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Effective interaction for one-hole states in K isotopes

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One-hole states of $A = 39$–47 potassium isotopes are examined with a model space spanning the $f_{7/2}{}^n(d_{3/2}s_{1/2})^{-1}$ and $f_{7/2}^{n-1}p_{3/2}d_{3/2}{}^{-1}$ configurations. The parameters of the neutron and particle-hole interaction are determined by least-squares fits to energy levels of Ca and K isotopes. The calculated spectra are in good agreement with experiment, and allow spin predictions to be made for the higher-mass K isotopes. The model wave functions are used to calculate magnetic moments and $M1$ transition rates, and measurements which would be useful in determining certain matrix elements of the effective dipole operator are suggested.

[NUCLEAR STRUCTURE Calculated one-hole spectra of K isotopes; effective interaction from least-squares fits; $M1$ transitions and magnetic moments.]

I. INTRODUCTION

With the development of large-scale shell-model computer programs such as the Oak Ridge and Glasgow codes, calculations in extremely large model spaces are now feasible. To a large extent, however, these advances in computational techniques have outstripped knowledge of the effective interaction in nuclei, and without a "good" set of two-body matrix elements exact diagonalizations in large spaces are of questionable value. In the *sd* shell it *has* been possible to derive effective interactions which justify such calculations, mainly because there are only 63 two-body matrix elements and the most important of these can be determined by iterative least-squares fits to the spectra of nuclei having only a few particles or holes in the major shell. The fact that the effective matrix elements obtained from fits at the start and end of the shell are rather different (the lower-shell interaction gives poor spectra for upper-shell nuclei, and vice versa) probably is due mainly to the effect of neglected fp-shell configurations, which for nuclei with $A \gtrsim 28$ undoubtedly lie lower in energy than a large fraction of the model space included in the $(sd)^{A-16}$ diagonalizations. Low-lying $\frac{7}{2}^-$ and $\frac{3}{2}^-$ levels appear in the spectra of odd-mass nuclei with $A \gtrsim 35$, and particle-transfer data suggest that even ground states in this mass region have appreciable $(fp)^2$ components. Any satisfactory treatment of nuclei at the start of the $f_{7/2}$ shell also must take into account these particle excitations from the *sd* shell, since their spectra contain many low-lying levels of predominantly one- and two-hole structure.

It seems clear that any shell-model calculation which attempts to successfully account for the spectra of a range of nuclei around mass 40 will have to use a model space spanning at least the $s_{1/2}$, $d_{3/2}$, $f_{7/2}$, and $p_{3/2}$ single-particle orbits. The $p_{3/2}$ lies only 2 MeV above the $f_{7/2}$, and it is unreasonable to believe that its effects on levels other than $\frac{3}{2}^-$ single-particle states can be represented adequately by a renormalization of the effective interaction. Holes in the $s_{1/2}$ shell may well be of minor importance for many nuclei, but because the $f_{7/2}s_{1/2}^{-1}$ interaction is less repulsive than that for $f_{7/2}d_{3/2}^{-1}$ (for the neutron-proton component, at least), their relative importance will increase with mass number beyond $A = 40$. A good example of this is seen in the spectra of potassium isotopes; the $l = 0$ hole strength moves down in energy steadily with increasing mass, until a $\frac{1}{2}^+$ state becomes the ground state for $A = 47$.

Hasper[1] has recently reported calculations for $A = 36$–39 nuclei using a model space based on the orbits listed above, and the results are encouraging. Even though fairly severe truncation had to be employed, because iterative least-squares fits were performed to determine effective interactions, the calculations successfully reproduce most features of the spectra up to quite high excitation energy. The only unsatisfactory feature of the work is that the number of empirical levels available for the fit was much smaller than the number of required two-body matrix elements, so that it was necessary to start from the best-fit modified surface delta interaction (MSDI) and include the change in magnitude of each matrix element following an iteration as an extra "error" to be minimized. With this method, it is quite possible that the final interaction obtained differs appreciably from the unconstrained best-fit interaction, if the latter is itself appreciably different from the MSDI approximation. The interaction found for even-parity levels differed from that required for odd-parity levels, and although not discussed in the text this difference must have been quite significant if it was deemed preferable to perform two separate fits rather than have the advantages of including all empirical levels in a single fit. It is not clear whether this is due mainly to the truncation scheme employed (which necessarily was different for even and odd parity levels) or to the iterative procedures inability to find a unique best-fit interaction because of the constraints imposed.

One of the objectives of the calculations reported in this paper was to obtain information about part of the effective interaction within $s_{1/2}$, $d_{3/2}$, $f_{7/2}$, and $p_{3/2}$ orbits, while being rather less ambitious than were the calculations of Hasper. The one-hole spectra of $A=39-47$ potassium isotopes are calculated taking into account $f_{7/2}^{n}d_{3/2}^{-1}$, $f_{7/2}^{n}s_{1/2}^{-1}$, and $f_{7/2}^{n-1}p_{3/2}d_{3/2}^{-1}$ configurations with $n=A-39$. It is clear from the $l=1$ single-particle strength observed for levels around 2 MeV in ^{40}K that the $p_{3/2}$ shell must be included, while the low-lying $l=0$ hole strength mentioned above obviously requires the inclusion of the $s_{1/2}$ shell. The $f_{7/2}^{n-1}p_{3/2}s_{1/2}^{-1}$ configuration is expected to have very minor importance for the levels discussed (most of which lie at below 2.5 MeV excitation energy), because its single-particle energy lies about 4.5 MeV above that of $f_{7/2}^{n}d_{3/2}^{-1}$, and its inclusion would probably give almost no information about the $p_{3/2}s_{1/2}^{-1}$ interaction. The omission of the $p_{1/2}$ and $f_{5/2}$ neutron orbits should not introduce significant errors, since they lie at about 4 and 6 MeV above the $f_{7/2}$, and if the Kuo-Brown[2] matrix elements are a valid guide the $f_{7/2}p_{1/2}$ and $f_{7/2}f_{5/2}$ $T=1$ interactions are very weakly attractive. The effects of $p_{3/2}^{2}$ configurations below the excitation energy at which they occur can largely be taken into account by suitable choice of $f_{7/2}^{2}$ and $f_{7/2}p_{3/2}$ effective two-body matrix elements, since perturbation theory suggests that, to leading order, these effects produce a mass-independent renormalization. (This is not true for one-particle excitations from the $f_{7/2}$ shell, which lead to both two- and three-body renormalizations even in lowest order.) The effect of neglecting three-hole configurations is more difficult to judge *a priori*, and can best be determined by a comparison of calculated and empirical spectra.

