g-shell admixtures and the ^{19}F(^{3}He,d)^{20}Ne*(4$^+$) reaction

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For the ^{19}F(^{3}He,d)^{20}Ne reaction we have studied the interplay between the one-step and multistep processes within the coupled-channel Born approximation. The data allow a wide variation in the contribution of the one-step 0g transfer and only yield an upper limit of 17% for the 0g intensity in the $K = 1/2$ Nilsson orbital.

[NUCLEAR REACTIONS ^{19}F(^{3}He, d)^{20}Ne*(4$^+$), E=10–23 MeV; coupled-channels Born approximation study of interplay between one-step and multistep processes.]

I. INTRODUCTION

The ^{19}F(^{3}He, d)^{20}Ne reaction to the 4.25-MeV 4$^+$ level of ^{20}Ne has been studied at bombarding energies of 10 MeV,[1] 16 MeV,[2] 21 and 23 MeV.[3] Data also exist at 18 MeV,[4] although we shall not discuss this further here, since only a seven point angular distribution was taken and it appears quite similar to the 16-MeV case. The particular interest in the 4$^+$ level arises from the fact that a direct one-step transfer from the $\frac{1}{2}^+$ ground state of ^{19}F is forbidden if ^{19}F and ^{20}Ne are assigned $(sd)^3$ and $(sd)^4$ configurations beyond the ^{16}O closed shell. In this case the reaction can only proceed via multistep processes (we imply two or more steps). It has been found[2,3] that by allowing for the inelastic excitation of the target and/or residual nucleus in the coupled channel Born approximation (CCBA) the observed data can be quite well explained. It is worth remarking that a similar conclusion was reached in studies[5,6] of the ^{19}F(α, t) ^{20}Ne and the ^{19}F(^{16}O, ^{15}N)^{20}Ne reactions leading to the 4$^+$ level.

Of course the actual wave functions of ^{19}F and ^{20}Ne will contain many complicated configurations and this will allow a direct one-step process to take place via the transfer of a $g_{9/2}$ or a $g_{7/2}$ proton. The calculations mentioned above suggest that the one-step process is of minor importance. However, Suzuki, Arima, and Kubo[7] have recently carried out distorted wave Born approximation (DWBA) calculations at $E_{^3\mathrm{He}} = 23$ MeV, including both one- and two-step processes and have concluded that they are of comparable importance.

The purpose of the present work is to reexamine the interplay of one-step and multistep processes in the CCBA. The one-step process was neglected in Ref. 3 and in Ref. 2 the form factors, i.e., wave functions, were generated in a spherical Woods-Saxon well. If the usual separation energy prescription is used, this requires a very deep well for a g particle which pulls the wave function in, thereby reducing its magnitude in the important surface region. This difficulty is obviated here by generating the appropriate Nilsson wave functions in a deformed Woods-Saxon well. The aim of the present work is to see whether the rather extensive data available place limits on the spectroscopic amplitude for the one-step transfer of a g particle. Such information is hard to obtain unambiguously elsewhere and would be valuable in nuclear structure work.

In Sec. II we give the details of the calculations. We do not employ spin-orbit terms in the optical potentials since they produce a small effect and since this allows the CCBA calculations to be greatly simplified. The formalism needed for the Penny-Satchler approach[8] has been discussed by Abdallah *et al.*[9] Here we give the equations needed for the alternative source-term method[10,11]; we remark that the (d, p) case is discussed in Ref. 10 using zero spin for the deuteron and proton, but this is for illustrative purposes and is not sufficiently general to be used in a practical case. In Sec. III we display our results and discuss their implications.

