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Projected Hartree-Fock calculations for ^{52,53}V

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The structure of the low-lying nuclear states in neutron rich isotopes of vanadium ^{52,53}V is investigated. The investigations are carried out in the framework of Hartree-Fock projection formalism by employing a realistic nucleon-nucleon interaction. All the valence nucleons outside the inert ^{40}Ca core are considered to be active in the configuration space of the full *fp* shell. The energy levels, static electromagnetic moments, and the electromagnetic transition probabilities are evaluated from the band-mixing calculations, wherein the lowest few energetically close intrinsic states of the nuclei are taken into account. The results of the present calculations are in fair agreement with the available experimental data.

[NUCLEAR STRUCTURE Vanadium isotopes: calculated energy levels, static moments, $B(E2)$ and $B(M1)$. Hartree-Fock projection formalism, realistic nucleon-nucleon interaction, band mixing.]

I. INTRODUCTION

Properties of the nuclei in the *fp*-shell region are currently of considerable experimental and theoretical interest. In recent years a large amount of experimental data, especially on high spin states, has become available through the study of heavy-ion induced fusion-evaporation type of reactions. Shell model calculations for the $20 \leq Z \leq 28$ and $N = 29$ isotones have been previously reported by Vervier[1,2] and by Horie and Ogawa,[3] and for the $N = 30$ isotones by Vervier,[2] McGrory,[4] and Horie and Ogawa.[5] None of these calculations, however, investigate the high spin states in the nuclei under consideration. Recently Nathan *et al.*[6] have reported the results of their shell model calculations on the yrast spectra of many nuclei in the range $20 < Z < 28$ and $28 < N < 40$. In all these calculations, an inert ^{48}Ca core was assumed, and the active protons were confined to the $0f_{7/2}$ orbital and the active neutrons to the $0f_{5/2}$, $1p_{3/2}$, and $1p_{1/2}$ orbitals, i.e., to the configuration space $(\pi 0f_{7/2})^n \times (\nu 1p_{3/2}, 0f_{5/2}, 1p_{1/2})^m$, where $n = Z - 20$ and $m = N - 28$. Moreover, all the shell model calculations mentioned above employ empirical nucleon-nucleon interactions. In the Horie and Ogawa[5] calculations for ^{53}V, the first six eigenstates are predominantly due to recoupling members of the $(f_{7/2})^3$ configuration. In that case, the dipole transitions between these states would be strictly forbidden. The observation[7] of $M1$ transitions between these states, however, indicates the presence of other admixtures, e.g., those resulting from the promotion of protons from the $f_{7/2}$ shell to the higher shells. In view of the above discussion, it is worthwhile to investigate the nuclear structure of these nuclei employing a realistic nucleon-nucleon (NN) interaction in a large configuration space of the full *fp* shell.

In the present work we are concerned with the nuclear structure of ^{52}V and ^{53}V. Results of our calculations for lighter vanadium isotopes have already been reported.[8] In the calculations reported here, an inert ^{40}Ca core is assumed and the single particle orbitals $0f_{7/2}$, $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$ are included in the active space. The calculations are performed in the framework of the Hartree-Fock (HF) projection formalism,[9] employing the modified[10] version of the Kuo-Brown effective NN interaction wherein the matrix elements in the $|(f_{7/2})^2 JT\rangle$ states are renormalized to account for the omission of the $g_{9/2}$ orbit included in the original model space of Kuo and Brown.[11] The same effective interaction has been used in our previous work on scandium,[12,13] vanadium,[8,13] and chromium[14] isotopes and was found to reproduce the energy spectra and electromagnetic properties reasonably well. The HF calculations with axially symmetric deformations show that there are many energetically close intrinsic states of the two vanadium nuclei under consideration. This necessitates a band mixing calculation to determine the admixture of various intrinsic states in the nuclear wave function. The nuclear wave functions are, therefore, obtained from the good angular momentum states projected from the individual intrinsic bands by the band-mixing prescription[13] outlined in Sec. II. In the present band-mixing calculations we have considered the prolate and oblate HF states and the intrinsic states obtained by elementary excitations from these HF states. The resulting band-mixed wave functions are then used to study the properties of low-lying levels in ^{52}V and ^{53}V. The results of the calculations are discussed in Sec. III and the conclusions are presented in Sec. IV.

II. DESCRIPTION OF THE CALCULATIONS

The calculations reported in the present work are carried out in the framework of the HF projection formalism[9] employing the modified[10] Kuo-Brown[11] effective interaction. All the valence nucleons outside an inert ^{40}Ca core are considered to be active in the configuration space of the full fp shell. The single particle energies of the basis states $0f_{7/2}$, $1p_{3/2}$, $1p_{1/2}$, and $0f_{5/2}$ are taken to be 0.0, 2.1, 3.9, and 6.5 MeV, respectively, as obtained from the experimental energy spectrum of ^{41}Ca. The HF calculations with axially symmetric deformations yield many energetically close intrinsic states. The energy E_K^{HF} of the intrinsic HF state ϕ_K is given by

$$E_K^{\mathrm{HF}} = \langle \phi_K | H | \phi_K \rangle , \tag{1}$$

where H is the nuclear Hamiltonian and K is the band quantum number. The good angular momentum states ψ^J projected from different intrinsic states may not be orthogonal. The nuclear wave function can then be expressed as

