PHYSICAL REVIEW C VOLUME 22, NUMBER 6 DECEMBER 1980

Electromagnetic properties of the deuteron and the Bethe-Salpeter equation with one-boson exchange

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(Received 1 May 1980)

The deuteron electromagnetic form factors are studied in a relativistic framework. Using the Bethe-Salpeter equation in the ladder approximation with a superposition of π, η, ϵ, δ, ρ, and ω exchange as the driving force, we calculate the deuteron vertex function. The construction of the deuteron current from the vertex function is described in some detail. In order to obtain numerical results an approximation is introduced in which parts of the boost effects are neglected and the corrections are discussed. The calculated form factors are compared with the nonrelativistic results obtained from the Reid soft-core interaction.

[NUCLEAR STRUCTURE Deuteron; relativistic calculation electromagnetic from factors; Bethe-Salpeter equation one-boson-exchange model.]

I. INTRODUCTION

In the description of nucleon-nucleon scattering at low energies it is usually assumed that the two-nucleon system behaves in a good approximation nonrelativistically. There are, however, indications that in certain physical situations such as elastic electron-deuteron scattering relativistic effects should also be included. In particular, it is expected that relativistic covariance and mesonic degrees of freedom are important. In the conventional approach these effects are treated as corrections to the nonrelativistic potential theory. The consistency of such calculations is, however, not at all clear. For this reason a genuine relativistic calculation is of interest.

Recently it was shown that the relativistic covariant Bethe-Salpeter equation (BSE) with one-boson exchanges as the driving force is capable of giving a reasonable description of the nucleon-nucleon system,[1] provided that an axial-vector coupling is used for the pion with the nucleons. In this paper we study the electromagnetic properties of the deuteron in this model. A prominent features of the BSE is that the so-called relativistic and meson-exchange effects[2] are consistently taken into account. Our results for elastic electron-deuteron scattering deviate markedly from the conventional perturbative approach. Using a quasipotential model this deviation was traced back to an inconsistent treatment of the dynamical effects of the mesonic degrees of freedom in the perturbative approach.[3] Here we give a detailed account of the BSE calculation. We will be concerned with the scattering amplitude for elastic electron-deuteron scattering. In the one-photon exchange approximation, shown schematically in Fig. 1, the amplitude is given by[4]

$$T_{fi}=\langle k'\lambda' \,|\, j_\mu^{\mathrm{el}} \,|k\lambda\rangle \frac{1}{q^2}\langle P'M' \,|\, J_D^\mu \,|PM\rangle\,, \tag{1.1}$$

where the electron current is

$$\langle k'\lambda' \,|\, j_\mu^{\mathrm{el}} \,|k\lambda\rangle = ie\bar{u}_{\lambda'}(\vec{k}')\gamma_\mu u_\lambda(\vec{k})\,. \tag{1.2}$$

As is well known, the deuteron current can be expressed in terms of invariant form factors. The important relations are summarized in Appendix A.

In order to determine the matrix elements of the deuteron current we need to know the deuteron wave function. Section II is devoted to the calculation of this wave function, using the homogeneous BSE, and its partial wave decomposition. In Sec. III we describe the deuteron current in the relativistic impulse approximation. The matrix elements of the isoscalar deuteron current have the interesting property that, for a ladder theory of the BSE, such as the one we are considering in this paper, gauge invariance holds. In Sec. IV we describe the results for the electromagnetic form factors in an approximation in which the negative-energy spinors and some effects of the boost transformations are neglected. Corrections to this so-called static calculation are studied in Sec. V. We find that the most important contribution comes from the boost on the one-particle propagator. Our final results for the form factors are very similar to the nonrelativistic calculations such as carried out for the Reid soft-core potential. This indicates that the relativistic and meson-exchange-current contributions to the isoscalar part of the electromagnetic current of nuclei are less important than is generally accepted. Some concluding remarks are made in Sec. VI.

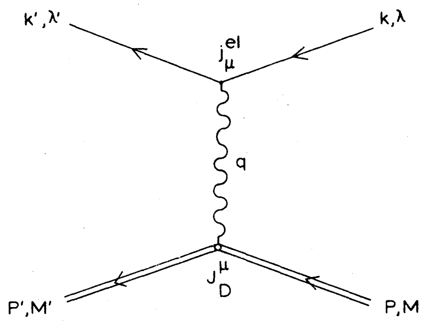

FIG. 1. Schematic representation of the scattering process; λ and M are the polarization of the electron and the deuteron and k and P are their respective momenta.

II. BOUND STATE WAVE FUNCTION

Our starting point is the BSE in the ladder approximation[5] for the two-nucleon system. It has the form

$$\phi(p',p;P)=K(p',p)-\frac{i}{4\pi^3}\int d^4p''K(p',p'')\times S(p'',P)\phi(p'',p;P)\,, \tag{2.1}$$

where ϕ is the T matrix, which depends on the initial and final relative momenta p and p' and the total momentum P. For the kernel K we take the one-boson-exchange terms of Ref. 1, consisting of the exchange of π, η, ϵ, δ, ρ, and ω mesons. The πN interaction is of the axial-vector type. For the two-nucleon propagator S we have

$$S(p,P)=\left[\frac{\not{P}^{(1)}}{2}+\not{p}^{(1)}-M_N\right]^{-1}\left[\frac{\not{P}^{(2)}}{2}-\not{p}^{(2)}-M_N\right]^{-1}\equiv S^{(1)}\left(\frac{P}{2}+p\right)S^{(2)}\left(\frac{P}{2}-p\right), \tag{2.2}$$

where the superscripts refer to the particles and M_N is the nucleon mass. The deuteron vertex function,[6] needed for the calculation of the matrix elements of the electromagnetic current, can in principle be constructed from the residue of the two-nucleon T matrix at the bound state pole $P^2=M_D{}^2$. Since the deuteron is a spin one particle characterized by a polarization M, the T matrix in the neighborhood of this pole takes the following form:

$$\phi(p',p;P)=\sum_M\frac{\psi^{(M)}(p';P)\tilde{\psi}^{(M)}(p;P)}{P^2-M_D{}^2}+\text{terms regular at } P^2=M_D{}^2. \tag{2.3}$$

The vertex functions ψ and $\tilde{\psi}$ satisfy the homogeneous BSE with the normalization condition[7]:

$$2P_\mu\delta_{MM'}=\frac{i}{4\pi^3}\int d^4p\,\tilde{\psi}^{(M)}(p;P)\left[\frac{\partial}{\partial P^\mu}S(p,P)\right]_{P^2=M_D{}^2}\psi^{(M')}(p;P)-\left(\frac{i}{4\pi^3}\right)^2\int d^4p\int d^4p'\tilde{\psi}^{(M)}(p';P)S(p',P)\left[\frac{\partial}{\partial P^\mu}K(p',p;P)\right]_{P^2=M_D{}^2}S(p,P)\psi^{(M)}(p;P)\,. \tag{2.4}$$

In a ladder theory, such as we are considering here, the second term on the right hand side of Eq. (2.4) vanishes since the kernel K is independent of the total momentum P. Taking the scalar product, in the center-of-momentum (c.m.) frame, of the vertex functions and a complete set of two-particle helicity states[8,9], we may define

$$\begin{aligned}\phi^{(M)}(p,a)&=\bar{V}^{\rho_1}_{\lambda_1}(\vec{p})\bar{V}^{\rho_2}_{\lambda_2}(\vec{p})\psi^{(M)}(p;P_{\text{c.m.}})\,,\\ \tilde{\phi}^{(M)}(p,a)&=\tilde{\psi}^{(M)}(p;P_{\text{c.m.}})V^{\rho_1}_{\lambda_1}(\vec{p})V^{\rho_2}_{\lambda_2}(\vec{p})\,.\end{aligned} \tag{2.5}$$

The helicity states are labeled by the relative momentum $\vec{p}$, the helicities λ_i, and the energy spins ρ_i of the particles. They are given by the positive- and negative-energy solutions of the Dirac equation and their explicit form is given for completeness in Appendix B. In Eq. (2.5) a denotes the helicity and energy-spin indices. The angular dependence, for a given angular momentum J, is of the form

$$\begin{aligned}\phi^{(M)}(p,a)&=[(2J+1)/2]^{1/2}D^{J*}_{M\lambda}(\Omega_{\vec{p}})\phi(|\vec{p}|,p_0,a)\,,\\ \tilde{\phi}^{(M)}(p,a)&=\tilde{\phi}(|\vec{p}|,p_0,a)D^{J}_{M\lambda}(\Omega_{\vec{p}})[(2J+1)/2]^{1/2}\,,\\ \lambda&=\lambda_1-\lambda_2\,.\end{aligned} \tag{2.6}$$

Owing to rotational invariance the M dependence is only present in the D functions.