II. CALCULATION

Our CCBA calculations are carried out using the source term method,[10,11] modified to take advantage of the fact that the optical potentials are independent of the spins of the projectile and ejectile. Consider the reaction $A + a \rightarrow B + b$, where $a = b + t$ and $B = A + t$. In the $A + a$ system we define wave functions

$$\psi^{J}_{\alpha A^M A^\pi A^\mu a} = \sum_{\mathfrak{A}} \frac{f^{J A^\pi A}_{\mathfrak{A};\alpha}(r_{Aa})}{r_{Aa}} \phi^{J}_{\mathfrak{A}\, A^M A^\pi A}(\hat{r}_{Aa}, \xi_A)\psi_{s_a \mu_a}, \tag{1}$$

where $\mathfrak{A} \equiv (I_A, l_a)$ labels the channels and α

$\equiv (I_\alpha, l_\alpha)$ indicates the channel in which there are incoming waves. The channel wave functions ϕ are given by

$$\phi_{\mathfrak{A}}^{J_A M_A \pi_A}(\hat{r}_{Aa}, \xi_A) = [\psi_{I_A}(\xi_A) i^{l_a} Y_{l_a}(\hat{r}_{Aa})]_{J_A M_A} , \tag{2}$$

where the square brackets indicate that I_A and l_a are coupled to J_A with component M_A; the rest of the notation should be obvious. Note that since the optical potentials are independent of the projectile spin s_a, this is not coupled to the remainder of the wave function. The radial wave functions f may be obtained by solving the usual set of homogeneous coupled equations for the inelastic scattering[12] of a spinless particle.

For the $B+b$ system the wave functions are defined in analogous fashion to Eqs. (1) and (2). It is necessary to solve the inhomogeneous equation

$$\left[-\frac{\hbar^2}{2\mu_{Bb}}\left(\frac{d^2}{dr_{Bb}^2} - \frac{l_b(l_b+1)}{r_{Bb}^2}\right) - E + \epsilon_B\right] f_{\mathfrak{B}:\alpha J_A \pi_A}^{J_B \pi_B j_{bt}}(r_{Bb})$$

$$+ \sum_{\mathfrak{B}'} \langle \phi_{\mathfrak{B}}^{J_B M_B \pi_B} | V | \phi_{\mathfrak{B}'}^{J_B M_B \pi_B} \rangle f_{\mathfrak{B}':\alpha J_A \pi_a}^{J_B \pi_B j_{bt}}(r_{Bb})$$

$$+ r_{Bb}\, \rho_{\mathfrak{B}:\alpha J_A \pi_A}^{J_B \pi_B j_{bt}}(r_{Bb}) = 0 . \tag{3}$$

Here μ_{Bb} is the reduced mass in the exit channel, ϵ_B is the excitation energy of nucleus B in the channel $\mathfrak{B}$, and j_{bt} is the angular momentum of the transferred particle t with respect to b. Now the source term ρ is given by

$$\bar{\rho}_{\mathfrak{B}:\alpha J_A \pi_A M_A \mu_a}^{J_B \pi_B j_{bt} M_B \mu_b}(r_{Bb}) = \sum_{\mathfrak{A}} \left\langle \phi_{\mathfrak{B}}^{J_B M_B \pi_B} \psi_{s_b \mu_b} \left| V_{\text{strip}} \right| \frac{f_{\mathfrak{A}:\alpha}^{J_A \pi_A}(r_{Aa})}{r_{Aa}} \phi_{\mathfrak{A}}^{J_A M_A \pi_A} \psi_{s_a \mu_a} \right\rangle$$

$$= \sum_{j_{bt}} C(s_b \mu_b j_{bt} m_{bt}; s_a \mu_a) C(J_A M_A j_{bt} m_{bt}; J_B M_B)\, \rho_{\mathfrak{B}:\alpha J_A \pi_A}^{J_B \pi_B j_{bt}}(r_{Bb}) . \tag{4}$$