Within the model space used, there are two single-particle energies and 10 two-body matrix elements for the valence neutrons, two single-hole energies for the proton hole, and 15 two-body matrix elements of the particle-hole interaction. Since there are only about 40 empirical levels in the $A=39-47$ K isotopes which can be included in the fit, the 12 neutron parameters were determined by a separate least-squares fit to selected levels of $A=42-49$ Ca isotopes. This calculation is discussed in Sec. II, and the calculation for K isotopes is discussed in Sec. III. Section IV is a discussion of information that can be gained from calculations of magnetic moments and $M1$ transition rates.

II. THE NEUTRON INTERACTION

There have been several attempts to account for the spectra of Ca isotopes within a truncated fp-shell model space. The fit carried out for the present work was similar to that of Engeland and Osnes,[3] who also considered $f_{7/2}^{n}$ and $f_{7/2}^{n-1}p_{3/2}$ configurations. The main difference concerns the empirical levels included as data to be fitted. More levels are now known in ^{45}Ca (the present fit included the first and second $\frac{3}{2}^-$ states, plus the lowest $\frac{9}{2}^-$ and $\frac{11}{2}^-$ states, in addition to the $\frac{7}{2}^-$ and $\frac{5}{2}^-$ states known to Engeland and Osnes), and the ^{49}Ca ground state can be included as the $f_{7/2}^{8}p_{3/2}$ state. The 2^+, (4^+), and $3^{(+)}$ levels now isolated at 3832, 4503, and 4612 keV in ^{48}Ca were assumed to be the $f_{7/2}^{7}p_{3/2}$ 2^+, 4^+, and 3^+ levels. It was deemed advisable to omit from the fitted data any levels which are known (or suspected) to be strongly perturbed by two-hole intruder states arising from sd-shell excitation; in particular, no 0^+ or 2^+ states of ^{42}Ca, 0^+, 2^+, or 4^+ states of ^{44}Ca, or 2^+ states of ^{46}Ca other than the lowest, were included. The lowest 4^+ and 6^+ states of ^{46}Ca were included, as was the lowest $\frac{15}{2}^-$ of ^{43}Ca.

The data selection described above resulted in a total of 28 levels for $A=42-49$. With single-particle energies taken from ^{41}Ca ($\epsilon_{7/2}=-8.363$ MeV, $\epsilon_{3/2}=-6.283$ MeV) the best-fit 10 parameters gave an rms error of 84 keV, while if the single-particle energies were treated as free parameters they changed only slightly to -8.400 and -6.347 MeV, the rms error being virtually unchanged. Two-body matrix elements given by the former calculation are listed in Table I. The diagonal $f_{7/2}p_{3/2}$ two-body matrix elements differ appreciably from those of Engeland and Osnes, although as might be expected their centroid remains almost the same. The excited states of ^{48}Ca play a crucial role in determining these matrix elements; the Engeland and Osnes parameters give a 2^+ in ^{48}Ca at 2.87 MeV, 1 MeV below the lowest state of this spin.

If any of the states in ^{42}Ca, ^{44}Ca, and ^{46}Ca mentioned above are included in the data to be fitted, the rms error becomes much larger. This is as expected, since if these states are either sd-shell excitations or $(fp)^n$ states strongly perturbed by such excitations they cannot be represented adequately in the present model space. As the $f_{7/2}$ shell fills, blocking effects will tend to decrease the importance of $(sd)^{-2}$ states, and their influence cannot simply be represented by an effective $f_{7/2}^{2}$

TABLE I. Neutron matrix elements $\langle f_{7/2}j_1 | v | f_{7/2}j_2\rangle_J$.

j_1j_2	$J=0$	$J=2$	$J=3$	$J=4$	$J=5$	$J=6$
$f_{7/2}f_{7/2}$	−2.953	−0.807		−0.166		0.115
$f_{7/2}p_{3/2}$		−1.074		−0.651		
$p_{3/2}p_{3/2}$		−1.425	0.032	0.159	0.925	

interaction. The present results give an estimate of their effect on the 0^+ ground states of ^{42}Ca and ^{44}Ca; the best-fit interaction within the model space used gives binding energies 160 and 320 keV too small.

III. ONE-HOLE STATES IN K ISOTOPES

With the one- and two-body parameters of the neutron interaction held fixed at the values of Table I, the two single-hole energies and 15 particle-hole matrix elements were determined by an unconstrained least-squares fit to energy levels of $A=39-47$ potassium isotopes. A total of 40 levels were included as data to be fitted. Most of these had spin assignments from experiment, but some other levels were included when preliminary calculations led to firm spin predictions which were not in conflict with any experimental data. The resulting best-fit parameters listed in Table II gave an rms error of 107 keV. The calculated spectra are compared with experiment in Figs. 1 and 2, and are discussed in some detail below.

