PHYSICAL REVIEW C VOLUME 22, NUMBER 6 DECEMBER 1980

Seagull terms in the Low equation for $\pi\ ^2H \to NN$

Roger H. Hackman
Hollins College, Hollins, Virginia 24020
and Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061
(Received 6 June 1980)

We derive a form for the seagull terms in the Low equation for $\pi\ ^2H \to NN$ which permits a Hamiltonian-independent evaluation. Our results disagree with previous interpretations of these equal-time commutators.

[NUCLEAR REACTIONS πNN-NN, study of seagull terms in Low equation, role in antisymmetrization and double counting.]

The Low equation provides an attractive, nonperturbative foundation for the investigation of scattering processes. Within the framework of the Lehmann-Symanzik-Zimmermann (LSZ) formalism, this equation may be conveniently developed for the reaction

$$\pi(q) + {}^2H(p_d) \to N(p_1) + N(p_2)$$

by first reducing the incident pion and one of the final state nucleons out of the S-matrix element. The saturation of the resulting time-ordered product with a complete set of physical intermediate states then leads to the Low equation of interest,

$$\begin{aligned} T_{\pi^2H\to NN} &= i\langle p_2|S_1|{}^2H\rangle \\ &- (2\pi)^3 \sum_n \frac{\delta^{(3)}(\vec{p}_1+\vec{p}_2-\vec{p}_n)}{p_{1_0}+p_{2_0}-p_{n_0}+i\epsilon}\langle p_2|J_1(0)|n\rangle_{\rm out\ out}\langle n|j_\pi(0)|{}^2H\rangle \\ &+ (2\pi)^3 \sum_n \frac{\delta^{(3)}(\vec{p}_1-\vec{p}_d+\vec{p}_n)}{p_{1_0}-p_{d_0}+p_{n_0}-i\epsilon}\langle p_2|j_\pi(0)|n\rangle_{\rm out\ out}\langle n|J_1(0)|{}^2H\rangle\,, \end{aligned} \tag{1}$$

where

$$J_1(x) = \mathfrak{D}_x\psi(x)\,,$$

$$\mathfrak{D}_x = \bar{u}(p_1)[-i\gamma\cdot\partial_x + m]\,,$$

$$S_1 = \int dx\, e^{ip_1\cdot x} S_1(x)\,,$$

$$S_1(x) = -i[\bar{u}(p_1)\gamma^0\psi(x)\,, j_\pi(0)]\delta(x_0)\,.$$

In the above, $\psi(x)$ is the interpolating nucleon field operator, $j_\pi(0)$ is the pion current operator, and $u(p_1)$ is a four-component Dirac spinor. In the following, we delete the "in" and "out" labels on physical states.

The matrix elements of the currents in the Low equation are readily interpretable in terms of off-mass-shell scattering processes and give the couplings of the reaction to other communicating channels. By way of contrast, the role of the equal-time commutators (seagull terms) which arise from taking the time derivatives of time-ordered products, is more subtle, and specific dynamical assumptions or reference to a particular interaction Lagrangian may be required for their evaluation.

Two previous sets of authors have applied this equation to the $\pi^2H \to NN$ reaction, each in a different context. Banerjee, Levinson, Shuster, and Zollman[1] (BLSZ) use soft-pion techniques in conjuction with the Low equation to determine the form of the transition operator to be used in a nonrelativistic distorted-wave Born approximation (DWBA) calculation. Alberg, Henley, Miller, and Walker[2] (AHMW), on the other hand, solve the Low equation (in some approximate sense), and obtain predictions for the differential and total cross sections. In the present paper, we show that in neither the AHMW nor the BLSZ work has the seagull term in Eq. (1) been properly evaluated. As a result, there have been a number of misconceptions regarding the role of the off-mass-shell amplitudes and the origin of the pion rescatterings in Eq. (1).