We make the usual parentage expansion

$$\psi_{I_B K_B} = \sum_{\substack{I_A l_{At} \\ j_{At}}} S_{l_{At} j_{At}}^{1/2}(I_B, I_A) \frac{U_{l_{At} j_{At}}(r_{At})}{r_{At}} [\psi_{I_A} [i^{l_{At}} Y_{l_{At}}(\hat{r}_{At}) \psi_{s_t}]_{j_{At}}]_{I_B K_B} , \tag{5}$$

where s_t is the spin of the transferred particle t. Making a similar expansion for the projectile-ejectile system, a-b, it is straightforward, though tedious, to show that

$$\rho_{\mathfrak{B}:\alpha J_A \pi_A}^{J_B \pi_B j_{bt}}(r_{Bb}) = \sum_{\substack{I_A l_a L \\ l_{bt} l_{At} j_{At}}} S_{l_{At} j_{At}}^{1/2}(I_B, I_A) S_{l_{bt} j_{bt}}^{1/2}(I_a, I_b) i^{l_{bt} - l_{At} - l_a - l_b}$$

$$\times (-1)^{j_{At} + s_t} \hat{L}^2 \hat{I}_B \hat{j}_{bt} \hat{j}_{At} \hat{J}_A \begin{Bmatrix} j_{At} & l_{At} & s_t \\ l_{bt} & j_{bt} & L \end{Bmatrix} \begin{pmatrix} I_B & l_b & J_B \\ I_A & l_a & J_A \\ j_{At} & L & j_{bt} \end{pmatrix}$$

$$\times \int \frac{U_{l_{At} j_{At}}(r_{At})}{r_{At}} V_{\text{strip}}(r_{bt}) \frac{f_{\mathfrak{A}:\alpha}^{J_A \pi_A}(r_{Aa})}{r_{Aa}} \frac{U_{l_{bt} j_{bt}}(r_{bt})}{r_{bt}}$$

$$\times [Y_{l_b}(\hat{r}_{Bb}) Y_{l_a}(\hat{r}_{Aa})]_{LM}^* [Y_{l_{At}}(\hat{r}_{At}) Y_{l_{bt}}(\hat{r}_{bt})]_{LM} d\vec{r}_{At}\, d\Omega_{Bb} , \tag{6}$$

where $\hat{J} = (2J+1)^{1/2}$. For heavy ions this expression may be evaluated exactly. However, we are concerned with the $(^3He, d)$ reaction, so it is reasonable to employ the usual zero-range approximation with $l_{bt} = 0$, $j_{bt} = s_t = \frac{1}{2}$. Expression (6) then simplifies to

$$\rho_{\mathfrak{B}:\alpha J_A \pi_A}^{J_B \pi_B 1/2}(r_{Bb}) = \sum_{\substack{I_A l_a \\ l_{At} j_{At}}} S_{l_{At} j_{At}}^{1/2}(I_B, I_A) S_{0,1/2}^{1/2}(I_a, I_b)\, i^{l_{At} + l_b - l_a} D_0 (4\pi)^{-1/2} \hat{l}_{At} \hat{l}_b \hat{I}_B \hat{j}_{At} \hat{J}_A C(l_b 0 l_{At} 0; l_a 0)$$

$$\times \begin{pmatrix} I_B & l_b & J_B \\ I_A & l_a & J_A \\ j_{At} & l_{At} & \frac{1}{2} \end{pmatrix} \frac{f_{\mathfrak{A}:\alpha}^{J_A \pi_A}(r_{Aa})}{r_{Aa}} \frac{U_{l_{At} j_{At}}(r_{Aa})}{r_{Aa}} , \tag{7}$$

where $r_{Aa} = (\mu_{Bb}/\mu_{Aa}) r_{Bb}$ and the zero-range strength D_0 is taken from Bassel.[13] Using this source term ρ, the solutions of Eq. (3) are obtained which match onto outgoing waves, i.e.,

$$f^{J_B \pi_B j_{bt}}_{\mathfrak{B}:\alpha J_A \pi_A}(r_{Bb}) \rightarrow M^{J_B \pi_B j_{bt}}_{\mathfrak{B}:\alpha J_A \pi_A}[F_{l_b}(k_B r_{Bb}) - iG_{l_b}(k_B r_{Bb})], \tag{8}$$

where F and G are the regular and irregular Coulomb functions.

