Collective shape transition in the low-lying spectra of nuclei near $N=28$ region

D. P. Ahalpara
Physical Research Laboratory, Ahmedabad-380009, India
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Deformed configuration mixing shell model calculations for the energy spectra and electromagnetic properties of low-lying collective states of ^{51}Cr, ^{53}Fe, ^{50}Ti, ^{52}Cr, ^{54}Fe, ^{53}Cr, and ^{55}Fe have been performed. For each of these nuclei these calculations give rise to a highly deformed band when a neutron is promoted from the predominantly $(f_{7/2})^n$ "spherical" ground intrinsic configuration to the unoccupied (pf) orbits. The K^π values of these excited bands are $1/2^-$, 4^+, and $7/2^-$ for the nuclei having $N=27$, 28, and 29, respectively. The energy spectra and electromagnetic properties of these calculated excited bands are compared with those experimentally observed. Following high-spin members of the collective bands have been predicted by the calculations: $J=11/2$, 13/2, 15/2, 17/2, and 19/2 levels at 2.7, 3.8, 4.36, 5.94, and 6.47 MeV, respectively, in ^{51}Cr, $J=9/2$, 11/2, 13/2, 15/2, and 17/2 levels at 2.94, 3.67, 4.95, 6.01, and 7.47 MeV, respectively, in ^{53}Fe, $J=15/2$, 17/2, 19/2, and 21/2 levels at 4.65, 5.77, 7.04, and 8.41 MeV, respectively, in ^{53}Cr, and $J=15/2$, 17/2, and 19/2 levels at 5.53, 6.92, and 8.33 MeV, respectively, in ^{55}Fe. In ^{50}Ti, ^{52}Cr, and ^{54}Fe the present model suggests an interesting possibility of $K=3^+$ collective band lying close in energy to the recently observed $K=4^+$ collective bands.

NUCLEAR STRUCTURE ^{50}Ti, ^{51,52,53}Cr, and ^{53,54,55}Fe. Calculated spectra, $B(E2)$ and $B(M1)$ transition strengths, $E2/M1$ mixing ratios, branching ratios and lifetimes in deformed configuration mixing shell model formalism in $(fp)^n$ model space. Modified Kuo-Brown interaction. Comparison with experiment and shell model and Nilsson model calculations.

I. INTRODUCTION

Numerous experimental studies in the past[1-30] have given rise to a wealth of experimental information regarding low lying high spin states in nuclei near the $N=28$ region showing interesting similarities in their energy spectra. A common feature is that besides the low lying spherical shell model states belonging predominantly to the $(f_{7/2})^n$ configuration, these nuclei show up low lying high spin states which form a well developed collective band structure. Conventional shell model calculations have been carried out in the past for the nuclei ^{50}Ti, ^{52}Cr, and ^{54}Fe (Refs. 16 and 31) and ^{51,53}Cr and ^{53,55}Fe (Refs. 4, 32–39) attributing these collective looking states to arise mainly from a $[(f_{7/2})^{n-1}(p_{3/2},p_{1/2},f_{5/2})^1]J$ configuration. The Nilsson model calculation for ^{51}Cr (Ref. 4) describes the low lying collective band to arise from a deformed intrinsic state in which a neutron is excited from the $k=\frac{7}{2}$ orbit to the lowest available $k=\frac{1}{2}$ orbit.

While considering the structure of the nuclei from the point of view of the self-consistent fields generated by the effective nucleon-nucleon interactions, we find that the promotion of a neutron from the highest occupied $k=\frac{7}{2}$ deformed orbit of the $f_{7/2}$ shell to the lowest unoccupied orbit involves a substantial change in the quadrupole moment of the promoted neutron. In the self-consistent field approach such a large change in the quadrupole moment of the excited neutron is expected to *polarize* the rest of the valence nucleons in the $f_{7/2}$ shell. This polarization of the valence nucleons may then lead to an intrinsic state with a deformation which is substantially larger than the deformation of the lowest energy Hartree-Fock (HF) intrinsic state. In the shell model calculations the structure of the valence nucleons is assumed to remain intact while promoting a nucleon to a higher orbit, and thus the cooperative polarization effect would be completely missing.

We have studied various isotones of $N=27$, 28, and 29 nuclei in the deformed configuration mixing shell model formalism. In the full (fp) shell configuration space the Hamiltonian is completely defined by the single particle energies relative to the ^{40}Ca core and the two-body effective interaction matrix elements. Empirical single particle energies from the ^{41}Ca spectrum have been used. Regarding the effective two-body interaction we note that the Kuo-Brown interaction[40] modified by McGrory *et al.*[41] (labeled MWH2) has been found to be quite satisfactory for describing the spectroscopic properties of (fp) shell nuclei with $N<28$.[42,43] However, around the $N\sim 28$ region this interaction gives rise to the single particle energies for the $p_{3/2}$, $p_{1/2}$, and $f_{5/2}$ orbits relative to ^{56}Ni which are slightly different from the observed energies in ^{57}Ni. The trimmed MWH2 interaction resulting after shifting the centroid energies of $(f_{7/2}\,p_{1/2})$ and $(f_{7/2}f_{5/2})$ multiplets by +0.061 MeV and +0.082 MeV, respectively, gives the correct single particle energies relative to ^{56}Ni. We have used the resulting set of the remodified Kuo-Brown effective interaction (labeled MWH3) in our calculations.

Deformed Hartree-Fock calculations are first carried out for the $N=27$ isotones ^{51}Cr and ^{53}Fe, $N=28$ isotones ^{50}Ti, ^{52}Cr, and ^{54}Fe, and $N=29$ isotones ^{53}Cr and ^{55}Fe using the MWH3 interaction to obtain the lowest energy HF intrinsic states.

A neutron from the highest occupied $k=\frac{7}{2}$ orbit is promoted to the $k=\frac{1}{2}$ orbit and a constrained HF calculation is carried out to obtain the lowest energy state with this excited configuration.