With the decomposition of the propagator, given in Appendix B, and Eqs. (2), (4), and (6) we find for the normalization condition in the c.m. frame

$$4M_D=\frac{i}{2\pi^2}\int dp_0d|\vec{p}|\,|\vec{p}|^2\sum_a\tilde{\phi}(|\vec{p}|,p_0,a)\times[\partial S(|\vec{p}|,p_0,a)/\partial E]_{E=M_D/2}\times\phi(|\vec{p}|,p_0,a)\,, \tag{2.7}$$

where we have taken $P=(\vec{0},2E)$. In the basis we employ here, the propagator is diagonal and depends only on the ρ-spin index (ρ_1,ρ_2):

$$S(+,+)=[(E-E_p)^2-p_0{}^2]^{-1},\quad S(-,-)=[(E+E_p)^2-p_0{}^2]^{-1},$$
$$S(+,-)=[E^2-(p_0-E_p)^2]^{-1},\quad S(-,+)=[E^2-(p_0+E_p)^2]^{-1}, \tag{2.8}$$

where $M_N{}^2$, and thus $E_p=(M_N{}^2+\vec{p}^2)^{1/2}$, has a small negative imaginary part. The equations for ϕ and $\tilde{\phi}$ can be found using the partial wave decomposition of the kernel:

$$K(p',a';p,a)=\frac{1}{2|\vec{p}'||\vec{p}|}\times\sum_{J,\mu}(2J+1)D^{J*}_{\mu,\lambda'}(\Omega_{\vec{p}'})\times K^J(|\vec{p}'|,p_0',a';|\vec{p}|,p_0,a)D^J_{\mu,\lambda}(\Omega_{\vec{p}}) \tag{2.9}$$

to be

$$\phi(|\vec{p}|,p_0,a)=-\frac{i}{2\pi^2}\int dp_0'd|\vec{p}'|\frac{|\vec{p}'|}{|\vec{p}|}\times\sum_b K^J(|\vec{p}|,p_0,a;|\vec{p}'|,p_0',b)\times S(|\vec{p}'|,p_0',b)\phi(|\vec{p}'|,p_0',b), \tag{2.10}$$

$$\tilde{\phi}(|\vec{p}|,p_0,a)=-\frac{i}{2\pi^2}\int dp_0'd|\vec{p}'|\frac{|\vec{p}'|}{|\vec{p}|}\times\sum_b \tilde{\phi}(|\vec{p}'|,p_0',b)S(|\vec{p}'|,p_0',b)\times K^J(|\vec{p}'|,p_0',b;|\vec{p}|,p_0,a).$$

Due to time-reversal invariance K^J is symmetric[1]:

$$K^J(|\vec{p}'|,p_0',a';|\vec{p}|,p_0,a)=K^J(|\vec{p}|,p_0,a;|\vec{p}'|,p_0',a'), \tag{2.11}$$

so that, in the case of a nondegenerate bound state, it follows that ϕ and $\tilde{\phi}$ differ by a phase factor only. In accordance with the normalization condition, and regarding the fact that the fermion field operators satisfy anticommutation relations, we take

$$\tilde{\phi}(|\vec{p}|,p_0,a)=-\phi(|\vec{p}|,p_0,a). \tag{2.12}$$

The basis states employed so far are labeled by J, λ_1, λ_2, ρ_1, and ρ_2. In the discussion of the deuteron it is more convenient to introduce states labeled by J, L, S, and ρ.[10] Adopting the spectroscopic notation ${}^{2S+1}L_J^\rho$ of Gammel *et al.*,[11] we have, in the deuteron, the following eight coupled states:

$$\begin{aligned}&1:\ {}^3S_1^+\quad 2:\ {}^3D_1^+\quad 3:\ {}^3S_1^-\quad 4:\ {}^3D_1^-\\ &5:\ {}^1P_1^e\quad 6:\ {}^3P_1^o\quad 7:\ {}^1P_1^o\quad 8:\ {}^3P_1^e\end{aligned} \tag{2.13}$$

which we denote by $\phi_n(|\vec{p}|,p_0)$, with $n=1,\ldots,8$. The first two states correspond to the nonrelativistic states. Due to the symmetry under the exchange of the particles in a state,[9] the first six channels are even in the relative energy, while the last two are odd.

The BSE in the bound state region is solved using similar techniques as described in Ref. 1. First a Wick rotation is applied, resulting in a nonsingular two-dimensional integral equation. However, since we are considering the bound state problem, there are no additional terms arising from the poles of the two-particle propagator. The Wick rotated equations with an inhomogeneous term have been solved very close to the bound state pole, using the scalar Padé approximant method to construct the solution. Although the [6/6] approximant was already convergent for most values of the argument of the vertex function, in the calculation a [10/10] approximant was used. The found solution was normalized using Eq. (2.7). As an additional test the computed vertex function was inserted in the homogeneous BSE and verified to be a correct solution indeed.

The coupling constants and the masses of the exchanged mesons are taken to be the same as in Ref. 1, except for the cutoff parameter Λ^2, which has been changed to 1.9 in order to fit the binding energy of the deuteron at 2.223 MeV. The phase shifts were recalculated and are given in Table I for the ${}^3S_1$-${}^3D_1$, ${}^1S_0$, and ${}^3P_0$ channels at various lab energies. We see that the phase shifts do not deviate significantly from the results for $\Lambda^2=1.8$ reported in Ref. 1. The positive-energy components of the wave function, i.e., $S_n(p,P)\phi_n(|\vec{p}|,p_0)$, can be compared with those obtained from a nonrelativistic calculation. In Figs. 2 and 3 these components are shown for different values of $p_4=-ip_0$, together with the wave functions of the Reid soft-core potential. The behavior as a function of p_4 is rather smooth, while the most pronounced p dependence occurs when p_4 vanishes. The structure for $p_4=0$ is very similar

TABLE I. Phase shifts with the same coupling parameters as in Ref. 1, except that Λ^2 has been changed to 1.9.

E (MeV)	${}^3S_1$	${}^3D_1$	ϵ_1	${}^1S_0$	${}^3P_0$
25	80.0	−2.96	1.60	51.9	8.93
50	61.1	−6.78	1.48	41.0	11.43
100	40.6	−12.82	1.03	26.6	8.48
150	27.3	−17.07	0.59	16.2	2.83
200	17.3	−20.08	0.287	7.9	−3.21
250	9.1	−22.21	0.034	0.9	−9.11

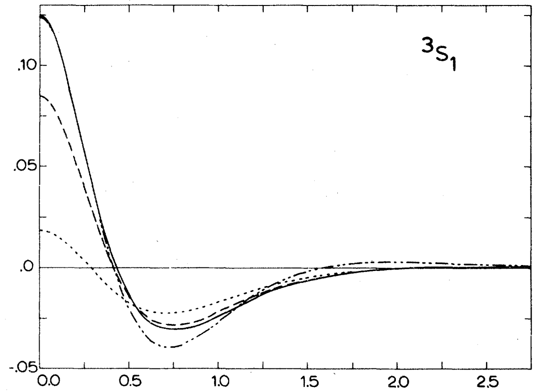

FIG. 2. The $^3S_1^+$ component of the deuteron wave function is shown for $p_4=0$ (———), $p_4=0.255$ (— — — —), $p_4=0.543$ (· · · ·), where p_4 is in units of nucleon mass. For comparison we also plotted the corresponding Reid wave function (— · · — · ·).

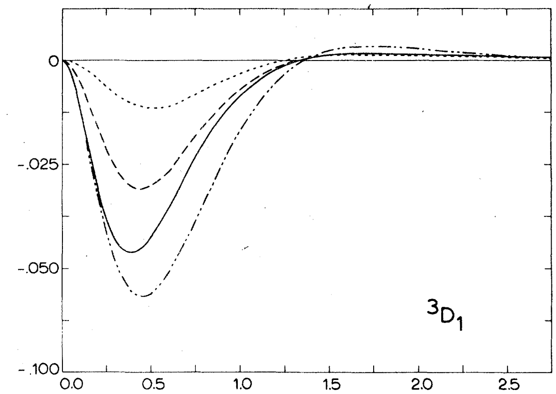

FIG. 3. The same as in Fig. 2 for the $^3D_1^+$ component of the wave function.

to that obtained for the Reid interaction, except that the d-wave component is smaller. Considering the relativistic components, the wave function is localized in p space around 0.3 like the d-wave component. This is shown in Figs. 4 and 5. Note that the wave function in the channels 7 and 8 vanishes for $p_4=0$, being an odd function of p_4.