Since it turns out that the central issue is the manner in which the antisymmetry of the two-nucleon system is embedded in the Low equation, it is useful to begin with a manifestly antisymmetric expression and relate that result to Eq. (1). To that end we modify the development of Eq. (1) by reducing out both final state nucleons, obtaining for the off-the-pion-mass-shell reaction matrix the expression

$$\langle p_2 p_1 | j_\pi(0) | ^2\mathrm{H}\rangle = -\int dx\, dy\, e^{ip_2\cdot x} e^{ip_1\cdot y} \mathfrak{D}_x \mathfrak{D}_y \times \langle 0 | T[\psi(x)\psi(y) j_\pi(0)] | ^2\mathrm{H}\rangle . \tag{2}$$

To develop the relationship between Eqs. (1) and (2) as straightforwardly as possible, we first act upon the T product in Eq. (2) with $\mathfrak{D}_y$, being careful to retain all nonvanishing equal-time commutators and anticommutators which result from the y_0 differentiation

$$\mathfrak{D}_x \mathfrak{D}_y T[\psi(x)\psi(y) j_\pi(0)] = \mathfrak{D}_x\{T[\psi(x) S_1(y)] + T[\psi(x) J_1(y) j_\pi(0)]\} . \tag{3}$$

Next, we rewrite the second time-ordered product in Eq. (3) in a more convenient form. Using the identity $\theta(x_0) = 1 - \theta(-x_0)$ and discarding superfluous theta functions when appropriate, we expand a part of this term in the form

$$\begin{aligned}\mathfrak{D}_x[\psi(x)J_1(y)j_\pi(0)\theta(x_0-y_0)\theta(y_0) - J_1(y)\psi(x)j_\pi(0)\theta(y_0-x_0)\theta(y_0)\theta(x_0) - J_1(y)j_\pi(0)\psi(x)\theta(y_0)\theta(-x_0)] \\ = \mathfrak{D}_x\{\psi(x)J_1(y)j_\pi(0)[1-\theta(y_0-x_0)]\theta(y_0) - J_1(y)j_\pi(0)\psi(x)\theta(y_0)\theta(-x_0) \\ - J_1(y)\psi(x)j_\pi(0)[1-\theta(x_0-y_0)]\theta(x_0)\theta(y_0)\} \\ = -\{\mathfrak{D}_x \overline{T}[\psi(x)J_1(y)]\}\theta(y_0)j_\pi(0) - J_1(y)\theta(y_0)\{\mathfrak{D}_x T[\psi(x)j_\pi(0)]\} + J_2(x)J_1(y)j_\pi(0)\theta(y_0) ,\end{aligned} \tag{4}$$

where $\overline{T}$ denotes the anti-time-ordered product.[3] Proceeding similarly with the remaining terms in this time-ordered product, we find

$$\begin{aligned}\mathfrak{D}_x[\psi(x)j_\pi(0)J_1(y)\theta(x_0)\theta(-y_0) + j_\pi(0)\psi(x)J_1(y)\theta(-x_0)\theta(-y_0)\theta(x_0-y_0) - j_\pi(0)J_1(y)\psi(x)\theta(-y_0)\theta(y_0-x_0)] \\ = -\{\mathfrak{D}_x \overline{T}[\psi(x)j_\pi(0)]\}J_1(y)\theta(-y_0) + j_\pi(0)\{\mathfrak{D}_x T[\psi(x)J_1(y)]\}\theta(-y_0) + J_2(x)j_\pi(0)J_1(y)\theta(-y_0) .\end{aligned} \tag{5}$$

This completes the rearrangement of the second term in Eq. (3).

The relationship between Eqs. (1) and (2) is now readily established. Consider first the contribution of the T and $\overline{T}$ products in Eq. (4) to Eq. (2). Inserting intermediate states and using translation invariance, this contribution takes the form

$$(2\pi)^3 \sum_n \left(\frac{\delta^{(3)}(\vec{p}_1 - \vec{p}_n)}{p_{1_0} - p_{n_0} + i\epsilon} \langle 0 | J_1(0) | n\rangle \langle n, p_2 | j_\pi(0) | ^2\mathrm{H}\rangle + i \frac{\delta^{(3)}(\vec{p}_1 + \vec{p}_2 - \vec{p}_n)}{p_{1_0} + p_{2_0} - p_{n_0} + i\epsilon} \int dx\, e^{ip_2\cdot x} \langle 0 | \{\mathfrak{D}_x \overline{T}[\psi(x)J_1(0)]\} | n\rangle\langle n | j_\pi(0) | ^2\mathrm{H}\rangle \right) . \tag{6}$$

