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Comments

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Comments on "Extension of the variable-moment-of-inertia model to high spins"

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In the procedure of fitting of the rotational spectra in high spins of even-even nuclei the assumption that there is equal uncertainty in the energy levels, overall, leads to more reasonable results than the assumption that each energy level should be weighted by the square of its inverse.

[NUCLEAR STRUCTURE Rotational spectra, VMI models, properties of ^{158,160}Dy, ^{162,164}Er, ^{168}Yb, ^{172,174,176}Hf.]

The article by Das and Banerjee[1] on the extension of the VMI model to high spins describes a four-parameter description (VMI234 model) of rotational spectra in even-even nuclei using the phenomenological energy equation

$$E=C_2(\mathscr{I}-\mathscr{I}_0)^2+C_3(\mathscr{I}-\mathscr{I}_0)^3+C_4(\mathscr{I}-\mathscr{I}_0)^4+\frac{I(I+1)}{2\mathscr{I}} \qquad (1)$$

and the equilibrium condition

$$\frac{\partial E}{\partial \mathscr{I}}=0=2C_2(\mathscr{I}-\mathscr{I}_0)+3C_3(\mathscr{I}-\mathscr{I}_0)^2+4C_4(\mathscr{I}-\mathscr{I}_0)^3-\frac{I(I+1)}{2\mathscr{I}^2}, \qquad (2)$$

where C_2, C_3, C_4, and $\mathscr{I}_0$ are the four parameters of the model.

Since the VMI234 model, as given by Eqs. (1) and (2), seems to be one of the most successful phenomenological models for even-even nuclei, we found it tempting to make some comments on the way the model is applied in Ref. 1, in order to greatly improve its generality and usefulness.

The way the model is applied, even to the sample of nuclei (^{158,160}Dy, ^{162,164}Er, ^{168}Yb, and ^{172,174,176}Hf) used for its demonstration, leads to some undesirable results. Specifically, Ref. 1 predicts a backbending for the nuclei ^{158,160}Dy, which is not justified by the experiments and also predicts that Eq. (2) has no physically meaningful solution for $\mathscr{I}$ beyond a certain I in the cases of all four nuclei ^{164}Er and ^{172,174,176}Hf, where $C_4<0$ we found.[1]

We reexamined here all eight nuclei of Ref. 1 and the results are given in Table I and shown in Fig. 1, together with those of Ref. 1 for comparison. As is apparent, the mentioned undesirable results of Ref. 1 are not present here for the available data of the nuclei ^{158,160}Dy, ^{164}Er, and ^{172,174,176}Hf. Also, the accuracy of the fit for the remaining nuclei, ^{162}Er and ^{168}Yb, as for all nuclei, is overall improved, as can be seen by comparison of either the individual levels or the rms deviations[3] listed in the last column of Table I. The only difference[4] between this work and the work in Ref. 1 is that in the least-square fit for determining the parameters C_n and $\mathscr{I}_0$ from Eqs. (1) and (2) we assumed here equal uncertainty for all level energies, instead of weighting each level energy by the square of its inverse (i.e., minimizing the relative error), as in Ref. 1.

The argument considered in Ref. 1 that the low-lying levels are measured more accurately and thus should be weighted more compared to the higher levels is more reasonable when we have to fit a smaller number of levels (e.g., up to $J^\pi \cong 10^+$). It seems, however, that in the cases where we have many levels, as in our case where the last to the first energies are in a ratio ~50:1, the minimization of relative error procedure seems to be a very severe requirement. The latter leads to a better fit for the first energy levels and a rather poor fit for the last levels, and, simultaneously, to bigger values of $\mathscr{I}$ for high spins, which may eventually show up as an unjus-

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TABLE I. For each element the experimental, calculated, and VMI model energies E_I are shown in the first three rows. The fourth and fifth give the calculated VMI234 model moment of inertia $2\mathcal{J}_I$, while the sixth and seventh give the calculated and VMI234 model square of the angular speed ω_I^2. The units of E, $\mathcal{J}$, and ω^2 are keV, (MeV)$^{-1}$, and (MeV)^2, respectively.

