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Equations-of-motion method and pairing correlations in odd-A nuclei

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An equations-of-motion method is presented for treating pairing correlations in odd-A nuclei. This method is the natural extension of a previous study of the pairing problem in doubly even nuclei, which provides the input data for the present work. It is shown that the spurious states arising from the use of an overcomplete set of basis vectors can be removed through a procedure analogous to that developed for the doubly even case. The energies, spectroscopic factors, and occupation probabilities of seniority-one states in the Ni isotopes are calculated in the first-order theory. By comparison with exact results, the accuracy of the method is seen to be very good.

[NUCLEAR STRUCTURE Equations-of-motion method in pairing-force theory.]

I. INTRODUCTION

The problem of treating pairing correlations both in even and odd nuclei by means of the equations-of-motion method has been investigated by various authors. In this context, the full treatment of the equations of motion for single-fermion operators has led to methods[1-3] involving iteration across even and odd nuclei simultaneously. As a consequence, this kind of approach is quite involved in application. By contrast, it has been shown[4-6] that the use of the equations of motion for pair creation operators leads to a simple theory which involves iteration only across even nuclei.

In a previous paper,[7] hereafter referred to as I, we presented a new formulation of this theory, which permits one to treat pairing correlations in systems of even numbers of identical nucleons with a limited amount of computational labor. The main point of I is that the spurious states arising from the use of an overcomplete set of basis vectors are identified and removed at each step of the iteration procedure. Applications of the theory at various orders of approximation[7,8] gave very accurate energies, occupation numbers, and two-particle transfer amplitudes in the test case of the even Ni isotopes.

In this paper we present a treatment of pairing correlations in odd nuclei, which is a straightforward extension of the method described in I. More precisely, we solve the equations of motion for an odd particle making use of the solution of the doubly even problem. The elimination of the spurious states is carried out through a procedure similar to that developed in I. The accuracy of the method, as tested by calculating the energies and spectroscopic factors of seniority-one states for the odd Ni isotopes, turns out to be excellent even at the lowest order of approximation (first-order theory).

II. FORMULATION OF THE THEORY

A. The equations of motion

We consider the pairing Hamiltonian

$$H=\sum_j \epsilon_j \hat{N}_j - \sum_{jj'} G_{jj'} A_j^\dagger A_{j'}, \tag{1}$$

where

$$\hat{N}_j = \sum_m a_{jm}^\dagger a_{jm}, \tag{2}$$

$$A_j^\dagger = \sum_{m>0} a_{jm}^\dagger a_{j\bar{m}}^\dagger, \tag{3}$$

and

$$a_{j\bar{m}}^\dagger = (-)^{j+m} a_{j-m}^\dagger .$$

We shall consider, in the following, states of seniority one in odd nuclei. The wave function for a system of an odd number $(N+1)$ of identical particles can then be related either to the seniority-zero states of the system with N particles or to the seniority-zero states of the system with $(N+2)$ particles. We have

$$|N+1, pjm\rangle = \sum_\beta c_{pj\beta}(N+1) a_{jm}^\dagger |N,\beta\rangle , \tag{4}$$

and

$$|N+1, pjm\rangle = \sum_\alpha g_{pj\alpha}(N+1) a_{j\bar{m}} |N+2,\alpha\rangle , \tag{5}$$

where β and α specify the states containing N and $(N+2)$ particles, respectively, and p stands for all the quantum numbers of the seniority-one states other than (jm). The coefficients $c_{pj\beta}(N+1)$ and $g_{pj\alpha}(N+1)$ are taken to be real.

The equations of motion for $a_{jm}^\dagger$ and $a_{j\bar{m}}$ are

$$[H, a_{jm}^\dagger] = (\epsilon_j - G_j) a_{jm}^\dagger - \sum_{j'} G_{jj'} a_{j\bar{m}} A_{j'}^\dagger , \tag{6}$$

$$[H, a_{j\bar{m}}] = -\epsilon_j a_{j\bar{m}} - \sum_{j'} G_{jj'} a_{jm}^\dagger A_{j'} , \tag{7}$$

where $G_j \equiv G_{jj}$. Taking matrix elements of (6) and (7) one has[2,3]

$$E_{pj}(N+1)S_{pj\beta}(N+1) = \mathcal{E}_{j\beta}(N)S_{pj\beta}(N+1) + \sum_{\alpha} \Delta_{j\alpha\beta}(N+2)T_{pj\alpha}(N+1), \tag{8}$$