Other interesting quantities are the probabilities of the various components. They cannot be defined in the basis (2.13) because the propagator is not diagonal, and we have to go back to the ρ-spin basis labeled by (ρ_1, ρ_2), which means that we diagonalize the propagator in the channels 5 through 8. The propagator is then the same as in (2.8). In view of the normalization condition (2.7), we define the n-state probability as

$$P_n=\frac{1}{8\pi^2 M_D}\int dp_4 dp\, p^2[\partial S(p,p_4,n)/\partial E]_{E=M_D/2}[\phi_n(p,p_4)]^2$$
$$=\frac{1}{4\pi^2 M_D}\int dp_4 dp\, p^2 \omega_n(E_p)[S(p,p_4,n)\phi_n(p,p_4)]^2\,, \tag{2.14}$$

where the weight function ω_n depends only on the ρ spin and the energy E_p:

$$\omega_{++}=E_p-E;\ \omega_{--}=-E_p-E;\ \omega_{+-}=\omega_{-+}=-E\,. \tag{2.15}$$

It is clear that the "probabilities" for the states containing negative-energy components are negative, which is readily understood if we realize that these numbers measure the effective charge of the state. As we will see in Sec. III, the normalization condition coincides with the matrix element of the charge operator at zero momentum transfer. The numerical values are given in Table II, together with the corresponding Reid results and experimental data. We find very small probabilities for the relativistic components, which can be attributed to the use of axial-vector coupling for the πN system. The d-state probability of the BSE wave function is lower than for the Reid wave function, but essentially in agreement with some recent quasipotential one-boson-exchange model calculations.[12] The calculation of the quadrupole moment yields essentially the same value for the Reid wave function and for the BSE, being too small as compared with experiment, while the magnetic moment is substantially higher for the BSE.

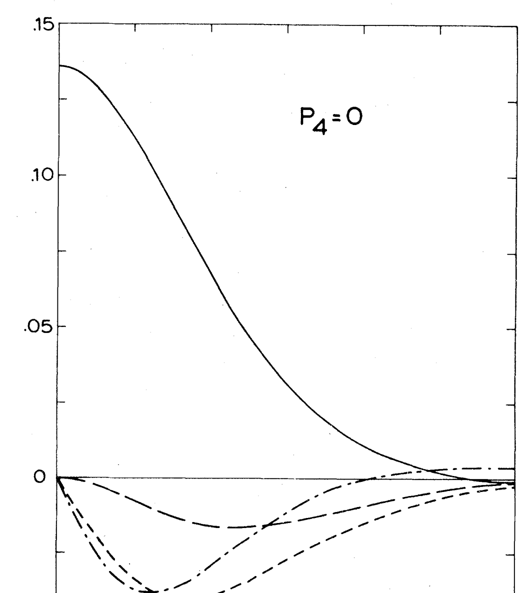

FIG. 4. The relativistic components of the deuteron wave function for $p_4=0$: $^3S_1^-$ (———), $^3D_1^-$ (— —), $^1P_1^e$ (— — — —), and $^3P_1^o$ (— · — ·). The $^1P_1^o$ and $^3P_1^e$ components are odd functions of p_4 so that they vanish in this case.

We have also determined the asymptotic D/S state ratio,[13] defined by

$$\rho_D = [\phi_2(p_4, p)/\phi_1(p_4, p)]_{E_p = M_D/2, p_4 = 0}, \tag{2.16}$$

i.e., the ratio of the $^3D_1^+$ over the $^3S_1^+$ component of the vertex function with both nucleons on the mass shell. We have to extrapolate this ratio to the unphysical point $\vec{p}^2/M_N^2 = -1 + M_D^2/4M_N^2 \cong -0.00237$. The calculated value is in good agreement with the recently obtained experimental value.

III. THE DEUTERON CURRENT

Having determined the bound state wave functions in the c.m. frame, we can in principle calculate the deuteron current matrix elements and as a result the electromagnetic (e.m.) form factors. In the impulse approximation, shown in Fig. 6, we have

$$\langle P'M' | J_\mu^D | PM\rangle = \frac{ie}{16\pi^3 M_D} \int d^4p \Big[\bar{\psi}^{(M')}(p'; P') S^{(1)}\left(\frac{P'}{2} + p'\right) \Gamma_\mu^{(1)}(q) S(p, P)\psi^{(M)}(p; P)$$
$$+ \bar{\psi}^{(M')}(p''; P') S^{(2)}\left(\frac{P'}{2} - p''\right) \Gamma_\mu^{(2)}(q) S(p, P)\psi^{(M)}(p; P)\Big], \tag{3.1}$$

where $p' = p + q/2$, $p'' = p - q/2$, $P' = P + q$. The photon-nucleon vertex function Γ_μ is assumed to be of the on-shell form:

$$\Gamma_\mu(q) = \gamma_\mu F_1(q^2) - \frac{1}{2M_N} \sigma_{\mu\nu} q^\nu F_2(q^2). \tag{3.2}$$

Since the deuteron is an $I = 0$ object, we only need the isoscalar part of the nucleon form factors. In the numerical calculations we have used the phenomenological dipole fit for the charge form factor,[14] and the fit by Iachello, Jackson, and Landé[15] to the magnetic form factor. Using the normalization condition for the deuteron vertex function, and the identity

$$\frac{\partial}{\partial P_\mu} S(p, P) = S(p, P)\left[\frac{\partial S^{-1}(p, P)}{\partial P_\mu}\right] S(p, P)$$
$$= \frac{1}{2}\Big[S^{(1)}\left(\frac{P}{2} + p\right)\gamma^{\mu(1)} S(p, P)$$
$$+ S^{(2)}\left(\frac{P}{2} - p\right)\gamma^{\mu(2)} S(p, P)\Big], \tag{3.3}$$

we find that, for $q \to 0$, the current is normalized as in Eq. (A4).

Due to the invariance of the Bethe-Salpeter equation for the exchange of particle one and particle two,[9] the two terms on the right hand side of Eq. (3.1) are identical, so that

$$\langle P'M' | J_\mu^D | PM\rangle$$
$$= \frac{ie}{8\pi^3 M_D} \int d^4p \Big[\bar{\psi}^{(M')}(p'; P') S^{(1)}\left(\frac{P'}{2} + p'\right)$$
$$\times \Gamma_\mu^{(1)}(q) S(p, P)\psi^{(M)}(p; P)\Big]. \tag{3.4}$$

It should be noted that the isoscalar current defined in this way is gauge invariant:

$$q^\mu \langle P'M' | J_\mu^D | PM\rangle = 0. \tag{3.5}$$

The proof is given in Appendix C.

From Eq. (3.4) it is clear that we need the deuteron vertex function in a general moving frame. The relation between vertex functions in different frames is given by[16]

$$\psi^{(M)}(p, P) = \Lambda^{(1)}(\mathfrak{L})\Lambda^{(2)}(\mathfrak{L})\psi^{(M)}(\mathfrak{L}^{-1}p, \mathfrak{L}^{-1}P), \tag{3.6}$$

where Λ is the operator for spin $\frac{1}{2}$ particles corresponding to the Lorentz transformation $\mathfrak{L}$. In practice we only need boost transformations along

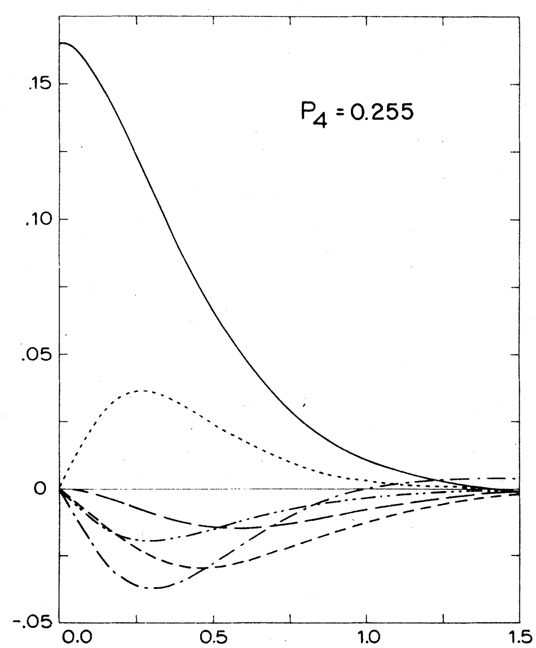

FIG. 5. The same as Fig. 4 for $p_4 = 0.255$, the $^1P_1^o$ (— •• — ••) and $^3P_1^e$ (• • • •) components are now different from zero.

TABLE II. Static properties of the deuteron in this model, compared with experiment and the results for the Reid interaction.