A. ^{40}K

The calculation suggests that the four levels below 900 keV are almost pure $f_{7/2}d_{3/2}^{-1}$ states, in agreement with data from the ^{39}K(d,p) and ^{41}Ca$(d,{}^3$He) reactions.[4,5] The 2^-, 3^-, 1^-, and 0^- levels at 2047, 2070, 2104, and 2626 keV are strongly excited by $l=1$ transfer in the former reaction, and must have large $p_{3/2}d_{3/2}^{-1}$ components. If one takes the view that the 0^- is pure $p_{3/2}d_{3/2}^{-1}$ (as given in the calculation), the observed stripping strengths suggest that the 1^- is also virtually pure, while the 2^- and 3^- states have $p_{3/2}d_{3/2}^{-1}$ components of about 70% and 50%, respectively. The calculation reproduces the 3^- component well, giving 52%, but gives a component of 96% in the 2^- state. It is one of the failures of the present model that it gives only two 2^- states, whereas at least one other 2^- (at 2419 keV) is observed below 3 MeV. It should be mentioned that the second 2^- in ^{40}K is the only state in the present calculation which gives large weight to the $p_{3/2}d_{3/2}^{-1}$ $J=2$ diagonal matrix element; if this state is omitted from the list of fitted data, the best-fit $J=2$ matrix element more than doubles in magnitude, while the other matrix elements in Table II change very little, and the rms error decreases slightly to 102 keV. The second 2^- in ^{40}K is then calculated to lie at 2.8 MeV, but it is clear that the $p_{3/2}d_{3/2}^{-1}$ $J=2$ strength can be positioned anywhere in the 2–3 MeV region without significantly effecting other results, and that a completely satisfactory treatment of 2^- states in ^{40}K requires the inclusion of 3p3h configurations.

The $l=1$ 3^- state at 2070 keV also has appreciable $l=0$ pickup strength,[5] as does the 4^- state at 2398 keV. The calculated strengths of these two levels are 0.41 and 1.13, experiment giving 0.39 and 1.28.

B. ^{41}K

With the exception of one or two levels, the calculation successfully reproduces the even parity spectrum of ^{41}K below 3 MeV. The calculated proton pickup strengths of the ground state and $\frac{1}{2}^+$ first excited state are 3.5 and 0.7, close to the values of 3.4 and 0.8 deduced from the ^{42}Ca$(d,{}^3$He) reaction.[6] The calculated strength of the 1.56 MeV $\frac{3}{2}{}_2^+$ state is 0.4, while experiment (which is subject to uncertainties because of the mixed-l

TABLE II. Particle-hole matrix elements $\langle j_1j_2^{-1} | v | j_3j_4^{-1}\rangle_J$ and single-hole energies.

$j_1j_2\ j_3j_4$	$J=0$	$J=1$	$J=2$	$J=3$	$J=4$	$J=5$
$f_{7/2}d_{3/2}\ f_{7/2}d_{3/2}$			1.185	0.678	0.558	1.392
$f_{7/2}d_{3/2}\ f_{7/2}s_{1/2}$				0.587	−0.085	
$f_{7/2}s_{1/2}\ f_{7/2}s_{1/2}$				1.159	0.423	
$p_{3/2}d_{3/2}\ p_{3/2}d_{3/2}$	1.029	0.524	0.452	1.534		
$p_{3/2}d_{3/2}\ f_{7/2}d_{3/2}$			−0.289	0.370		
$p_{3/2}d_{3/2}\ f_{7/2}s_{1/2}$				1.070		
$\epsilon(d_{3/2})=$ 8.410 MeV						
$\epsilon(s_{1/2})=$ 10.869 MeV						

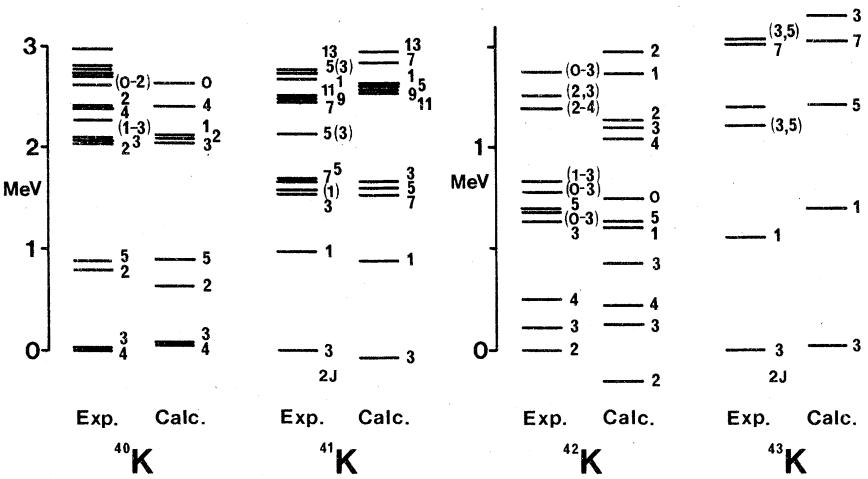

FIG. 1. Calculated one-hole spectra of $A = 40–43$ K isotopes. Levels which are known to be of parity $(-1)^A$ are omitted from the experimental spectra.

fits required at this energy) gives about 0.25 for $d_{3/2}$ transfer. The second $\frac{1}{2}^+$ is calculated to lie at 2.65 MeV, and can be associated with the level at 2.67 MeV which is excited by $l=0$ in pickup reactions, but its calculated strength of 1.1 appears to be about 70% too large. An $l=0$ component has been reported for the triplet at 1.6 MeV,[6] but if there is another $\frac{1}{2}^+$ state here it must lie outside the present model space. The same appears to be true for the $\frac{5}{2}(\frac{3}{2})^+$ level at 2143 keV; the second model $\frac{5}{2}^+$ lies at 2.6 MeV, and can most reasonably be associated with the 2.76 MeV state, while the third model $\frac{3}{2}^+$ lies at 3.7 MeV.

