Energies and branching ratios of γ transitions in ^{13}C

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The energies and relative intensities of the γ-ray transitions connecting the bound levels of ^{13}C are determined. Level energies and γ-ray branching ratios are derived from these measurements. It is proposed that a mixed ^{10}B-Pu source—emitting ^{13}C γ rays of 3683.921±0.023 and 3853.170±0.022 keV—can provide a valuable γ-ray calibration standard. The measured branching ratios are combined with previous lifetime measurements to give electromagnetic matrix elements which are compared to the predictions of a large basis shell-model calculation.

NUCLEAR REACTIONS ^{10}B(α, p)^{13}C, ^{12}C(d, p)^{13}C, E_α=1.66 MeV, E_d=2.51 MeV. measured E_γ; deduced level energies; measured branching ratios; deduced $|M(ML)|^2$ and $|M(EL)|^2$. Ge(Li) detectors.

I. INTRODUCTION

The motives for the research described in this report were twofold. First, recent calculations by one of us (D.J.M.) have treated on an equal footing the $1s$, $1p$, $2s$, $1d$, $2p$, and $1f$ shells in a shell-model description of the lighter ($A \lesssim 21$) nuclei.[1] Mass 13, and particularly ^{13}C, provides an especially convenient primary test of the wave functions generated assuming various interactions between the nucleons. The bound levels of ^{13}C and their electromagnetic decays are illustrated in Fig. 1.[2-5] In collecting the available experimental information on the ^{13}C electromagnetic matrix elements, it became clear that those corresponding to the less intense decays of the bound states of ^{13}C (the 595- and 764-keV transitions of Fig. 1) were known with considerably less precision than could be attained with current conventional techniques. Thus, we were interested in sharpening the experimental values for these more poorly known ^{13}C electromagnetic matrix elements and, then, in comparing experiment and theory for all the known matrix elements connecting the bound states.

Second, in many areas of nuclear physics and technology, there is a need for a consistent set of precisely measured γ-ray energies available from conveniently accessible sources. The work of Helmer, Greenwood, and Gehrke[6-8] and of others[9] has provided energy standards which are generally known to $\lesssim$ 10 eV and which are fairly well spaced (logarithmically) up to 3.5 MeV. Above 3.5 MeV, however, there are few available standards—especially emanating from long-lived sources.

It occurred to us that the decay of the ^{13}C 3854-keV level, producing γ rays of 3684 and 3854 keV, could provide an important addition to the available pool of γ-ray energy standards. It was shown[10] some time ago that a useful source of ^{13}C 3854- and 3684-keV γ rays could be fabricated from a mixture of ^{10}B with a long-lived α emitter such as ^{239}Pu or ^{241}Am. ^{13}C is then produced from the ^{10}B(α,p)^{13}C reaction (Q_0=4.063 MeV). Such a procedure is also used to provide a source of ^{12}C 4.44-MeV γ rays via the ^{9}Be(α,n)^{12}C reaction and of ^{16}O 6.13-MeV γ rays via the ^{13}C(α,n)^{16}O reaction. The ^{12}C 4.44-MeV level has a mean life τ of 0.061 ps[2] and is thus short-lived compared to the stopping time of nuclei in matter (of the order of 0.5–2

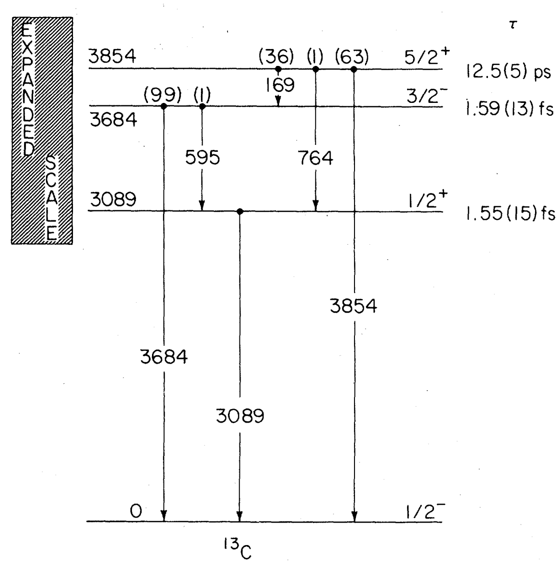

FIG. 1. The bound levels of ^{13}C and the γ transitions between them. The excitation energies and branching ratios are from the present study. The spin-parity assignments and mean lifetimes τ are from Ref. 2 except for the 3854-keV level, which is discussed in the text.

ps). For this reason the ^{12}C 4.44-MeV γ rays from, say a Be-Pu source, are Doppler broadened (by an amount ~1% or ~45 keV) and thus are not usable as an energy standard in Ge(Li) spectroscopy for which the energy resolution is ~2–4 keV at 3–6 MeV. By contrast, the ^{16}O 6.13-MeV level and the ^{13}C 3.85-MeV level have mean lives of 27 and 12 ps, respectively,[2] and thus, for properly prepared sources (no voids), Doppler broadening effects for the γ rays emitted by these levels following nuclear reactions are essentially negligible. Thus ^{10}B-Pu, as well as ^{13}C-Pu, is a potentially valuable long-lived γ-ray energy standard.