States with definite angular momenta are projected from these deformed intrinsic states to obtain the low lying energy spectra. The projected states are then used to calculate various electromagnetic properties such as $B(E2)$ and $B(M1)$ transition strengths, $E2/M1$ multipole mixing ratios, and lifetimes of the states. For calculating the $E2$ transition strengths we have used the effective charges $e_p=1.33\ e$ and $e_n=0.64\ e$ as obtained by Dhar and Bhatt[42] through a least squares fit to the observed $E2$ transitions in various (fp) shell nuclei.

In Sec. II we describe the intrinsic HF states. In Sec. III we present the results of energy spectra and electromagnetic properties. Section IV contains a brief summary.

II. STRUCTURE OF THE INTRINSIC STATES

Deformed HF calculations in full $(fp)^n$ space are first carried out to obtain the lowest energy deformed HF intrinsic states of ^{50}Ti, ^{51,52,53}Cr, and ^{53,54,55}Fe with prolate deformation. The effective MWH3 interaction tends to fill up the $f_{7/2}$ shell for neutrons and gives rise to spherical $(f_{7/2})^n$ configuration with little admixture of other higher lying (fp) shell orbits. In Fig. 1(a) we show a schematic diagram of the intrinsic state of ^{51}Cr with the ground configuration having total $K=\frac{7}{2}$ and corresponding to a small deformation δ_1. In the third and fifth columns of Table I the energy and quadrupole moment of the intrinsic HF states of nuclei with ground configuration are given. We next obtain the 1 particle-1 hole intrinsic HF states in which a neutron from the $k=\frac{7}{2}$ deformed orbit of $f_{7/2}$ shell is promoted to the second $k=\frac{1}{2}$ deformed orbit. As illustrated in Fig. 1(b) the excited 1p-1h intrinsic state of ^{51}Cr having $K=\frac{1}{2}$ corresponds to filling up of orbits at a deformation δ_2 (beyond the

TABLE I. The energies and quadrupole moments of the ground and the 1p-1h intrinsic HF states of isotones of $N=27$, 28, and 29 nuclei.

N	Nucleus	$E_{\rm HF}$ (MeV)	$E_{\rm 1p\text{-}1h}$ (MeV)	$Q_{\rm HF}$ (b^2)	$Q_{\rm 1p\text{-}1h}$ (b^2)
27	^{51}Cr	−30.42	−30.40	18.3	29.8
	^{53}Fe	−41.97	−40.99	15.5	27.7
28	^{50}Ti	−18.7	−15.55	8.6	19.3
	^{52}Cr	−32.49	−30.49	13.0	25.3
	^{54}Fe	−45.5	−42.44	10.5	23.1
29	^{53}Cr	−31.85	−31.53	20.1	30.0
	^{55}Fe	−45.25	−44.38	17.9	28.4

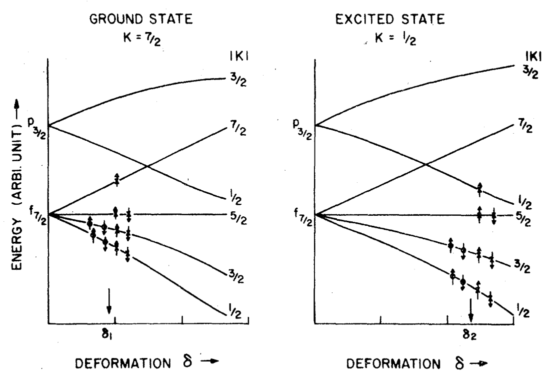

FIG. 1. The schematic Nilsson diagram for the intrinsic states of ^{51}Cr having the ground and the excited 1 particle-1 hole configurations with prolate deformation. The ground and the excited intrinsic states correspond to filling of the deformed orbitals before the crossing (at deformation δ_1) and after the crossing (at deformation δ_2) respectively of the $k=\frac{7}{2}$ and the second $k=\frac{1}{2}$ deformed orbits.

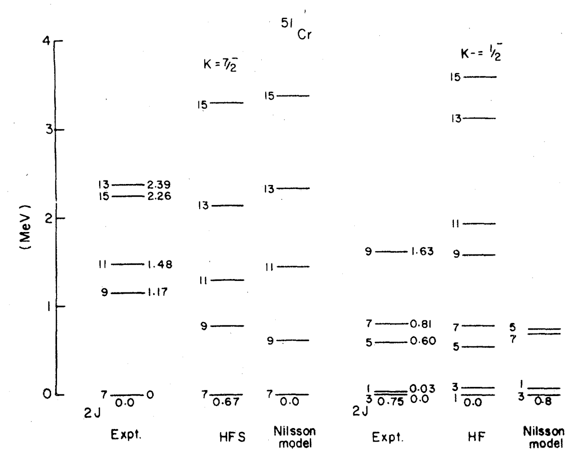

FIG. 2. Comparison of the experimental and calculated level spectra of the ground $K=\frac{7}{2}^-$ and 1p-1h $K=\frac{1}{2}^-$ collective bands in ^{51}Cr. The spectra are drawn relative to the band head states with $J=\frac{7}{2}^-$ and $\frac{3}{2}^-$. The figures near these states indicate their excitation energies relative to the $J=\frac{7}{2}^-$ ground state.

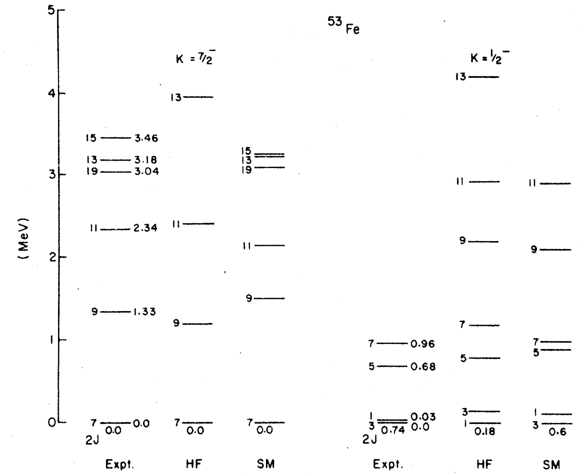

FIG. 3. Comparison of the experimental and calculated level spectra of the ground $K=\frac{7}{2}^-$ and 1p-1h $K=\frac{1}{2}^-$ collective bands in ^{53}Fe. The spectra are drawn relative to the band head states with $J=\frac{7}{2}^-$ and $\frac{3}{2}^-$. The figures near these states indicate their excitation energies relative to the $J=\frac{7}{2}^-$ ground state.