Nucleus	E (keV)$^{I^\pi}$	2^+	4^+	6^+	8^+	10^+	12^+	14^+	16^+	18^+	20^+	22^+	rms deviations
^{158}Dy	$E_{\mathrm{exp}}{}^{a}$	98.94	317.26	637.88	1044.02	1519.9	2049.2	2612.5	3190.5	3781.5	4407.3	5085.4	
	E_{cal}	95.6	311.7	634.3	1045.1	1524.6	2052.2	2608.9	3184.8	3784.2	4414.6	5080.4	4.45
	$E_{\mathrm{VMI234}}{}^{b}$	98.49	318.1	640.5	1046	1518	2042	2604	3193	3796	4399		6.52
	$2\mathcal{J}_{\mathrm{cal}}$	63.41	66.14	70.29	75.80	82.84	91.74	102.38	112.59	120.53	126.61	131.50	
	$2\mathcal{J}_{\mathrm{VMI234}}{}^{b}$	61.82	65.62	70.82	77.00	83.96	91.72	100.42	110.34	122.20	137.36		
	$\omega_{\mathrm{cal}}{}^{2}$	0.006	0.018	0.034	0.050	0.064	0.074	0.080	0.086	0.094	0.105	0.117	
	$\omega_{\mathrm{VMI234}}{}^{2b}$	0.006	0.019	0.033	0.049	0.062	0.074	0.083	0.089	0.092	0.089		
^{160}Dy	E_{exp}	86.79	283.79	581.03	966.71	1428.59	1951.4	2514.9	3091.6	3672.1			
	E_{cal}	85.8	282.1	580.0	967.4	1430.0	1951.6	2513.4	3092.4	3671.9	(4264.5)^{c}	(4884.3)	1.07
	E_{VMI234}	86.59	284.0	582.3	968.8	1429	1949	2511	3097	3671	(4194)		2.53
	$2\mathcal{J}_{\mathrm{cal}}$	70.34	72.33	75.39	79.53	84.85	91.71	100.94	113.88	126.97	135.60	141.58	
	$2\mathcal{J}_{\mathrm{VMI234}}$	69.80	72.04	75.44	79.84	85.30	91.92	100.34	112.06	136.04	157.76		
	$\omega_{\mathrm{cal}}{}^{2}$	0.005	0.015	0.030	0.046	0.061	0.074	0.082	0.084	0.085	0.091	0.101	
	$\omega_{\mathrm{VMI234}}{}^{2}$	0.005	0.015	0.030	0.045	0.061	0.074	0.083	0.087	0.074	0.067		
^{162}Er	E_{exp}	102.08	329.63	666.76	1096.8	1602.9	2165.1	2745.7	3292.3	3846.5	4462.8		
	E_{cal}	99.0	324.2	662.6	1096.9	1607.0	2168.0	2741.4	3289.7	3855.6	4457.7	(5100.4)	4.64
	E_{VMI234}	101.5	330.2	670.0	1101	1604	2157	2736	3304	3841	(4390)		6.30
	$2\mathcal{J}_{\mathrm{cal}}$	61.08	63.31	66.75	71.45	77.74	86.82	103.81	119.77	126.96	131.81	135.65	
	$2\mathcal{J}_{\mathrm{VMI234}}$	59.78	62.64	66.86	72.30	79.04	87.62	99.66	120.72	137.54	145.86		
	$\omega_{\mathrm{cal}}{}^{2}$	0.006	0.020	0.038	0.056	0.073	0.083	0.078	0.076	0.085	0.097	0.110	
	$\omega_{\mathrm{VMI234}}{}^{2}$	0.007	0.020	0.038	0.055	0.070	0.081	0.085	0.075	0.072	0.079		
^{164}Er	E_{exp}	91.39	299.47	614.34	1024.3	1517.8	2082.3	2700.8	3261.3	3766.8			
	E_{cal}	89.8	295.9	610.7	1023.4	1520.5	2085.9	2698.0	3261.8	3766.4	(4309.5)	(4893.7)	2.54
	$2\mathcal{J}_{\mathrm{cal}}$	67.17	68.70	71.08	74.34	78.62	84.30	92.83	135.21	141.49	145.53	148.74	
	$\omega_{\mathrm{cal}}{}^{2}$	0.005	0.017	0.033	0.052	0.071	0.088	0.097	0.060	0.068	0.079	0.091	
^{168}Yb	E_{exp}	87.73	286.55	585.30	970.05	1425.4	1935.9	2488.5	3073.0	3686.8	4336.8		
	E_{cal}	87.3	286.0	585.0	970.2	1425.8	1936.2	2487.9	3072.4	3687.9	4336.4	(5020.0)	0.54
	E_{VMI234}	87.68	286.7	585.6	970.0	1425	1935	2489	3075	3686	(4322)		0.81
	$2\mathcal{J}_{\mathrm{cal}}$	69.25	71.70	75.43	80.37	86.51	93.81	101.95	110.03	117.17	123.17	128.23	
	$2\mathcal{J}_{\mathrm{VMI234}}$	69.02	71.64	75.54	80.56	86.60	93.62	101.56	110.12	118.64	126.38		
	$\omega_{\mathrm{cal}}{}^{2}$	0.005	0.016	0.030	0.045	0.059	0.071	0.081	0.090	0.100	0.111	0.123	
	$\omega_{\mathrm{VMI234}}{}^{2}$	0.005	0.016	0.029	0.044	0.059	0.071	0.081	0.090	0.097	0.105		
^{172}Hf	E_{exp}	95.17	309.2	628.2	1037.2	1520.9	2064.3	2653.6	3276.7	3918.8			
	E_{cal}	94.7	308.8	628.4	1037.6	1520.9	2064.0	2653.6	3276.8	3918.8	(4556.9)		0.28
	$2\mathcal{J}_{\mathrm{cal}}$	63.99	66.81	70.83	75.76	81.48	87.95	95.30	103.89	114.66	132.29		
	$\omega_{\mathrm{cal}}{}^{2}$	0.006	0.018	0.033	0.050	0.066	0.081	0.092	0.101	0.104	0.096		