The first few contributing states for each term are depicted in Figs. 1(a), 1(b), and 1(c). The first term in this expression is simply the contribution from the disconnected piece of $\langle p_2 | J_1(0) | n\rangle_{\mathrm{out}}$ in Eq. (1), while the second term is the contribution from the connected piece. In a completely analogous fashion, the contribution of the T and $\overline{T}$ products in Eq. (5) to Eq. (2) [see Figs. 1(d), 1(e), and 1(f)]

$$(2\pi)^3 \sum_n \left(\frac{\delta^{(3)}(\vec{p}_1 + \vec{p}_2 + \vec{p}_n - \vec{p}_d)}{p_{1_0} + p_{2_0} + p_{n_0} - p_{d_0} - i\epsilon} \langle 0 | j_\pi(0) | n\rangle \langle n, p_2 | J_1(0) | ^2\mathrm{H}\rangle - i \frac{\delta^{(3)}(\vec{p}_1 - \vec{p}_d + \vec{p}_n)}{p_{1_0} - p_{d_0} + p_{n_0} - i\epsilon} \int dx\, e^{ip_2\cdot x} \langle 0 | \{\mathfrak{D}_x \overline{T}[\psi(x) j_\pi(0)]\} | n\rangle \langle n | J_1(0) | ^2\mathrm{H}\rangle \right) , \tag{7}$$

may be identified with the contributions from the connected and disconnected pieces of $\langle p_2 | j_\pi(0) | n\rangle_{\mathrm{out}}$ in the crossed term in Eq. (1). The remaining terms in Eqs. (4) and (5) do not contribute to Eq. (2) due to the unrestricted x_0 integrations.[4] It follows that

$$i\langle p_2 | S_1 | ^2\mathrm{H}\rangle = -\int dx\, dy\, e^{ip_2\cdot x} e^{ip_1\cdot y} \times \langle 0 | \{\mathfrak{D}_x T[\psi(x) S_1(y)]\} | ^2\mathrm{H}\rangle . \tag{8}$$

This is just the expression which would have resulted from the formal application of reduction techniques to $N(p_2)$ in the matrix elements of S_1.

Before proceeding, it is instructive to consider the lowest order contributions to Eq. (1) in view of the manifest antisymmetry of Eq. (2) and in light of the above developments. We note first that the direct absorption of the pion by nucleon $N(p_2)$ follows from the $n = N$ intermediate state contribution to the second term in Eq. (7) [see

Fig. 1(e)]. We further note that, since to lowest order (in pion rescatterings) ${}_{\rm out}\langle\pi(q')N(p')N(p_2)\,|j_\pi(0)\,|{}^2\mathrm{H}\rangle$ involves the elastic scattering of the pion from $N(p')$ and $N(p_2)$, the $n=\pi N$ contribution to the first intermediate state sum in Eq. (6) provides for the elastic scattering of the pion by $N(p_2)$ followed by absorption by $N(p_1)$ [see Fig. 1(a)]. And similarly, the $n=\pi N$ contribution to the second term in Eq. (7) [Fig. 1(f)] involves the backwards (in time) elastic scattering of the pion by $N(p_2)$ with absorption by $N(p_1)$. Now, however, we have a dilemma: Where are the graphs involving the direct absorption of the pion by $N(p_1)$ and the elastic rescattering of the pion by $N(p_1)$ with absorption by $N(p_2)$, whose presence necessarily follows from the antisymmetry of Eq. (2)? While it may be tempting to interpret the matrix element of S_1 in terms of the direct absorption by $N(p_1)$,[5] this would not resolve the issue of the "missing" rescattering graphs.

The simplest way to proceed is with a direct evaluation of Eq. (8). If we assume that the equal-time anticommutator $[\psi_2, S_1]_+$ vanishes,[6] we obtain

$$i\langle p_2\,|S_1\,|{}^2\mathrm{H}\rangle = -i(2\pi)^3\sum_n\left(\frac{\delta^{(3)}(\vec{p}_2-\vec{p}_n)}{p_{2_0}-p_{n_0}+i\epsilon}\langle 0\,|J_2(0)\,|n\rangle\langle n\,|S_1\,|{}^2\mathrm{H}\rangle+\frac{\delta^{(3)}(\vec{p}_2-\vec{p}_d+\vec{p}_n)}{p_{2_0}-p_{d_0}+p_{n_0}-i\epsilon}\langle 0\,|S_1\,|n\rangle\langle n\,|J_2(0)\,|{}^2\mathrm{H}\rangle\right). \tag{9}$$