The measurement of the ^{13}C γ-ray energies is described in Sec. II. Branching ratio measurements for the 3684- and 3854-keV levels are described in Sec. III and the shell-model calculations are described and compared to experiment in Sec. IV.

II. ENERGY MEASURMENTS

A. General considerations

There are several different possible routes to precision energy measurements for the second- and third-excited states of ^{13}C at 3684 and 3854 keV. We have chosen the use of cascade-crossover relationships utilized so effectively by Greenwood *et al.*[7] In this method the crossover energy is computed from the energies of the cascade γ rays, thus enabling an extension to higher energies from known standards.

The source of all ^{13}C rays in these measurements was the ^{13}C 3854-keV level formed via ^{10}B + α or ^{12}C + d. Since the ^{13}C first and second excited states at 3089 and 3684 keV have mean lives of 1.55 ± 0.15 and 1.59 ± 0.13 fs, respectively,[2] the γ rays emitted by them after direct feeding in a nuclear reaction have the Doppler shifts and broadenings appropriate to the kinematical conditions and essentially independent of the stopping medium for the recoiling ^{13}C nuclei. In contrast, the ^{13}C 3854-keV level, with a lifetime long compared to the stopping time of ^{13}C nuclei in matter, emits γ rays with very small Doppler effects. (However, there are some and these will be considered.) So we direct our attention to those γ transitions resulting from decay of the 3854-keV level. In both ^{10}B + α and ^{12}C + d there are resonances at which the ^{13}C 3854-keV level is preferentially formed. For ^{12}C + d a deuteron beam of energy 2.51 MeV was used together with an unenriched ^{12}C target 40-keV thick to the incident deuterons in order to take advantage of the high yield from the ^{12}C(d,p_3)^{13}C resonance at 2.49 MeV, which has a center-of-mass resonance width of 37 ± 4 keV.[2,11] The target was deposited on a 0.002 54-cm thick molybdenum backing. Beam currents were in the range 10–40 nA. The ^{12}C + d resonance was quite useful for several aspects of this study but the ^{10}B + α resonance at 1.645 MeV,[2] previously used for ^{13}C branching ratio measurements at this laboratory,[12,13] produced a considerably cleaner source of radiations from the 3854-keV level.[14] Consequently most of the energy measurements were made using a target of 20- or 30-μg/cm^2 ^{10}B deposited on ^{27}Al and a beam of 1.66- or 1.72-MeV α particles with an intensity of 0.5–0.8 μA. At E_α = 1.72 MeV the yield was integrated over resonances at 1.645 and 1.68 MeV.[2]

Of the three cascade transitions shown in Fig. 1 (those of 169, 595, and 764 keV), the energies of the 169- and 764-keV transitions were measured. That at 595-keV was unresolved from the ^{74}Ge 596-keV line induced in the Ge(Li) detector by the neutrons produced in the ^{10}B + α and ^{12}C + d reactions used to form ^{13}C.

The level energies were derived from the energies of these two cascades and from a measurement of the ground-state decay of the 3089-keV level. All measurements were made with an ORTEC 50-cm^3 true coaxial Ge(Li) detector operated at a bias of 4000 V. Signals were amplified with an ORTEC 472A spectroscopy amplifier and sorted by an 8192-channel analog-to-digital converter (ADC) operated with a digital offset of 4096 to 5632 channels. Counting rates were kept within the range 3–8 kHz and the beam currents were kept constant so as to avoid systematic errors due to the dependence of detector gain on counting rate.

The mixed source technique was used in which radioactive calibration standards were recorded simultaneously with the ^{13}C γ rays. Care was taken to position the sources at nearly the same position as the beam spot on the ^{12}C or ^{10}B target to avoid systematic uncertainties from differing source-detector positions.[15] Peak positions were determined with the program SAMPO.[16] Essentially, a Gaussian with exponential tails and a parabolic background are simultaneously fitted to a given peak. The shape of the peak (width and tail parameters) is predetermined and any or all of the background parameters can be predetermined. With care in its use, this program is found to give very reliable peak positions.

Gamma-ray energies were determined from a least-squares fit to a polynomial expansion of the peak channel position x via