TABLE I. (*Continued.*)

Nucleus	E (keV) I^{π}	2^+	4^+	6^+	8^+	10^+	12^+	14^+	16^+	18^+	20^+	22^+	rms deviations
^{174}Hf	E_{exp}	91.01	297.45	608.37	1009.42	1485.9	2020.6	2596.8					0.14
	E_{cal}	90.8	297.4	608.4	1009.6	1485.7	2020.7	2596.8					
	$2\mathcal{J}_{\mathrm{cal}}$	66.63	68.95	72.46	77.10	82.85	89.67	97.29					
	ω_{cal}^2	0.005	0.017	0.032	0.048	0.064	0.078	0.089					
^{176}Hf	E_{exp}	88.35	290.17	596.95	997.94	1481.27	2034.9	2646.8					0.05
	E_{cal}	88.3	290.1	597.0	998.0	1481.2	2034.9	2646.8	(3302.3)				
	$2\mathcal{J}_{\mathrm{cal}}$	68.40	70.29	73.07	76.56	80.70	85.48	91.12	98.39				
	ω_{cal}^2	0.005	0.016	0.031	0.049	0.068	0.085	0.101	0.112				

[a] All experimental energies come from Ref. 2.

[b] All information for the VMI234 model come from Ref. 1.

[c] Numbers in parentheses stand for extrapolated values.