Consider the matrix elements of S_1 appearing on the right hand side of Eq. (9). Let α and β be two arbitrary states. Using completeness in the commutator

$$\begin{aligned}i\int d\vec{x}\,e^{-i\vec{p}\cdot\vec{x}}\langle\alpha\,|[\psi(0,\vec{x}),j_\pi(0)]\,|\beta\rangle\\ =(2\pi)^3 i\sum_m[\delta^{(3)}(\vec{p}+\vec{p}_\alpha-\vec{p}_m)\langle\alpha\,|\psi(0)\,|m\rangle\langle m\,|j_\pi(0)\,|\beta\rangle-\delta^{(3)}(\vec{p}+\vec{p}_m-\vec{p}_\beta)\langle\alpha\,|j_\pi(0)\,|m\rangle\langle m\,|\psi(0)\,|\beta\rangle]\,.\end{aligned} \tag{10}$$

these matrix elements may be written in the form

$$\begin{aligned}i\langle\alpha\,|S_1\,|\beta\rangle=\langle\alpha,p_1\,|j_\pi(0)\,|\beta\rangle^c+(2\pi)^3\sum_m\Bigg(&\frac{\delta^{(3)}(\vec{p}_1+\vec{p}_\alpha-\vec{p}_m)}{p_{1_0}+p_{\alpha_0}-p_{m_0}+i\epsilon}\langle\alpha\,|J_1(0)\,|m\rangle\langle m\,|j_\pi(0)\,|\beta\rangle\\ &-\frac{\delta^{(3)}(\vec{p}_1+\vec{p}_m-\vec{p}_\beta)}{p_{1_0}+p_{m_0}-p_{\beta_0}-i\epsilon}\langle\alpha\,|j_\pi(0)\,|m\rangle\langle m\,|J_1(0)\,|\beta\rangle\Bigg)\,,\end{aligned} \tag{11}$$

where the superscript c on the first term denotes a fully connected matrix element (in ψ). The disconnected-ψ contribution to Eq. (10) gives the first term in this equation, while the remaining terms follow from the connected contribution.[7]

Using Eq. (11) in conjuction with Eq. (9), we have

$$\begin{aligned}i\langle p_2\,|S_1\,|{}^2\mathrm{H}\rangle=&-(2\pi)^3\sum_n\frac{\delta^{(3)}(\vec{p}_2-\vec{p}_n)}{p_{2_0}-p_{n_0}+i\epsilon}\langle 0\,|J_2(0)\,|n\rangle\langle n,p_1\,|j_\pi(0)\,|{}^2\mathrm{H}\rangle^c\\ &-(2\pi)^6\sum_{m,n}\frac{\delta^{(3)}(\vec{p}_2-\vec{p}_n)}{p_{2_0}-p_{n_0}+i\epsilon}\,\frac{\delta^{(3)}(\vec{p}_1+\vec{p}_n-\vec{p}_m)}{p_{1_0}+p_{n_0}-p_{m_0}+i\epsilon}\langle 0\,|J_2(0)\,|n\rangle\langle n\,|J_1(0)\,|m\rangle\langle m\,|j_\pi(0)\,|{}^2\mathrm{H}\rangle\\ &+(2\pi)^6\sum_{m,n}\frac{\delta^{(3)}(\vec{p}_2-\vec{p}_n)}{p_{2_0}-p_{n_0}+i\epsilon}\,\frac{\delta^{(3)}(\vec{p}_1+\vec{p}_m-\vec{p}_d)}{p_{1_0}+p_{m_0}-p_{d_0}-i\epsilon}\langle 0\,|J_2(0)\,|n\rangle\langle n\,|j_\pi(0)\,|m\rangle\langle m\,|J_1(0)\,|{}^2\mathrm{H}\rangle\\ &-(2\pi)^3\sum_n\frac{\delta^{(3)}(\vec{p}_2-\vec{p}_d+\vec{p}_n)}{p_{2_0}-p_{d_0}+p_{n_0}-i\epsilon}\langle p_1\,|j_\pi(0)\,|n\rangle^c\langle n\,|J_2(0)\,|{}^2\mathrm{H}\rangle\\ &+(2\pi)^6\sum_{m,n}\frac{\delta^{(3)}(\vec{p}_2-\vec{p}_d+\vec{p}_n)}{p_{2_0}-p_{d_0}+p_{n_0}-i\epsilon}\,\frac{\delta^{(3)}(\vec{p}_1+\vec{p}_m-\vec{p}_n)}{p_{1_0}+p_{m_0}-p_{n_0}+i\epsilon}\langle 0\,|j_\pi(0)\,|m\rangle\langle m\,|J_1(0)\,|n\rangle\langle n\,|J_2(0)\,|{}^2\mathrm{H}\rangle\\ &-(2\pi)^6\sum_{m,n}\frac{\delta^{(3)}(\vec{p}_2-\vec{p}_d+\vec{p}_n)}{p_{2_0}-p_{d_0}+p_{n_0}-i\epsilon}\,\frac{\delta^{(3)}(\vec{p}_1-\vec{p}_m)}{p_{1_0}-p_{m_0}-i\epsilon}\langle 0\,|J_1(0)\,|m\rangle\langle m\,|j_\pi(0)\,|n\rangle\langle n\,|J_2(0)\,|{}^2\mathrm{H}\rangle\,.\end{aligned} \tag{12}$$