$$\Psi^J = \sum_i C_i^J \psi_i^J . \tag{2}$$

The nuclear Hamiltonian is diagonalized in the basis space of the good J states projected from the various intrinsic states. The band-mixing coefficients C_i and the energy ϵ of the nuclear state Ψ can be obtained from the equation

$$\sum_j (\langle \psi_i | H | \psi_j \rangle - \epsilon \langle \psi_i | \psi_j \rangle) C_j = 0 \tag{3}$$

for each angular momentum J. Thus, energy of each nuclear state is determined by solving the determinantal equation[13]

$$| \langle \psi_i | H | \psi_j \rangle - \epsilon \langle \psi_i | \psi_j \rangle | = 0 . \tag{4}$$

The static electromagnetic moments and transition probabilities are then obtained by evaluating[15] the matrix elements of the relevant multipole operator between the initial and final nuclear states [Eq. (1)].

III. RESULTS AND DISCUSSION

The band-mixed wave functions[13] obtained from the self-consistent HF projection formalism are employed in the nuclear structure calculations of the two neutron rich vanadium isotopes ^{52}V and ^{53}V reported here. The magnetic moment μ and the reduced transition probabilities $B(M1)$ are calculated by employing the bare magnetic dipole operator corresponding to a free nucleon, whereas the electric quadrupole moment Q and the reduced transition probabilities $B(E2)$ are evaluated by employing an effective quadrupole operator with effective charges assigned to the valence nucleons. The effective charges $e_p = 1.33e$ for protons and $e_n = 0.64e$ for neutrons employed in the present calculations were obtained[16] by the least squares fit to the observed $B(E2)$ values in Ti, V, Cr, and Fe isotopes in the mass region $44 \leq A \leq 54$. A slightly different set of effective charges $e_p = 1.25e$ and $e_n = 0.47e$, obtained by Kuo and Osnes[34] in a microscopic calculation, was also used for comparison.

The band quantum number K, the energy E_K^{HF}, and the mass quadrupole moments $Q_K^{\mathrm{HF}}(p)$ for protons and $Q_K^{\mathrm{HF}}(n)$ for neutrons, of the intrinsic states employed in the present band-mixing calculations in ^{52}V and ^{53}V are shown in Table I. It should be noted that the prolate HF state is energetically the lowest for both the nuclei, although the energy difference between the lowest prolate and the lowest oblate states in ^{52}V is rather small ($\simeq$0.5 MeV). It is also clear from Table I that the mass quadrupole moment $Q_K^{\mathrm{HF}}(p)$ in the lowest intrinsic HF state increases slightly by $\sim 6\%$ from ^{52}V to ^{53}V, whereas the corresponding mass quadrupole moment $Q_K^{\mathrm{HF}}(n)$ changes drastically (increases by $\sim 50\%$) as we go from ^{52}V to ^{53}V. This, however, is understandable in the sense that an increase in the number of neutrons outside the N

TABLE I. Intrinsic states of ^{52}V and ^{53}V included in the present band-mixing calculations. The band quantum numbers K, energy E_K^{HF}, and mass quadrupole moments $Q_K^{\mathrm{HF}}(p)$ of proton and $Q_K^{\mathrm{HF}}(n)$ of neutrons, are tabulated.

Nucleus	$\lvert K\rvert$	E_K^{HF} (MeV)	$Q_K^{\mathrm{HF}}(p)$ (fm^2)	$Q_K^{\mathrm{HF}}(n)$ (fm^2)
^{52}V	2.0	−33.58	31.92	29.60
	1.0	−33.33	31.62	29.07
	4.0	−33.05	−26.61	−21.06
	1.0	−32.77	−26.83	−20.84
	3.0	−31.96	23.48	28.97
	2.0	−31.81	23.69	29.45
^{53}V	1.5	−33.18	35.19	45.14
	2.5	−31.55	−26.84	−29.50
	2.5	−31.22	26.60	45.00
	0.5	−30.81	−26.84	−28.16
	1.5	−30.58	−21.08	−29.39
	2.5	−30.27	−18.65	−29.39

$=28$ closed shell will deform the nuclei more and more. We find[8,13] that $Q_K^{\mathrm{HF}}(n)$ decreases first slowly from ^{47}V to ^{49}V (because of the filling of neutrons in the $f_{7/2}$ subshell, deformation decreases) and then drastically from ^{49}V to ^{51}V, where the neutron $0f_{7/2}$ subshell is completely full. As we start putting particles outside the $N=28$ closed subshell, $Q_K^{\mathrm{HF}}(n)$ starts increasing rapidly again. The change in $Q_K^{\mathrm{HF}}(p)$ is, however, not so drastic as we go from ^{47}V to ^{53}V (see Refs. 8 and 13). The lowest HF intrinsic states employed in the present calculations correspond to a deformation $\delta \simeq 0.09$ and 0.11 in ^{52}V and ^{53}V, respectively.