crossing of the $k=\frac{7}{2}$ and second $k=\frac{1}{2}$ deformed orbits) which is larger than the ground state deformation δ_1. In the fourth and sixth columns of Table I the energies and quadrupole moments of the 1p-1h intrinsic states are given. We find that the quadrupole moments of the 1p-1h intrinsic states are substantially larger than those of the ground intrinsic states. This large change in the quadrupole moment arises as a result of polarization of the valence $f_{7/2}$ shell nucleons by the promoted neutron. It is thus clear that our 1p-1h states are much more complex than the states with $(f_{7/2})^{n-1}(p_{3/2}\,p_{1/2}f_{5/2})^1$ configuration employed in conventional shell model calculations in which it is rather difficult to include the effects of such a large configuration mixing.

TABLE III. The calculated $E2/M1$ multipole mixing ratios for transitions in the $K=\frac{7}{2}$ and $K=\frac{1}{2}$ bands of states in ^{51}Cr.

J_i	J_f	$K=\frac{1}{2}$ ground band HF	$K=\frac{7}{2}$ collective band HF
$\frac{1}{2}$	$\frac{3}{2}$	0.01	
$\frac{5}{2}$	$\frac{3}{2}$	0.25	
$\frac{7}{2}$	$\frac{5}{2}$	0.02	
$\frac{9}{2}$	$\frac{7}{2}$	0.19	0.14
$\frac{11}{2}$	$\frac{9}{2}$		0.03
$\frac{13}{2}$	$\frac{11}{2}$		0.07

TABLE II. The electromagnetic transition strengths in the low lying levels of ^{51}Cr.

J_i	J_f	$B(E2)$ in W.u. Expt. (Refs. 4, 32)	$B(E2)$ HF	$B(E2)$ Nilsson Model (Ref. 4)	$B(M1)$ in W.u. Expt. (Refs. 4, 32)	$B(M1)$ HF	$B(M1)$ Nilsson Model (Ref. 4)
$K=\frac{7}{2}$ ground band							
$\frac{9}{2}$	$\frac{7}{2}$		19.3	17.5	0.26	0.58	0.09
$\frac{11}{2}$	$\frac{9}{2}$		18.4	17.7	0.68	0.89	0.13
	$\frac{7}{2}$	7.4	4.4	3.8			
$\frac{13}{2}$	$\frac{11}{2}$		15.0	15.2	0.45	1.02	0.16
	$\frac{9}{2}$		7.4	7.0			
$\frac{15}{2}$	$\frac{13}{2}$		11.4			1.05	
	$\frac{11}{2}$	<23	9.5	9.4			
$K=\frac{1}{2}$ collective band							
$\frac{1}{2}$	$\frac{3}{2}$		47.3	20.6	0.20	0.35	0.61
$\frac{5}{2}$	$\frac{3}{2}$	$10.3^{+5.5}_{-4.4}$	6.7	3.0	0.15	0.02	0.01
	$\frac{1}{2}$	$13.6^{+8.0}_{-5.5}$	23.6	10.3			
$\frac{7}{2}$	$\frac{5}{2}$		3.3	1.5	<0.05	0.21	0.39
	$\frac{3}{2}$	$10.0^{+5.0}_{-5.7}$	29.8	13.2			
$\frac{9}{2}$	$\frac{7}{2}$		2.0			0.02	
	$\frac{5}{2}$	$8.4^{+4.2}_{-5.3}$	32.9				

TABLE IV. The lifetimes of low lying levels in ^{51}Cr. The starred levels belong to the collective $K=\frac{1}{2}$ band.

J	Exc En (MeV)	Lifetimes τ (ps) Expt. (Ref. 4)	HF
$*\frac{1}{2}$	0.78	8×10^3	$3.3\times10^{3*}$
$\frac{9}{2}$	1.17	0.08 ± 0.03	0.03
$*\frac{5}{2}$	1.35	$1.5^{+2.5}_{-0.5}$	7.1*
$\frac{11}{2}$	1.48	$0.8^{+0.5}_{-0.3}$	0.79
$*\frac{7}{2}$	1.56	>4.0	4.93*
$\frac{15}{2}$	2.26	>1.0	
$\frac{13}{2}$	2.39	$0.1^{+0.03}_{-0.01}$	0.04

TABLE VI. The $E2/M1$ multipole mixing ratios for transitions in the $K=\frac{7}{2}$ and $K=\frac{1}{2}$ bands of states in ^{53}Fe.

$J_i\rightarrow J_f$		$K=\frac{7}{2}$ ground band Expt. (Ref. 6)	HF	$K=\frac{1}{2}$ collective band Expt. (Ref. 6)	HF
$\frac{1}{2}$	$\frac{3}{2}$				0.01
$\frac{5}{2}$	$\frac{3}{2}$			$-0.49^{+0.1}_{-0.11}$	-0.33
$\frac{7}{2}$	$\frac{5}{2}$				0.02
$\frac{9}{2}$	$\frac{7}{2}$	0.11 ± 0.002	0.16		
$\frac{11}{2}$	$\frac{9}{2}$	0.11 ± 0.02	0.09		
$\frac{13}{2}$	$\frac{11}{2}$	$0.0\ \pm0.03$	0.06		
$\frac{15}{2}$	$\frac{13}{2}$	0.01 ± 0.04	0.02		

III. ENERGY SPECTRA AND ELECTROMAGNETIC PROPERTIES

$N = 27$ isotones ^{51}Cr and ^{53}Fe

The low lying levels of ^{51}Cr and ^{53}Fe have been investigated through a variety of experiments.[1-8] Theoretical calculations have been done in the framework of the Nilsson model[4] and the shell model.[32-35] The calculated and experimental spectra are compared in Figs. 2 and 3. The spectra are drawn relative to the band head states $J=\frac{7}{2}$ and $\frac{1}{2}$.