In order to obtain the cross section, we first need to take the linear combination of wave functions (1) which represents a Coulomb distorted plane wave along the z axis plus outgoing spherical waves. Specifically,

$$\psi^{\mathrm{tot}}_{K_\alpha \mu_a} = \sum_{\alpha J_A \pi_A} (4\pi)^{-1/2} \frac{\hat{l}_\alpha}{k_\alpha} e^{i\sigma_\alpha} \times C(I_\alpha K_\alpha l_\alpha 0; J_A M_A) \psi^{J_A M_A \pi_A \mu_a}_\alpha, \tag{9}$$

where σ_α is the Coulomb phase shift. Carrying through the factors in this equation along with the Clebsch-Gordan coefficients from Eq. (4), the cross section may be written

$$\left(\frac{d\sigma}{d\Omega}\right)_{I_\alpha \to I_B} = (\hat{I}_\alpha \hat{j}_{bt})^{-2} \sum_{\substack{K_B K_\alpha \\ j_{bt} m_{bt}}} |g^{j_{bt} m_{bt}}_{I_B K_B I_\alpha K_\alpha}|^2,$$

where

$$g^{j_{bt} m_{bt}}_{I_B K_B I_\alpha K_\alpha} = \frac{1}{ik_\alpha} \left(\frac{v_B}{v_\alpha}\right)^{1/2} \sum_{\substack{l_\alpha l_b \\ J_A J_B}} C(I_\alpha K_\alpha l_\alpha 0; J_A M_A) C(I_B K_B l_b \lambda_b; J_B M_B) C(J_A M_A j_{bt} m_{bt}; J_B M_B) \times M^{J_B \pi_B j_{bt}}_{\mathfrak{B}:\alpha J_A \pi_A} e^{i(\sigma_\alpha + \sigma_{\mathfrak{B}})} \hat{l}_\alpha \hat{l}_b (-1)^{(\lambda_b + |\lambda_b|)} \left[\frac{(l_b - |\lambda_b|)!}{(l_b + |\lambda_b|)!}\right]^{1/2} P^{|\lambda_b|}_{l_b}(\cos\theta), \tag{10}$$

and v is the relative velocity of the particles in the channel specified. The above formalism eliminates specific reference to the spins s_a and s_b of the projectile and ejectile. This reduces the number of channels which need to be coupled, and therefore the time required for the computations is significantly decreased.

In our CCBA calculations we allow inelastic scattering among three members of the $K = \frac{1}{2}$ and $K = 0$ ground state rotational bands in ^{19}F and ^{20}Ne, respectively. Specifically, we include the $\frac{1}{2}^+$ ground state, 0.20-MeV $\frac{5}{2}^+$ and 1.56-MeV $\frac{3}{2}^+$ levels in ^{19}F and the 0^+ ground state, 1.63-MeV 2^+ and 4.25-MeV 4^+ levels in ^{20}Ne. The inelastic scattering is described by the standard deformed optical potential which is used in conjunction with macroscopic rotational wave functions. The necessary parameters were obtained[2] from fits to the elastic and inelastic data and are listed in Table 1 of Ref. 2. The real well depths here are appropriate to 16-MeV bombarding energy; for the other energies the depths were modified slightly according to the prescriptions in Refs. 14 and 15. As we have mentioned, the spin-orbit terms in the optical potentials were not included. According to Ref. 2 the shape of the 4^+ angular distribution is insenstive to the spin-orbit terms—they can change the magnitude somewhat, but this is also sensitive to several of the other optical parameters.

The spectroscopic amplitudes needed for the transfer process were taken from the rotational model expression[16]

$$S^{1/2}_{l_{At} j_{At}}(I_B, I_A) = \sqrt{2}\, \frac{\hat{I}_A}{\hat{I}_B} C(I_A \tfrac{1}{2} j_{At} - \tfrac{1}{2}; I_B 0)\, c_{n_{At} l_{At} j_{At}}(-\tfrac{1}{2}). \tag{11}$$