and

$$E_{pj}(N+1)T_{pj\alpha}(N+1) = \mathcal{E}_{j\alpha}(N+2)T_{pj\alpha}(N+1) + \sum_{\beta}\Delta_{j\alpha\beta}(N+2)S_{pj\beta}(N+1), \tag{9}$$

with

$$\mathcal{E}_{j\beta}(N) = \epsilon_j + E_{\beta}(N) - G_j, \tag{10}$$

$$\mathcal{E}_{j\alpha}(N+2) = E_{\alpha}(N+2) - \epsilon_j, \tag{11}$$

$$\Delta_{j\alpha\beta}(N+2) = -\sum_{j'} G_{jj'}X_{j'\alpha\beta}(N+2), \tag{12}$$

where

$$X_{j\alpha\beta}(N+2) = \langle N+2, \alpha | A_j^{\dagger} | N, \beta\rangle. \tag{13}$$

The spectroscopic amplitudes $S_{pj\beta}(N+1)$ and $T_{pj\alpha}(N+1)$ are defined as

$$S_{pj\beta}(N+1) = \langle N+1, pjm | a_{jm}^{\dagger} | N, \beta\rangle, \tag{14}$$

$$T_{pj\alpha}(N+1) = \langle N+1, pjm | a_{jm}^{-} | N+2, \alpha\rangle. \tag{15}$$

Of course, when the sums on α and β run over all existing seniority-zero states, either Eq. (8) or Eq. (9) permits one to solve the problem exactly. It should be noted, however, that different numbers of core states (n_α, n_β) are involved in these two equations. As a consequence, when approximations are made, one should use Eq. (9) in conjunction with the wave function (4) for $1 \leq (N+1) \leq \Omega$ (where $\Omega = \sum_j \Omega_j$ is the total pair degeneracy of the system) and Eq. (8) in conjunction with (5) for $\Omega \leq (N+1) \leq 2\Omega - 1$. This is not quite true, however, when the core states are restricted to only one state. Indeed, this case requires a special discussion which will be given in the next section.

In this section we shall present the formulation of the theory which is based on the use of Eqs. (4) and (9). In other words, we shall consider explicitly systems in which the number of particles outside closed shells is not larger than half of the total number of available states. The formalism for systems with holes inside closed shells can be readily derived along the same lines.[8]

The one-particle transfer amplitudes $S_{pj\beta}(N+1)$ and $T_{pj\alpha}(N+1)$ are related to the coefficients $c_{pj\beta}(N+1)$ of (4) through

$$S_{pj\beta}(N+1) = \sum_{\beta'} c_{pj\beta'}(N+1)R_{j\beta'\beta}(N), \tag{16}$$

$$T_{pj\alpha}(N+1) = -\frac{1}{\Omega_j}\sum_{\beta} c_{pj\beta}(N+1)X_{j\alpha\beta}(N+2), \tag{17}$$

where

$$R_{j\beta'\beta}(N) = \langle N, \beta' | a_{jm}a_{jm}^{\dagger} | N, \beta\rangle = \delta_{\beta\beta'} - \rho_{j\beta'\beta}(N) \tag{18}$$

are the elements of the metric matrix of the $(N+1)$-particle states $a_{jm}^{\dagger}|N,\beta\rangle$. From the normalization condition $\langle N+1, pjm | N+1, pjm\rangle = 1$ it follows that

$$\sum_{\beta} c_{pj\beta}(N+1)S_{pj\beta}(N+1) = 1. \tag{19}$$

Inverting any set of n_β linearly independent equations extracted from the n_α $(n_\alpha \geq n_\beta)$ relations (17), we obtain

$$c_{pj\beta}(N+1) = -\Omega_j \sum_{[\alpha']} \chi_{j\alpha'\beta}(N+2)T_{pj\alpha'}(N+1), \tag{20}$$

where $\chi_{j\alpha'\beta}$ is the cofactor of $X_{j\alpha'\beta}$ divided by $\det X$, and the notation $[\alpha']$ indicates a sum restricted to n_β values. Substitution of expression (20) for $c_{pj\beta}$ in (16) yields the following relation between the amplitudes $S_{pj\beta}$ and $T_{pj\alpha}$:

$$S_{pj\beta}(N+1) = \sum_{[\alpha']} K_{j\alpha'\beta}T_{pj\alpha'}(N+1), \tag{21}$$

with

$$K_{j\alpha'\beta} = -\Omega_j \sum_{\beta'} \chi_{j\alpha'\beta'}(N+2)R_{j\beta'\beta}(N). \tag{22}$$