It should be mentioned here that the projected energies obtained from the seventh and higher intrinsic states (not listed in Table I) are substantially higher than the corresponding energies obtained from the intrinsic states listed in Table I. Consequently, these intrinsic states do not affect the energy spectrum obtained by considering the band-mixing between the lowest six intrinsic states only. Although the effect of the higher intrinsic states on the energy spectra of the two nuclei under consideration is not very significant, their effect on the $B(M1)$ and $B(E2)$ values may not be negligible, because the electromagnetic transitions are more sensitive to even the small admixtures of the intrinsic states. The band-mixing calculations with the lowest four intrinsic states show that the inclusion of the fifth and sixth intrinsic states change the $B(M1)$ and $B(E2)$ values by less than 10% for various transitions in the two nuclei under consideration. The effect of the still higher intrinsic states is expected to be even smaller. Therefore, only the intrinsic states listed in Table I have been included in the present calculations. The results of the nuclear structure calculations in ^{52}V and ^{53}V obtained by explicitly including the band-mixing between the lowest six states (Table I) are discussed below.

^{52}V

The low-lying positive parity states in the odd-odd nucleus ^{52}V have been extensively studied through (n,γ),[17-23] (d,p),[24-27] and (^{3}He$,p$) (Ref. 28) reactions and β-decay studies.[29] Recently, the high spin states up to $J^\pi=9^+$ have been identified and studied by Brown *et al.*[30] and Nathan *et al.*,[6] populated in heavy-ion reactions ^{48}Ca(^{7}Li$, 3n$)^{52}V and ^{48}Ca(^{11}B$, \alpha 3n$)^{52}V, respectively. The first structure calculations for this nucleus were performed by Vervier[2] by restricting the valence particles outside the ^{48}Ca core to the configuration space $(\pi 0f_{7/2})^3 \times (\nu 1p_{3/2}, 0f_{5/2}, 1p_{1/2})^2$. In these calculations, the neutron-proton interaction was taken to be a modified δ interaction. These shell model calculations[2] do not calculate yrast states with $J^\pi > 5^+$ and, in general, the agreement between the calculated and the experimental spectra of low-lying states in ^{52}V is not very good. Moreover, the transition rates and the static moments are not investigated in this work.[1] Horie and Ogawa[3] carried out shell-model calculations in the same configuration space as that of Vervier[2] by employing an np interaction determined by a least square fit to spectra of $N=29$ nuclei. These calculations gave a fairly good description of the low-lying states up to $J=9^+$ in ^{52}V. These authors,[3] however, employ very large effective charges ($e_p=1.9e$ and $e_n=1.0e$) to calculate the reduced transition probabilities. The low-lying states in ^{52}V are excited in the (d,p) reaction[24-27] with strong $1_n=1$ transitions and therefore have dominant $\pi(f_{7/2})_{7/2}{}^3\ \nu(p_{3/2})$ character. The states with the lowest proton sen-

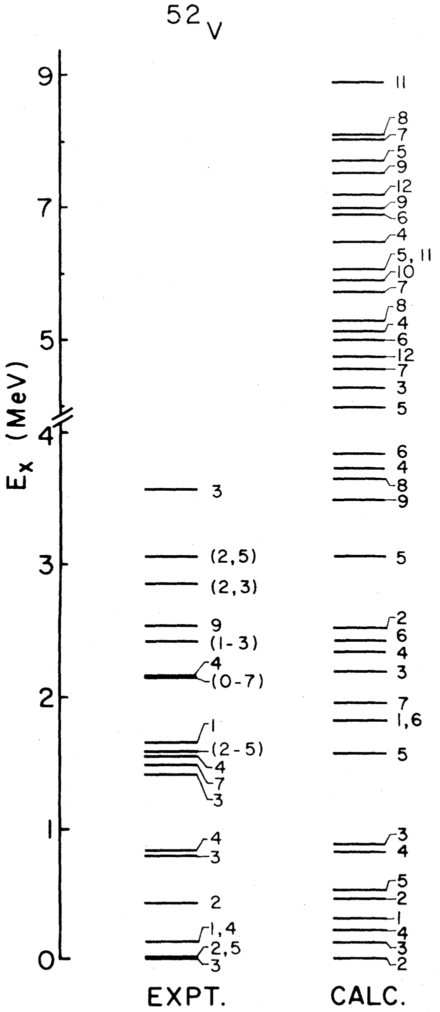

FIG. 1. Comparison of experimental and theoretical spectra for positive-parity levels of ^{52}V. The numbers on the right of the levels indicate J values.

TABLE II. Quadrupole and magnetic moments of the low-lying positive parity states in ^{52}V. The effective charges employed in Calc. I and Calc. II are $e_p = 1.33e$, $e_n = 0.64e$; and $e_p = 1.25e$, $e_n = 0.47e$, respectively.

	Q (e fm^2)		μ (μ_N)	
J	Calc. I	Calc. II	Calc.	SM[a]
1	4.3	3.8	2.602	
2	18.9	16.7	2.181	
3	−0.3	−0.1	2.295	3.930
4	−24.4	−21.9	2.841	
5	−14.9	−12.8	3.677	

[a] Reference 3. These shell-model calculations are carried out in the configuration space $(\pi 0f_{7/2})^3 \times (\nu 1p_{3/2}, 0f_{5/2}, 1p_{1/2})^1$ with $g_p = 5.85\mu_N$ and $g_n = -1.91\mu_N$.

iority form a multiplet of four yrast states with spins 2^+, 3^+, 4^+, and 5^+. However, the (^{3}He,p) studies[28] indicate the admixtures due to other configurations even at the lowest ^{52}V excitation energies. It is desirable, therefore, to pursue the nuclear structure studies in the configuration space of the full fp shell.