Here $c_{nlj}(-\frac{1}{2}) = (-1)^{l+1/2-j} c_{nlj}(\frac{1}{2})$ is the coefficient of the spherical state specified by n, l, and j in the $K = -\frac{1}{2}$ Nilsson orbital. These coefficients, along with the corresponding radial wave functions, which are needed in Eq. (7), were obtained by solving the coupled equations for a proton bound in a deformed Woods-Saxon potential. The numerical solution was obtained with the computer code BOUND,[17] which employs a relaxation technique.[18] Standard parameters were employed in the calculation: radius parameter $r_0 = 1.25$ fm, diffuseness $a = 0.65$ fm, and a spin-orbit potential of derivative Woods-Saxon form with a strength of 7.5 MeV. Deformations of $\beta_2 = 0.40$ and $\beta_4 = 0.15$ were taken from a fit to ^{20}Ne(d, d') data.[19] The real well depth (62 MeV) was obtained by fitting the binding energy of the proton. (A binding energy of 13.15 MeV was deduced for the difference in intrinsic energies between the bands in ^{19}F and ^{20}Ne using the usual rotational model formulas.[20])

The values of $|c_{nlj}|$ thus obtained are given in Table I. We predict about 4% for the $0g$ intensity, which is comparable to the figure of 3% estimated previously,[19] but rather smaller than the value of 7.5% deduced from the work of Suzuki

TABLE I. Magnitudes of the Nilsson coefficients for the $K=\frac{1}{2}$ orbital (number 6).

nlj	$1s_{1/2}$	$0d_{5/2}$	$0d_{3/2}$	$0g_{9/2}$	$0g_{7/2}$
$\lvert c_{nlj}(\frac{1}{2})\rvert$	0.4820	0.7970	0.2983	0.1792	0.0756

et al.[7] The 1*s* and 0*d* radial wave functions are very similar to those obtained using a spherical Woods-Saxon well of standard geometry with the well depth fitted to the observed binding energy. This is not the case for the 0*g* functions. The deformed-well wave functions are larger than those obtained in a spherical well in the important surface region—by a factor of ≈ 1.7 at 5 fm. This enhances the one-step cross section by a factor of ≈ 2.7 at forward angles. The wave functions obtained from the deformed well can, in fact, be reproduced quite well by a spherical well calculation provided that the radius parameter r_0 is increased to ≈ 1.5 fm. This is in agreement with the study of Suzuki *et al.*[7]

III. RESULTS

We shall concentrate here on the $^{19}F(^3He,d)^{20}Ne$ reaction to the 4.25-MeV 4^+ level of ^{20}Ne, since we have little to add to previous discussions[2,3] of the 0^+ and 2^+ angular distributions. The results obtained for the various bombarding energies between 10 and 23 MeV are shown in Fig. 1. Here our CCBA predictions are indicated by dashes, while the dot-dash curves give the angular distributions for the one-step transfer of a 0*g* proton. (Note that these curves were computed in the CCBA with only the $\frac{1}{2}^+$ to 4^+ spectroscopic amplitudes nonzero. The "pure" DWBA with a single level in each channel and appropriate optical parameters gives quite similar results.) It is clear that at 10 and 16 MeV the one-step process poorly predicts both the shape and magnitude of the data, whereas the CCBA yields quite reasonable results. At the higher energies the shape and magnitude of the one-step and CCBA predictions become more similar, but nevertheless the CCBA is obviously favored. Clearly multistep processes play a significant role here. We wish to ask what limitations the data place on the relative importance of the one-step process.

To simplify matters we note that the only 0*g* transfer of significance here is the direct one between the $\frac{1}{2}^+$ and 4^+ levels. Further, there is little sensitivity to whether a $0g_{9/2}$ or a $0g_{7/2}$ transfer is involved. Therefore we define a modified Nilsson amplitude $c'_{0g_{9/2}} \equiv c' = 0.20$, which includes the effect of the $0g_{7/2}$ transfer. We have

