Pion reactions in deuterium and the muon-capture rate and weak form factors

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The muon-capture rate for the reaction $\mu^- + {}^2\mathrm{H} \to n + n + \nu_\mu$ is calculated by the use of the elementary particle model. The form factors describing the axial current matrix element are determined from the pion photoproduction reaction $\gamma + {}^2\mathrm{H} \to n + n + \pi^+$ via the partially conserved axial vector current hypothesis. The form factors describing the vector current matrix element are obtained from the reactions $\gamma + {}^2\mathrm{H} \to p + n$ and $e + {}^2\mathrm{H} \to p + n + e$. A result of $\Gamma = 155\ \mathrm{sec}^{-1}$ is obtained. In addition a capture rate for the reaction $\pi^- + d \to n + n + \gamma$ of $\Gamma = 3.41 \times 10^{14}\ \mathrm{sec}^{-1}$ is obtained.

[NUCLEAR REACTIONS Muon-capture ${}^2\mathrm{H}(\mu^-, \nu_\mu)nn$ and pion-capture ${}^2\mathrm{H}(\pi^-, \gamma)nn$ rates are calculated using the elementary particle model.]

I. INTRODUCTION

It is of interest to obtain theoretical results for the muon capture process $\mu^- + {}^2\mathrm{H} \to n + n + \nu_\mu$. The deuteron is the simplest of complex nuclei and it is important that theoretical and experimental predictions be in agreement for this case. Furthermore, weak form factors used in obtaining the muon-capture result, namely those describing the charged current matrix elements, can be used to obtain the matrix elements for the neutral weak current. These in turn can be used to obtain[1] cross sections for the neutral current neutrino reactions $\nu_\mu + {}^2\mathrm{H} \to n + p + \nu_\mu$ and $\nu_e + {}^2\mathrm{H} \to n + p + \nu_e$, thus serving as a useful test of the Weinberg-Salam[2] model.

In this calculation we make use of the partially conserved axial current (PCAC) hypothesis[3] and a soft[4] pion approximation to obtain the form factors describing the weak axial vector current matrix element from pion photoproduction data, $\gamma + {}^2\mathrm{H} \to n + n + \pi^+$. The form factors describing the weak vector current matrix element are obtained via the conserved vector current hypothesis from photodisintegration and electrodisintegration data, namely $\gamma + {}^2\mathrm{H} \to p + n$ and $e + {}^2\mathrm{H} \to p + n + e$.

We make use here of the elementary particle model.[5] In this approach the initial and final nuclear states are treated as elementary particles of appropriate spin and parity. Form factors describing the matrix elements are obtained by the use of the above mentioned processes and appropriate SU(2) current commutation relations. The advantage of this treatment over conventional ones is that it avoids the use of nuclear wave functions which are often not well known but to which cross sections are very sensitive.

In Sec. II of this paper we exhibit and discuss the general form of the weak current matrix element and obtain the form factors necessary to describe them. In Sec. III of this paper we calculate the total muon-capture rate and also the total pion-capture rate. In Sec. IV we discuss the results of the calculations presented.

II. THE WEAK CURRENT MATRIX ELEMENT

The matrix element for the muon-capture process $\mu^- + {}^2\mathrm{H} \to n + n + \nu_\mu$ is given to the lowest order in $G(= 1.02 \times 10^{-5} m_p^{-2})$, the weak coupling constant, by

$$\langle n, n, \nu | H_w(0) {}^2\mathrm{H}, \mu^- \rangle = \frac{G}{\sqrt{2}} \cos\theta_c \bar{u}_\nu \gamma^\lambda (1 - \gamma_5) u_\mu \times \langle nn | J_\lambda^\dagger(0) | {}^2\mathrm{H} \rangle . \tag{1}$$

Here θ_c is the Cabbibo angle ($\cos\theta_c = 0.98$), and

$$J^\lambda(0) = V^\lambda(0) - A^\lambda(0) \tag{2}$$

is the weak hadronic current with V^λ and A^λ the vector current and axial current, respectively. Thus the problem of determining the capture rate reduces to obtaining the matrix elements $\langle nn | V_\lambda^\dagger(0) | {}^2\mathrm{H} \rangle$ and $\langle nn | A_\lambda^\dagger(0) | {}^2\mathrm{H} \rangle$.