High-spin states are well reproduced by the calculation. The lowest $\frac{11}{2}^+$, $\frac{13}{2}^+$, and $\frac{15}{2}^+$ levels are given at 2.59, 2.96, and 3.90 MeV, and are observed at 2.53, 2.77, and 3.90 MeV. The second $\frac{11}{2}^+$ is predicted to lie at 3.1 MeV.

C. ^{42}K

^{42}K provides a rigorous test of any theoretical model, since there are at least 11 odd parity levels lying below 1.5 MeV. To a first approximation these arise from a $d_{3/2}$ hole coupled to the lowest three states of ^{43}Ca, but because the lowest $\frac{5}{2}^-$ and $\frac{3}{2}^-$ in ^{43}Ca have appreciable $p_{3/2}$ components more than the $f_{7/2}d_{3/2}^{-1}$ interaction is involved. Information about the ^{43}Ca ground state component comes from the ^{43}Ca$(d, {}^3$He$)$ reaction,[7] and Table III shows a comparison of calculated and empirical strengths. Agreement is good for $d_{3/2}^{-1}$ strength in the lowest levels. The calculation suggests that there should be measurable $l=2$ and $l=0$ strength in the 639 keV 3_2^- state, but experiments performed to date have not had sufficient resolution to distinguish this from strength to the 699 keV 5_1^- state. Better resolution would also be useful in isolating the 2_2^- level, since this is predicted to have an $l=2$ strength of 0.25. Some $l=0$ strength is observed at 1.2 MeV, and the calculation can reproduce this if the 1194 $(2-4)^-$ and 1255 keV $(2,3)^-$ levels are the 4_2^- and 3_3^- states; these have calculated strengths

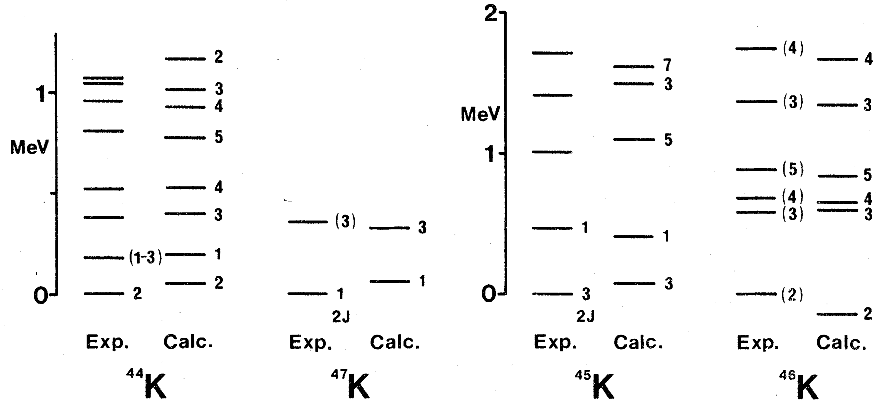

FIG. 2. Calculated one-hole spectra of $A = 44–47$ K isotopes. Levels which are known to be of parity $(-1)^A$ are omitted from the experimental spectra.

TABLE III. ^{43}Ca$(d,{}^3$He$)^{42}$K proton pickup strengths C^2S, and ^{41}K$(d,p)^{42}$K neutron stripping strengths $(2J+1)C^2S/4$.

	Pickup			Stripping			
	Calc		Exp (Ref. 7)	Calc		Exp (Ref. 8)	
Level (keV)	$l=0$	$l=2$	$l=2$	$l=1$	$l=3$	$l=1$	$l=3$
0_1(785?)				0.03		0.02	
1_1(682?)				0.00			
2_1(0)		0.33	0.36	0.02	1.03		0.43
3_1(107)	0.00	0.65	0.70	0.07	1.00	0.03	0.38
3_2(639)	0.07	0.10		0.01	0.08	0.02	0.05
4_1(258)	0.00	0.86	0.92		1.20		0.48
5_1(699)		1.14	1.07		2.23		1.1

of 0.28 and 0.13, respectively, while the total empirical strength is 0.53.

Neutron stripping strengths from the ^{41}K(d,p) reaction[8] also are given in Table III. The total empirical $l=3$ strength is only 40% of the expected $f_{7/2}$ strength of 6, and it appears that it should be renormalized by a factor of 2.5; if this is done, there is fair agreement between theory and experiment for the 2_1^-, 3_1^-, 4_1^-, and 5_1^- states. The calculation also agrees with experiment in giving much greater strength to the 3_1^- than to the 3_2^-. In this paper on the (d,p) reaction, it is claimed by the authors that the $f_{7/2}^n d_{3/2}^{-1}$ wave functions of Dieperink and Brussard[9] completely fail to reproduce the observed relative spectroscopic factors; we find, however, that the numbers they give are in error, and that the Dieperink and Brussard model in fact gives $l=3$ strengths quite similar to those of the present calculation, except that the strength to the lowest two 3^- states is distributed in the ratio of only 2.8 : 1.

The present calculation predicts that there are low-lying 0^- and 1^- levels, and the states observed at 682 and 785 keV are the most likely candidates. Based on current empirical information, each could have spin 0 or 1. The fact that the 0^- is calculated to have moderately large $l=1$ neutron strength, while the calculated strength of the 1^- is vanishingly small, supports the assignment of 1^- to the 682 keV level and 0^- to the 785 keV level.