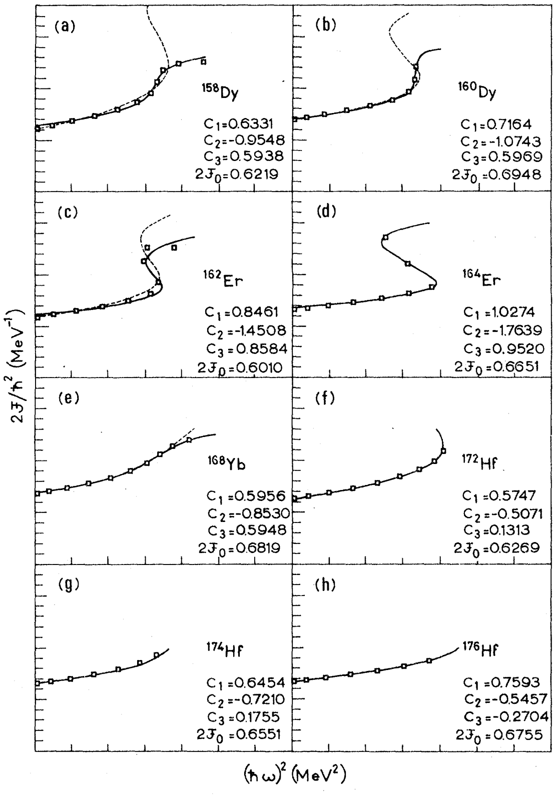

FIG. 1. Plots of $2\mathcal{J}/\hbar^2$ vs $(\hbar\omega)^2$ for the nuclei ^{158,160}Dy, ^{162,164}Er, ^{168}Yb, and ^{172,174,176}Hf. In each block of the figure the calculations of the present work for one of the nuclei examined is shown with a solid line. Squares stand for the experimental points, while broken lines stand for the results of Ref. 1. For ^{164}Er and ^{172,174,176}Hf, Ref. 1 predicts complex $\mathcal{J}$ beyond a certain value of I. In each block the parameters of Eqs. (1) and (2) determined are also given in units: $\mathcal{J}_0 = 10^{-1}$ keV^{-1}; C_2: 10^{-3} keV^3, C_3: 10^{-5} keV^4, and C_4: 10^{-7} keV^5.

tified backbending.

Given that the main utility of the phenomenological energy expressions from the experimentalist's point of view is to provide guidance about the extension of the known domains by extrapolation,[5] the better fit of the higher spin levels provided by the method followed in this paper is more desirable since it leads to more reliable extrapolation[6] results. The rather poor power of extrapolations of the VMI234 model becomes apparent by comparing its predictions (levels in parentheses in Table I) with the experimental values. Also from the theorist's point of view, any physical meaning of the parameters involved is very questionable under the present explanations of the backbending phenomenon,[7-10] which consider that high spin state behavior involves physical phenomena which are not present in the very low-lying rotational levels.

As one can see from Fig. 1 the parameter C_4 is negative here only for the case of ^{176}Hf (and not for ^{164}Er and ^{172,174}Hf, as in Ref. 1). No physically meaningful solutions for $\mathscr{I}$, however, start from $I^\pi = 18^+$, i.e., beyond the available experimental data. We also remark that even for the cases of ^{172,174}Hf, where here $C_4 > 0$, we have no physically meaningful solutions for $\mathscr{I}$ beyond the available experimental data (i.e., from $I^\pi = 22^+$ and 16^+, respectively). Thus the fact that there are no physically meaningful solutions $\mathscr{I}$ beyond a certain I does not necessarily coincide with $C_4 < 0$ as reported in Ref. 1. Finally, we remark that here for ^{164}Er no such levels appear up to $I^\pi = 22^+$.

Finally, we may say that in the procedure of the fitting of the spectra with high spins for even-even nuclei, the assumption that there is equal uncertainty in the energy levels, overall, leads to more reasonable results than the assumption that each energy level should be weighted by the square of its inverse. Specifically, the procedure applied here is superior to that of the VMI234 model in three ways: the region of applicability (^{164}Er, ^{172,174,176}Hf nuclei cannot even be considered in the VMI234 model), the quantitative fit to the experimental energies (see rms deviations in Table I), and the high spin behavior (see Fig. 1).

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[3] Also, in Ref. 5 rms deviations from the experimental values were used for the comparison of two models for yrast band level energies in even-even nuclei.

[4] In Ref. 6 a very interesting discussion on the estimates of uncertainties in the parameters of an angular velocity expansion of nuclear rotational energies was given. In this discussion the influence of the number of levels considered in the fit (degrees of freedom), and the uncertainties in the experimentally measured energies were estimated. No discussion of fittings, as here, assuming equal uncertainty in the energy levels, or minimizing the relative error, was made. This discussion becomes meaningful when the fitting involves high spin states and particularly the backbending region, while Ref. 6 is limited to the region of moderate spin and clearly below the backbending region.

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