The form of these matrix elements has been obtained by the author in earlier papers[6-9] and is given by

$$\langle nn | V_\mu^\dagger(0) | {}^2\mathrm{H} \rangle = \eta \bar{u}(p_1) \left(\frac{F_1}{M_d^2} \epsilon_{\mu\nu\rho\sigma} \xi^\nu Q^\rho d^\sigma + \frac{F_2}{M_d^2} \epsilon_{\nu\rho\sigma\mu} \xi^\rho q^\sigma \right) \gamma_5 v(p_2) , \tag{3a}$$

$$\langle nn | A_\mu^\dagger(0) | {}^2\mathrm{H} \rangle = \eta u(p_1) \left(F_A \xi_\mu + F_P \frac{\xi \cdot Q q_\mu}{M_d^2} \right) \gamma_5 v(p_2) , \tag{3b}$$

where $\eta = [m^2/(E_1 E_2)]^{1/2} (2\pi)^{-1/2} (2d_0)^{-1/2}$, m and M_d are the neutron and deuteron masses, respectively, d_μ is the deuteron four-momentum, E_1 and E_2 are the neutron energies, ξ_μ is the deuteron polarization vector, n_{1_μ} and n_{2_μ} are the neutron four momenta, and

$$
\begin{aligned}
Q_\mu &= n_{1_\mu} + n_{2_\mu} , \\
q_\mu &= n_{1_\mu} + n_{2_\mu} - d_\mu , \\
P_\mu &= n_{1_\mu} - n_{2_\mu} .
\end{aligned} \tag{4}
$$

Thus if F_A, F_P, F_1, and F_2 can be determined, $\langle nn | J_\lambda^\dagger(0) | d\rangle$ is entirely determined and the muon-capture rate may be calculated. In particular, F_A[10] dominates the muon- and pion-capture rates and so must be determined as carefully as possible.

We make use of the Kroll-Ruderman theorem[4] and follow the derivation of Griffiths and Kim[11] using PCAC and minimal coupling

$$(\partial_\mu \pm iea_\mu(x))A^{\mu}(x)^{(\pm)} = \sqrt{2}\, m_\pi^2 f_\pi \Phi_\pi^\pm(x) , \tag{5}$$

where a_μ is the photon field, $A^{\mu(\pm)}$ the charge raising or lowering axial vector current, and $\Phi_\pi^\pm$ the positive or negative pion field. Taking matrix elements between the initial and final states one can write, returning to the notation of this paper,

$$
\begin{aligned}
\langle nn | \partial_\mu A^\mu(x)^\dagger | \gamma d\rangle - ie\epsilon_\mu \langle nn | A^\mu(x) | d\rangle\langle 0 | a_\mu(x) | \gamma\rangle \\
= \sqrt{2} f_\pi m_\pi^2 \langle nn | \Phi_\pi^{(-)}(x) | \gamma d\rangle ,
\end{aligned} \tag{6}
$$

leading to

$$
\begin{aligned}
-i(p_\pi)_\mu \langle nn | A^\mu(0)^\dagger | \gamma d\rangle - ie\epsilon_\mu \langle nn | A^\mu(0)^\dagger | d\rangle [(2\pi)^3 2k_0]^{-1/2} \\
= \frac{\sqrt{2}\, m_\pi^2 f_\pi}{q^2 - m_\pi^2} \langle nn\pi^+ | \gamma d\rangle .
\end{aligned} \tag{7}
$$

Letting $p_{\pi\mu} = (p_\gamma + p_d - p_{n_1} - p_{n_2})_\mu$ be small, since we set $m_\pi = 0$ and consider processes near threshold, we obtain

$$ie\epsilon_\mu \langle nn | A^\mu(0)^\dagger | d\rangle \simeq \langle nn\pi^+ | \gamma d\rangle [(2\pi)^3 2k_0]^{1/2} f_\pi \sqrt{2} . \tag{8}$$

Making use of Eq. (3b) we obtain

$$
\begin{aligned}
\epsilon_\mu \langle nn | A^\mu(0)^\dagger | d\rangle = \eta \bar{u}(p_1) \left(F_A \xi \cdot \epsilon + F_P \xi \cdot \frac{Qq\cdot\epsilon}{M_d^2} \right) \\
\times \gamma_5 v(p_2) .
\end{aligned} \tag{9}
$$