$$E_\gamma = \sum_{n=0}^{m} a_n x^n . \tag{1}$$

In all cases m = 2 or 3 was sufficient to obtain a

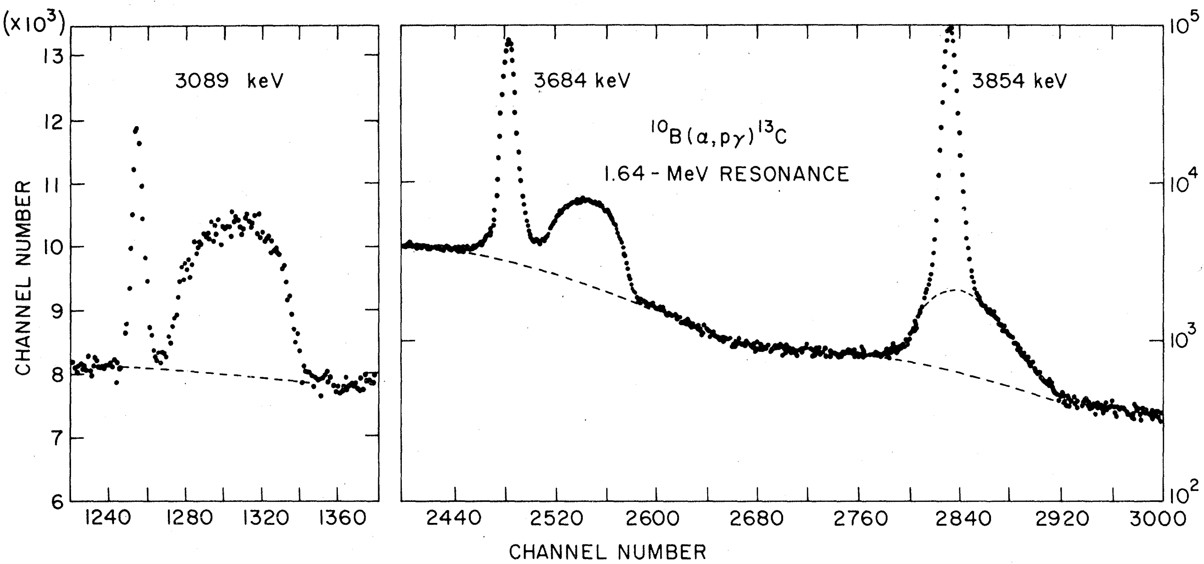

FIG. 2. High energy portions of a 0° spectrum from $^{10}B+\alpha$ at $E_\alpha=1.72$ MeV and a 30-μg/cm^2 ^{10}B target. The energy dispersion is 0.484 keV/channel. Line shapes due to ground-state transitions from the three bound levels of ^{13}C are shown. The expected shape of the background under these line shapes is indicated by dashed lines.

satisfactory fit. By a satisfactory fit is meant either a χ_D^2 (chi squared divided by the degrees of freedom) of ≤ 1.0 or a χ_D^2 not decreasing with increasing m.

B. Doppler effects

Line shapes[17] observed at 0° to the beam for the ground-state transitions of the three bound levels of ^{13}C are shown in Fig. 2. For the 3854-keV level the line shape is greatly attenuated by the long lifetime of the level. It can be usefully characterized as a part corresponding to a γ ray of energy 3854 keV (92.5% in intensity) due to emission of γ rays after the recoils come to rest and a shifted distribution (7.5% in intensity) due to decays from moving recoils. These two parts of the line shape are separated by the dashed curve in Fig. 2.

The 3684- and 3089-keV line shapes contain a portion due to feeding through the 3854-keV level. This portion will have the identical shape (scaled by the transition energy) as that of the 3854-keV line. They will also have a portion due to direct feeding of the 3684- and 3089-keV levels. This latter portion is unaffected by the level lifetimes and has a shape determined by the kinematics of the reaction and the angular distribution of the recoils.[18]

The energies of the 169- and 764-keV transitions from the decay of the 3854-keV level were measured at 90° to the beam and the question raised is: What is the uncertainty in this measurement due to the shifted portion of the line shape? At 0° the mean energy (centroid) of the shifted portion of the 3854-keV line shape is 8.9 keV above the 3854-keV line. To first order, then, the mean energy of the shifted portion of the line shape of all three γ rays originating from the 3854-keV level has the dependence on the γ-ray emission angle θ_γ:

$$\langle E_\gamma\rangle = E_{\gamma 0}(1+0.002\,31\cos\theta_\gamma)\,. \quad (2)$$

At 90° to the beam in $^{10}B+\alpha$ the full width at half maximum (FWHM) of the shifted portion of the 3854-keV line was observed to be ~8 keV, which corresponds to ~1.6 keV and 0.35 keV for the 764- and 169-keV transitions, respectively. These widths are comparable to or less than the detector resolution so that at 90° the shifted portion is effectively incorporated into the line shape. From Eq. (2) it is obvious then that the error introduced into the energy measurement depends on the deviation from 90° and for the 169-, 764-, and 3854-keV lines we have uncertainties of ~0.45, 2.3, and 11.6 eV for a 1° error in setting θ_γ where we take the shifted portion to be 7.5% of the total intensity. It is relatively easy to set the detector to within ±1° at 90°. Also these uncertainties represent upper limits since the shifted portion has a long tail and the line shape is determined from nearby radioactive lines with the result that a sizable fraction of the shifted line shape is absorbed into the background. Nevertheless, these uncertainties were added in quadrature in determining the final energies of the 169- and 764-keV transitions.

The energy of the 3089-keV line was measured twice. The first measurement was made at 0° to the beam where the portion due to decay of the 3854-keV level was best separated from that due to direct feeding of the 3684- and 3089-keV levels. The spectra were similar to that of Fig. 2. In these measurements the γ-ray line shape at 3089 keV was carefully determined by interpolation of the ^{56}Co lines at higher and lower energies and the fit was confined to the region for which the 3089-keV intensity was $\geq 10\%$ of the peak intensity with the background constrained to be linear. Tests on the 3854-keV peak showed that with this procedure the uncertainty in peak position due to the shifted portion of the line shape was 10 eV at most.