	P_{D^+}	P_{D^-}	$P_{^3P_1}$	$P_{^1P_1}$	$\rho_{D/S}$	Q (mb)	$\mu_D\left(\frac{e}{2M_N}\right)$
exp					0.0263(13)	2.86	0.857406
BSE	4.8	-6×10^{-4}	-8.6×10^{-6}	-2.5×10^{-2}	0.0258	2.76	0.8655
Peid	6.4				0.0260	2.77	0.8380

the z axis, with momentum P.

$$\Lambda(\mathfrak{L})=\left(\frac{E_D+M_D}{2M_D}\right)^{1/2}\left(1+\gamma^0\gamma^3\frac{P}{E_D+M_D}\right),$$
$$E_D=(M_D{}^2+P^2)^{1/2}. \tag{3.7}$$

Together with the transformation properties of the kernel and the propagators:

$$K(p',p)=\Lambda(\mathfrak{L})K(\mathfrak{L}^{-1}p',\mathfrak{L}^{-1}p)\Lambda^{-1}(\mathfrak{L}),$$
$$S(p,P)=\Lambda(\mathfrak{L})S(\mathfrak{L}^{-1}p,\mathfrak{L}^{-1}P)\Lambda^{-1}(\mathfrak{L}), \tag{3.8}$$
$$\Lambda(\mathfrak{L})=\Lambda^{(1)}(\mathfrak{L})\Lambda^{(2)}(\mathfrak{L}),$$

it is straightforward to show that the BSE is covariant.

The matrix elements of the deuteron current are calculated in the Breit frame, i.e., $\vec{P}'+\vec{P}=0$. This means that we have $\vec{P}'=-\vec{P}=\vec{q}/2$, and $q_0=0$. We choose $\vec{q}$ along the (positive) z axis and use Eqs. (3.6) through (3.8) to rewrite the current in terms of c.m. quantities:

$$\langle P',M'|J_\mu^D|P,M\rangle=\frac{ie}{8\pi^3M_D}\int d^4k\bar{\psi}^{(M')}(k';P_{\text{c.m.}})S^{(1)}\left(\frac{P_{\text{c.m.}}}{2}+k'\right)\times\tilde{\Gamma}_\mu(q)S(k,P_{\text{c.m.}})\psi^{(M)}(k;P_{\text{c.m.}}), \tag{3.9}$$

where

$$\tilde{\Gamma}_\mu(q)=\Lambda^{-1}(\mathfrak{L}')\Gamma_\mu^{(1)}(q)\Lambda(\mathfrak{L}). \tag{3.10}$$

In Eq. (3.9) we have introduced $k'=\mathfrak{L}'^{-1}p'$, $k=\mathfrak{L}^{-1}p$ and changed the integration variable to k, using $|\det\mathfrak{L}|=1$. $\mathfrak{L}'$ and $\mathfrak{L}$ are defined by

$$\mathfrak{L}'^{-1}P'=\mathfrak{L}^{-1}P=(M_D,\vec{0})=P_{\text{c.m.}}. \tag{3.11}$$

In the Breit frame $\mathfrak{L}'$ equals $\mathfrak{L}^{-1}$ and $\mathfrak{L}$ can be written as

$$\mathfrak{L}_\mu{}^\nu=\begin{bmatrix}(1+\eta)^{1/2} & 0 & 0 & -\sqrt{\eta}\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ -\sqrt{\eta} & 0 & 0 & (1+\eta)^{1/2}\end{bmatrix} \tag{3.12}$$

with $\eta=\vec{q}^2/4M_D{}^2=-q^2/4M_D{}^2$. From this we find

$$k_0'=(1+2\eta)k_0-2[\eta(1+\eta)]^{1/2}k_3-\sqrt{\eta}q/2,$$
$$k_1'=k_1,\quad k_2'=k_2, \tag{3.13}$$
$$k_3'=(1+2\eta)k_3-2[\eta(1+\eta)]^{1/2}k_0+(1+\eta)^{1/2}q/2.$$

We proceed by performing the partial-wave reduction of (3.9). This is straightforward but tedious algebra, and is performed using the algebraic program SCHOONSCHIP.[17] Details are given in Appendix D. The resulting expression, in terms of the eight states introduced in the previous section, has the form

$$\langle P'M'|J_\mu^D|PM\rangle=\frac{-ie}{4\pi^2M_D}\sum_{\substack{n_1n_2\\n_3n_4}}\int_{-\infty}^{\infty}dk_0\int_0^{\infty}dk\,k^2\int_{-1}^{1}d\cos\theta[\phi_{n_1}(k',k_0')S^{(1)}_{n_1n_2}(k',k_0')\times\tilde{\Gamma}_{\mu,n_2n_3}(k',k,q)S_{n_3n_4}(k,k_0)\phi_{n_4}(k,k_0)], \tag{3.14}$$

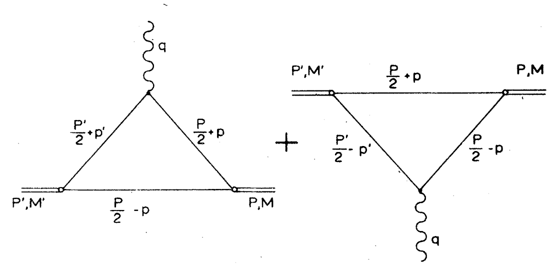

FIG. 6. The impulse approximation to the deuteron current.

where $\tilde{\Gamma}$ is given by Eq. (D4). Due to the fact that k' is a function of $\cos\theta$, the integration over $\cos\theta$ cannot be carried out analytically, and as a consequence we have to evaluate numerically a three-dimensional integral. The remaining sections are concerned with the evaluation of Eq. (3.14).

IV. STATIC APPROXIMATION

Solving the Bethe-Salpeter equation we performed a Wick rotation in the relative-energy plane. This implies that we obtain a vertex function along the imaginary relative energy axis. Now we can proceed in various ways. For example, one can calculate the vertex function along the real p_0 axis, but this is a rather tedious procedure to carry out numerically because the vertex function has an infinite number of branch points along the real axis. From the BSE it can readily be seen that the branch points of the c.m. vertex function are located in the intervals

$$\{-\infty, M_D/2 - [(M_N+\mu)^2+\vec{p}^2]^{1/2}\}$$

and

$$\{-M_D/2+[(M_N+\mu)^2+\vec{p}^2]^{1/2}, \infty\},$$

where μ is the pion mass and $\vec{p}$ the relative momentum. Another possibility is to perform a Wick rotation in the expression for the deuteron current. To see whether this is feasible or not, we have to analyze the singularities in the integrand of Eq. (3.14). These singularities occur in the propagators and the deuteron vertex function. For real $|\vec{k}|$ values the propagators have poles on the real k_0 axis. The position of the poles in the two-particle propagator $S(k, P_{c.m.})$ are given by

$$k_0 = \pm M_D/2 + (M_N^2+\vec{k}^2)^{1/2} - i\epsilon$$

and

$$k_0 = \pm M_D/2 - (M_N^2+\vec{k}^2)^{1/2} + i\epsilon \tag{4.1}$$

while the one-particle propagator $S^{(1)}(P_{c.m.}/2+k')$ has poles at

$$k_0 = -(1+4\eta)\frac{M_D}{2} \pm [(M_N^2+\vec{k}^2+2\alpha M_D k_3+\alpha^2 M_D^2)^{1/2} - i\epsilon] \tag{4.2}$$

with $\alpha = 2[\eta(1+\eta)]^{1/2}$.

Taking the boost transformation in the arguments of the outgoing vertex functions into account, we have additional branch points in the k_0 variable occurring in the intervals

$$\left\{-\infty, \frac{M_D}{2} - [(M_N+\mu)^2+\vec{k}^2]^{1/2}\right\}$$

and

$$\left\{-(1+4\eta)\frac{M_D}{2} + [(M_N+\mu)^2+\vec{k}^2+2\alpha M_D k_3+\alpha^2 M_D^2]^{1/2}, \infty\right\}. \tag{4.3}$$

From the location of the singularities we see that the right hand pole of the one-particle propagator and the right hand cut of the outgoing vertex function can cross the imaginary k_0 axis for certain values of q^2. All other singularities are always on the same side of the imaginary axis.

First of all we note that there are no pinching singularities. This can be seen in the following way. The lowest right hand singularity that crosses the imaginary axis is the pole in the one-particle propagator. If this does not pinch with the highest left hand singularity the pole at $k_0 = M_D/2 - (M_N^2+\vec{k}^2)^{1/2}$ of the two-particle propagator, there are no further possibilities for pinching. The condition for the poles to coincide is

$$(1+2\eta)M_D - (M_N^2+\vec{k}^2)^{1/2} - (M_N^2+\vec{k}^2+2\alpha M_D k_3+\alpha^2 M_D^2)^{1/2} = 0. \tag{4.4}$$

However, since

$$(M_N^2+\vec{k}^2)^{1/2} + (M_N^2+\vec{k}^2+2\alpha M_D k_3+\alpha^2 M_D^2)^{1/2} \geq (4M_N^2+\alpha^2 M_D^2)^{1/2} > (1+2\eta)M_D, \tag{4.5}$$

this condition is never fulfilled.