Inserting now (22) into (9), we obtain the eigenvalue equation

$$\sum_{[\alpha']} M_{j\alpha\alpha'}T_{pj\alpha'}(N+1) = E_{pj}(N+1)T_{pj\alpha}(N+1), \tag{23}$$

with

$$M_{j\alpha\alpha'} = \mathcal{E}_{j\alpha}(N+2)\delta_{\alpha\alpha'} + \sum_{\beta}\Delta_{j\alpha\beta}(N+2)K_{j\alpha'\beta}. \tag{24}$$

The single-particle occupation probabilities $v_{pj}^2(N+1)$, defined by

$$v_{pj}^2(N+1) = \frac{1}{2\Omega_j}\langle N+1, pjm | \hat{N}_j | N+1, pjm\rangle, \tag{25}$$

are obtained from the relation

$$v_{pj}^2(N+1) = \frac{1}{2\Omega_j} + \sum_{\beta\beta'} c_{pj\beta}(N+1) \times S_{pj\beta'}(N+1)\rho_{j\beta'\beta}(N), \tag{26}$$

which can be readily derived by making use of the expansion (4).

From the above it is clear that the solution for the odd nuclei can be found in a very simple way using as input the results for the doubly even problem.[7]

B. Elimination of the spurious states

The use of an overcomplete set of basis vectors $a^\dagger_{jm}|N,\beta\rangle$ in (4) gives rise to spurious states in the problem of odd nuclei. Here we present a procedure to resolve the above difficulty, which is quite close to that developed in I for the case of doubly even nuclei. We start by diagonalizing the $n_\beta \times n_\beta$ metric matrix (18) for each value of the angular momentum j (for simplicity we neglect the possibility of changes in the principal quantum number). The overcompleteness of the set of basis vectors implies that some eigenvalues must vanish. This entails (we employ the abbreviation $|j\beta\rangle \equiv a^\dagger_{jm}|N,\beta\rangle$)

$$\sum_\beta b^{(i)}_{j\beta}|j\beta\rangle = 0\,, \quad i = 1 \text{ to } n_s\,, \tag{27}$$

where the $b^{(i)}_{j\beta}$ are the coefficients of the decomposition of ith eigenvector of vanishing norm in terms of the overcomplete set $|j\beta\rangle$, and n_s is the number of spurious states. Making use of the n_s relations (27), it is a simple matter to determine the coefficients of the expansion of the n_s spurious vectors $|(j\beta)_s\rangle$ in terms of the n_g good vectors $|(j\beta)_g\rangle$,

$$|(j\beta)_s\rangle = \sum_{(j\beta)_g} k_{(j\beta)_s(j\beta)_g}\,|(j\beta)_g\rangle\,. \tag{28}$$

Then each of the n_β vectors $|j\beta\rangle$ may be written as

$$|j\beta\rangle = \sum_{\beta'} \xi_{j\beta\beta'}|j\beta'\rangle\,, \tag{29}$$

where the $n_\beta \times n_\beta$ matrix ξ is defined by

$$\xi_{j\beta\beta'} = \begin{cases} \delta_{\beta\beta'} & \text{if } |j\beta\rangle \text{ is nonspurious}\,, \\ k_{j\beta j\beta'} & \text{if } |j\beta\rangle \text{ is spurious and } |j\beta'\rangle \\ & \quad \text{nonspurious}\,, \\ 0 & \text{if both } |j\beta\rangle \text{ and } |j\beta'\rangle \text{ are spurious}\,. \end{cases} \tag{30}$$

Assuming now that the pairs of indices $(j\beta)$, $(j\beta')$ refer to nonspurious states and using Eqs. (16), (17), and (29), we obtain

$$S_{pj\beta}(N+1) = \sum_{\beta'} \bar{c}_{pj\beta'}(N+1)R_{j\beta'\beta}(N)\,, \tag{31}$$

$$T_{pj\alpha}(N+1) = -\frac{1}{\Omega_j}\sum_{\beta'} \bar{c}_{pj\beta'}(N+1)X_{j\alpha\beta'}(N+2)\,, \tag{32}$$

where

$$\bar{c}_{pj\beta'}(N+1) = \sum_{\beta''} c_{pj\beta''}(N+1)\xi_{j\beta''\beta'}\,. \tag{33}$$

Elimination of the coefficients $\bar{c}_{pj\beta'}$ from (31) and (32) yields

$$S_{pj\beta}(N+1) = \sum_{\{\alpha'\}} K_{j\alpha'\beta}T_{pj\alpha'}(N+1)\,, \tag{34}$$

where $K_{j\alpha'\beta}$ is defined in Eq. (22), and the notation $\{\alpha'\}$ indicates a sum restricted to n_g values.