Seven states have been isolated between 1.1 and 1.4 MeV, with the 1143, 1268, and 1376 keV levels being assigned even parity. As discussed above, the 1194 and 1255 keV levels could well be the 4_2^- and 3_3^- states given by the calculation, while the 1378 keV $(0–3)^-$ level with $l=1$ neutron strength may be the 1_2^-. This leaves only the 2_2^- state unassigned. The 844 keV $(1–3)^-$ level is a possibility, although the calculated excitation energy would then be in error by 300 keV. Another candidate is the level at 1113 keV, given a $(3,4)^+$ assignment by Endt and Van der Leun.[10] This assignment is based on the fact that a level at about this energy is excited by $l=4$ in the ^{44}Ca(d,α) reaction,[11] but the 1113 keV level observed in ^{41}K(n,γ) decays[12] 50% to the 682 keV (1^-) state so is unlikely to be the state excited in (d,α).

D. ^{43}K

Much of what is known about the spectrum of ^{43}K has come from the ^{44}Ca$(d,{}^3$He) and ^{44}Ca(t,α) reactions.[6,13] Spectroscopic factors extracted in the various experiments are not in good agreement, but the most recent $(d,{}^3$He) work was performed with $E_d=52$ MeV and might be expected to give the most reliable results. The calculated strengths of 3.3 and 1.05 for the ground state and 561 keV $\frac{1}{2}^+$ state are in good agreement with the empirical values of 3.15 and 1.15. The second $\frac{1}{2}^+$ state is placed close to the correct excitation energy of 2.45 MeV, but the calculated $l=0$ strength of 0.5 appears to be about 50% too large.

The model spectrum appears to support $\frac{5}{2}^+$ and $\frac{3}{2}^+$ assignments to the $l=2$ levels located at 1111 and 1546 keV. The calculated strength of the $\frac{3}{2}{}_2^+$ is 0.45, while the empirical value for the 1546 keV level is about 0.35 if it has spin $\frac{3}{2}$. The 1111 keV level has the unexpectedly large strength of 0.36 if it has spin $\frac{5}{2}$, and it is possible that this and the 1546 keV level are both $\frac{3}{2}^+$ states, with a 6p3h intruder state being responsible for the extra level; the ^{44}Ca spectrum is well known to be strongly perturbed by 6p2h admixtures. The 1205 keV level excited in the (t,α) reaction, and assigned weak $l=2$ strength in one experiment,[13] could then be the $\frac{5}{2}{}_1^+$ state.

The ^{40}Ar$(\alpha,p\gamma)$ reaction[14] has been used recently in an attempt to locate high-spin states in ^{43}K. A $\frac{7}{2}^+$ was located at 1510 keV, very close to the calculated energy of 1.54 MeV for the $\frac{7}{2}{}_1$ state, and other levels observed at 2048, 2509, and 3115 keV could well be the lowest $\frac{9}{2}^+$, $\frac{11}{2}^+$, and $\frac{13}{2}^+$ states, calculated to lie at 2.00, 2.53, and 3.14 MeV. The yrast $\frac{15}{2}^+$, $\frac{17}{2}^+$, at $\frac{19}{2}^+$ levels are predicted to lie at 3.5, 5.2, and 5.8 MeV.

E. ^{44}K

The only level below 1.5 MeV in ^{44}K for which the spin has been determined with any certainty is the ground state, but the present calculation leads to quite confident predictions for the spins of several levels. The 183 keV level, which is excited in the gamma decay of the 1886 keV 1^+ state, is almost certainly the lowest 1^- state. This level is not excited in the ^{44}Ca(t, ^{3}He) reaction,[15] and this is readily understood if the reaction is dominated by a $(t,\alpha)(\alpha,{}^3\text{He})$ two-step process; the 1^- would then be excited only through its $f^4_{7/2}(0)p_{3/2}d^{-1}_{3/2}$ component, and although the calculation gives $p_{3/2}$ components which sum to 20% this particular component is only about 0.2% of the wave function. Over 90% of the wave function is a $d_{3/2}$ hole coupled to the lowest $\frac{5}{2}^-$ of ^{45}Ca.

The 383, 520, 812, and 969 keV levels are excited in the ^{44}Ca(t, ^{3}He) reaction, and appear to be the 3^-_1, 4^-_1, 5^-_1, and 4^-_2 states. The 3^- and 4^- states have large components of both $f^5_{7/2}(\frac{7}{2})d^{-1}_{3/2}$ and $f^5_{7/2}(\frac{7}{2})s^{-1}_{1/2}$, each of which can contribute to the (t, ^{3}He) strength. The 1051 and 1077 keV levels observed[16] following the β decay of ^{44}Ar may be the 3^-_2 and 2^-_2 states. The latter, but not the former, state is fed by gamma decay of the 1886 keV 1^+ level.

The ^{48}Ca(p, αn) reaction[17] has been used recently to isolate states at 1013, 1241, and 1368 keV. Their gamma decays suggest fairly high spins, for none of them decays to either the 2^- ground state or the 183 keV (1^-) level. Although this is purely conjecture without further experimental work, it is possible that these states are 4^+, 5^+, and 6^+ levels of 6p2h structure, arising from the coupling of two proton holes to the low-lying states of ^{46}Sc. A $^{46}\text{Sc}\times{}^{38}\text{Ar}$ weak-coupling model gives such states at about 2 MeV if Bansal and French[18] parameters are used for the particle-hole interaction (or close to 3 MeV if those of Zamick[19] are chosen), but it is probable that a treatment including $s_{1/2}$ holes would lower this energy because the centroid of the $f_{7/2}s^{-1}_{1/2}$ neutron-proton interaction is over 200 keV less repulsive than that of $f_{7/2}d^{-1}_{3/2}$.