Equation (12) is the foundation for an explicit model-independent evaluation of the seagull term. Several features of this equation are particularly worthy of comment. First, we note that the $n=N$ contribution to the fourth term gives the $N(p_1)$ direct absorption graph [the $1\leftrightarrow 2$ analog of Fig. 1(e)] and that the $n=\pi N$ contribution from the first and fourth terms [the $1\leftrightarrow 2$ analog of Figs. 1(a) and 1(f), respectively)] provide the missing graphs involving pion rescattering from $N(p_1)$ and absorption by $N(p_2)$. Consider next the second term in Eq. (12). To lowest order, this contribution involves the breakup reaction followed by a pion exchange [Fig. 1(g)]. Combining this term with the first term in Eq. (6) and neglecting the equal-time anticommutator $[\psi_2, J_1]_+$, we obtain

$$(2\pi)^3 \sum_n \frac{\delta^{(3)}(\vec{p}_1+\vec{p}_2-\vec{p}_n)}{p_{1_0}+p_{2_0}-p_{n_0}+i\epsilon} \int dx \langle 0 | [J_2(0)J_1(x)e^{ip_1\cdot x} - J_1(0)J_2(x)e^{ip_2\cdot x}]\,\theta(x_0)\,|n\rangle\langle n|j_\pi(0)|{}^2\mathrm{H}\rangle\,, \tag{13}$$

where $p_1{}^2 = p_2{}^2 = m^2$. Thus, the second term in Eq. (12) antisymmetrizes the final interaction and puts the nucleon associated with $J_1(0)$ back on the mass shell. In many studies in theoretical nuclear physics, it is necessary to work with nonrelativistic wave functions. We note in this context that the matrix element in Eq. (13) appears to be a more satisfactory object to relate to the nonrelativistic, off-energy-shell, nucleon-nucleon scattering amplitude than does $\langle p_2|J_1(0)|n\rangle_{\mathrm{out}}$.[8]

The lowest order graphs contributing to the third term in Eq. (12) (we note that the sixth term is its $J_1 \leftrightarrow J_2$ analog) are depicted in Figs. 1(h) and 1(i). For the present, we note that the graph in Fig. 1(h) is included in both the direct absorption graph and the $\pi^2\mathrm{H}\to\pi NN\to NN$ intermediate state contribution, and that is possible to show that these terms are required to prevent double counting difficulties.[9] The remaining term in Eq. (12) involves $N\bar{N}$ contributions, which we shall neglect here.

We are now in a position to comment on the work of AHMW and BLSZ. We consider first the more recent work. AHMW evaluate the Low equation in the one meson truncation and, as first approximation, consider only those processes in which a pion is produced by one nucleon and scattered by the second nucleon. In this work, the seagull term is taken to have the form

$$\langle p_2|[B_1, j_k]|{}^2\mathrm{H}\rangle = \langle p_1|j_k B_2|{}^2\mathrm{H}\rangle\,, \tag{14}$$