The energy levels obtained in the present calculations are displayed in Fig. 1 along with the experimental energy spectrum. The low-lying levels of this odd-odd nucleus are very closely spaced and the approximations used in deriving the effective NN interaction make it impossible to predict the energy levels with an accuracy of better than a few hundred keV. The experimental yrast states with $J^\pi = 1^+$, 2^+, 4^+, and 5^+ lie within a range of 150 keV above the 3^+ ground state. The 2^+ yrast state is only 17 keV above the ground state. The (n,γ) studies[17-23] indicate the presence of two doublet levels at 17 and 142 keV spaced 6 keV apart. The present calculations give a fairly good description of the yrast states with $J^\pi = 1^+$, 2^+, 3^+, and 4^+, except that the positions of the close lying $J^\pi = 3^+$ and $J^\pi = 2^+$ states are interchanged. Our calculated 5^+ yrast state is about 500 keV higher than the observed 5^+ yrast level at 23 keV. The high spin yrast states with $J^\pi = 7^+$ and 9^+ have been recently located through heavy-ion reactions[6,30] at excitation energies of 1492 and 2542 keV, respectively. The observed 7^+ state is fairly well reproduced by the present calculations. The calculated excitation energy of the 9^+ state is about 1 MeV too high compared with the corresponding observed excitation energy of this state. It may be pointed out here that the recent shell-model calculations of Nathan *et al.*[6] also predict the 9^+ yrast state at an excitation energy which is pretty close to our prediction. The present calculations predict the high spin states 6^+, 8^+, 10^+, 11^+, and 12^+ to occur

TABLE III. The $B(E2)$ and $B(M1)$ values for transitions in ^{52}V between the positive parity yrast states J_i and J_f. Other details are the same as in Table II.

		$B(E2; J_i \rightarrow J_f)$ (e^2 fm^4)				$B(M1; J_i \rightarrow J_f)$ (μ_N^2)	
J_i	J_f	Expt.[a]	Calc. I	Calc. II	SM[b]	Expt.[d]	Calc.
1	2		15.4	12.9			1.66
	3		55.8	44.4			
2	3		196.4	154.0		1.18 ± 0.59	0.33
4	2		25.3	19.2			
	3		52.3	39.0			0.012
	5		122.6	97.0			0.16
6	5		11.6	8.6			0.023
	4		10.0	7.7			
	7		7.5	5.7			0.12
7	5	107.2 − 26.5	110.0	86.0	104		
8	7		1.6	1.1			3×10^{-5}
	6		124.5	94.8			
	9		2.5	1.9			0.39
9	7	83 ± 5[c]	44.4	33.9	85		
10	8		103.5	76.6			
	9		2.2	1.5			0.10
11	9		42.2	29.7			
	10		3.1	2.1			0.12
11	12		30.5	22.5			4.61
10	12		7.5	3.9			

[a] Reference 6.

[b] Reference 3. These shell-model calculations are carried out with $e_p = 1.9e$ and $e_n = 1.0e$.

[c] Reference 30.

[d] Reference 18.

TABLE IV. $B(E2)$ and $B(M1)$ values for the γ transitions between the positive parity excited states in ^{52}V. Other details are the same as in Table II.

		$B(E2)$ $e^2\text{fm}^4$		$B(M1)$ $\mu_N{}^2$			$B(E2)$ $e^2\text{fm}^4$		$B(M1)$ $\mu_N{}^2$
J_i	J_f	Calc. I	Calc. II	Calc.	J_i	J_f	Calc. I	Calc. II	Calc.
2_1	1	141.7	111.0	0.26	6_1	4	37.3	29.3	
	3	42.3	33.9	0.004		5	15.0	12.5	0.11
	2	0.0003	0.007	0.77		6	0.03	0.02	0.047
3_1	1	46.6	34.5			7	48.4	36.3	0.33
	3	0.65	0.43	0.055		4_1	4.0	3.1	
	2_1	9.0	6.6	0.03		5_1	0.5	0.5	0.52
	4_1	108.1	87.3	0.14	7_1	5	0.02	0.01	
4_1	2	28.9	22.3			6	0.97	0.50	0.065
	3	12.2	9.2	0.0		7	9.5	6.7	13.9
	5	0.2	0.1	1.59		8	20.6	15.2	0.29
	4	22.0	18.4			9	6.8	4.7	
	2_1	72.4	55.7			5_1	75.4	58.2	
5_1	3	0.2	0.2			6_1	8.2	5.9	0.005
	5	0.2	0.1	0.01		6_2	9.7	6.9	2.22
	3_1	74.4	55.7		7	5_1	3.6	2.8	
	4_1	15.1	10.7	0.04	8	6_1	0.05	0.04	
6	4_1	117.0	89.8		1_1	2	9.4	6.9	0.21
	5_1	51.9	41.3	0.16		3	2.3	2.4	
1_1	1	9.4	7.0	0.13	4_2	2	3.8	2.9	
2_2	1	1.2	1.3	0.31		3	14.1	10.8	0.24
	2	6.6	4.7	0.02		4	3.5	2.3	0.032
	1_1	66.5	50.5	0.09		2_1	2.7	2.4	
	2_1	12.7	8.7	0.099		3_1	0.7	0.6	0.39
	4	0.08	0.2			4_1	8.9	6.7	0.23
	4_1	2.6	2.3			6_1	3.5	3.3	
	3_2	3.5	1.7	0.12		3_2	9.7	7.6	1.57
	4_2	94.0	74.3		5_2	3	22.3	17.8	
3_2	1	4.8	3.5			5	4.3	3.1	0.85
	4	8.4	6.6	0.001		7	3.3	2.2	
	5	1.9	1.5			3_1	2.8	2.1	
	3	6.2	4.2	0.068		5_1	0.002	0.02	1.46
	1_1	49.6	38.7			6_1	68.0	51.6	0.05
	3_1	7.5	6.6	0.95		3_2	28.7	22.1	
	4_1	11.7	11.0	0.16		4_2	0.29	0.31	0.78
	5_1	0.06	0.10		6_2	4_1	4.2	3.2	
6_2	4	0.8	0.7			5_1	1.6	1.2	0.005
	5	10.9	7.8	0.001		6_1	0.25	0.59	0.073
	6	1.54	1.26	0.27		4_2	72.4	54.6	
	7	1.8	2.2	0.67		5_2	1.2	1.2	1.0
	8	0.3	0.3						