Finally, the eigenvalue problem (23) can be reformulated as follows:

$$\sum_{\{\alpha'\}} \bar{M}_{j\alpha\alpha'}T_{pj\alpha'}(N+1) = E_{pj}(N+1)\boldsymbol{T}_{pj\alpha}(N+1)\,, \tag{35}$$

where $\bar{M}$ is the $n_g \times n_g$ matrix

$$\bar{M}_{j\alpha\alpha'} = \mathcal{E}_{j\alpha}(N+2)\delta_{\alpha\alpha'} + \sum_{\beta''\beta'''} \Delta_{j\alpha\beta''}(N+2)K_{j\alpha'\beta'''}\xi_{j\beta''\beta'''}\,. \tag{36}$$

Although $\bar{M}$ is not symmetric, it can be shown[8] that all solutions of Eq. (35) have real energies.

It should be noted that once the coefficients $\bar{c}_{pj\beta'}(N+1)$ have been determined, Eq. (31) gives only n_g amplitudes $S_{pj\beta}(N+1)$. The remaining $(n_\beta - n_g)$ amplitudes can be obtained from the relation

$$S_{pj\beta''}(N+1) = \sum_{\beta'''} \xi_{j\beta''\beta'''}S_{pj\beta'''}(N+1)\,. \tag{37}$$

III. FIRST-ORDER THEORY AND ITS APPLICATION TO Ni ISOTOPES

We now turn our attention to the first-order theory, wherein the cores states are restricted to one state. In this case the procedure described in Sec. II is greatly simplified and the equations of motion are trivially solvable.

The wave function $|N+1, jm\rangle$ can be written in two ways, either as

$$|N+1, jm\rangle = c_j(N+1)a^\dagger_{jm}|N\rangle\,, \tag{38}$$

or as

$$|N+1, jm\rangle = g_j(N+1)a_{j\bar{m}}|\boldsymbol{N}+2\rangle\,, \tag{39}$$

where $|\boldsymbol{N}\rangle$ and $|N+2\rangle$ are the ground states of the $\boldsymbol{N}$- and $(N+2)$-particle systems, respectively (from now on we drop the quantum numbers α, β, and p). Since the two vectors $a^\dagger_{jm}|N\rangle$ and $a_{j\bar{m}}|N+2\rangle$ correspond (approximately) to only a single state, it follows that one eigenvalue of the metric matrix,

$$\mathcal{R} = \begin{pmatrix} 1-\rho_j(N) & -\dfrac{X_j(N+2)}{\Omega_j} \\ -\dfrac{X_j(N+2)}{\Omega_j} & \rho_j(N+2) \end{pmatrix}, \tag{40}$$

is $\simeq 0$. Thus, in a good approximation, we have

$$b_j a^{\dagger}_{jm}|N\rangle + f_j a_{j\bar{m}}|N+2\rangle = 0\,, \tag{41}$$

from which we obtain

$$S_j(N+1) = k_j(N+1)T_j(N+1)\,, \tag{42}$$

where

$$k_j(N+1) = -\frac{f_j}{b_j} = \frac{\mathcal{R}_{11}}{\mathcal{R}_{12}} = -\frac{\Omega_j[1-\rho_j(N)]}{X_j(N+2)}. \tag{43}$$

Then, by use of the relation (42), the eigenvalue equation (23) reduces to

$$E_j(N+1) = \mathcal{E}_j(N+2) + k_j(N+1)\Delta_j(N+2)\,. \tag{44}$$

Now the question arises as to which one of the two wave functions (38) and (39) represents a better approximation to the low-lying states of the $(N+1)$-particle system. Clearly, neither of them takes account of the blocking effect of the pairing force. It is, however, easy to see that the spectroscopic amplitudes $S_j(N+1)$, $T_j(N+1)$ and the occupation probabilities $v_j^2(N+1)$ are not insensitive to the choice (38) or (39). Therefore we give a simple criterion of how to choose the wave function $|N+1, jm\rangle$ which leads to the best results for the above quantities. One chooses as core basis state the vector which has the smaller weight in (41), namely $a^{\dagger}_{jm}|N\rangle$ if $|k_j|>1$, and $a_{j\bar{m}}|N+2\rangle$ if $|k_j|<1$. The reason is that this vector is the one which is less affected by the lack of antisymmetrization between the core and the extra particle.