At first sight it might seem that there are also singularities in $\cos\theta' = k_3'/|\vec{k}'|$ and $E_{k'} = (M_N^2+\vec{k}'^2)^{1/2}$, which arise when k_3' and k' become complex. These singularities are spurious, however, since they arise from the introduction of a complete set of helicity spinors. In particular, in view of

$$\sum_{\rho\lambda} V_\lambda^\rho(\vec{k}) V_\lambda^\rho(\vec{k}) = 1 \tag{4.6}$$

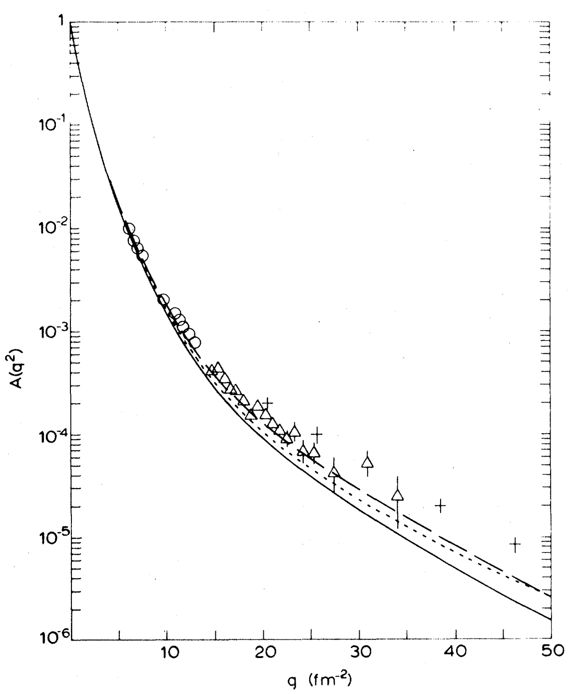

FIG. 7. $A(q^2)$, Eq. (A8), in the static approximation (· · · ·) and the corrections included (——). For comparison also the Reid result is shown (— —). The data labeled by ○, △, and + are from Refs. 18, 19, and 20, respectively.

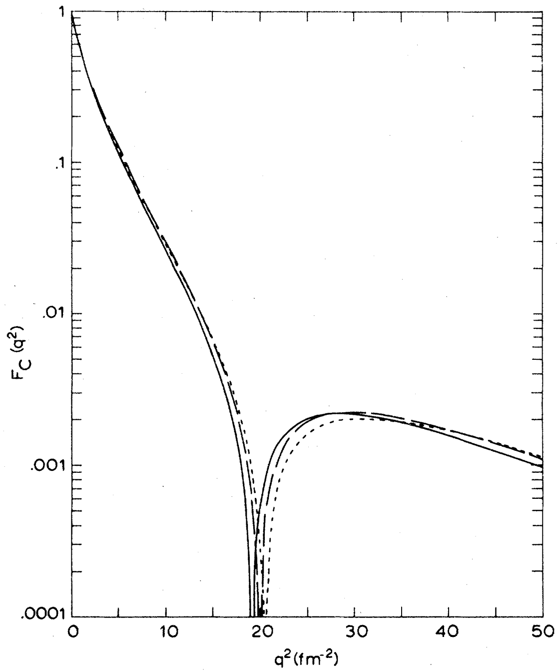

FIG. 8. $F_C(q^2)$, the same as in Fig. 7.

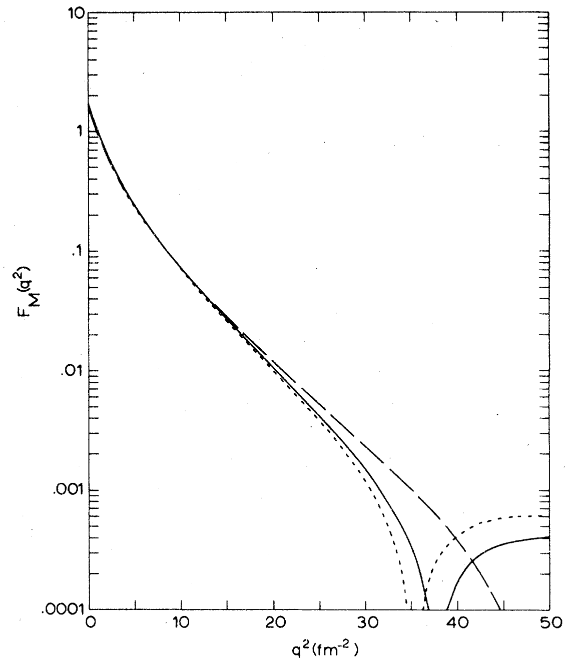

FIG. 10. $F_M(q^2)$ in units of $1/M_N$, the same as in Fig. 7.

all these singularities cancel when the summation over all channels is carried out, and as a result we do not have to consider them.

From the above discussion we conclude that as far as the singularities are concerned the Wick rotation can be carried out. Rotating the integration contour to the imaginary k_0 axis gives rise to additional contributions from the poles and branch cuts crossing the imaginary axis. As a first approximation we neglect the effect of the boost transformation on the arguments k' given by Eq. (3.13). In so doing, no singularities cross the imaginary axis anymore and we may simply replace k_0 by ik_4 in Eq. (3.14), with k_4 real. The so-called "static approximation" is obtained by neglecting the negative-energy-spin states, i.e., keeping only the ${}^3S_1^+$ and ${}^3D_1^+$ channels. Our results, calculated in this approximation, can be compared with a nonrelativistic calculation, such as for the Reid potential. As can be seen from Figs. 7–10 our results for the various form factors are very similar to the nonrelativistic ones.

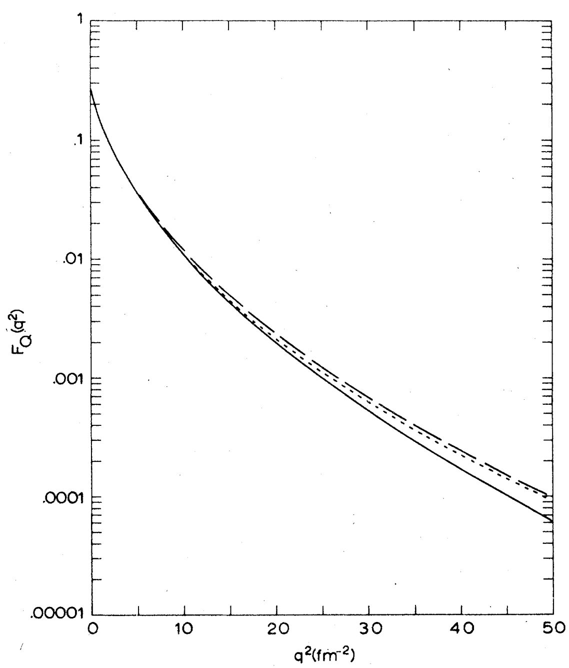

FIG. 9. $F_Q(q^2)/M_D{}^2$ in fm^2, the same as in Fig. 7.

It should be noted that not all effects of the Lorentz transformations have been neglected, since the boost operators Λ are still included in the e.m. vertex. This is done because we find a strong interference effect between the corrections from the negative-energy states and the boost operators,[21] which can be understood by noting that the boost operator mixes the different states.

Figure 11 shows the tensor polarization[22] for electron-deuteron scattering:

$$P(q^2)=\frac{4\sqrt{2}}{3}\,\eta\,\frac{F_C F_Q+\frac{1}{3}\eta F_Q{}^2}{F_C{}^2+\frac{8}{9}\eta^2 F_Q{}^2}\;. \tag{4.7}$$

The behavior in the static approximation is sim-

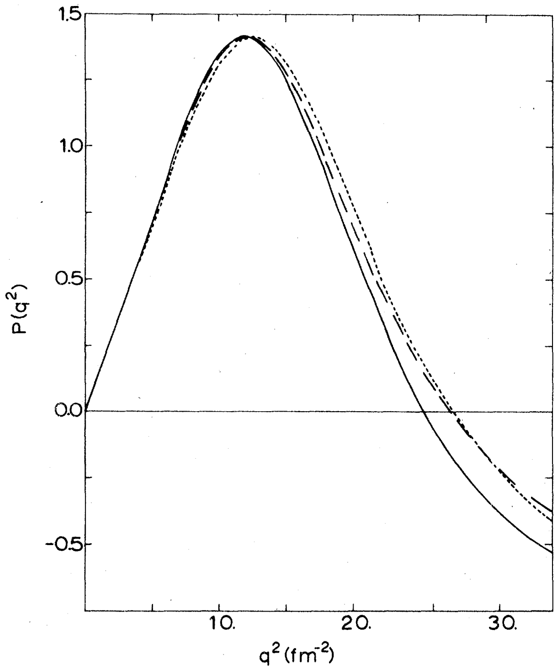

FIG. 11. The tensor polarization $P(q^2)$, the same as in Fig. 7.