at 1835, 3659, 5943, 6105, and 4772 keV excitation energy, respectively. The predicted energies of the 6^+ and 8^+ yrast states agree well with those obtained from the $(\pi 0f_{7/2})^3 \times (\nu 1p_{3/2}, 0f_{5/2}, 1p_{1/2})^1$ shell-model calculations of Nathan *et al.*[6] These shell-model calculations[6] predict the 10^+ yrast state about 500 keV too high compared with the corresponding excitation energy obtained from the present calculations. These authors[6] did not investigate the $J^\pi = 11^+$ and 12^+ yrast states. The observed 1^+, 2^+, 3^+, and 4^+ (Refs. 28, 19, 21, 27, 25) excited states at excitation energies of 1665, 437, 794, and 846 keV are very well reproduced by the present calculations. The level at 1418 keV was assigned a spin-parity of 3^+ previously in the (n,γ) studies,[19,21] whereas the recent $(^3\text{He},p)$ work of Caldwell *et al.*[28] assigns a spin-parity of $(2^+–5^+)$ to this level. Our calculations predict a 3^+ state at 2193 keV, which is about 700 keV too high if we believe that the 1418 keV level is a 3^+ state. The level at 2435 keV has been assigned a spin-parity of $(1^+–3^+)$ in the $(^3\text{He},p)$ work[28] and $(2^+–5^+)$ in the (d,p) work.[25] The excitation energy of this level agrees quite well with our calculated 3^+ level at 2193 keV. We therefore conclude that our calculated level at 2193 keV corresponds to the experimental level at 2435 keV, and on this basis we suggest a spin-parity of 3^+ for the observed level at 2435 keV. Our predicted 3^+ level at 4296 keV may very well correspond to the observed[28] 3^+

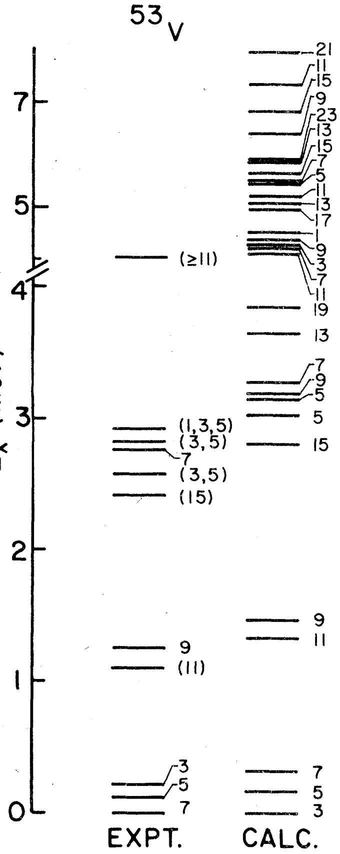

FIG. 2. The experimental and the calculated energy spectrum of negative-parity states in ^{53}V. The numbers on the right of the levels are the $2J$ values.

TABLE V. The electric quadrupole moments and the magnetic dipole moments of the low-lying negative parity states in ^{53}V. Other details are the same as in Table II.

		Q ($e\,\mathrm{fm}^2$)		μ	(μ_N)
$2J$	Calc. I	Calc. II	SM^a	Calc.	SM^a
3	18.3	15.9		2.235	
5	−10.5	−9.4		3.054	
7	−12.8	−10.9	−18.0	4.277	5.350
9	−27.9	−24.6		4.162	

[a] Reference 5. These shell model calculations are done using configurations of the form $(\pi 0f_{7/2})^3 \times (\nu 1p_{3/2}, 0f_{5/2}, 1p_{1/2})^2$ with $e_p = 1.9e$ and $e_n = 1.0e$.