Ground band $K=\frac{7}{2}$

In ^{51}Cr the calculated spectra agree qualitatively with experiment. The structures of intrinsic states in HF and Nilsson models are quite similar and the energy spectra obtained in these models also are similar to each other. In ^{53}Fe the states with $J>\frac{13}{2}$ have not been shown in the HF spectrum, as the contents of these states in the ground intrinsic state are too weak. The shell model gives a better description of the ground band.

TABLE V. The electromagnetic transition strengths in the low lying levels of ^{53}Fe.

J_i	J_f	$B(E2)$ in W.u. Expt. (Refs. 6, 7)	HF	$B(M1)$ in W.u. Expt. (Refs. 6, 7)	HF
$K=\frac{7}{2}$ ground band					
$\frac{9}{2}$	$\frac{7}{2}$	15.0	14.8	0.53	0.48
$\frac{11}{2}$	$\frac{9}{2}$	2.7–8.1	13.0	0.2–0.6	0.70
	$\frac{7}{2}$	1.9–5.7	3.5		
$\frac{13}{2}$	$\frac{11}{2}$		9.4		0.73
	$\frac{9}{2}$		5.9		
$K=\frac{1}{2}$ collective band					
$\frac{3}{2}$	$\frac{1}{2}$		19.8		0.2
$\frac{5}{2}$	$\frac{3}{2}$	23^{+17}_{-11}	5.6	0.02 ± 0.007	0.01
	$\frac{1}{2}$		19.8		
$\frac{7}{2}$	$\frac{5}{2}$		2.7		0.27
	$\frac{3}{2}$		24.9		
$\frac{9}{2}$	$\frac{7}{2}$		1.7		0.01
	$\frac{5}{2}$		27.8		
$\frac{11}{2}$	$\frac{9}{2}$		1.0		0.24
	$\frac{7}{2}$		28.3		

TABLE VII. The lifetimes of low lying levels in ^{53}Fe. The starred levels belong to the $K=\frac{1}{2}$ collective band.

J	Exc En (MeV)	Lifetimes τ (ps) Expt. (Refs. 6–8)	HF
$*\frac{1}{2}$	0.77	$(2.9\pm0.3)\times10^3$	2.5×10^{3}*
$\frac{9}{2}$	1.33		0.03
$*\frac{5}{2}$	1.42	4.0 ± 1.0	6.4*
$*\frac{7}{2}$	1.70		2.1*
$\frac{11}{2}$	2.34		0.04
$\frac{13}{2}$	3.18		0.06
$\frac{15}{2}$	3.46		1.3

Excited band $K=\frac{1}{2}$

The excited bands in ^{51}Cr and ^{53}Fe are fairly well reproduced in HF, Nilsson, and shell model calculations. The observed bunching of states with $J=\frac{1}{2}$, $\frac{3}{2}$ and with $J=\frac{5}{2}$, $\frac{7}{2}$ is also well reproduced. It may be noted that this type of bunching of states is typically found in the rotation-particle-coupling model through the Coriolis coupling term. Based on the good agreement for the deformed $K=\frac{1}{2}$ bands the present model predicts the following high spin states: $J=\frac{11}{2}$, $\frac{13}{2}$, $\frac{15}{2}$, $\frac{17}{2}$, and $\frac{19}{2}$ states at 2.7, 3.8, 4.36, 5.94, and 6.47 MeV, respectively, in ^{51}Cr, and $J=\frac{9}{2}$, $\frac{11}{2}$, $\frac{13}{2}$, $\frac{15}{2}$, and $\frac{17}{2}$ states at 2.94, 3.67, 4.95, 6.01, and 7.47 MeV, respectively, in ^{53}Fe.

The $B(E2)$ and $B(M1)$ values, the $E2/M1$ mixing ratios, and the lifetimes of states in ^{51}Cr and ^{53}Fe are compared with experiment and with the results of the Nilsson model calculation in Tables II through VII. In the $K=\frac{1}{2}$ deformed band we get enhanced $E2$ transitions compared to experiment indicating a larger deformation obtained in the present calculation. The $M1$ transitions are qualitatively reproduced in HF and Nilsson models.

The $E2/M1$ multipole mixing ratios for the transitions in the low lying levels of ^{51}Cr have not been measured so far. The calculated lifetimes are generally well reproduced. The calculated magnetic moments of the ground state $J=\frac{7}{2}$ and the closely spaced collective states $J=\frac{3}{2}$ (0.75 MeV) and $\frac{1}{2}$ (0.78 MeV) in ^{51}Cr are $-0.53\ \mu_N$, $-0.33\ \mu_N$, and $0.89\ \mu_N$, respectively.

The calculated $E2$ and $M1$ transitions in the $K=\frac{7}{2}$ ground band of ^{53}Fe agree well with the available experimental data. Not much information is available for the transitions in the $K=\frac{1}{2}$ deformed band. The $E2/M1$ mixing ratios are in good agreement with experiment. The agreement for the lifetimes is good. The calculated magnetic moments of the ground state $J=\frac{7}{2}$ and the excited $J=\frac{3}{2}$ (0.74 MeV)

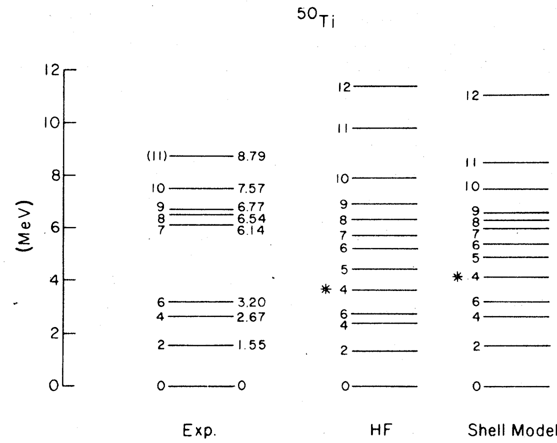

FIG. 4. Comparison of the experimental and calculated level spectra of the ground $K=0^+$ and 1p-1h $K=4^+$ collective bands in ^{50}Ti. The starred $J=4^+$ states in ^{50}Ti are the band head states of the $K=4^+$ collective band.

and $\frac{1}{2}$ (0.77 MeV) states are $-0.66\ \mu_N$, $-0.69\ \mu_N$, and $0.84\ \mu_N$, respectively. It would be interesting to measure these magnetic moments to compare with the predicted values.