We first consider the case $|k_j|>1$. Then from Eq. (38) it follows that

$$S_j(N+1) = c_j(N+1)[1-\rho_j(N)]\,, \tag{45}$$

$$T_j(N+1) = -c_j(N+1)\frac{X_j(N+2)}{\Omega_j}. \tag{46}$$

Making use of the normalization condition,

$$c_j(N+1)S_j(N+1) = 1\,, \tag{47}$$

we immediately find that

$$S_j^2(N+1) = 1-\rho_j(N)\,, \tag{48}$$

$$T_j^2(N+1) = \frac{X_j^2(N+2)}{\Omega_j^2[1-\rho_j(N)]}\,. \tag{49}$$

Note that Eqs. (45) and (46) imply a relation between the $S_j(N+1)$ and $T_j(N+1)$ which is just the same as (42). The occupation probabilities are obtained by using Eq. (26) which reduces to

$$v_j^2(N+1) = \rho_j(N) + \frac{1}{2\Omega_j}\,. \tag{50}$$

We turn next to the case $|k_j|<1$. Then the choice of the wave function (39) leads to

$$T_j(N+1) = g_j(N+1)\rho_j(N+2)\,, \tag{51}$$

$$S_j(N+1) = -g_j(N+1)\frac{X_j(N+2)}{\Omega_j}\,, \tag{52}$$

and the normalization condition becomes

$$g_j(N+1)T_j(N+1) = 1\,. \tag{53}$$

From Eqs. (51), (52), and (53) we obtain

$$T_j^2(N+1) = \rho_j(N+2)\,, \tag{54}$$

$$S_j^2(N+1) = \frac{X_j^2(N+2)}{\Omega_j^2\rho_j(N+2)}\,. \tag{55}$$

Using Eq. (39), we find for the occupation probabilities

$$v_j^2(N+1) = \rho_j(N+2) - \frac{1}{2\Omega_j}\,. \tag{56}$$

In contrast to the previous case, Eqs. (51) and (52) yield a relation between the amplitudes $S_j(N+1)$ and $T_j(N+1)$ which is different from (42). This is because the choice of the wave function (38) or (39) implies a different approximation for these quantities. Actually, it can be easily verified

TABLE I. Values of the energies E_j $(N+1)$ of the various seniority-one states for the Ni isotopes (MeV). The columns labeled P.W. are the results of the present work.

j \ $N+1$	3			5			7			9			11		
	Exact	BCS	P.W.	Exact	BCS	P.W.	Exact	BCS	P.W.	Exact	BCS	P.W.	Exact	BCS	P.W.
$\frac{3}{2}$	−0.93	−0.48	−0.86	−0.74	−0.21	−0.68	0.34	0.90	0.37	2.34	3.14	2.32	5.55	6.19	5.57
$\frac{5}{2}$	−0.47	−0.14	−0.50	−0.83	−0.20	−0.83	−0.07	0.63	−0.11	1.69	2.62	1.62	4.80	5.50	4.71
$\frac{1}{2}$	0.23	0.48	0.22	−0.23	0.23	−0.20	0.30	0.83	0.32	1.73	2.47	1.65	4.07	4.96	4.22
$\frac{9}{2}$	3.10	3.28	3.10	2.55	2.90	2.60	2.95	3.33	3.01	4.23	4.69	4.29	6.43	6.70	6.48

TABLE II. Values of the spectroscopic factors $S_j^2(N+1)$ for the odd Ni isotopes.

j \ $N+1$	3		5		7		9		11	
	Exact	P.W.	Exact	P.W.	Exact	P.W.	Exact	P.W.	Exact	P.W.
$\frac{3}{2}$	0.644	0.657	0.366	0.339	0.229	0.197	0.132	0.120	0.057	0.041
$\frac{5}{2}$	0.917	0.918	0.795	0.787	0.589	0.666	0.356	0.397	0.132	0.099
$\frac{1}{2}$	0.964	0.964	0.916	0.910	0.845	0.831	0.745	0.722	0.584	0.402
$\frac{9}{2}$	0.993	0.993	0.986	0.984	0.979	0.973	0.972	0.960	0.968	0.943

that the difference

$$\delta k_j(N+1) = \left| \frac{X_j(N+2)}{\Omega_j \rho_j(N+2)} - \frac{\Omega_j[1-\rho_j(N)]}{X_j(N+2)} \right|, \quad (57)$$

which is exactly equal to zero when $N=0$, tends to increase with N. Clearly the first-order theory is exact for $N+1=1$, since in this case there is only one core state, i.e., the vacuum $|0\rangle$.