Odd parity yrast levels with spins 6 to 9 are predicted to lie at 2.2, 2.6, 3.6, and 4.1 MeV.

F. ^{45}K

Most information on the ^{45}K spectrum comes from the β decay of ^{45}Ar and the ^{46}Ca(t, α) reaction.[20,13] In the latter reaction the $\frac{3}{2}^+$ ground state and $\frac{1}{2}^+$ first excited state are clearly excited by a direct process, with $l=2$ and $l=0$ pickup strengths in the ratio 2.4 : 1. The calculated strengths are 3.3 and 1.35, giving a ratio of 2.5 : 1.

The 1424 keV level has been assigned weak $l=0$ strength in the (t, α) reaction, but the calculation places the second $\frac{1}{2}^+$ at 2.3 MeV. It seems clear from information given in the experimental paper that the evidence for $l=0$ pickup is rather weak, and it may be that the 1424 keV state is actually the second $\frac{3}{2}^+$, which is predicted to have an $l=2$ pickup strength of 0.38. This $\frac{3}{2}{}^+_2$ is calculated to have roughly equal $M1$ branches to the $\frac{3}{2}{}^+_1$ and $\frac{1}{2}{}^+_1$ states, while the 1424 keV level is observed to have branches of 60% and 40%. Evidence from the β decay of ^{45}Ar is consistent with this level having spin $\frac{1}{2}$, $\frac{3}{2}$, or $\frac{5}{2}$.

It seems clear that the $\frac{5}{2}{}^+_1$ is the level at 1020 keV, while the $\frac{7}{2}{}^+_1$ is probably the 1639 keV ($\frac{3}{2}-\frac{7}{2}$) level. This latter state has gamma decay branches to the $\frac{7}{2}^-$ at 1020 keV (15%), the ground state (64%), and the $\frac{5}{2}{}^+_1$ state (21%). Based on the calculated $\frac{7}{2}\to\frac{5}{2}$ $M1$ strength, this requires the $E1$ transition to the $\frac{7}{2}^-$ to be 10^{-3} Weisskopf units (Wu), and the ground state transition to have the reasonable $E2$ strength of 9 Wu.

Yrast levels of spins $\frac{9}{2}^+-\frac{15}{2}^+$ are predicted to lie at 2.25, 2.7, 3.0, and 3.3 MeV. The $\frac{9}{2}\to\frac{7}{2}$ and $\frac{13}{2}\to\frac{11}{2}$ $M1$ transition strengths are calculated to be very weak.

G. ^{46}K

The calculation successfully reproduces the six odd parity levels which lie below 2 MeV in ^{46}K. There is strong mixing of $s_{1/2}$ and $d_{3/2}$ holes in the 3^- and 4^- states, with the major components being

$$|3^-_1\rangle = 0.845\ |f^7_{7/2}d^{-1}_{3/2}\rangle + 0.383\ |f^7_{7/2}s^{-1}_{1/2}\rangle + \cdots,$$

$$|3^-_2\rangle = 0.416\ |f^7_{7/2}d^{-1}_{3/2}\rangle - 0.745\ |f^7_{7/2}s^{-1}_{1/2}\rangle + \cdots,$$

$$|4^-_1\rangle = 0.537\ |f^7_{7/2}d^{-1}_{3/2}\rangle - 0.766\ |f^7_{7/2}s^{-1}_{1/2}\rangle + \cdots,$$

$$|4^-_2\rangle = 0.807\ |f^7_{7/2}d^{-1}_{3/2}\rangle + 0.482\ |f^7_{7/2}s^{-1}_{1/2}\rangle + \cdots.$$

Daehnick *et al.*[21] have used experimental ^{48}Ca(d, α) and ^{48}Ca(p, ^{3}He) cross sections in an attempt to deduce the 3^- and 4^- wave functions, assuming that ^{48}Ca is a good closed-shell nucleus and that the reactions proceed (at least at forward angles) by one-step deuteron transfer. From data given in their paper it is possible to deduce the ratio of $(f_{7/2}s_{1/2})$ to $(f_{7/2}d_{3/2})$ (d, α) transfer amplitudes given by their choice of optical model parameters, and hence to calculate ratios of cross sections for the wave functions listed above. We find that R_4' [the ratio of (d, α) cross sections for the two 4^- states, corrected for Q dependence] is 20, while R_3' is 3.3. The former result is in good agreement with experiment, since the 4^-_2 state is very weakly excited. The calculated ratio for 3^- states lies between the values observed for 17 and 80 MeV deuterons.

H. ^{47}K

Very little is known about the ^{47}K spectrum, except that there is a probable $\frac{3}{2}^+$ state lying 360 keV above the $\frac{1}{2}^+$ ground state. The calculation reproduces these two levels, giving $l=2$ and $l=0$ proton pickup strengths of 3.9 and 1.59. The strength ratio of 2.45 is midway between the ratios found in the ^{48}Ca(d, ^{3}He) and ^{48}Ca(t, α) reactions.[22,13] Although the ground state is mainly an $s_{1/2}$ hole it has a sizable component of a $d_{3/2}$ hole coupled to the 2^+ state of ^{48}Ca, and 0.4 of the $l=0$ strength is predicted to lie in a $\frac{1}{2}^+$ state at 5 MeV.