$N=28$ isotones ^{50}Ti, ^{52}Cr, and ^{54}Fe

There have been numerous experimental studies for the low lying levels in ^{50}Ti, ^{52}Cr, and ^{54}Fe.[9-21] In ^{52}Cr the high spin states show a distinct rotational-like band structure starting with the $J=4^+$ state at 3.42 MeV excitation energy. The present HF model provides a very simple explanation for the occurrence of the band of high spin states which arise from the 1p-1h deformed $K=4^+$ intrinsic HF state. The 1p-1h intrinsic HF states in ^{50}Ti, ^{52}Cr, and ^{54}Fe are highly deformed states incorporating a substantial amount of polarization of the valence

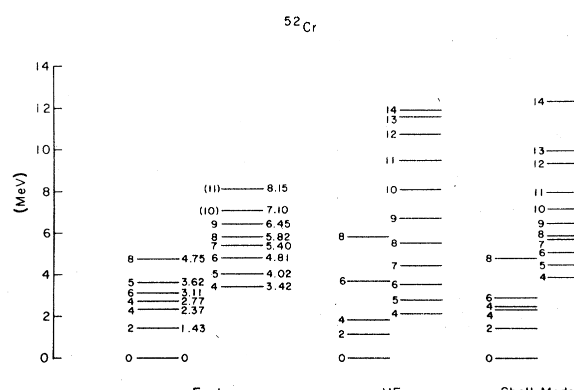

FIG. 5. Comparison of the experimental and calculated level spectra of the ground $K=0^+$ and 1p-1h $K=4^+$ collective bands in ^{52}Cr.

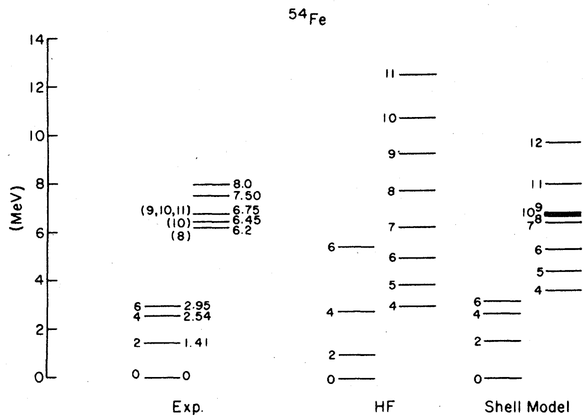

FIG. 6. Comparison of the experimental and calculated level spectra of the gound $K=0^+$ and 1p-1h $K=4^+$ collective bands in ^{54}Fe.

nucleons due to the promoted neutron in the deformed $k=\frac{1}{2}$ orbit. The mass quadrupole moments of the $K=4^+$ 1p-1h intrinsic states of the above nuclei are 19.3 b^2, 25.3 b^2, and 23.1 b^2, respectively, as compared to 8.6 b^2, 13.0 b^2, and 10.5 b^2 for the ground $K=0^+$ intrinsic states. The calculated spectra of these nuclei are compared with experiment and with the shell model[16,31] in Figs. 4, 5, and 6. In spite of the large difference in the structure of the states the HF spectrum shows a close resemblance with the shell model spectrum.

Ground band $K=0^+$

The calculated spectra in the HF and shell models agree fairly well with experiment. In ^{54}Fe the content of the $J=6^+$ state in the intrinsic HF state is rather low and hence its excitation energy is not accurately calculated.

Excited band $K=4^+$

We note that the HF model gives rise to two deformed 1p-1h intrinsic states with $K=4^+$ and 3^+ lying very close in energy arising from the coupling of odd neutrons in $k=\frac{7}{2}$ and $k=\frac{1}{2}$ deformed orbits with total $K=\frac{7}{2}\pm\frac{1}{2}$. The structures of the two deformed states are very similar to each other. We have also carried out the band mixing calculation for the $K=4^+$ and $K=3^+$ bands of states and find that the $K=3^+$ band lies very close in energy to the $K=4^+$ band. The $K=3^+$ band proposed in the present calculation has not been observed so far. We point out that the observed members of the high spin states in the three nuclei have been populated following a heavy ion reaction. It is generally observed that such a cascade feeds only the yrast band of states. It is quite likely therefore that the $K=3^+$ band may be very weakly populated in these reactions. A strikingly similar situation exists in the low lying spectrum of ^{48}V (Refs. 44 and 45), where two low lying negative parity bands with $K=4^-$ and 1^- lying close in energy have been observed. The yrast band of states with $K=1^-$ was easily detected in a heavy ion reaction[44] but the excited band with $K=4^-$ was detected with difficulty.[45] Thus it would be very interesting to look for the proposed $K=3^+$ bands lying close in energy to the observed $K=4^+$ bands in ^{50}Ti, ^{52}Cr, and ^{54}Fe.

TABLE VIII. The electromagnetic transition strengths in the low lying levels of ^{50}Ti.

		$B(E2)$ in e^2 fm^4			
$J_i \to J_f$		Expt. (Ref. 13)	HF	Shell model (Ref. 16)	$B(M1)$ in μ_N^2 HF
$K=0^+$ ground band					
2	0	66 ± 8	63	83	
4	2	60^{+14}_{-10}	67	82	
6	4	34.2 ± 1.2	46	39	
$K=4^+$ collective band					
5	4		180		1.0
6	5		148		1.68
	4		49		
7	6		123		2.16
	5		70		
8	7		99		2.32
	6		81		
9	8		71		2.21
	7		82		
10	9		55		2.0
	8		68		

TABLE IX. The branching ratios and the $E2/M1$ multipole mixing ratios for the transitions in the low lying levels of ^{50}Ti.