level at 3586 keV populated in the (^{3}He,p) reaction. The level reported[17-23] at 2859 keV has been assigned the spins (2^+, 3^+) tentatively; whereas the present calculations predict a 2^+ level at an excitation of 2517 keV. We therefore suggest a spin-parity of 2^+ for this level. Our calculations do not reproduce the observed[19,21] 4^+ level at 1560 keV. The present calculations predict a 4^+ level at 2340 keV, which agrees fairly well with the experimental[17-23] 4^+ level at 2169 keV. The present calculations predict a 5^+ level at 1578 keV, and this excitation energy agrees very well with the experimentally observed level at 1580 keV. The 1580 keV level was assigned a spin-parity of (2^+–5^+) in the (d,p) work of Catala *et al.*[25] On the basis of the present calculations, we therefore suggest a spin-parity of 5^+ for this state. The level reported at 2152 keV was assigned a spin-parity of (0^+–7^+) in (d,p) studies.[25] The present work predicts a 6^+ level at 2431 keV, and we therefore suggest a spin-parity of 6^+ for the observed level at 2152 keV.

The excitation energy of the experimental level at 3066 keV is very well reproduced by the present calculations. This level was tentatively assigned a spin-parity of (2^+–5^+) in the (d,p) measurements,[25] whereas the present calculations predict a spin-parity of 5^+ to this level.

The present calculations predict a few excited states which have not been experimentally identified until now. The excitation energies (in keV) of these states are 4^+(3745, 5168, 6508), 5^+(4011, 6091, 7758), 6^+(3839, 5029, 6921), 7^+(4594, 5739, 8053), 8^+(5332, 8139), 9^+(7023, 7564), 11^+(8929), and 12^+(7201).

The experimental data on static moments and the transition rates is very scanty. The electric quadrupole moments and the magnetic moments of the low-lying positive parity states are presented in Table II. The experimental values are not available for comparison for even the ground state. The shell-model calculations[3] predict substantially large values for the magnetic moment of the ground state, as seen from Table II. The earlier theoretical investigations of Vervier[2] and of Horie and Ogawa[3] do not report any results on the quadrupole moments of the low-lying levels in ^{52}V. The calculated $B(M1)$ and $B(E2)$ values for the electromagnetic transitions between the yrast states are tabulated in Table III. The experimental data as well as the results of the shell model calculations[3] are available for only a few γ transitions. The observed $B(M1)$ value for the $2^+ \rightarrow 3^+$ transition is quite uncertain due to large errors. Our calculated value for this transition is smaller by a factor of ~4 than the experimental value. The calculated $B(E2)$ value for the $7^+ \rightarrow 5^+$ transition agrees very well with the shell-model results.[3] It can be seen from Table III that the result of the present calculations for the $7^+ \rightarrow 5^+$ transition agrees very well with the upper limit of the observed[6] value. The present calculations, however, do not reproduce the experimental $B(E2)$ value for the $9^+ \rightarrow 7^+$ transition. The calculated value is larger than the corresponding experimental value by a factor of 2.

The calculated $B(E2)$ and $B(M1)$ values for the transitions between the excited states are given in Table IV. The experimental data or the results from any other structure calculation are not available for comparison. It would be, therefore, very desirable to measure the transition rates between the low-lying states in ^{52}V.

^{53}V

The low-lying states of this nucleus have been studied through a variety of nuclear reactions.[6,7,31-33] Recently Nathan *et al.*[6] have investigated the high spin states up to $J^\pi = \frac{15}{2}^-$ populated in the heavy-ion reaction ^{48}Ca(^{11}B, $\alpha 2n$)^{53}V. The earlier theoretical calculations for ^{53}V were carried out by McGrory,[4] and Horie and Ogawa.[5] These shell-model calculations[4,5] restrict the particles outside the inert ^{48}Ca core to the configuration space of the type $(\pi 0f_{7/2})^3 \times (\nu 1p_{3/2}, 0f_{5/2}, 1p_{1/2})^2$ and employ an effective interaction obtained by fitting the experimental spectra of $N=29$ nuclei. Both of these calculations[4,5] predict a level sequence which is typical of $(f_{7/2})^{\pm 3}$ nuclei. However, $M1$ transitions are forbidden between states which are members of a pure $(f_{7/2})^3$ multiplet. The observation[7] of $M1$ transitions between the low-lying states of this nucleus indicates the presence of other admixtures. Both of these calculations[4,5] do not report any results on transition rates in ^{53}V. It is, therefore, desirable to study the structure of the low-lying states in this nucleus in detail by employing realistic NN interaction in the configuration space of the complete *fp* shell.

The calculated and experimental spectra are compared in Fig. 2. The low-lying states of this nucleus are reproduced fairly well by the present calculations, except that the positions of the close-lying levels $J^\pi = \frac{7}{2}^-$ and $J^\pi = \frac{3}{2}^-$ are interchanged. The present calculations predict the high spin states $\frac{13}{2}^-$, $\frac{17}{2}^-$, $\frac{19}{2}^-$, $\frac{21}{2}^-$, and $\frac{23}{2}^-$ at excitation energies of 3646, 4985, 3852, 7957, and 5918 keV, respectively. It is interesting to note that the excitation energies of the $\frac{13}{2}^-$, $\frac{17}{2}^-$, and $\frac{21}{2}^-$ are close to

TABLE VI. The transition strengths for the γ transitions between the negative parity yrast states in ^{53}V. Other details are same as in Table II.