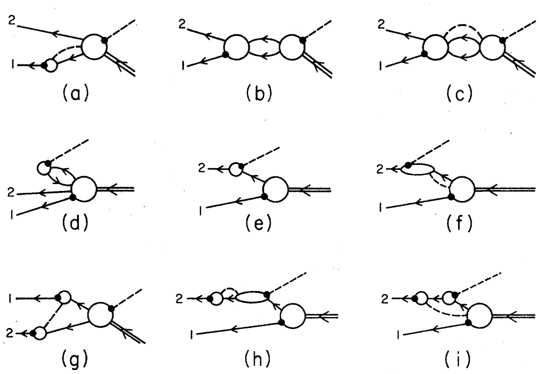

FIG. 1. Intermediate state contributions to Eqs. (6), (7), and (12). Heavy lines refer to nucleons or antinucleons, double lines to deuterons, and dashed lines to pions. The heavy dot at the intersection of an external line with an interaction bubble denotes an off-mass-shell particle. All interaction bubbles are fully connected.

where B_1 is the physical nucleon annihilation operator and j_k is the pion current operator. The right-hand side of this equation, after the insertion of intermediate states, just gives the fourth term in Eq. (12) (in the limit that $N\bar{N}$ pairs are neglected). Thus the formalism of AHMW is missing the graph, which is the $1\leftrightarrow 2$ analog of Fig. 1(a), which includes forward rescattering by $N(p_1)$ and absorption by $N(p_2)$. Presumably these authors have included this contribution by mistakenly attributing to $\langle p_1|J_2(0)|\pi NN\rangle$ a disconnected piece wherein a physical nucleon $N(p_2)$ propagates freely and $N(p_1)$ emits a pion. It is worth noting that if one follows this assumption through to its conclusion and solves the resulting integral equations, the graphs depicted in Figs. 1(h) and 1(i) will be double counted. Thus, the basic formalism of AHMW must be altered in accordance with Eq. (12).

In the second-mentioned work, BLSZ make the identification

$$\langle p_2|S_1|{}^2\mathrm{H}\rangle = (\chi_\alpha|[V_\pi, C]|{}^2\mathrm{H}) \tag{15}$$

at the physical pion production threshold, where χ_α is a plane-wave two-nucleon state, V_π is the pion production potential, and C is the pion annihilation operator. At the physical threshold, as noted above, the seagull term not only antisymmetrizes the final state interaction, but is involved in the pion rescatterings as well. The discussion in the BLSZ work of the two-nucleon singularity of Eq. (1) may easily be generalized to allow for the antisymmetrization of the final state interaction by the seagull term. However, it must be remembered that the soft pion result of these authors lacks the basic antisymmetry appropriate to the physical $\pi^2\mathrm{H}\to NN$ process [i.e., nucleon $N_1(p_d+q-p_2)$ goes off the mass shell as $q\to 0$ in the BLSZ work]. Thus the correctness of their procedure of indentifying $T(q_0=0)$ with the nonrelativistic production amplitude is questionable. Furthermore, when pion scattering is considered, these authors are missing the $1\leftrightarrow 2$ analogs of Figs. 1(a) and 1(f). Consequently the connection that these authors make between Eq. (1) and the nonrelativistic corrections to the DWBA is misleading, as they do not have the full complement of rescattering graphs, nor do they have the double-counting corrections of Figs. 1(h) and 1(i).[10]

In summary, our formalism provides the basis for a Hamiltonian-independent evaluation of the seagull terms in the Low equation for $\pi^2\mathrm{H} \to NN$. These equal-time commutators play a more complicated and important role in the description of this reaction than has been previously appreciated. We emphasize that the present theory does not have a double-counting problem. However, it is precisely the correct treatment of the seagull term which alleviates this difficulty. The results of this paper are consistent with the generalization of the Low equation obtained by the present author in a previous paper,[11] although the role of Figs. 1(h) and 1(i) was overlooked in that publication.

This work was supported in part by the National Science Foundation.

---

[1] M. K. Banergee, C. A. Levinson, M. D. Shuster, and D. A. Zollman, Phys. Rev. C 3, 509 (1971); 13, 2444 (1976).

[2] M. A. Alberg, E. M. Henley, G. A. Miller, and J. F. Walker, Nucl. Phys. A306, 447 (1978).

[3] For a discussion of this product, see J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields*, (McGraw-Hill, N. Y., 1965), p. 272.