A second measurement was made at 90° to the beam using a 20-μg/cm ^{10}B target and $E_\alpha = 1.66$ MeV. The change of beam and target conditions from those of Fig. 2 resulted in a much larger fraction of the total 3089-keV line shape being due to feeding via the 3854-keV level. Thus the shifted portion of the line shape was barely discernible above background and was broad enough to be considered part of this background.

C. Results

The 169-keV 3854 → 3684 transition

The energy of the ^{13}C 169-keV γ ray was measured relative to γ rays from the decay of 115 d ^{182}Ta. Eight spectra were accumulated, each with different gain and digital offset. The energy dispersion varied from 25 to 40 eV/channel and the energy calibration utilized either eight or nine of the most intense ^{182}Ta lines between 84 and 230 keV. These γ rays are all known[6] with uncertainties less than 1 eV. The detector was at 90° to the beam and 14 cm from the target. For the ^{13}C 169-keV line the energy resolution (i.e., FWHM), was 1.1 keV and the peak/background intensity ratio was ~18:1. In a typical run, the maximum deviation of the fit to Eq. (1) with $m = 2$ and nine ^{182}Ta energies was 4 eV and the rms deviation was 1.3 eV. These excellent fits were achieved without any corrections for local nonlinearities[6] in the ADC-PHA system and served to limit any such effects and associated uncertainties for these and subsequent measurements with the same system.

The energy of the ^{13}C 169-keV line was extracted relative to that of the ^{182}Ta γ ray of 179.3948 ± 0.0005 keV. The average result from the eight runs for the energy difference of the 179- and 169-keV lines, Δ(179-169), was 10.094 $\pm$ 0.0032 keV, where the uncertainty was taken from the external error since χ_D^2 for the fit to obtain the average was greater than unity. We allow a systematic uncertainty of 2.4 eV and quote a ^{13}C energy of 169.300 ± 0.004 keV.

The most accurate previous measurement[19] of this energy, 169.25 ± 0.04 keV, was made at this laboratory using the same technique. The factor of 10 greater accuracy in the present study is due to several sources, the most important of which are the use of greater energy dispersion—25–40 eV/channel as opposed to 220 eV/channel, and eight measurements instead of two. In addition, in the previous work a peak was observed at 167.6 keV in the ^{182}Ta spectrum with an intensity ~10% of the ^{13}C 169-keV line. Since the two peaks were not completely resolved, their separation introduced some systematic uncertainty. We identified the 168-keV peak as summing of ^{182}Ta 68- and 100-keV γ rays. It was reduced to a negligible intensity by using a relatively large source-detector distance and interposing lead foil between the ^{182}Ta source and the detector.

The 764-keV 3854 → 3089 transition

The energy of this transition was inferred from the separation between it and the 779-keV γ ray from 1.34 y ^{152}Eu. Several secondary standards were used to provide the energy calibration. These included the 867-keV line from ^{152}Eu, the ^{137}Cs line at 661.660 ± 0.003 keV,[6] annihilation radiation, and the ^{54}Mn 834.843 ± 0.013 keV γ ray.[6] Less important were several secondary standards from room background such as the ^{214}Bi line at 609.312 ± 0.012 keV,[20] and, for the $^{12}C + d$ measurements, two beam-induced γ rays—the ^{17}F 495.33 ± 0.10 keV line[2] and the ^{17}O $0.87 \rightarrow 0$ transition which was measured in the present studies to have an energy of 870.725 ± 0.020 keV. A partial spectrum from one of the six measurements is shown in Fig. 3. Considering that the $3854 \rightarrow 3089$ transition is only a 1.1% branch (see Sec. III) the peak-background intensity ratio of ~5:1 for the 764-keV peak is even better than was anticipated.

The weighted average of the six measurements of the separation for the ^{152}Eu 779-keV and ^{13}C 764-keV lines was 14.587 ± 0.008 keV. In obtaining this result a correction was made for the presence of a ^{152}Eu peak at 764.905 ± 0.009 keV (Ref. 9) and 1.4% as intense as the ^{152}Eu 779-keV line. The correction amounted to 12 eV and could be made with an error negligible compared to other sources.

Both Helmer *et al.*[6] and Meyer[9] quote accurate values for the ^{152}Eu 368- and 411-keV γ rays which cascade in competion with the 779-keV crossover. From their energies we obtain 778.803(6) keV for the crossover and this we adopt.