We eliminate F_p by making use of a PCAC result[12]

$$F_P = -M_d^2 F_A / (q^2 - m_\pi^2) . \tag{10}$$

Substituting into (9), squaring the result, and summing over all spins we obtain

$$
\begin{aligned}
| \langle nn\pi | \gamma d\rangle |^2 = \frac{e^2}{f_\pi^2 2} \frac{\eta^2 \eta_1^2}{m^2} \left[3 + \frac{2\vec{q}\cdot\vec{Q}}{q^2 - m_\pi^2} - \frac{q^2 \vec{Q}^2}{(q^2 - m_\pi^2)^2} \right] \\
\times (p_1 \cdot p_2 + m^2) F_A^2 ,
\end{aligned} \tag{11}
$$

where f_π ($f_\pi \cos\theta \simeq 96$ MeV) is the pion decay constant and $\eta_1 = [(2\pi)^3 2k_0]^{-1/2}$. We are now ready to obtain F_A from threshold pion-photoproduction data.[13]

The results are[14]

$$|F_A|^2 = |\mathfrak{F}_A|^2 f_A^2 ,$$

where

$$
\begin{aligned}
|\mathfrak{F}_A|^2 = \frac{(3.61 \times 10^1 + 6.13 \times 10^{-1} Q_0)}{[(Q_0 - 0.11)^2 + 6.76 \times 10^{-2}]} \\
\times [1.0 - e^{-6.7\times 10^{-7}(q^2 + 1.6\times 10^4)^2}] \\
\times [1.0 + 1.57 \times 10^2 e^{-9.49\times 10^{-10}(q^2 + 0.97\times 10^5)^2}] \\
\times [1.0 - 1.71 e^{-2.83\times 10^{10}(q^2 + 1.05\times 10^5)^2}]
\end{aligned} \tag{12}
$$

with

$$f_A = (1.0 - q^2/M_A^2)^{-2}$$

and

$$M_A = 0.912 \text{ GeV}.$$

In Fig. 1 we show the cross section resulting from this form of F_A and the experimental situation.

It is necessary to make a comment on the expected accuracy of the procedure we have followed here. Corrections to Eq. (8) are[15] of the order of m_π/m_i or about 7–8 % in this case. For the nucleon case it is higher,[9] approximately $2m_\pi/m_p$ (or 30%). We should therefore expect something in the range of 10–20 % corrections, which would be tolerable.

The remaining form factors F_1 and F_2 have been

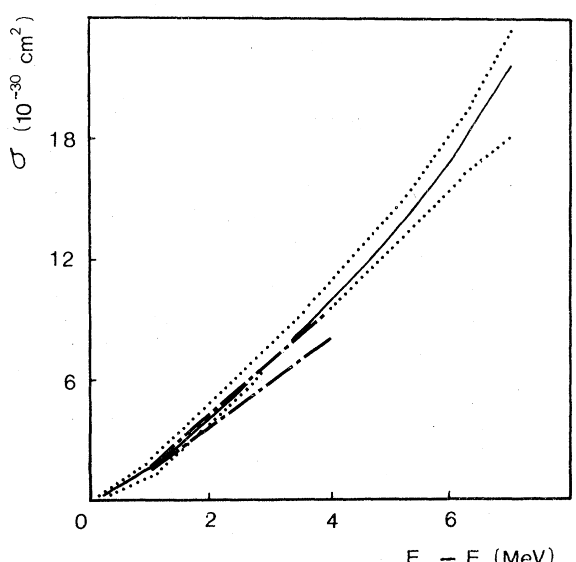

FIG. 1. Plot of the pion-photoproduction cross section for the reaction $\gamma + d \to n + n + \pi^+$ as a function of photon energy above threshold. The solid line represents the fit used in this paper. The dotted lines represent the experimental range determined by Booth *et al.* and the dashed line represents that of Audit *et al.*

obtained by the author in a previous work.[9] They enter into the weak current matrix elements in the form $(F_1 - F_2)^2$, and are obtained from analysis of the reactions $\gamma + d \to n + p$ and $e + d \to n + p + e'$ by the use of the conserved vector current (CVC).[16] The results are quoted below:

$$|F_1 - F_2|^2 = [f_1(q^2)]^2(\mathfrak{F}_1 - \mathfrak{F}_2)^2 \tag{13a}$$

$$f_1(q^2) = (1 - q^2/M_v^2),\quad M_v = 0.84\text{ GeV}, \tag{13b}$$

$$|\mathfrak{F}_1 - \mathfrak{F}_2|^2 = \frac{\sqrt{2}\,(1.05 + 1.41\times 10^{-4} q\cdot d}{(5.21\times 10^{-4} q\cdot d - 2.26)^2 + 1.8}\left\{1 - \frac{1}{M_{d^4}}\left[0.75 q^2 - q\cdot d - \left(\frac{q\cdot d}{M_d}\right)^2\right]^2\right\} R(q^2, \cos\theta), \tag{13c}$$

where

$$R(q^2,\cos\theta) = [1.0 + 2.9\cos^2\theta + q^2(1.73\times 10^{-5} + 5.02\cos^2\theta) + q^4(3.27 - 10^{-9} + 9.48\times 10^{-9}\cos^2\theta)]$$
$$\times \frac{1 + 0.12\exp[-9.8\times 10^{-9}(q^2 + 0.02\times 10^6)^2]}{1 + 9.90\times 10^2 + 2.2\times 10^3[1 - \exp(-5.5\times 10^{-12} q^4)]q^4}, \tag{13d}$$

and θ is the angle between n_1 and q (see Ref. 9). From previous experience,[17] the contribution of the vector part of the current to relatively low energy processes is very small, on the level of a few percent.

Thus we have obtained the form factors F_1, F_2, F_A, and F_P and therefore the weak current matrix element. We should point out that the forms given here are certainly not unique. Since our purpose here is merely to transfer data from one process to another it is merely necessary that the fits be good ones. We are now ready to obtain the muon-capture and pion-capture rates.

III. CALCULATION OF THE MUON-CAPTURE AND PION-CAPTURE RATES

The matrix element squared for the process $\mu^- + {}^2\text{H} \to n + n + \nu_\mu$ is given by

$$|M|^2 = \frac{2}{6 m_\mu m^2}\times\left[\frac{F_A^2(m_\mu + M_d)(m_\mu + M_d - \nu)}{2}\times\left(3 m_\mu \nu + \frac{2 m_\mu^2 \nu^2}{m_\mu^2 - 2\nu m_\mu - m_\pi^2} + \frac{\nu^3 m_\mu^3}{(m_\mu^2 - 2\nu m_\mu - m_\pi^2)^2}\right) + (F_1 - F_2)^2 m_\mu \nu^3\right]. \tag{14}$$

Using the F_A, F_P, F_1, and F_2 obtained in Eqs. (10), (12), and (13) we obtain a value of $\Gamma = 155\text{ sec}^{-1}$ for the muon capture rate. Using the projection operators[18] developed in Ref. 6, we obtain values for the doublet and quartet capture rates, $\Gamma_d = 450.4\text{ sec}^{-1}$ and $\Gamma_q = 7.3\text{ sec}^{-1}$, respectively.

The data that we have can also be used to obtain the radiative pion-capture rate for the reaction $\pi^- + d \to n + n + \gamma$. The matrix element squared for this reaction, M^2, is given by

$$|M_\pi|^2 = \frac{1}{3m^2}\left\{\left[3 + \frac{2E\gamma^2}{m_\pi(m_\pi - 2E\gamma) - m_\pi^2} - \frac{E_\gamma^2[m_\pi(m_\pi - 2E_\gamma]}{[m_\pi(m_\pi - 2E_\gamma) - m_\pi^2]^2}\right]\times(m_\pi + M_d)\left(\frac{m_\pi + M_d - E_\gamma}{2}\right)F_A^2\right\}\frac{e^2}{2f_\pi^2}. \tag{15}$$

This leads to a result of $\Gamma = 3.41\times 10^{14}\text{ sec}^{-1}$.

IV. CONCLUSION

There exist three experimental results for Γ_d, the doublet capture rate,[19] namely one due to Wang *et al.*,

$$\Gamma_d = 365 \pm 96\text{ sec}^{-1}; \tag{16}$$

one due to Bertin *et al.*,

$$\Gamma_d = 451 \pm 70\text{ sec}^{-1}; \tag{17}$$

and finally one due to Placci *et al.*,

$$\Gamma_d = 445 \pm 60\text{ sec}^{-1}. \tag{18}$$

Theoretical calculations yield[20] a Γ_q of about $\Gamma_q \simeq 6\sim 10\text{ sec}^{-1}$.