		Branching ratios in %			$E2/M1$ mixing ratios		
$J_i \to J_f$		Expt. (Ref. 16)	HF	Shell model (Ref. 16)	Expt. (Ref. 16)	HF	Shell model (Ref. 16)
$K=4^+$ collective band							
7	6		2.16	0.03			
	5		0.06	0.72			
8	7	94 ± 3	97.4	93.7	0.02 ± 0.01	0.02	0.01
	6		2.4	2.5			
9	8	100	99.6	98.8	0.04 ± 0.02	0.01	0.01
	7	<19	0.4	1.2			
10	9	100	99.9	99.9	0.04 ± 0.02	0.04	0.03
	8	<15	0.09	0.09			
11	10	100	99.6	99.3	0.17 ± 0.10	0.02	0.05
	9	<12	0.4	0.7			

Various electromagnetic properties of low lying levels in the $N=28$ isotones are given in Tables VIII through XII. It is seen that the $E2$ transitions in the ground $K=0^+$ bands are fairly well described The $E2$ and $M1$ transitions in the excited $K=4^+$ bands have not been measured so far. It would be very interesting to experimentally observe these transitions so as to provide a measure for the deformation of $K=4^+$ collective band. The calculated branching ratios and $E2/M1$ mixing ratios in ^{50}Ti agree fairly well with the experimental values. The lifetimes of the excited 2^+ and 4^+ states in ^{52}Cr are in good agreement with experiment. The static quadrupole moment of the first excited 2^+ state in ^{52}Cr is calculated to be $-17\ e\,\mathrm{fm}^2$, which agrees well with the experimental value[20] $-14 \pm 8\ e\,\mathrm{fm}^2$.

In ^{54}Fe the lifetimes of the states with $J=2^+$ (1.41 MeV), 4^+ (2.54 MeV), and 6^+ (2.95 MeV) are predicted as 1.96, 4.7, and 0.9 ps, respectively. The calculated static quadrupole moment of the first excited 2^+ state is $-15.7\ e\,\mathrm{fm}^2$.

The calculated magnetic moments of the band head states in ^{50}Ti, ^{52}Cr, and ^{54}Fe are $-1.49\ \mu_N$, $-1.06\ \mu_N$, and $-1.46\ \mu_N$, respectively.

TABLE X. The electromagnetic transition strengths in the low lying levels of ^{52}Cr.

		$B(E2)$ in $e^2\mathrm{fm}^4$		$B(M1)$ in μ_N^2
$J_i \to J_f$		Expt. (Ref. 13)	HF	HF
$K=0^+$ ground band				
2	0	119 ± 7	93	
4	2	83 ± 17	117	
6	4	59.5 ± 3.4	116	
8	6		110	
$K=4^+$ collective band				
5	4		211	1.1
6	5		222	1.7
	4		37	
7	6		193	1.9
	5		67	
8	7		160	2.0
	6		87	
9	8		128	2.0
	7		98	
10	9		100	1.9
	8		102	

$N=29$ isotones ^{53}Cr and ^{55}Fe

The low lying levels in ^{53}Cr and ^{55}Fe have been studied in the past by numerous reaction experiments.[7,22-30] As in the $N=27$ isotones, the level schemes in the low lying spectra of ^{53}Cr and ^{55}Fe are closely similar to each other. Besides the ground $K=\frac{1}{2}$ band the two nuclei show up $K=\frac{7}{2}$ deformed excited band of states starting at 1.54 and 1.41 MeV excitation energy, respectively. Theoretically, the excited deformed states in ^{53}Cr and ^{55}Fe have been studied using the shell model[34-36,39] within the neutron hole configurations $(f_{7/2})^{-n_1}(p_{3/2}p_{1/2}f_{5/2})^{n_2}$ with $(n_1, n_2) = (5, 2)$ and $(3, 2)$, respectively. The calculated low lying energy spectra of ^{53}Cr and ^{55}Fe are compared with experiment and shell model results in Figs. 7 and

TABLE XI. The lifetimes of low lying levels in ^{52}Cr.

J(exc en)	Expt. (Refs. 13, 14)	Calc.
$2^+(1.43)$	1.27 ± 0.14	1.48
4^+ (2.37)	$13.7^{+3.6}_{-2.3}$	9.52

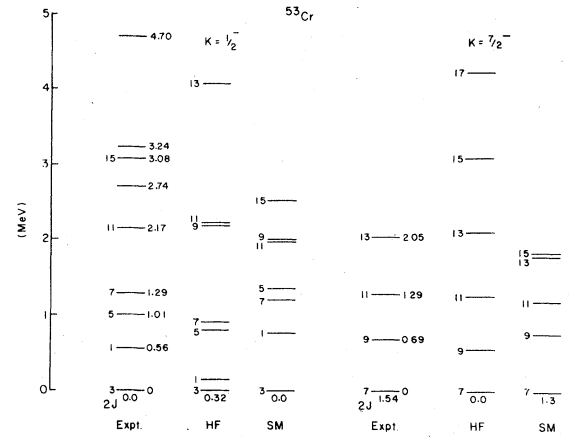

FIG. 7. Comparison of the experimental and calculated level spectra of the ground $K=\frac{1}{2}^-$ and 1p-1h $K=\frac{7}{2}^-$ collective bands in ^{53}Cr. The spectra are drawn relative to the band head states with $J=\frac{1}{2}^-$ and $\frac{7}{2}^-$. The figures near these states indicate their excitation energies relative to the $J=\frac{3}{2}^-$ ground state.

8, respectively. As such, the shell model calculation gives many levels below 4 MeV excitation energy. We have shown in the figures only the yrast band of states belonging to ground and 1p-1h configurations to compare with our calculated $K=\frac{1}{2}$ and $K=\frac{7}{2}$ bands of states.

Ground band $K=\frac{1}{2}$

The low lying ground band of states are qualitatively reproduced in the HF model. The calculated spectra show up bunching of levels which is typically found in the rotation-particle-coupling model through the Coriolis coupling term. The experimental spectra, however, do not show pronounced bunching of levels. In the ground band of ^{53}Cr, a $J=\frac{9}{2}$ state is obtained in both the calculated spectra very close to the $J=\frac{11}{2}$ state. In the experimental spectrum the $J=\frac{9}{2}$ state is not identified as yet. Looking at the striking similarity between the experimental spectrum of ^{53}Cr and ^{55}Fe, it would be interesting to look for the $J=\frac{9}{2}$ state predicted in the HF and the shell model calculations.