Above the lowest $\frac{3}{2}^+$, the calculated spectrum has a large energy gap before there is a $\frac{5}{2}^+$ state at 3.8 MeV, with $\frac{7}{2}^+$, $\frac{3}{2}^+$, and second $\frac{5}{2}^+$ states at 3.85, 4.1, and 4.4 MeV. The two $\frac{5}{2}^+$ states, perturbed by $d_{5/2}$ hole admixtures, could be the levels excited with $l=2$ in the ^{48}Ca(d, ^{3}He) reaction[23] at close to 4 MeV.

IV. MAGNETIC MOMENTS AND $M1$ TRANSITIONS

Within the model space used for the present calculations the magnetic dipole operator is specified by just five effective single-particle matrix elements. These are the diagonal $f_{7/2}$ and $p_{3/2}$ matrix elements for neutrons, the diagonal $d_{3/2}$ and $s_{1/2}$ matrix elements for proton holes, and the l-forbidden matrix element linking $s_{1/2}$ with $d_{3/2}$. Values for three of these matrix elements are readily obtained; the magnetic moments of ^{41}Ca and ^{39}K give

$$M_{ff}=(f_{7/2}\,||\,\mu\,||\,f_{7/2})=-5.11\mu_{\rm N}\,,$$

$$M_{dd}=(d_{3/2}^{-1}\,||\,\mu\,||\,d_{3/2}^{-1})=1.01\mu_{\rm N}\,,$$

and if the 0^- (2626 keV) and 1^- (2104 keV) states of ^{40}K are assumed to be pure $p_{3/2}d_{3/2}^{-1}$ (see Sec. III) the $0^-\to1^-$ reduced $M1$ transition rate of 470 ± 40 mW.u. then gives

$$M_{pp}=(p_{3/2}\,||\,\mu\,||\,p_{3/2})=-2.74\pm0.18\mu_{\rm N}$$

or

$$+4.75\pm0.18\mu_{\rm N}\,.$$

The former value is chosen, because it is closer to the Schmidt value of $-4.94\mu_{\rm N}$ and to the effective value of $-2.33\mu_{\rm N}$ found for $N=29$ nuclei.[24]

The *magnitude* of the matrix element connecting $d_{3/2}$ and $s_{1/2}$ holes can be deduced from the $M1$ lifetime of the lowest $\frac{1}{2}^+$ state in ^{39}K, which gives

$$M_{ds}=(d_{3/2}^{-1}\,||\,\mu\,||\,s_{1/2}^{-1})=\pm0.5\pm0.1\mu_{\rm N}\,.$$

The $E2/M1$ mixing ratio has an unknown sign and cannot be used to determine the phase of M_{ds}. The magnetic moments of ^{29}P and ^{31}P provide estimates:

$$M_{ss}=(s_{1/2}^{-1}\,||\,\mu\,||\,s_{1/2}^{-1})=\pm3.02\mu_{\rm N}\quad(^{29}\text{P})$$

$$=+2.77\mu_{\rm N}\quad(^{31}\text{P})\,,$$

but since these nuclei have cores which are different from the ^{40}Ca core of potassium isotopes these values may not be appropriate. For the present calculations M_{ss} was set equal to $3\mu_{\rm N}$, but to determine the sensitivity of the results to this matrix element the calculations were repeated using the Schmidt value of $6.8\mu_{\rm N}$.

With experimental data currently available it is frustratingly difficult to fix either the sign of M_{ds} or a well-defined value for M_{ss}. Table IV shows a comparison of experimental[10] and calculated magnetic moments. The first column of calculated values is for a negative sign for M_{ds}, the second column is for a positive sign. Although agreement with experiment is better in all cases with the negative sign, the differences are not sufficiently great to justify a sign selection. Changes with the Schmidt value for M_{ss} are small, being $-0.033\mu_{\rm N}$ in the case of ^{40}K and less than $0.01\mu_{\rm N}$ for the other nuclei. Table V lists $M1$ transition rates in ^{40}K, ^{41}K, and ^{42}K, and again the data is not very helpful in determining the sign of M_{ds}. The 4^-(2397 keV) $\to 3_1^-$ transition in ^{40}K is the only reasonably strong transition which seems to clearly favor a particular sign, but this transition rate is expecially sensitive to M_{ss} and increasing this to $5\mu_{\rm N}$ would bring the calculated value with positive M_{ds} into agreement with experiment. It is obviously not valid to use weak transitions as a basis for parameter fixing, since even small admixtures of states lying outside the model space could give corrections of the same order of magnitude.

Schreider *et al.*[25] have considered the lifetime of the lowest $\frac{1}{2}^+$ state in ^{41}K and used this to predict a positive sign for M_{ds}. The $\frac{1}{2}^+\to\frac{3}{2}^+$ transition rate is indeed very sensitive to the sign of M_{ds}, as are the analagous transition rates in ^{43}K and ^{45}K, but there is controversy in the literature as to the correct lifetime of the $\frac{1}{2}^+$ state. Lister *et al.*[26] have defended their lifetime of 5 ps [which gives $B(M1)=5\pm2$ mW.u. when combined with the measured $B(E2\uparrow)$ value], even though there are

TABLE IV. Magnetic moments calculated with (A) M_{ds} negative and (B) M_{ds} positive.

	Exp	Calc (A)	Calc (B)
^{40}K(4^-)	−1.298	−1.261	−1.235
^{40}K(3^-)	−1.29(9)	−1.332	−1.459
^{41}K($\frac{3}{2}^+$)	+0.215	0.259	0.281
^{42}K(2^-)	−1.143	−1.224	−1.263
^{43}K($\frac{3}{2}^+$)	±0.163	0.229	0.249
^{44}K(2^-)		−1.381	−1.387
^{45}K($\frac{3}{2}^+$)	±0.173	0.244	0.256

TABLE V. Reduced $M1$ transition rates (mW.u.) calculated with (A) M_{ds} negative and (B) M_{ds} positive.