A best fit to the pion photoproduction data as mentioned above yields

$$\Gamma = 155\text{ sec}^{-1}, \tag{19a}$$

and

$$\Gamma_d = 450.4\text{ sec}^{-1}, \tag{19b}$$

$$\Gamma_q = 7.3\text{ sec}^{-1} \tag{19c}$$

for the doublet and quartet rates, respectively. This is close to the upper range of experimental values [Eqs. (20) and (21)], but certainly in agreement with experiment considering the expected error.

A number of theoretical calculations of this rate have been made.[21] They are all impulse approximation based calculations except for an earlier calculation by the author. All of these calculations yield rates corresponding to $\Gamma \simeq 130\ \text{sec}^{-1}$.

The difference in these calculations may be more apparent than real because of the expected error associated with the use of the Kroll-Ruderman theorem in the calculation presented here. However, there are some real differences in the calculations.

In the elementary particle model approach we make use of SU(2) (isospin) commutation relations. Thus final state strong interactions are incorporated but not in general final state electromagnetic interactions. However, because we here have used the reaction $\gamma + d \to \pi^+ + n + n$ to obtain the $\langle nn | A_\mu^\dagger | d \rangle$ matrix element, the error introduced may be relatively small since the final state contains two neutrons in both cases.

The other difference comes about because the elementary particle model treatment, in principle, would contain all possible Feynman diagrams to the first order in G for the interaction of the muon with the initial nucleus leading to the specific final state. In practice, approximations reduce this, but it seems likely that at least some meson exchange contributions, etc., are taken into account. A careful study of this point would be very useful.

Finally, we have calculated the pion-capture rate to be as mentioned above, $\Gamma = 3.41 \times 10^{+14}$ sec^{-1}. No experimental capture rates have been obtained for this reaction but several theoretical calculations exist,[22] one by Sotona and Truhlik being somewhat representative and done for several different assumptions. Their results vary from $\Gamma = 2.96 \times 10^{+14}\ \text{sec}^{-1}$ for the case of the soft-pion limit to $\Gamma = 4.0 \times 10^{14}\ \text{sec}^{-1}$ for the case of direct use of pion-photoproduction data. They also offer a result of $\Gamma = 3.83 \times 10^{14}\ \text{sec}^{-1}$ for the soft-pion limit case plus corrections. This is in reasonable agreement with our result of $3.41 \times 10^{14}\ \text{sec}^{-1}$ in the soft-pion limit but fitted to photoproduction data. Again most of the comments made for muon capture apply here.

Thus the soft-pion approach combined with elementary particle techniques and pion photoproduction data appears to yield muon-capture rates in reasonable agreement with experiment and other calculated rates and pion-capture rates in reasonable agreement with other calculated rates. This gives us some confidence that the form factors obtained here are reasonable and may be used in the calculation of other weak processes in deuterium.

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[1] By the use of SU(2) current commutation relations, elements $\langle np | A_\mu^{(3)} | d \rangle$ and $\langle np | V_\mu^{(3)} | d \rangle$ may be obtained directly from $\langle nn | A_\mu^\dagger | d \rangle$, $\langle nn | V_\mu^\dagger | d \rangle$. Charge symmetry may be used to obtain $\langle pp | A_\mu | d \rangle$ and $\langle pp | V_\mu | d \rangle$.

[2] S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8)* edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[3] M. Gell-Mann and M. Levy, Nuovo Cimento 16, 705 (1960); Y. Nambu, Phys. Rev. Lett. 4, 380 (1960).

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[5] For details of the elementary-particle model, see, for example, C. W. Kim and H. Primakoff, Phys. Rev. 139, B1447 (1965); 139, B556 (1965). For other processes, see, for example, C. W. Kim and S. L. Mintz, Phys. Lett. 31B, 503 (1970).

[6] S. L. Mintz, Phys. Rev. D 8, 2946 (1973).

[7] S. L. Mintz, Phys. Rev. D 10, 3017 (1974).

[8] S. L. Mintz, Phys. Rev. D 13, 639 (1976).

[9] S. L. Mintz, Phys. Rev. D 18, 3158 (1978).