At this point it is worth mentioning that a wave function for the low-lying states of an odd nucleus constructed by adding a particle and a hole to the ground states of the adjacent doubly even nuclei has been proposed in Ref. 1. The above discussion, however, makes it clear that the explicit use of such a wave function is neither necessary nor useful, as it would only give rise to spurious effects which are not inherent in our equations-of-motion approach.

As an illustration and application of our theory, we calculate the energies, spectroscopic factors, and occupation probabilities for the odd Ni isotopes using as input data the results of I for the even isotopes (the single-particle energies and the coupling strength G are the same as those used in Ref. 9). In Tables I–IV we compare the results obtained from the first-order theory with the exact ones; in Table I we also give the Bardeen-Cooper-Schrieffer (BCS) energies.[9] It appears that our first-order energies are remarkably close to the exact ones and much better than those obtained by the usual quasiparticle approximation. From Tables II and III, we see that very good results are also achieved in first order for the one-particle transfer intensities $S_j^2(N+1)$ and $T_j^2(N+1)$. It can be easily verified that they are superior to the BCS ones.

Concerning the occupation probabilities, Table IV shows that they are not overall as good as the energies and spectroscopic factors. This is not surprising, as these quantities are most sensitive to the blocking effect. It should be noted, however, that this effect is minimized by our choice of the wave function $|N+1, jm\rangle$. Better results are easily obtained from higher-order applications of the theory.[8]

IV. CONCLUSIONS

We have presented an equations-of-motion method to treat pairing correlations in odd-A nuclei conserving the number of particles. This method takes full advantage of the possibility of finding solutions to the pairing problem completely within the subspace of even nuclei.[7] The results obtained from the first-order theory show that our procedure provides a numerically simple and remarkably accurate treatment of states of seniority one.

Some of the advantages of the equations-of-motion method over the usual BCS treatment have been stressed in I. Here we want to remark that our first-order theory leads consistently to results which are vastly improved over the BCS ones for

TABLE III. Values of the spectroscopic factors $T_j^2(N+1)$ for the odd Ni isotopes.

j \ $N+1$	3		5		7		9		11	
	Exact	P.W.	Exact	P.W.	Exact	P.W.	Exact	P.W.	Exact	P.W.
$\frac{3}{2}$	0.624	0.591	0.763	0.729	0.858	0.812	0.932	0.876	0.981	0.922
$\frac{5}{2}$	0.198	0.220	0.397	0.471	0.627	0.632	0.850	0.832	0.972	0.947
$\frac{1}{2}$	0.081	0.086	0.150	0.161	0.249	0.252	0.405	0.432	0.953	0.919
$\frac{9}{2}$	0.013	0.014	0.020	0.021	0.027	0.026	0.030	0.029	0.030	0.019

TABLE IV. Values of the single-particle occupation probabilities $v_j^2(N+1)$ for the odd Ni isotopes.

j \ $N+1$	3		5		7		9		11	
	Exact	P.W.	Exact	P.W.	Exact	P.W.	Exact	P.W.	Exact	P.W.
$\frac{3}{2}$	0.579	0.593	0.641	0.479	0.685	0.562	0.721	0.626	0.742	0.671
$\frac{5}{2}$	0.218	0.248	0.294	0.380	0.514	0.465	0.747	0.666	0.818	0.780
$\frac{1}{2}$	0.500	0.536	0.500	0.590	0.500	0.669	0.500	0.778	0.500	0.419
$\frac{9}{2}$	0.105	0.107	0.110	0.116	0.116	0.127	0.120	0.141	0.122	0.157

both even and odd nuclei. The results of higher-order calculations will be included in a next paper.[8]

Finally, we wish to point out that our use[7,8,10] of the equations-of-motion method in pairing-force theory indicates its usefulness in treating more complicated problems of nuclear structure. An extension of the theory described in I to include states of angular momentum different from zero is in progress.

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