Nucleus	Transition	Calc (A)	Calc (B)	Exp
^{40}K	$2_1 \rightarrow 3_1$	150	130	185 ± 25
	$5_1 \rightarrow 4_1$	73	69	40 ± 4
	$2_2 \rightarrow 3_1$	10	9	2.4 ± 0.2
	$\rightarrow 2_1$	11	11	13 ± 1
	$3_2 \rightarrow 4_1$	9	0.0	2.2 ± 0.5
	$\rightarrow 3_1$	0.2	15	2.9 ± 0.6
	$\rightarrow 2_1$	6	1.0	1.7 ± 0.4
	$1_1 \rightarrow 2_1$	10	10	5 ± 1
	$4_2 \rightarrow 4_1$	3	7	1.9 ± 1.5 or 13 ± 6 [a]
	$\rightarrow 3_1$	29	8	32 ± 13
	$0_1 \rightarrow 1_1$	470	470	470 ± 40 [b]
^{41}K	$\frac{1}{2}_1 \rightarrow \frac{3}{2}_1$	2.0	42	5 ± 2 or 75 ± 20
	$\frac{3}{2}_2 \rightarrow \frac{3}{2}_1$	13	11	11 ± 2
	$\rightarrow \frac{1}{2}_1$	63	41	48 ± 9
	$\frac{5}{2}_1 \rightarrow \frac{3}{2}_1$	7	5	0.5 ± 0.2
	$\frac{7}{2}_2 \rightarrow \frac{7}{2}_1$	18	16	100 ± 60
	$\frac{9}{2}_1 \rightarrow \frac{7}{2}_1$	11	8	<5.5
	$\frac{13}{2}_1 \rightarrow \frac{11}{2}_1$	88	94	31 ± 1
^{42}K	$3_1 \rightarrow 2_1$	77	87	63 ± 9
	$4_1 \rightarrow 3_1$	46	46	47 ± 3
	$5_1 \rightarrow 4_1$	65	72	6 ± 1 or >18

[a] $\delta = -2.4 \pm 0.5$ or 0.32 ± 0.12.
[b] Used to determine the effective matrix element M_{pp}.

other measurements which give a lifetime of about 0.5 ps and $B(M1) = 75 \pm 20$ mW. u.]. This transition is totally insensitive to the value of M_{ss}, so if agreement can be reached on the lifetime it would appear to allow an unambiguous choice of sign.

The largest discrepancies between calculated and empirical transition rates are for the $\frac{7}{2}_2 \rightarrow \frac{7}{2}_1$ and $\frac{13}{2}_1 \rightarrow \frac{11}{2}_1$ transitions in ^{41}K and the $5_1 \rightarrow 4_1$ transition in ^{42}K. The calculated value for the first of these may not be as bad as it appears, since there has been only one experimental measurement and this has a large uncertainty. The lifetime of the 5^- in ^{42}K comes from analysis of a gamma cascade following heavy-ion excitation,[27] and the 59 ps delay observed could be due to some higher-lying level; in support of this interpretation is the other measurement[28] of < 20 ps for the 5^- lifetime. The lowest $\frac{13}{2}^+$ in ^{41}K is calculated to be almost entirely a $d_{3/2}$ hole coupled to the 6^+ state of ^{42}Ca, while the lowest $\frac{11}{2}^+$ has large components of a $d_{3/2}$ hole coupled to both the 6^+ and 4^+ states. The empirical $\frac{13}{2} \rightarrow \frac{11}{2}$ transition rate appears to require the lowest $\frac{11}{2}^+$ state to have a $f_{7/2}^2(6)d_{3/2}^{-1}$ component of only 25% rather than the 74% given by the calculation, but since the second $\frac{11}{2}^+$ (which is mainly 4^+ coupled) lies only 500 keV higher it is clear that this would require only minor changes to the interaction.

Lifetime measurements for ^{46}K would be useful for determining both the sign of M_{ds} and the magnitude of M_{ss}, because even the lowest 3^- and 4^- states have large $s_{1/2}^{-1}$ components. The decay scheme of the 3_2^- state at 1370 keV would be especially informative. The $3_2 \rightarrow 4_1$ transition rate is calculated to be large and to depend strongly on M_{ss}, $B(M1)$ being 0.25 W. u. if $M_{ss} = 3\mu_N$ and 1.0 W. u. if M_{ss} has the Schmidt value; the sign of M_{ds} has less than a 10% effect. On the other hand the

$3_2 \rightarrow$ ground state transition rate does not depend on M_{ss} but is sensitive to the sign of M_{ds}; $B(M1)$ is 36 mW.u. and 6 mW.u. for negative and positive sign, respectively.

V. SUMMARY

The model space chosen for the present calculations, spanning the $f^n_{7/2}(d_{3/2}s_{1/2})^{-1}$ and $f^{n-1}_{7/2}p_{3/2}d^{-1}_{3/2}$ configurations, appears to be sufficiently large to give spectra in good agreement with experiment for the $A=39-47$ potassium isotopes. Although there is some evidence for a few three-hole intruder states, it appears that these are not as important as might have been expected *a priori*, and almost all known levels of parity $(-1)^n$ lying below 2 MeV can be reproduced with a suitable effective interaction. Spin predictions can be made for levels in the higher-mass isotopes.

Further information about the structure of these states which could be gained from $M1$ transition rates is rather limited at present, mainly due to the scarcity of experimental data for $A > 42$. Certain data which would be useful in fixing matrix elements of the effective magnetic dipole operator have been pointed out.

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