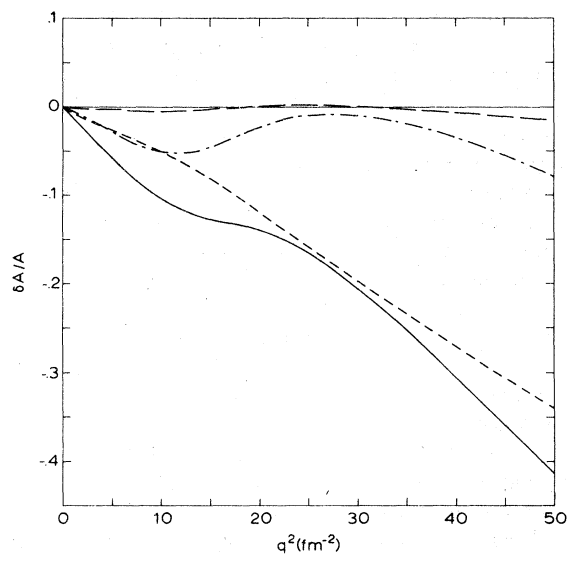

FIG. 12. The relative corrections to $A(q^2)$ in the static approximation. The correction from the channels containing negative-energy states (— —) is small. The dominant corrections come from the boost on the one-particle propagator (— — — —), while for low momentum transfer also the boost on the argument of the wave function gives a significant contribution (— • — •).

ilar to the result for the Reid potential, and the corrections to the static approximation, which are discussed in the next section, do not alter this for momentum transfers up to 20 fm^{-2}.

V. CORRECTIONS TO THE STATIC APPROXIMATION

In this section we investigate the effects of the approximations made in the previous section. Three kinds of corrections to the static approximation can be distinguished: (i) contributions from the negative-energy spinor states, (ii) corrections to the arguments of the e.m. and deuteron vertex functions due to the boost transformation, and (iii) boost effects on the one-particle propagator.

First we consider the negative-energy states. For the electric form factor $A(q^2)$ the relative correction is less than 5% up to $q^2 \simeq 50$ fm^{-2} as is shown in Fig. 12. The contributions from the negative-energy states can also be determined by constructing the effective two-body e.m. operator using a lowest order perturbation calculation for the case of axial-vector πN coupling. The effective two-body charge operator due to the $N\bar{N}$ states, has the following form in the static limit:

$$j_0^{N\bar{N}} = \frac{g^2}{16 M_N^3 \omega^2}\left[(\vec{\sigma}\cdot\vec{k}_\pi)\frac{q^2}{M_N^2} + (\vec{\sigma}\cdot\vec{q})\frac{k_\pi^2}{M_N^2} - (\vec{\sigma}\cdot\vec{q})\frac{\vec{k}_\pi\cdot\vec{q}}{M_N^2}\right]^{(1)} (\vec{\sigma}\cdot\vec{k}_\pi)^{(2)} \tau^{(1)}\cdot\tau^{(2)} ,$$

where ω, g, $\vec{k}_\pi$, and $\vec{q}$ are, respectively, the pion energy, the pion-nucleon coupling constant, the pion momentum, and the momentum transferred to the two-nucleon system by the electron. This expression is only valid for isospin zero initial and final states. In other cases there will also be a contribution from the contact term which is due to the use of a derivative pion-nucleon coupling. The charge operator was calculated using the prescriptions of Chemtob and Rho.[23]

The results from both calculations are virtually in agreement, showing that the contributions from the negative-energy spinors are small in an axial-vector theory.

To account for the effects of the boost on the arguments we expand the vertex function and the matrix elements of the e.m. operator in a Taylor series around $\vec{k}' = \vec{k} + \vec{q}/2$ and $k_0' = k_0$, which is in fact an expression in powers of $q/2M_D$. We only calculated the corrections using the ${}^3S_1^+$ and ${}^3D_1^+$ channels, because the contributions from the other channels are already small. For the expansion of the vertex function up to first order we have

$$\psi_\alpha(k_0',k')=\psi_\alpha(k_0,|\vec{k}+\vec{q}/2|)+\left[2\eta k_0\frac{\partial}{\partial k_0}\psi_\alpha(k_0,k)+\left(2\eta k_3+\frac{\eta}{1+(1+\eta)^{1/2}}\frac{q}{2}\right)\cos\theta'\frac{\partial}{\partial k}\psi_\alpha(k_0,k)\right]_{k=|\vec{k}+\vec{q}/2|}$$
$$+\left\{-(\alpha k_3+2\eta E)\frac{\partial}{\partial k_0}\psi_\alpha(k_0,k)-\alpha k_0\cos\theta'\frac{\partial}{\partial k}\psi_\alpha(k_0,k)\right\}_{k=|\vec{k}+\vec{q}/2|}. \tag{5.1}$$

The terms in square brackets are even in k_0, while the terms in curly brackets are odd. Similarly we find for the e.m. vertex operator

$$\Gamma_\mu(\vec{k}',\vec{k})=\Gamma_\mu(\vec{k}+\vec{q}/2,\vec{k})+\left[2\eta k_3+\frac{\eta}{1+(1+\eta)^{1/2}}\frac{q}{2}\right]\frac{\partial\Gamma_\mu}{\partial k_3'}\bigg|_{k_3'=k_3+q/2}+\{-\alpha k_0\}\frac{\partial\Gamma_\mu}{\partial k_3'}\bigg|_{k_3'=k_3+q/2}. \tag{5.2}$$

The separation in even and odd parts in k_0 has been done because the integration over k_0 in Eq. (3.14) can, using Eqs. (5.1) and (5.2) and the decomposition of the one-particle propagator in even and odd parts, readily be reduced to an integration over only positive values of k_0 in the two-channel approximation. In the expressions (5.1) and (5.2) we need to know the derivatives of Γ_μ and ψ. For Γ_μ this is calculated numerically by taking the difference:

$$\frac{\partial\Gamma_\mu}{\partial k_3'}\bigg|_{k_3'=k_3+q/2}=\Gamma_\mu\left(\vec{k}+\frac{\vec{q}}{2}+\epsilon\hat{z},\vec{k}\right)-\Gamma_\mu\left(\vec{k}+\frac{\vec{q}}{2},\vec{k}\right)\Big/\epsilon. \tag{5.3}$$

Since we know the solution ψ of the BSE only at discretized meshpoints, its derivatives have been calculated by taking the derivatives of the kernel with respect to the initial momenta and then iterating the integral equation over; i.e., ψ' is calculated using $\psi'=K'S\psi$ rather than by interpolation of the wave function. In this way the derivatives could be determined with a much higher accuracy. Our results were stable for $\epsilon\simeq10^{-3}$ with a numerical error of less than 1% due to the evaluation of the derivatives.

The relative correction from Eqs. (5.1) and (5.2) to $A(q^2)$ is shown in Fig. 12. We find that the contribution is significant for low q^2, but at higher q^2 it is much smaller than the additional contributions from the corrections to the one-particle propagator. For $q^2<(2M_N-M_D)M_D\simeq0.107$ fm^{-2} the latter corrections can be calculated simply by taking the correct one-particle propagator into account in the Wick-rotated integral for the deuteron current. For higher values of q^2 the pole of the one-particle propagator can cross the imaginary axis in the k_0 plane. To calculate the residue of the pole, we assume that we can expand the deuteron vertex function in a Taylor series around $k_0=0$, and keep only the first term.

It should be noted that the same pole also causes an integrable singularity in the Wick-rotated integral. In order to get numerically stable results we performed a subtraction of the pole. This is done by subtracting and adding a term to the Wick-rotated integrand of Eq. (3.14) of the form

$$(ik_4-k_p^0)^{-1}[(E-E_k)^2+k_4^2]^{-1}\left[\sum_{\alpha',\alpha}\psi_{\alpha'}\left(0,\left|\vec{k}+\frac{\vec{q}}{2}\right|\right)\mathrm{Res}(k_p^0)\Gamma^\mu_{\alpha',\alpha}\left(\vec{k}+\frac{\vec{q}}{2},\vec{k}\right)\psi_\alpha(0,|\vec{k}|)\right], \tag{5.4}$$

where k_p^0 and $\mathrm{Res}(k_p^0)$ are the position and residue of the one-particle propagator pole. The added term gives rise to an integral over k_4, which can be evaluated analytically. It is given by

$$\int_{-\infty}^{\infty}dk_4\{(ik_4-k_p^0)[(E-E_k)^2+k_4^2]\}^{-1}=-\mathrm{sgn}(k_p^0)\frac{\pi}{(E_k-E)(E_k-E+|k_p^0|)}. \tag{5.5}$$

The correction to $A(q^2)$ from the full one-particle propagator is shown in Fig. 12. We see that of the three corrections it is the dominant one. At $q^2\simeq50$ fm^{-2} it is about -50%. In our calculations we have neglected the corrections arising from the intersection of the cuts of the outgoing vertex function with the imaginary k_0 axis. They are expected to be small since a simple estimate shows that they occur for $k>4$ fm^{-1} so that they are suppressed by the incomcing vertex function. The pole of the one-particle propagator on the other hand already begins to contribute for $k>0.3$ fm^{-1}, where the vertex function is still appreciable.