J_i	J_f	$B(E2; J_i \to J_f)$	$e^2\,\mathrm{fm}^4$		$B(M1, J_i \to J_f)$ μ_N^2
		Expt.[a]	Calc. I	Calc. II	Calc.
$\frac{3}{2}$	$\frac{5}{2}$		297.6	220.4	0.13
	$\frac{7}{2}$		196.6	149.2	
$\frac{5}{2}$	$\frac{7}{2}$		236.5	179.7	0.16
$\frac{9}{2}$	$\frac{5}{2}$	≤ 73.3[b]	104.9	76.9	
	$\frac{7}{2}$		58.0	40.8	0.20
	$\frac{11}{2}$		103.9	79.1	0.45
$\frac{11}{2}$	$\frac{7}{2}$	184.5 ± 26.0	173.4	129.3	
$\frac{13}{2}$	$\frac{11}{2}$		22.8	15.2	0.005
	$\frac{9}{2}$		151.8	113.0	
	$\frac{15}{2}$		19.5	15.1	0.13
$\frac{15}{2}$	$\frac{11}{2}$	147.8 ± 35.5	182.0	135.0	
$\frac{17}{2}$	$\frac{15}{2}$		11.2	6.2	0.04
	$\frac{13}{2}$		66.5	50.9	
	$\frac{19}{2}$		16.7	12.4	0.86
$\frac{19}{2}$	$\frac{15}{2}$		0.4	0.3	
$\frac{21}{2}$	$\frac{19}{2}$		22.0	15.8	4.72
	$\frac{17}{2}$		7.1	5.0	
	$\frac{23}{2}$		13.2	8.9	11.9
$\frac{23}{2}$	$\frac{19}{2}$		11.2	7.0	

[a] Reference 6.

[b] Reference 7.

the recent shell-model results of Nathan *et al.*[6] These calculations,[6] however, predict the $\frac{19}{2}^-$ and $\frac{23}{2}^-$ states at energies which are substantially higher than the excitation energy of these states obtained in our calculations. The measurements of Nathan *et al.*[6] show a state at 4085 keV, which decays to a $\frac{15}{2}^-$ level at 2420 keV. The level at 4085 keV has been tentatively assigned[6] a spin of ($\geq \frac{11}{2}$). The present calculations also predict a $\frac{11}{2}^-$ excited state at 4136 keV. So the observed level at 4085

TABLE VII. $B(E2)$ and $B(M1)$ values for the γ transitions between the excited negative parity states in ^{53}V. Other details are same as in Table II.

J_i	J_f	$B(E2)$ $e^2\,\mathrm{fm}^4$ Calc. I	$B(E2)$ $e^2\,\mathrm{fm}^4$ Calc. II	$B(M1)$ $\mu_N{}^2$ Calc.
$\frac{7}{2}_1$	$\frac{3}{2}$	11.8	7.9	
	$\frac{5}{2}$	0.002	3×10^{-5}	0.41
	$\frac{7}{2}$	3.0	1.5	0.43
	$\frac{9}{2}$	9.8	8.6	0.19
	$\frac{11}{2}$	6.7	5.7	
	$\frac{5}{2}_1$	161.4	123.4	0.018
	$\frac{9}{2}_1$	66.7	48.5	0.069
	$\frac{5}{2}_2$	1.5	1.1	0.089
$\frac{5}{2}_1$	$\frac{3}{2}$	25.0	16.8	0.075
	$\frac{5}{2}$	0.6	0.8	0.49
	$\frac{7}{2}$	0.07	0.01	2.75
	$\frac{9}{2}$	23.6	20.2	
$\frac{3}{2}_1$	$\frac{3}{2}$	0.003	0.002	0.044
	$\frac{5}{2}$	2.9	1.8	1.1
	$\frac{7}{2}$	4.8	3.0	
	$\frac{5}{2}_1$	1.9	1.0	0.53
	$\frac{7}{2}_1$	13.2	9.3	
	$\frac{5}{2}_2$	7.4	7.4	5.81
	$\frac{7}{2}_2$	5.4	5.4	
$\frac{9}{2}_1$	$\frac{5}{2}$	2.8	2.0	
$\frac{5}{2}_2$	$\frac{3}{2}$	1.9	1.9	0.28
	$\frac{5}{2}$	19.2	12.1	0.26
	$\frac{7}{2}$	0.01	0.07	9.80
	$\frac{9}{2}$	9.1	7.2	
	$\frac{5}{2}_1$	132.8	100.2	0.002
$\frac{7}{2}_2$	$\frac{5}{2}$	0.3	0.2	0.29
	$\frac{7}{2}$	0.3	0.9	0.35
	$\frac{3}{2}$	0.3	0.4	
	$\frac{9}{2}$	6.2	3.5	0.017
	$\frac{11}{2}$	1.4	0.7	
	$\frac{5}{2}_2$	153.4	115.3	0.023
	$\frac{7}{2}_1$	3.1	2.3	2×10^{-4}
	$\frac{11}{2}_1$	2×10^{-5}	9×10^{-5}	
	$\frac{9}{2}_1$	32.7	26.1	0.75
$\frac{9}{2}_1$	$\frac{9}{2}$	0.7	0.7	0.002
	$\frac{11}{2}$	6.4	5.9	0.13
	$\frac{5}{2}_1$	16.9	13.0	
	$\frac{5}{2}_2$	22.1	16.7	
$\frac{11}{2}_1$	$\frac{7}{2}$	0.15	0.03	
	$\frac{9}{2}$	4.5	3.7	0.11
	$\frac{13}{2}$	0.03	3×10^{-5}	0.11
	$\frac{15}{2}$	12.3	10.2	
	$\frac{7}{2}_1$	84.3	64.4	
	$\frac{9}{2}_1$	43.8	33.6	0.077
	$\frac{11}{2}$	11.6	7.7	0.74
$\frac{13}{2}_1$	$\frac{9}{2}$	1.98	1.24	
	$\frac{11}{2}$	2.00	1.28	0.058
	$\frac{13}{2}$	1.0	0.7	0.084
	$\frac{15}{2}$	30.9	24.1	0.16
	$\frac{17}{2}$	5.5	4.3	
	$\frac{9}{2}_1$	66.8	48.8	
	$\frac{11}{2}_1$	21.6	15.0	2×10^{-6}
	$\frac{9}{2}_2$	6.9	5.7	
$\frac{13}{2}$	$\frac{9}{2}_1$	2.6	2.1	
$\frac{9}{2}_2$	$\frac{5}{2}$	1.1	1.5	
	$\frac{7}{2}$	0.06	0.10	0.41
	$\frac{11}{2}$	0.08	0.04	0.37
	$\frac{13}{2}$	4.2	2.5	
	$\frac{7}{2}_1$	36.9	27.5	0.34
	$\frac{11}{2}_1$	47.6	36.4	0.003
	$\frac{5}{2}_2$	306.4	221.5	
	$\frac{7}{2}_2$	82.0	60.9	0.012