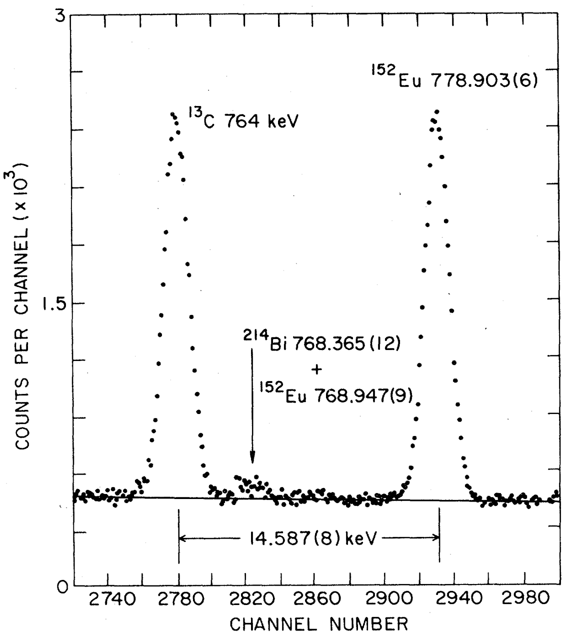

FIG. 3. Partial spectrum of $^{10}B+\alpha$ at the 1.64-MeV resonance. The Ge(Li) detector was at 0° to the beam and 18 cm from the target and a ^{152}Eu source. The weak ^{214}Bi line is from room background. A weak ^{152}Eu peak (see Ref. 9) lies under the ^{13}C 764-keV peak. The energy dispersion is 97 eV/channel and the energy resolution at 779 keV is 1.62 keV FWHM.

(Note: We shall sometimes note the uncertainty in the last significant figure by a following number in parenthesis.) We neglect the value of 778.925(13) keV given for the crossover by Helmer *et al.*[6] From our measurement of Δ(799-764), we then obtain 764.316(10) keV for the ^{13}C 3854→3089 γ ray.

The 3089 → 0 transitions

The first measurement of the energy of the 3089-keV γ ray was made with the $^{10}B+\alpha$ reaction and the conditions of Fig. 2. Seven spectra were recorded at 0° to the beam, all with different energy dispersion and digital offset. The measurements all were relative to ^{56}Co γ rays[7] with the γ ray of energy[7] 3201.954(14) keV providing the main calibrations. The precautions used in extracting the peak positions have been outlined in Sec. II B. The weighted mean of the separation of the ^{56}Co 3202-keV peak and the ^{13}C 3089-keV peak in this measurement was Δ(3202-3089) = 112.889(20) keV, where the uncertainty includes ± 10 eV from the Doppler line shape.

As outlined in Sec. II B, a second measurement was made at 90° to the beam. Ten spectra were collected. A novel feature of this measurement was the use of two ADC-PHA units, i.e., the amplified signal was split and sent to two independent analyzers. In this way it was hoped that the uncertainty due to nonlinearities in the ADC-PHA system could be isolated and also reduced. Thus ten pairs of spectra were analyzed. The average difference in the energy of the 3089-keV line from the two ADC-PHA systems was 10 eV. However, the difference for the individual pairs was of the order of the total uncertainties. Thus, local nonlinearities in the ADC-PHA system appeared to dominate the overall uncertainty of the measurement. The value of Δ(3202-3089) adopted for this measurement was the unweighted average of the two sets, namely Δ(3202-3089) = 112.917(17) keV.

The average of the 0° and 90° determinations is Δ(3202-3089) = 112.905(17), which gives 3089.049(20) keV for the ^{13}C energy. The uncertainty includes 14 eV in the ^{56}Co 3202-keV line.

The γ-ray energy measurements are summarized in Table I. Also shown in Table I are the remaining transition energies (see Fig. 1) and the level energies derived from the three measured γ-ray energies.

III. GAMMA-RAY BRANCHING RATIOS

A straightforward measurement of the γ-ray branching ratios of the ^{13}C 3854-keV level would utilize a measurement of the relative intensities of the 169-, 764-, and 3854-keV transitions. Because the relative efficiencies for detecting 169- and 3854-keV γ rays is rather difficult to obtain accurately, we wished to avoid measurements on the 169-keV transition. This was possible because of the separation via the Doppler effect of the portions of the γ-ray line shapes originating from the 3854-keV level (see Fig. 2). If we denote the ratio of the intensity of the γ ray of energy x

TABLE I. Level and γ-ray energies for ^{13}C.

E_i (keV)	E_f (keV)	E_R^a (eV)	E_γ (keV)
3089.443(20)	0	394	3089.049(20)[b]
3684.482(23)	0	561	3683.921(23)
	3089	15	595.013(11)
3853.783(22)	0	613	3853.170(22)
	3089	24	764.316(10)[b]
	3684	1	169.300(4)[b]

[a] Recoil energy, $E_R = 536.9\,[E_\gamma\,(\mathrm{MeV})]^2/A$ eV. $E_i - E_f = E_\gamma + E_R$.

[b] Measured value. All other energies are derived from these three.

to that of the 3854-keV γ ray as $R(x) \equiv I(x)/I(3854)$ and $B(x)$ as the branching ratio for that γ ray in the decay of its emitting state, then equating the incoming and outgoing γ-ray flux gives

$$B(3854) = [1 + R(3684) + R(3089)]^{-1}, \quad B(764) = R(764) \cdot B(3854), \quad B(169) = 1 - B(3854) - B(764). \tag{3}$$

We see that the branching ratios for the 3854-keV level can be obtained from three ratios without requiring a measurement of the intensity of the 169-keV γ ray. For the 3684-keV level we used two alternate approaches to obtain $R(595)$ and thus the branching ratios, either

$$R(595) = R(3089) - R(764) \tag{4a}$$

or

$$R(595) = R(765) \cdot [I(595)/I(764)]. \tag{4b}$$

In either case we have

$$B(3684) = [1 + R(595)/R(3684)], \quad B(595) = 1 - B(3684). \tag{5}$$

The relationship of Eq. (4a) demands the subtraction of two small quantities and is therefore especially susceptible to systematic uncertainties. For this reason the additional measurement of $I(595)/I(764)$ was made so that Eq. (4b) could be applied.