VI. CONCLUDING REMARKS

In the previous sections we have presented the results of a relativistic covariant calculation of the e.m. form factors of the deuteron. It was shown that the boost transformation in the arguments of the final-state vertex function and the negative-energy spin states give relatively small corrections to the static calculation. As a result the corrections can in principle be treated in a perturbative way. On the other hand, the contribution from the one-particle propagator is not very small. Hence one should consider more reliable possibilities of taking this contribution into account. In particular, the zeroth order approximation of the wave function needed to calculate the pole contribution should be improved on. Also a good estimate of the contribution from the displaced cuts of the wave function is needed. One possibility is to carry out the Wick rotation in such a way that the path of integration in the k_0 plane is shifted with respect to the imaginary axis, so that the singularities are circumvented. It is clear that such a calculation will be complicated because of the choice of the integration path is $\vec{k}$ dependent and the vertex function will be needed for complex arguments.

Concerning the deuteron e.m. form factors, we find that our results are analogous to the nonrelativistic results. Although in our calculations we have included the relativistic effects and pair-excitation contributions, our results are distinct different from the lowest order perturbation analysis of the isoscalar exchange-current effects. In a previous paper[3] it was argued, in a relativistic quasipotential model, that these differences can be ascribed to the neglect of the consistent treatment of the dynamics of the two-particle system and the e.m. properties, in the conventional approach, using the nonrelativistic situation as a starting point. Also the shift of the dip of the magnetic form factor is opposite to the result of Gari and Hyuga,[24] who find that the dip at 40–45 fm^{-2} disappears completely, due to mesonic corrections. In this connection it would be interesting to determine the magnetic form factor in the region of 35–60 fm^{-2}, since exchange effects seem to be significant in this momentum transfer region.

Although in our calculations of the parameters of the nucleon-nucleon interaction we have not performed a detailed χ^2 fit to the scattering data, we believe that this does not invalidate our conclusion that the pair-excitation term is not as important as generally accepted. To find agreement between, for example, the theoretically determined value of $A(q^2)$ and experiment, especially at higher momentum transfer, rather different mechanisms are needed than the ones considered here.

The work of M. J. Z. was supported by de Stichting voor Fundamenteel Onderzoek der Materie (FOM).

APPENDIX A. THE MATRIX ELEMENTS OF THE DEUTERON CURRENT

The procedure to relate the deuteron-current matrix elements to the invariant form factors is well known. From covariance considerations one immediately infers that the current should have the following form:

$$\langle P', M' | J_\mu^D | P, M\rangle = -\frac{e}{2M_D} e_\rho^*(\vec{P}', M') J_\mu^{\rho\sigma} e_\sigma(\vec{P}, M), \tag{A1}$$

where M_D is the deuteron mass and the spin 1 polarization vectors are defined by the relations

$$\begin{aligned} &e_\mu^*(\vec{P}, M) e^\mu(\vec{P}, M') = -\delta_{MM'}, \\ &\sum_M e_\mu^*(\vec{P}, M) e_\nu(\vec{P}, M) = -g_{\mu\nu} + \frac{P_\mu P_\nu}{M_D^2}, \\ &P_\mu e^\mu(\vec{P}, M) = 0. \end{aligned} \tag{A2}$$

From Lorentz covariance and time-reversal invariance we get

$$\begin{aligned} J_\mu^{\rho\sigma} = {} & (P'_\mu + P_\mu)\left[g^{\rho\sigma} F_1(q^2) - \frac{q^\rho q^\sigma}{2M_D^2} F_2(q^2)\right] \\ & + I_{\mu\nu}^{\rho\sigma} q^\nu G_1(q^2) \end{aligned} \tag{A3}$$

where F_1, F_2, and G_1 are invariant functions of the momentum transfer q, and $I_{\mu\nu}^{\rho\sigma}$ is the infinitesimal generator of the Lorentz transformations. For $q \to 0$ the current matrix elements are normalized as

$$\langle P, M' | J_\mu^D | P, M\rangle = e\frac{P_\mu}{M_D} \delta_{M'M}. \tag{A4}$$

Instead of the form factors F_1, F_2, and G_1 used in Eq. (A3), one often introduces the charge, quadrupole, and magnetic form factors, given by

$$\begin{aligned} &F_C = F_1 + \tfrac{2}{3}\eta(F_1 + (1+\eta)F_2 + G_1), \\ &F_Q = F_1 + (1+\eta)F_2 + G_1, \\ &F_M = G_1, \end{aligned} \tag{A5}$$

where $\eta = -q^2/4M_D^2$. They have the property that

$$F_C(0) = 1, \quad F_Q(0) = M_D^2 Q, \quad F_M(0) = \mu_D \frac{M_D}{M_N} \tag{A6}$$

where Q and μ_D are the quadrupole and magnetic moments of the deuteron. For the scattering process of unpolarized electrons on deuterons only some combinations of these form factors enter. In the limit of vanishing electron mass, the cross section is given by the well-known Rosenbluth formula

$$d\sigma = d\sigma_{\text{Mott}}\left[A(q^2) + B(q^2)\tan^2\frac{\theta}{2}\right] \tag{A7}$$

with

$$\begin{aligned} &A(q^2) = F_C^2 + \tfrac{8}{9}\eta^2 F_Q^2 + \tfrac{2}{3}\eta F_M^2, \\ &B(q^2) = \tfrac{4}{3}\eta(1+\eta) F_M^2. \end{aligned} \tag{A8}$$

The deuteron-current matrix elements are most conveniently evaluated in the Breit frame, which is defined by

$$\vec{P}+\vec{P}'=0. \tag{A9}$$

Taking $\vec{q}$ along the z-axis, Eqs. (A1)–(A3) yield

$$\langle M'|J_0^D|M\rangle = e(1+\eta)^{1/2}\{F_1\delta_{MM'}+2\eta[F_1+(1+\eta)F_2+G_1]\delta_{M',0}\delta_{M,0}\},$$

$$\langle M'|J_1^D|M\rangle = e\frac{q}{2M_D}\left(\frac{1+\eta}{2}\right)^{1/2}G_1(\delta_{M',M+1}-\delta_{M',M-1}), \tag{A10}$$

$$\langle M'|J_2^D|M\rangle = -ie\frac{q}{2M_D}\left(\frac{1+\eta}{2}\right)^{1/2}G_1(\delta_{M',M+1}+\delta_{M',M-1}),$$

$$\langle M'|J_3^D|M\rangle = 0.$$

APPENDIX B. TWO-PARTICLE HELICITY STATES

A complete basis of two-particle helicity states can be constructed from the positive- and negative-energy solutions $u_\lambda(p)$ and $v_\lambda(p)$ of the Dirac equation following the conventions of Jacob and Wick,[8] as described by Kubis[9] and by Fleischer.[25] The convention for γ matrices is that of Bjørken and Drell.[26]

We define for particle 1:

$$U_{\lambda_1}(p)=(m/E_p)^{1/2}u_{\lambda_1}(p),$$
$$W_{\lambda_1}(p)=(-1)^{1/2+\lambda_1}(m/E_p)^{1/2}\Lambda_{31}(\pi)v_{\lambda_1}(p) \tag{B1}$$

and for particle 2:

$$U_{\lambda_2}(p)=(-1)^{1/2-\lambda_2}(m/E_p)^{1/2}\Lambda_{31}(\pi)u_{\lambda_2}(p),$$
$$W_{\lambda_2}(p)=(m/E_p)^{1/2}v_{\lambda_2}(p)$$

for particles moving along the z axis. For particles moving in a direction (θ,φ), we employ a rotation:

$$R(\theta,\varphi)=\Lambda_{12}(\varphi)\Lambda_{31}(\theta)\Lambda_{12}^{-1}(\varphi), \tag{B2}$$

where Λ_{ab} is a rotation from the a axis to the b axis:

$$\Lambda_{ab}(\alpha)=\cos\alpha/2+\gamma^a\gamma^b\sin\alpha/2. \tag{B3}$$

Using the ρ-spin notation $V_\lambda^\rho(p)$, with $V_\lambda^+(p)=U_\lambda(p)$ and $V_\lambda^-(p)=W_\lambda(p)$, the normalization of the helicity states can be written as

$$V_\lambda^{\rho\dagger}(\vec{p})V_{\lambda'}^{\rho'}(\vec{p})=\delta_{\rho\rho'}\delta_{\lambda\lambda'}. \tag{B4}$$

They form a complete set:

$$\sum_{\rho,\lambda}V_\lambda^\rho(\vec{p})V_\lambda^{\rho\dagger}(\vec{p})=1. \tag{B5}$$

One readily verifies the following identity:

$$S^{(1)}(p)=(\not{p}^{(1)}-M_N+i\epsilon)^{-1} = \frac{\sum_{\lambda_1}U_{\lambda_1}(\vec{p})\bar{U}_{\lambda_1}(\vec{p})}{p_0-E_p+i\epsilon}+\frac{\sum_{\lambda_1}W_{\lambda_1}(\vec{p})\bar{W}_{\lambda_1}(\vec{p})}{p_0+E_p-i\epsilon}. \tag{B6}$$

A similar relation holds for particle two.