keV may either correspond to the $\frac{19}{2}^-$ level at 3852 keV or the $\frac{11}{2}^-$ level at 4136 keV. Therefore, we suggest a spin-parity of ($\frac{11}{2}^-$ or $\frac{19}{2}^-$) for this level. The measurements of Parks *et al.*[32] assign a spin-parity of ($\frac{3}{2}^-, \frac{5}{2}^-$) and ($\frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$) to the levels at 2829 and 2931 keV, respectively. The present calculations predict two $\frac{5}{2}^-$ levels at 3027 and 3140 keV, thus favoring a spin-parity of $\frac{5}{2}^-$ for both the experimental levels at 2829 and 2931 keV. Our calculated $\frac{7}{2}^-$ state at 3273 keV is about 500 keV higher in energy compared with the corresponding observed[33] level. The present calculations also predict a few excited states. The excitation energies (in keV) of these levels are $\frac{5}{2}^-$ (5440), $\frac{7}{2}^-$ (4243, 5494), $\frac{9}{2}^-$ (3190, 4415, 6402), $\frac{11}{2}^-$ (5245, 7346), $\frac{13}{2}^-$ (5100, 5878), and $\frac{15}{2}^-$ (5679, 6839).

The calculated electric quadrupole moments and the magnetic dipole moments are shown in Table V. The experimental values are not available for comparison. The shell-model results of Horie and Ogawa[5] for the $J^\pi = \frac{7}{2}^-$ state are considerably higher than the values obtained from the present calculations. The $B(E2)$ and $B(M1)$ values for the γ transitions between the low-lying negative parity yrast states in ^{53}V are presented in Table VI. The experimental data is available only in a few cases. The experimental $B(E2)$ values for the γ transitions $\frac{9}{2}^- \to \frac{5}{2}^-$, $\frac{11}{2}^- \to \frac{7}{2}^-$, and $\frac{15}{2}^- \to \frac{11}{2}^-$ are very well reproduced by the present calculations. The shell-model calculations of McGrory,[4] and Horie and Ogawa[5] do not report any results on the transition strengths. The $B(E2)$ and $B(M1)$ values for transitions between the excited states are listed in Table VII.

IV. CONCLUSIONS

The properties of the nuclear levels in ^{52}V and ^{53}V are investigated in the framework of the Hartree-Fock projection formalism by employing the effective NN interaction.[10,11] All the nucleons outside the inert ^{40}Ca core are explicitly treated in the configuration space of the complete *fp* shell. The energy levels and the electromagnetic properties of the low-lying levels in ^{52,53}V are calculated by employing band-mixed wave functions. The observed nuclear levels, including those with high angular momentum, are fairly well reproduced by the present calculations. The calculations predict many excited states in both of these nuclei. The quadrupole and the magnetic moments for the low-lying states in ^{52,53}V have not yet been measured. The shell-model values for the static moments of the ground state in both the nuclei are substantially higher than our calculated values. The data on transition strengths in these nuclei are very sparse. The computed $B(E2)$ values agree quite well with the experimental values, except for the $9^+ \to 7^+$ transition in ^{52}V. The computed $B(M1)$ value for the $2^+ \to 3^+$ transition in ^{52}V is smaller by a factor of about 4 compared with the only available experimental measurement.[30] The transition strengths for a large number of transitions between the excited states have been calculated. It will be, therefore, interesting to look for the predicted levels in ^{52,53}V through heavy-ion and other types of reactions, and to measure the electromagnetic properties of the low-lying levels in these nuclei.

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