[4] For example, consider the contribution of the last term of Eq. (4) to Eq. (2). With an appropriate use of completeness, we have

$$\int dx\,dy\, e^{i(p_2\cdot x+p_1\cdot y)}\, \langle 0\,|\,J_2(x)J_1(y)j_\pi(0)\theta(y_0)\,|\,{}^2\mathrm{H}\rangle = (2\pi)^4 \sum_n \delta^{(4)}(p_n - p_2)\,\langle 0\,|\,J_2(0)\,|\,n\rangle \times \langle n\,|\int dy\, e^{ip_1 y} J_1(y) j_\pi(0)\theta(y_0)\,|\,{}^2\mathrm{H}\rangle .$$

Due to the energy-momentum delta function, only the single-nucleon state contributes to the intermediate state sum; however, this vanishes, since $\langle 0\,|\,J_2(0) \times |\,N(p_n)\rangle = 0$.

[5] This is the procedure followed in the preprint version of Ref. 2.

[6] In this paper, we neglect the equal-time anticommutators $[\psi(\vec{x},0), J(\vec{y},0)]_+$ and $[\psi(\vec{x},0), S(\vec{y},0)]_+$. This is equivalent to assuming that there are no dinucleon couplings in the interaction Lagrangian and does not affect any of the conclusions of this article.

[7] The separation of the matrix elements of ψ into connected and disconnected parts is most easily effected by noting that

$$\pm\langle m\,|\,a(p)^{\text{out}}_{\text{in}}\,|\,n\rangle = \int dx\,\frac{\partial}{\partial x^0}\left[e^{ip\cdot x}\langle m\,|\,\bar{u}(p)\gamma_0\psi(x)\,|\,n\rangle\,\theta(\pm x_0)\right] = \pm\int d\vec{x}\, e^{-i\vec{p}\cdot\vec{x}}\langle m\,|\,\bar{u}(p)\gamma_0\psi(\vec{x},0)\,|\,n\rangle + i\int dx\, e^{ip\cdot x}\langle m\,|\,J(x)\,|\,n\rangle\,\theta(\pm x_0).$$

[$a(p)^{\text{out}}_{\text{in}}$ is the asymptotic nucleon annihilation operator.] Thus, the sign of $i\epsilon$ in the denominator of Eq. (11) depends upon the asymptotic nature of the states involved.

[8] To identify the nonrelativistic limit of a field theoretic amplitude with an off-energy-shell, quantum-mechanical amplitude $\mathcal{T}_{\alpha\beta}$, one must show that in some suitable approximation, they satisfy the same equation. We note that in this instance (n = two-nucleon state), $\mathcal{T}_{\alpha\beta}$ satisfies the manifestly antisymmetric, nonlinear integral equation

$$\mathcal{T}_{\alpha\beta} = V_{\alpha\beta} + \sum_\lambda \mathcal{T}_{\alpha\lambda}(E_\beta - E_\lambda + i\eta)^{-1}\mathcal{T}_{\beta\lambda}{}^* .$$

[See M. L. Goldberger and K. M. Watson, *Collision Theory* (Krieger, New York, 1975)], where $V_{\alpha\beta} = ((H - E_\alpha)\chi_\alpha, \chi_\beta)$, χ is a plane wave state, and H is the Hamiltonian. Both the matrix element defined in Eq. (13) (T_{0E}) and the matrix element

$$\mathcal{T}_{0M} = \bar{u}(p_1)\,\langle p_2\,|\,(-i\gamma\cdot\partial + m)\psi(0)\,|\,n\rangle_{\text{out}}$$

satisfy nonlinear Low equations, but only the Low equation for T_{0E} shares the manifest antisymmetry of the above equation. Thus it is a straightforward matter to relate T_{0E} and $\mathcal{T}$ through a suitable definition of V, but T_{0M} has a basic asymmetry which is untenable in a potential theory. This identification of T_{0E} as an off-energy-shell amplitude is quite natural since in Eq. (13), $\vec{p}_n = \vec{p}_1 + \vec{p}_2$, while in general $E_n \neq E_1 + E_2$. We note that in T_{0M}, the "missing momentum" to be associated with ψ is defined through translation invariance, i.e., $p_1' \equiv p_n - p_2$, hence both energy and three-momentum are automatically conserved [p_1' should not be confused with the momentum p_1 of a final state nucleon which appears in $\bar{u}(p_1)$], however $(p_n - p_2)^2 \neq m^2$ in general.

[9] This point will be discussed more thoroughly in a manuscript currently in preparation.

[10] We note that there are some subtleties involved in the direct application of Eq. (12) to BLSZ, as these authors study the pion production amplitude with a nucleon off the mass shell. However, these comments are still appropriate at the physical threshold.

[11] R. H. Hackman, Phys. Rev. C 19, 1873 (1978).