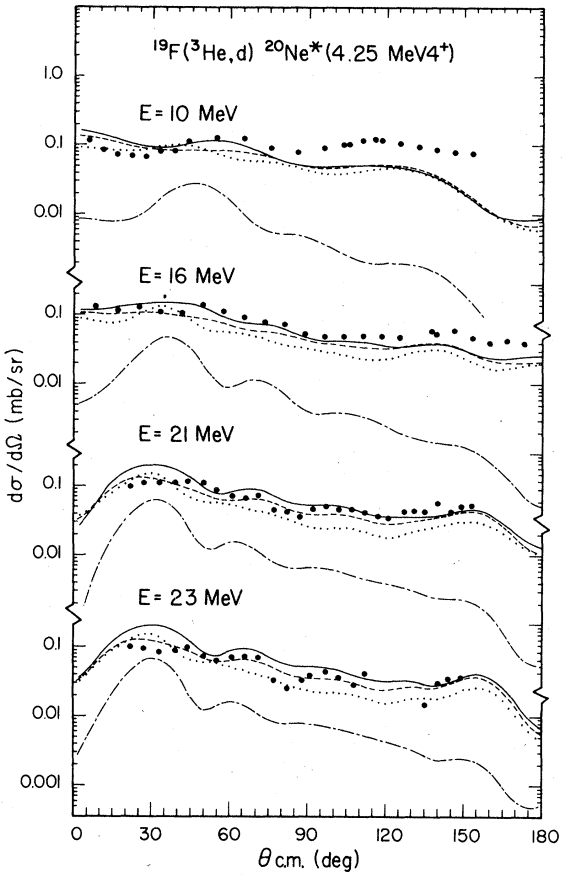

FIG. 1. Angular distributions for the $^{19}F(^3He,d)^{20}Ne^*(4^+)$ reaction at bombarding energies of 10–23 MeV. The data are compared to CCBA calculations with various values of the modified Nilsson amplitude c' appropriate to the $0g_{9/2}$ orbital: for the full curve c' is 0.0, for the dashed curve c' is 0.20, and for the dotted curve c' is 0.41. The dot-dash curve gives the angular distribution for the pure one-step transfer process using a c' of 0.20.

then varied c' and examined the effect on the angular distribution. Two extreme examples are shown in Fig. 1. In the first case (full line) we set $c'=0$ so that the reaction can only proceed by multistep processes. The form factors were actually computed using a deformed Woods-Saxon well with the basis restricted to 1*s* and 0*d* orbitals. However, the Nilsson coefficients and wave functions differ little from those discussed above. Further, very similar angular distributions are obtained if bound state wave functions from a spherical well are employed with either Nilsson model or shell model[21] spectroscopic amplitudes. Thus the nuclear structure uncertainties in the multistep predictions are slight. For the second case we have increased c' until, in our view, the fit to the data became unacceptable. This

required a c' of 0.41; the CCBA results are indicated by the dotted curves in Fig. 1.

We see from Fig. 1 that as c' is increased in the CCBA calculations, i.e., as the one-step contribution is enhanced, the predicted angular distribution changes slowly. This makes it impossible to extract a precise value by comparing to the data, particularly since the overall magnitude of the cross sections can be altered by changing the optical parameters.[2] Small values of c' between 0 and 0.20 do seem to be preferred and give a reasonably good account of the data. Certainly by the time c' has reached 0.41 the agreement with the data has significantly deteriorated with regard to both the phasing and the emergence of an unwanted forward angle peak. For this value of c' the magnitudes of the one-step and multistep processes are comparable; however, the interference between the two does not appear to allow a simple interpretation.

In their calculations Suzuki *et al.*[7] obtained an effective c' of 0.30 so that their one- and two-step DWBA cross sections were comparable. However, as we have seen, this is not required by the data, which only places an upper limit of 17% on the intensity of $0g$ configurations admixed into the $K=\frac{1}{2}$ Nilsson orbital. This conclusion was unchanged in brief studies which we carried out for the $^{19}\mathrm{F}(\alpha,t)^{20}\mathrm{Ne}$ reaction and for the present $(^3\mathrm{He},d)$ reaction at a higher bombarding energy (70 MeV).

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