[10] This has been noted by a number of authors. See, for example, H. Uberall and L. Wolfenstein, Nuovo Cimento 10, 136 (1958).

[11] D. Griffiths and C. W. Kim, Nucl. Phys. B6, 49 (1968).

[12] We use here essentially an argument due to Y. Nambu, Phys. Rev. Lett. 4, 380 (1960).

[13] G. Audit *et al.*, Phys. Rev. C 16, 1517 (1977); E. C. Booth *et al.*, Phys. Lett. 66B, 236 (1977).

[14] We note that because $\mathfrak{F}_A$ is a scalar form factor used in describing the matrix element $\langle nn | A_\mu^+(0) | d \rangle$, it must be a scalar function of the two neutron four-momentum n_{1_μ} and n_{2_μ} as well as the deuteron four-momentum d_μ. We use the combinations q^2 and $Q \cdot d$ (equal to $Q_0 M_d$ in the laboratory frame) given by Eq. (4), in line with the earlier work in Refs. 6–9. We also note that it is possible to use a substantially simplified form of the fit given by Eq. (12), for example,

$$|\mathfrak{F}_A|^2 = [(3.61 \times 10^1 + 6.13 \times 10^{-1} Q_0)/(Q_0 - 0.11)^2] \times [1.0 - e^{-6.7 \times 10^{-7}(q^2 + 1.6 \times 10^4)^2}]\,(1 + 4.703 \times 10^{-14} q^6),$$

and still be within experimental error. The results for muon capture and radiative pion capture are not particularly sensitive to this change and are within 7% of their values for the more elaborate fit used here.

[15] See Ref. 12 and G. W. Gaffney, Phys. Rev. 161, 1599 (1967).

[16] R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

[17] See Ref. 6. In particular a very crude idea of the relative sizes of the axial vector and vector contributions may be obtained by looking at the ratio of the quartet state transition matrix element and the doublet state transition matrix element squared. The axial contribution is greatly suppressed in the former. The result is just 2–3%.

[18] The doublet and quartet projection operators are, respectively, $P_d = \frac{1}{3}[(\delta^{\alpha\beta}) - \frac{1}{2}\sigma^{ij}S_{ij}^{\alpha\beta}]$ and $P_q = \frac{2}{3}[(\delta)^{\alpha\beta} + \frac{1}{4}\sigma^{ij} S_{ij}^{\alpha\beta}]$, where $S_{ij}^{\alpha\beta} = g_i^{\alpha} g_j^{\beta} - g_i^{\beta} g_j^{\alpha}$ and $\sigma^{\mu\nu} = (i/2)[\gamma^{\mu}, \gamma^{\nu}]$. See Ref. 6 for their use.

[19] I.-T. Wang *et al.*, Phys. Rev. 139, B1528 (1965) give Eq. (16); A. Placci *et al.*, Phys. Rev. Lett. 25, 475 (1970) give Eq. (17). Their measurement is not actually a measurement of the doublet rate, but they estimate under the conditions of the experiment that it is effectively a doublet capture rate measurement. Finally A. Bertin *et al.*, Phys. Rev. D 11, 3774 (1973) give Eq. (18), to which the above comment also applies.

[20] See, for example, Ref. 6 or Q. Ho-Kim, J. P. Lavine, and H. S. Picker, Phys. Rev. C 13, 1966 (1976).

[21] See Refs. 6, 10, and 20; I.-T. Wang, Phys. Rev. 139, B1539 (1965); E. Cremmer, Nucl. Phys. B2, 409 (1967); P. Pascual, R. Tarrach, and F. Vidal, Nuovo Cimento 11A, 241 (1972); E. Truhlik, Nucl. Phys. B45, 303 (1972); M. Sotona and E. Truhlik, *ibid.* A229, 471 (1974); E. Ivanov and E. Truhlik, *ibid.* A316, 451 (1979).

[22] A. Reitan, Nucl. Phys. 87, 232 (1966); M. Sotona and E. Truhlik, *ibid.* A262, 400 (1976). For work relating to a_{nn} see B. F. Gibson and G. J. Stephenson, Los Alamos Report No. LA-5424-MS and W. B. Gibbs, B. F. Gibson, and G. J. Stephenson, Phys. Rev. C 11, 90 (1975).