The Dirac spinors, in a direct-product representation, are simply found by applying a boost transformation along the z direction to the rest frame spinors:

$$u_\lambda(p)=\Lambda(p)u_\lambda(0),\quad v_\lambda(p)=\Lambda(p)v_\lambda(0),$$
$$u_\lambda(0)=\begin{pmatrix}1\\0\end{pmatrix}\chi_\lambda,\quad v_\lambda(0)=\begin{pmatrix}0\\1\end{pmatrix}\chi_{-\lambda}, \tag{B7}$$

where χ_λ is the usual Pauli spinor and $\Lambda(p)$ is given by Eq. (3.7).

APPENDIX C. GAUGE INVARIANCE OF THE DEUTERON CURRENT

In principle it is not trivial that the e.m. deuteron current, in the ladder approximation, will be conserved. We will give the proof for the isoscalar case; for the isovector case the proof goes along similar lines but one finds then, as expected, that it is necessary to include additional terms into the current to preserve gauge invariance. These terms are the γbb current (b is an isospin 1 boson) and seagull terms due to derivative couplings of the bosons and due to the use of strong meson-nucleon form factors. They arise because the τ_3 of the electromagnetic vertex operator does not commute with the isospin operator on the bNN vertex.

In the isoscalar case the proof is simple. We define

$$\phi^{(M)}(p;P)=S^{(1)}\left(\frac{P}{2}+p\right)S^{(2)}\left(\frac{P}{2}-p\right)\psi^{(M)}(p;P) \tag{C1}$$

and analogous for $\tilde{\phi}^{(M)}$. The homogeneous Bethe-Salpeter equation is then written as

$$\psi^{(M)}(p;P)=-\frac{i}{4\pi^3}\int d^4p'K(p-p')\phi^{(M)}(p';P), \tag{C2}$$

where we used the fact that, in the ladder approximation, the kernel is a function of $p-p'$ only. The inclusion of strong form factors used in our actual calculations does not alter this dependence. We note that

$$q^\mu\Gamma_\mu=F_1\not{q}=F_1\left[S^{(1)^{-1}}\left(\frac{P}{2}+p+q\right)-S^{(1)^{-1}}\left(\frac{P}{2}+p\right)\right]. \tag{C3}$$

Inserting (3.4) into (3.5) and using (C3) we find

$$q^\mu\langle M'|J_\mu^D|M\rangle = \frac{ie}{8\pi^3M_D}\int d^4p\left[\tilde{\psi}^{(M')}\left(p+\frac{q}{2};P+q\right)\phi^{(M)}(p;P) - \tilde{\phi}^{(M')}\left(p+\frac{q}{2};P+q\right)\psi^{(M)}(p;P)\right]F_1. \tag{C4}$$

In the second term we insert (C2) and use

$$-\frac{i}{4\pi^3}\int d^4p\,\tilde{\phi}^{(M')}\left(p+\frac{q}{2};P+q\right)K(p-p') = -\frac{i}{4\pi^3}\int d^4k\,\tilde{\phi}^{(M')}(k;P+q)K\left(k-p'-\frac{q}{2}\right)$$
$$=\tilde{\psi}^{(M)}\left(p'+\frac{q}{2};P+q\right), \tag{C5}$$

which proves Eq. (3.5).

APPENDIX D

In this appendix we present some details of the derivation of Eq. (3.14) for the deuteron-current matrix elements. We start from the equation for the current (3.4), use the decomposition of the propagator (B6), and insert the unit operator (B5) for particle two between $\tilde{\psi}$ and $\tilde{\Gamma}$:

$$\langle P'M'|J_\mu^D|PM\rangle=\frac{ie}{8\pi^3M_D}\int d^4k\,\tilde{\phi}^{(M')}(k',a')S^{(1)}(k_0',k',a')\tilde{\Gamma}_{\mu,a',a}(\vec{k}',\vec{k},q)S(k_0,k,a)\phi^{(M)}(k,a), \tag{D1}$$

where

$$\tilde{\Gamma}_{\mu,a',a}(\vec{k}',\vec{k},q)=\bar{V}^{\rho_1'}_{\lambda_1'}(\vec{k}')V^{\rho_2'\dagger}_{\lambda_2'}(\vec{k}')\Gamma_\mu(q)V^{\rho_1}_{\lambda_1}(\vec{k})V^{\rho_2}_{\lambda_2}(\vec{k}). \tag{D2}$$

Next we replace $\tilde{\phi}$ and ϕ by the expressions (2.6) and change to the new basis states (2.13), using the unitary transformation matrix[8]:

$$\langle J\lambda_1\lambda_2|JLS\rangle=\left(\frac{2L+1}{2J+1}\right)^{1/2}C^{LSJ}_{0\lambda\lambda}C^{1/2,1/2S}_{\lambda_1-\lambda_2\lambda}. \tag{D3}$$

We define

$$\tilde{\Gamma}_{\mu,n',n}(k',k,q)=\frac{[(2L'+1)(2L+1)]^{1/2}}{4\pi}\sum_{\substack{\lambda_1'\lambda_2'\\ \lambda_1\lambda_2}}\int d\varphi[D^{1}_{M',\lambda'}(\Omega_{\vec{k}'})C^{L'S'J}_{0\lambda'\lambda'}C^{1/2,1/2S'}_{\lambda_1'-\lambda_2'\lambda'}$$
$$\times\tilde{\Gamma}_{\mu,a',a}(\vec{k}',\vec{k},q)C^{LSJ}_{0\lambda\lambda}C^{1/2,1/2S}_{\lambda_1-\lambda_2\lambda}D^{1*}_{M,\lambda}(\Omega_{\vec{k}})]. \tag{D4}$$

Here we also have included the linear combinations ρ for the negative energy states (labeled by ρ_1 and ρ_2).

The φ integration can then be carried out immediately. For this purpose we note

$$V^{\rho_1}_{\lambda_1}(\vec{k})=\Lambda_{12}(\varphi)\Lambda_{31}(\theta)e^{i\lambda_1\varphi}V^{\rho_1}_{\lambda_1}(k),$$
$$V^{\rho_2}_{\lambda_2}(\vec{k})=\Lambda_{12}(\varphi)\Lambda_{31}(\theta)e^{-i\lambda_2\varphi}V^{\rho_2}_{\lambda_2}(k), \tag{D5}$$

and analogous for the conjugate spinors; furthermore,

$$D^J_{\mu',\mu}(\Omega)=e^{-i\mu'\varphi}d^J_{\mu',\mu}(\theta)e^{i\mu\varphi}. \tag{D6}$$

Collecting all exponentials we find $\exp[i(M-M')\varphi]$. The other φ-dependent factors can be written as

$$\Lambda_{12}^{-1}(\varphi)\tilde{\Gamma}_\mu(q)\Lambda_{12}(\varphi)=\Lambda^{-1}(\mathcal{L}')\Lambda_{12}^{-1}(\varphi)\Gamma_\mu(q)\Lambda_{12}(\varphi)\Lambda(\mathcal{L}) \tag{D7}$$

since the boosts along the z axis commute with the rotation over φ. Using Eq. (3.2) we obtain

$$\Lambda_{12}^{-1}(\varphi)\Gamma_\mu(q)\Lambda_{12}(\varphi)=\Gamma_\mu(q),\quad \mu=0,3$$
$$\Lambda_{12}^{-1}(\varphi)\Gamma_\mu(q)\Lambda_{12}(\varphi)=\Gamma_\mu(q)\Lambda_{12}^{2}(\varphi),\quad \mu=1,2$$
$$\Lambda_{12}^{2}(\varphi)=\cos\varphi+\gamma^1\gamma^2\sin\varphi. \tag{D8}$$

So that the φ integration results in

$$2\pi\delta_{M',M},\quad \text{for } \mu=0,3$$
$$2\pi\left[\left(\frac{\delta_{M',M+1}+\delta_{M',M-1}}{2}\right)-i\gamma^1\gamma^2\left(\frac{\delta_{M',M+1}-\delta_{M',M-1}}{2}\right)\right]$$
$$\text{for } \mu=1,2. \tag{D9}$$

The SCHOONSCHIP programs are organized as follows. First we calculate (D2), and perform the φ integration of (D4). Next we introduce the states $|J,L,S,\rho\rangle=|n\rangle$ as in (D4) and finally we specify M' and M to get the matrix elements needed in Eq. (3.4). In view of (D9) the expressions for the invariant functions F_i and G_1 can then immediately be obtained using the general form (A10) for the deuteron-current matrix elements.

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