Excited band $K=\frac{7}{2}$

For the excited $K=\frac{7}{2}$ band in ^{53}Cr and ^{55}Fe the excitation energies of states with $J=\frac{9}{2}$, $\frac{11}{2}$, and $\frac{13}{2}$ relative to the band head state $J=\frac{7}{2}$ are fairly well described. In ^{53}Cr two higher lying states with $J=\frac{15}{2}$ and $\frac{17}{2}$ are predicted in the present calculation. It would be worth noting that the excited $K=\frac{7}{2}$ band shows a better rotational sequence as against the ground state band.

Various electromagnetic properties of the low lying levels in ^{53}Cr and ^{55}Fe are shown in Tables XIII through XVIII. In ^{53}Cr the $E2$ and $M1$ transitions are fairly well described. Considering the transitions in the $K=\frac{1}{2}$ ground band of states we note that there is an ambiguity in the experimental value for the $\frac{11}{2}\rightarrow\frac{7}{2}$ $E2$ transition. Poletti *et al.*[25] have quoted two possible values: 227^{+190}_{-170} $e^2\,\mathrm{fm}^4$ and 32 ± 10 $e^2\,\mathrm{fm}^4$. The former value seems more possible because of the expected strong inband transition. The observed $\frac{9}{2}\rightarrow\frac{7}{2}$ $E2$ transition in the $K=\frac{7}{2}$ deformed band is quite enhanced and provides a

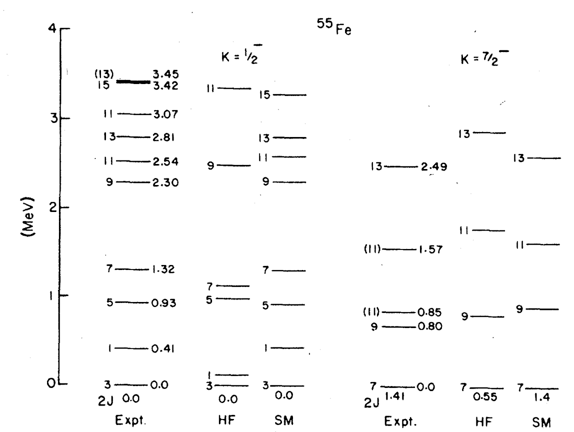

FIG. 8. Comparison of the experimental and calculated level spectra of the ground $K=\frac{1}{2}^-$ and 1p-1h $K=\frac{7}{2}^-$ collective bands in ^{55}Fe. The spectra are drawn relative to the band head states with $J=\frac{1}{2}^-$ and $\frac{7}{2}^-$. The figures near these states indicate their excitation energies relative to the $J=\frac{3}{2}^-$ ground state.

TABLE XII. The electromagnetic transition strengths in the low lying levels of ^{54}Fe.

$J_i \rightarrow J_f$		Expt. (Ref. 13)	HF	$B(M1)$ in μ_N^2 HF
$K=0^+$ ground band				
2	0	102 ± 4	75	
4	2	78 ± 16	94	
6	4	39.5 ± 0.5	79	
$K=4^+$ collective band				
5	4		296	0.78
6	5		282	1.17
	4		58	
7	6		217	1.35
	5		102	
8	7		155	1.41
	6		126	
9	8		105	1.41
	7		134	

TABLE XIII. The electromagnetic transition strengths in the low lying levels of ^{53}Cr.

$J_i \rightarrow J_f$		$B(E2)$ in e^2fm^4 Expt. (Ref. 25)	HF	Shell model (Ref. 39)	$B(M1)$ in $\mu_N{}^2$ Expt. (Ref. 25)	HF	Shell model (Ref. 39)
$K=\frac{1}{2}^-$ ground band							
$\frac{3}{2}$	$\frac{1}{2}$		154			0.63	
$\frac{5}{2}$	$\frac{3}{2}$	~36	45	69	~0.02	0.017	0.016
	$\frac{1}{2}$		155				
$\frac{7}{2}$	$\frac{5}{2}$	<6	21	0.01	0.978±0.012	0.70	1.43
	$\frac{3}{2}$	136±17	194	147			
$\frac{9}{2}$	$\frac{7}{2}$		13			0.016	
	$\frac{5}{2}$		213				
$\frac{11}{2}$	$\frac{9}{2}$		8			0.56	
	$\frac{7}{2}$	227^{+190}_{-70}	213	165			
$\frac{13}{2}$	$\frac{11}{2}$		6			0.01	
	$\frac{9}{2}$		219				
$K=\frac{7}{2}^-$ collective band							
$\frac{9}{2}$	$\frac{7}{2}$	350±200	467		0.41±0.04	0.54	
$\frac{11}{2}$	$\frac{9}{2}$	>50	466		>25	0.82	
	$\frac{7}{2}$		97				
$\frac{13}{2}$	$\frac{11}{2}$		393			0.97	
	$\frac{9}{2}$		175				
$\frac{15}{2}$	$\frac{13}{2}$		320			1.06	
	$\frac{11}{2}$		227				

clue for the large deformation of the $K=\frac{7}{2}$ collective band. The calculated stronger transition indicates that HF calculation leads to a slightly larger deformation for the $K=\frac{7}{2}$ band compared to the one implied by experiment. It would be interesting to measure other inband transitions.

TABLE XIV. The $E2/M1$ multipole mixing ratios for transitions in the $K=\frac{1}{2}$ and $K=\frac{7}{2}$ bands of states in ^{53}Cr.