The four intensity ratios needed to obtain the branching ratios of the 3854- and 3684-keV levels were obtained from a multitude of spectra taken in the course of this investigation. The most accurate results were obtained from 0° spectra for which the separation in any given γ-ray line of the component due to feeding via the 3854-keV level was most accurate. Figure 4 illustrates one such 0° spectrum. The mean values from eleven measurements of $R(3684)$ and seven measurements of $R(3089)$ were 0.675 ± 0.015 and $(2.34 \pm 0.08) \times 10^{-2}$, respectively. The ratio $R(764)$ was obtained from four spectra taken at 0°, 30°, 45°, and 90° to the beam for the primary purpose of obtaining angular distributions in the $^{10}B + \alpha$ reaction, and from two 0° $^{12}C + d$ spectra. Results for $R(764)$ of $(1.943 \pm 0.060) \times 10^{-2}$ and $(1.889 \pm 0.060) \times 10^{-2}$ were obtained from the two reactions. We adopt $(1.916 \pm 0.060) \times 10^{-2}$.

The intensity ratio $I(764)/I(595)$ was obtained from coincidence spectra taken with 595- and 764-keV γ rays detected at 0° in the Ge(Li) detector in coincidence with 3089-keV γ rays detected in a 5×5-in NaI(Tl) detector at 100° to the beam. The $^{10}B + \alpha$ reaction was used. Since the 3089-keV level has $J = \frac{1}{2}$, there is no spatial correlation in-

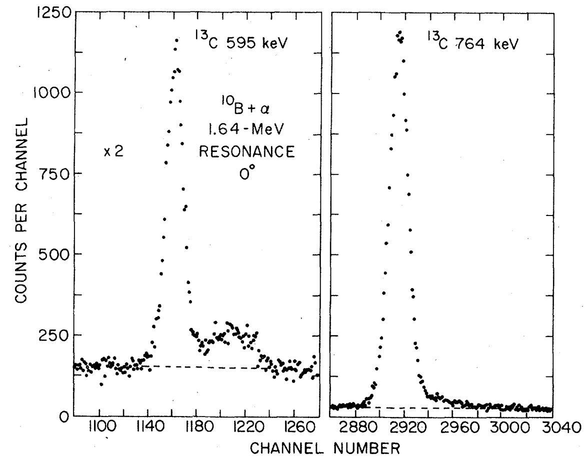

FIG. 4. Partial 0° spectrum from $^{10}B + \alpha$ showing the 595- and 764-keV peaks detected in coincidence with the 3089-keV transition. The Doppler line shapes, clearly apparent, should be identical (when scaled by the γ-ray energy) to those of the 3684- and 3854-keV lines, respectively, in Fig. 2.

volved in the $\gamma\gamma$ coincidence and the intensity ratio could be extracted exactly as for a singles measurement. Three spectra were recorded at a beam current of 0.8 μA and a total elapsed time of 34 h. The 595- and 764-keV regions of one of these are shown in Fig. 4. From these spectra we obtained $I(595)/I(764) = 0.230 \pm 0.011$, which, from Eq. (4b), gives $R(595) = (0.441 \pm 0.024) \times 10^{-2}$. On the other hand, Eq. (4a) gives $R(595) = (2.34 \pm 0.08) \times 10^{-2} - (1.916 \pm 0.050) \times 10^{-2} = (0.424 \pm 0.023) \times 10^{-2}$. The two results are in good agreement and our adopted average is $R(595) = (0.433 \pm 0.020) \times 10^{-2}$.

To obtain the four intensity ratios presented here from the raw relative intensities, two corrections were made; namely, that due to the dependence of detection efficiency on γ-ray energy and that due to the different angular distributions of the γ rays. The uncertainties given so far for the four ratios do not include contributions from these sources. The relative detector efficiency was determined from ^{152}Eu and ^{56}Co spectra taken with the sources *in situ* for the same source-detector distances and combinations of absorbers as in the intensity measurements. These two sources give γ-ray peaks of known intensity from 122 to 3548 keV.[9] At the higher energies the efficiency versus γ-ray energy, E_γ, has the form E_γ^{-k} with $k \simeq 0.4$–0.9, depending on the absorbers used. From previous experience it is known that the extrapolation of this function to higher γ rays (3854 keV) can be made accurately. It is estimated that the uncertainties in $R(3684)$ and $R(3089)$ due to those in the relative efficiencies are < 1% while those in $R(764)$ and $R(595)$ are < 1.5%.

