_{ij}G_0^+V_{jk}G_0^+V_{jk} + \cdots , \quad (51)$$

and it is obvious that, except for the leading term V_{ij}, τ_{ijk} has the structure of an effective three-body interaction. *Therefore, it follows from Eq. (20) that* $\tau_{ijk}(E-H_0+i\epsilon)^{-1}V_{jk} = \tau_{ijk} - V_{ij}$ *is an effective three-body interaction.* Now, if we back substitute Eqs. (26)–(27) into Eq. (25), we obtain for T_{11} the equation

$$T_{11} = (\tau_{213}+\tau_{231})G_0^+(\tau_{312}+\tau_{321})G_0^+ \times [(V_{12}+V_{13}) + (\tau_{123}+\tau_{132})G_0^+T_{11}]. \quad (52)$$

The kernel of this equation is

$$\mathcal{K} = (\tau_{213}+\tau_{231})G_0^+(\tau_{312}+\tau_{321})G_0^+(\tau_{123}+\tau_{132})G_0^+ \quad (53)$$

$$= (V_{12}+V_{23}+\bar{\tau}_{213}+\bar{\tau}_{231})G_0^+(V_{13}+V_{23}+\bar{\tau}_{312}+\bar{\tau}_{321}) \times G_0^+(V_{12}+V_{13}+\bar{\tau}_{123}+\bar{\tau}_{132})G_0^+ , \quad (54)$$

where $\bar{\tau}_{ijk}$ is the three-body portion of τ_{ijk}. Now any product containing at least *one* of the $\bar{\tau}_{ijk}$ will be connected since multiplying by additional interactions cannot disconnect something which is connected. Therefore, the connectivity of Eq. (34) is determined solely by the factors $(V_{12}+V_{23})(V_{13}+V_{23}) \times (V_{12}+V_{13})$, and comparison with Eqs. (14)–(15) shows that this is also connected. Therefore, summing the graphs in writing $V_iG_i^+$ in terms of the τ_{ikj} does not alter the connectivity structure of the BKLT equations. It is, further, quite interesting that the effective interactions τ_{ijk} satisfy equations which themselves possess only connected graphs after iteration. The equation satisfied by the τ_{ijk}, Eq. (20), is very similar to that satisfied by the effective interactions in the Faddeev equations written in the form of Eqs. (28)–(30). There, one encounters

$$t_{ij} = V_{ij} + t_{ij}(E-H_0+i\epsilon)^{-1}V_{ij} , \quad (55)$$

where i, j range over the particles and $i \neq j$. Of course, in this case, particle k is purely a spectator except for the fact that it takes part of the energy and momentum, but cannot transfer any to particles i and j. Thus, t_{ij} is a disconnected operator even after subtracting out V_{ij}. By contrast, as noted above, τ_{ijk} becomes an effective three-body operator and is therefore completely connected after V_{ij} is subtracted out. We shall discuss this in more detail in the next section.

III. DISCUSSION

The resulting equations (25)–(27) are quite similar to those of Faddeev when the latter are taken in the form of Eqs. (40)–(42). It is seen that in both sets of equations, the partially interacting Green's functions are replaced by the totally free Green's function G_0^+. From the standpoint of applications to nuclear rearrangements, this is an advantage because the breakup arrangement is then more easily taken into account. Furthermore, both the BKLT equations and the Faddeev-type equations involve the new effective potentials τ_{ijk}. These operators can be interpreted as describing the scattering of particle i by particle j bound to (or interacting with) particle k. Similarly, τ_{ikj} describes the particle i colliding with k which is bound to (or dissociated from but interacting with) particle j. However, particle i interacts only indirectly with k in τ_{ijk} and with j in τ_{ikj}. Our notation has been chosen to reflect this property of the τ_{ijk}. The first two indices i, j indicate that the free particle i interacts with j which has first interacted with particle k. Similarly then, τ_{ikj} reflects the interaction of i with k after k has interacted with j. The fact that particle j in τ_{ijk} also feels the presence of k in addition to i results in $\tau_{ijk} - V_{ij}$ being effectively a three-body "interaction." As a result, $\tau_{ijk} - V_{ij}$ involves only connected graphs when iterated. Unlike the T_{ij}, which satisfy connected kernel equations only by virtue of the cyclic coupling of the various arrangement channels,[15,16] the τ_{ijk} satisfy *uncoupled* connected equations which are amenable to calculations using, e.g., the homogeneous integral solution method developed by Sams and Kouri.[22] It would be of interest to carry out calculations of τ_{ijk} in order to see how it behaves. However, we can gain additional insight by rewriting Eq. (20) as

$$\tau_{ijk} = V_{ij}(1-G_0^+V_{jk})^{-1} \quad (56)$$

$$= V_{ij} + V_{ij}(E-H_0-V_{jk}+i\epsilon)^{-1}V_{jk} . \quad (57)$$

Then using the fact that

$$(E-H_0-V_{jk}+i\epsilon)^{-1}V_{jk} = (E-H_0+i\epsilon)^{-1}t_{jk} , \quad (58)$$

we obtain

$$\tau_{ijk} = V_{ij} + V_{ij}G_0^+t_{jk} . \quad (59)$$

Equations (57) and (59) are formal solutions rather than integral equations for the τ_{ijk} operator. The latter is particularly revealing and shows that τ_{ijk} describes the collision of j and k via the standard effective two-body interaction (with i as a pure spectator) followed by free propagation and finally a simple impulsive interaction between particles i and j. In the forms Eqs. (57) and (59) it is again quite clear that $V_{ij}G_0^+t_{jk} = \tau_{ijk} - V_{ij}$ is connected and is a sort of effective three-body interaction.

It is also worth noting that the computational effort involved in calculating the τ_{ijk} will be greater than that required to calculate the t_{jk}. Indeed, Eq. (59) shows that the τ_{ijk} contain the t_{jk} as effective interactions. Therefore, the effort involved in using Eqs. (25)–(27) will be greater than that for the Faddeev-Lovelace equations, Eqs. (28)–(30). The main point is that if one has reason to feel that the arrangement channel coupling sequence of the BKLT equations is more appropriate than that of Eqs. (28)–(30), and the process involves breakup, one should use the form Eqs. (25)–(27) rather than Eqs. (11)–(13).

Now we remark that using the Born approximation to the τ_{ijk} in Eqs. (25)–(27) leads to

$$T_{11} = (V_{12} + V_{23})G_0^+ T_{21} , \quad (60)$$

$$T_{21} = (V_{13} + V_{23})G_0^+ T_{31} , \quad (61)$$

$$T_{31} = (V_{12} + V_{13}) + (V_{12} + V_{13})G_0^+ T_{11} . \quad (62)$$

This is compared to the Born approximation to Faddeev's equations [in the form Eqs. (40)–(42)],

$$T_{11} = V_{23}G_0^+ T_{21} + V_{23}G_0^+ T_{31} , \quad (63)$$

$$T_{21} = V_{13} + V_{13}G_0^+ T_{11} + V_{13}G_0^+ T_{31} , \quad (64)$$

and

$$T_{31} = V_{12} + V_{12}G_0^+ T_{11} + V_{12}G_0^+ T_{21} . \quad (65)$$

These expressions ought to apply at sufficiently high energies. It will be interesting to compare these alternate approximate equations.

Finally, we remark that whether one writes BKLT equations in the form of Eqs. (13)–(15) or (25)–(27) has no effect so far as the occurrence of spurious solutions. Such spurious solutions will occur in general for the BKLT equations as has been discussed in detail by Vanzani,[23] Chandler,[24] Adhikari and Glöckle,[25] and Kowalski.[26]

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