$J_i \rightarrow J_f$		$K=\frac{1}{2}$ ground band Expt. (Ref. 23)	HF	$K=\frac{7}{2}$ collective band Expt. (Ref. 23)	HF
$\frac{3}{2}$	$\frac{1}{2}$		0.10		
$\frac{5}{2}$	$\frac{3}{2}$	−0.36±0.02	−0.43		
$\frac{7}{2}$	$\frac{5}{2}$	0.0 ±0.02	0.01		
$\frac{9}{2}$	$\frac{7}{2}$			0.17±0.03	0.17
$\frac{11}{2}$	$\frac{9}{2}$			0.11±0.04	0.12
$\frac{13}{2}$	$\frac{11}{2}$			0.14±0.05	0.13

IV. SUMMARY

The present deformed configuration mixing HF calculations strongly suggest that the low lying collective bands in the nuclei with $N=27$, 28, and 29 are likely to be very deformed collective states in

TABLE XV. The lifetimes of low lying levels in ^{53}Cr. The starred levels belong to the collective $K=\frac{7}{2}$ band.

J	Exc En (MeV)	Lifetimes τ (ps) Expt. (Refs. 7, 22)	HF
$\frac{1}{2}$	0.56	0.9±0.35	0.25
$\frac{5}{2}$	1.01	2.5	2.7
$\frac{7}{2}$	1.29	1.6±0.2	0.9
$\frac{11}{2}$	2.17	6.7±3.1	7.3
*$\frac{9}{2}$	2.23	$0.4^{+0.4}_{-0.15}$	0.31*
*$\frac{11}{2}$	2.83	<1	0.28*
$\frac{13}{2}$	3.59		0.12

TABLE XVI. The electromagnetic transition strengths in the low lying levels of ^{55}Fe.

$J_i \to J_f$		$B(E2)$ in e^2 fm^4 Expt. (Ref. 25)	HF	Shell model (Ref. 39)	$B(M1)$ in $(\mu_N)^2$ Expt. (Ref. 25)	HF	Shell model (Ref. 39)
$K=\frac{1}{2}^-$ ground band							
$\frac{3}{2}$	$\frac{1}{2}$		131			0.85	
$\frac{5}{2}$	$\frac{3}{2}$		38			0.019	
	$\frac{1}{2}$		134				
$\frac{7}{2}$	$\frac{5}{2}$	26^{+26}_{-18}	16	6	$0.05^{+0.018}_{-0.026}$	0.86	1.01
	$\frac{3}{2}$	210 ± 90	164	132			
$\frac{9}{2}$	$\frac{7}{2}$		11	13	$(53\pm 21)\times 10^{-4}$	0.02	0.07
	$\frac{5}{2}$	170 ± 65	187	38			
$\frac{11}{2}$	$\frac{9}{2}$		5	0.4	0.01 ± 0.004	0.53	1.47
	$\frac{7}{2}$	17 ± 6	183	108			
$K=\frac{7}{2}^-$ collective band							
$\frac{9}{2}$	$\frac{7}{2}$	140 ± 70	423		0.14 ± 0.07	0.41	
$\frac{11}{2}$	$\frac{9}{2}$		418			0.62	
	$\frac{7}{2}$		91				
$\frac{13}{2}$	$\frac{11}{2}$		349			0.75	
	$\frac{9}{2}$		165				

contrast to the ground state bands which have a predominant $(f_{7/2})^n$ component. We also note that this is a local phenomenon in that such collective bands are not expected to occur in nuclei with $N \geq 30$. The collective bands arise as a result of promotion of a neutron from the highest occupied $k=\frac{7}{2}$ to the second $k=\frac{1}{2}$ deformed orbit, allowing a large change in quadrupole moment and resulting in core polarization which would not be possible if the latter deformed orbit is completely filled in the ground configuration.

The results of the present calculations give a fair description of the spectroscopic properties of the spherical ground band of states as well as the deformed collective band of excited high spin states. The deformed collective bands of states in these nuclei are attributed to the states projected from the lowest energy deformed intrinsic HF states with 1 particle-1 hole configuration. The results for the energy spectra and various electromagnetic properties in general show an improve-

TABLE XVII. The $E2/M1$ multipole mixing ratios for transitions in the $K=\frac{1}{2}$ and $K=\frac{7}{2}$ bands of states in ^{55}Fe.

$J_i \to J_f$		$K=\frac{1}{2}$ ground band Expt. (Ref. 27)	HF	$K=\frac{7}{2}$ collective band Expt. (Ref. 27)	HF
$\frac{3}{2}$	$\frac{1}{2}$		0.04		
$\frac{5}{2}$	$\frac{3}{2}$	-0.32 ± 0.02	-0.35		
$\frac{7}{2}$	$\frac{5}{2}$	0.07 ± 0.03	0.02		
$\frac{9}{2}$	$\frac{7}{2}$		0.21	0.21 ± 0.02	0.21
$\frac{11}{2}$	$\frac{9}{2}$	0.0 ± 0.02	0.01	0.16 ± 0.03	0.19
$\frac{13}{2}$	$\frac{11}{2}$	0.02 ± 0.02	0.04		0.17

TABLE XVIII. The lifetimes of low lying levels in ^{55}Fe. The starred levels belong to the collective $K=\frac{7}{2}$ band.

J	Exc En	Lifetimes τ (ps) Expt. (Ref. 25)	HF
$\frac{1}{2}$	0.41	≈ 6	0.5
$\frac{5}{2}$	0.93	≈ 9	3.3
$\frac{7}{2}$	1.32	$0.92^{+0.87}_{-0.25}$	0.98
$*\frac{9}{2}$	2.21	0.75 ± 0.40	0.26*
$\frac{9}{2}$	2.30	$0.90^{+0.70}_{-0.23}$	0.73
$\frac{11}{2}$	2.54	17 ± 6	1.4
$*\frac{11}{2}$	2.98		0.12*
$*\frac{13}{2}$	3.90		0.07*

ment over the previous Nilsson model and shell model descriptions. A limitation of the model, however, is that the excitation energy of the band head state of the 1p-1h collective band is not well reproduced. Certain high spin members of the collective bands are predicted. The present model also suggests an interesting possibility of a $K=3^+$ band in the low lying spectra of ^{50}Ti, ^{52}Cr, and ^{54}Fe lying close in energy to the observed $K=4^+$ band. It would be interesting to look experimentally for such a collective band. The present calculation also suggests the need for data on the electromagnetic transitions within the excited deformed band of states which will provide a clue for the deformation of these collective band of states.

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