Previous measurements of the $^{12}C(d,p)^{13}C$ reaction at and near the 2.49-MeV resonance had determined the population parameters at the resonance so that any of the ^{13}C γ-ray angular distributions could be calculated given their multipole mixing ratio. The $E3/M2$ and $M2/E1$ mixing ratios of the 3854 → 0 and 3684 → 0 transitions are known,[21] while for both reactions we assumed the 595- and 764-keV transitions to be pure $E1$ and $E2$, respectively. As it turns out, both transitions are too fast to contain enough ML radiation to affect this assumption if we take reasonable limits[22] for the ML strengths. The angular distributions of the 3854- and 3684-keV γ rays were measured for the $^{10}B(\alpha,p)^{13}C$ reaction with the beam and target conditions ($E_\alpha = 1.72$ MeV, 30-μg/cm^2 target) used for the intensity measurements. The results for the coefficients in $W(\theta) = 1 + A_2 P_2(\cos\theta) + A_4 P_4(\cos\theta)$ were A_2, $A_4 = 0.113 \pm 0.004$, 0.003 ± 0.008 for the 3854 → 0 transition and $A_2 = 0.064 \pm 0.002$ for the 3684 → 0 transition. The angular distributions of the 595- and 764-keV γ rays were calculated from these results. A valuable check on the reliability of the angular distribution corrections was that the two reactions gave strongly differing angular distributions (e.g., A_2, $A_4 = +0.380$, -0.252 for the 3854 → 0 transition[21] in $^{12}C + d$ as opposed to -0.113, $+0.003$ in $^{10}B + \alpha$) and yet the two reactions gave the very closely agreeing values for $R(3684)$, $R(3089)$, and $R(764)$. Uncertainties in the intensity ratios due to uncertainties in the relative angular distributions were ~1–2% for both reactions including the effect of the finite solid angle subtended by the Ge(Li) detector. The final branching ratios are given in Table II, where they are compared to previous measurements.

In addition to the desirability of increased accuracy, the present study was motivated by the discrepancy in the two most recent measurements for the branching ratio of the 3854 → 3089 transition [columns (b) and (c) of Table II] and in the only two previous measurements for the 3684 → 3089 branching ratio. During the course of this work it became clear that the values quoted for these two branching ratios by Tryti *et al.*[11] were liable to a large systematic error due to the presence of the $^{74}Ge(n,n')$ 596-keV γ ray which was quite noticeably present in the $^{12}C + d$ reaction used by Tryti *et al.* as well as by us. The results of Pixley *et al.*[12] for the 3854 → 3089 branching ratio are based on their measurement of $I(764)/I(169)$ and the measurement of $I(169)/I(3854)$ of Mackin, Mills, and Thirion.[23] Taking our branching ratio for the 169-keV transition rather than that of Mackin *et al.* modifies the branching ratio given by Pixley *et al.* for the 3854 → 3089 transition to 1.4 ± 0.3 %, which is in good agreement with our result. In conclusion, all branching ratio measurements for the γ decay of the bound states of ^{13}C are in agreement except for the early results of Mackin *et al.*[23] and the results of Tryti *et al.*,[11] for which the discrepancy is not surprising. The present results are considerably more accurate than those previously obtained and meet the objective of obtaining accurate electromagnetic matrix elements for the two weaker transitions.

IV. SHELL MODEL CALCULATIONS AND COMPARISON TO EXPERIMENT

Comparison between theory and experiment for the electromagnetic decays of the bound levels of

TABLE II. Resumé of branching ratio results for ^{13}C.

E_i (keV)	E_f (keV)	Branching ratio (%) (a)	(b)	(c)	(d)	(f)
3854	0	62.5 ± 0.6	63.4 ± 0.8	63.5 ± 1.2	75.2 ± 4.0	62.0 ± 4.0
	3089	1.20 ± 0.04	0.6 ± 0.2	1.6 ± 0.3	0.93 ± 0.20	(1.0)
	3684	36.3 ± 0.6	36.0 ± 0.7	34.9 ± 1.2	23.8 ± 4.0	37.0 ± 4.0
					(e)	
3684	0	99.25 ± 0.04	98.4 ± 0.3		99.35 ± 0.10	
	3089	0.75 ± 0.04	1.6 ± 0.3		0.65 ± 0.10	

[a] Present
[b] Reference 11.
[c] Reference 10.
[d] References 12 and 23.
[e] Reference 13.
[f] S. Gorodetzky, R. M. Freeman, A. Gallmann, and F. Haas, Phys. Rev. 149, 801 (1966).

TABLE III. Comparison of the shell-model predictions to experiment for electromagnetic transitions in ^{13}C.

Transition $J_i \to J_f$	Quantity[a]	^{13}C Theory[b]	Exp.[c]
$\frac{5}{2}^+ \to \frac{1}{2}^-$	$B(M2)$	0.526(+0.013)[d]	0.47 ± 0.02
	$B(E3)$	2.15(−1.54)	9.80 ± 5.0
	$x(E3/M2)$	+0.05	$+0.12 \pm 0.03$[e]
	τ (ps)	18.0	20.0 ± 0.8
$\frac{5}{2}^+ \to \frac{1}{2}^+$	$B(E2)$	2.16(−1.36)	1.65 ± 0.09
	τ (ns)	0.80	1.04 ± 0.05
$\frac{5}{2}^+ \to \frac{3}{2}^-$	$B(E1)$	0.53×10^{-2}	$(1.06 \pm 0.05) \times 10^{-2}$
	τ (ps)	68.6	34.4 ± 1.5
$\frac{3}{2}^- \to \frac{1}{2}^-$	$B(M1)$	0.63(−0.083)	0.39 ± 0.03
	$B(E2)$	6.79(+6.5)	3.63 ± 0.40[f]
	$x(E2/M1)$	−0.100	-0.094 ± 0.009[g]
	τ (fs)	1.00	1.60 ± 0.13
$\frac{3}{2}^- \to \frac{1}{2}^+$	$B(E1)$	1.38×10^{-2}	$(3.95 \pm 0.4) \times 10^{-2}$
	τ (ps)	0.61	0.21 ± 0.02
$\frac{1}{2}^+ \to \frac{1}{2}^-$	$B(E1)$	2.56×10^{-2}	$(3.9 \pm 0.4) \times 10^{-2}$
	τ (fs)	2.33	1.55 ± 0.15

[a] The transition strengths $B(EL)$ and $B(ML)$ are in Weisskopf units (Ref. 24), the mixing ratios $x[(L+1)/L]$ are dimensionless, and the units for the mean lifetimes τ are given.

[b] Bare g factors are used for all magnetic transitions. For $E2$ and $E3$ transitions the isoscalar effective charge, $(e_p + e_n)/2$ is double the bare value (0.5e). No effective charge is used for $E1$ transitions. The radial matrix elements are computed using oscillator wave functions with $b = 1.684$ fm, corresponding to $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$.

[c] From the mean lives given in Fig. 1, the presently determined branching ratios of Table II and the other information as noted below.

[d] The bracketed quantities are ratios of isoscalar to isovector matrix elements using bare charges or g factors.

[e] Reference 21.

[f] From the ^{13}C(e, e') results of Ref. 25.

[g] The $B(E2)$ value from the ^{13}C(e, e') results of Ref. 25 together with the adopted mean life of 1.59(13)fs for the 3684-keV level gives $x = 0.093 \pm 0.009$. The value from Ref. 21 (with an uncertainty corresponding to one standard deviation) is -0.096 ± 0.020. The weighted mean of the magnitudes is given while the sign is from the latter.

^{13}C is given in Table III. The experimental transition strengths are given in Weisskopf units (W.u.)[24] and utilize the lifetimes listed in Fig. 1, the currently measured branching ratios (Table II), and mixing ratios from the literature.[21,25] The sign convention for the mixing ratios is that of Rose and Brink.[26] The theoretical transition strengths are calculated using the experimental excitation energies.

A. Transitions between the odd-parity states

The $\frac{3}{2}^- \to \frac{1}{2}^-$ transition strengths and mixing ratio are adequately described by calculations restricted to p-shell configurations provided that effective charges are used in computing the $E2$ matrix element. The $B(M1)$ and $B(E2)$ values listed in Table III for this transition were calculated using Cohen and Kurath's (8–16)2BME effective interaction.[27] Other p-shell calculations, for example those which use a realistic effective interaction,[28] give similar values for the transition strengths. In fact, the essential features of the $\frac{3}{2}^- \to \frac{1}{2}^-$ decay are obtained in the LS limit[29]; for the (8–16)2BME interaction the intensities of $[f] = [4^3 1]$ $L=1$ $S=\frac{1}{2}$ in the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ wave functions are 70.6% and 87.7%, respectively.

In extended basis calculations the dominant $2\hbar\omega$ admixtures occur through a component of the central force which transforms as an SU(4) scalar and as a (20) tensor under SU(3). These admixtures are of 1p-1h $[s^4p^8(pf)$ and $s^3p^9(sd)]$ and 2p − 2h $[s^4p^7(sd)^2$ and $s^2p^{11}]$ character, both types of configuration being necessary to ensure that no spurious center of mass components are present in the wave functions. They have the same $[4^3 1]$ $L=1$ $S=\frac{1}{2}$ quantum numbers as the dominant $0\hbar\omega$ configuration and have SU(3) quantum numbers (23), (12), or (01) [from the product (03) × (20)] with the (23) configuration most important. For example, in a calculation[30] which includes all (23) and (12) configurations restricted to $S=\frac{1}{2}$ only, the percentages of total $2\hbar\omega$, 2p-2h, 1p − 1h, and (23) content in the $\frac{1}{2}^-_1$ and $\frac{3}{2}^-_1$ wave functions are (13.14, 8.22, 4.93, 10.99) and (17.04, 10.64, 6.40, 14.77), respectively. The 1p-1h configurations lead to an enhancement of the $E2$ matrix element equivalent to that given by $\Delta N = 2$ admixtures[29] in the Nilsson model [the $E2$ operator for $\Delta\hbar\omega = +2$ transitions transforms as (20) under SU(3)]; the 1p-1h → 1p-1h and 2p-2h → 2p-2h contributions to the $E2$ matrix element are an additional source of enhancement. In going from the $0\hbar\omega$ space to the $(0+2)\hbar\omega$ space the $B(E2)$ increases from 1.95 to 3.11 W.u., i.e., the experimental $B(E2)$ is reproduced without recourse to an effective charge. On the other hand, the $M1$ matrix element should be almost unchanged by $2\hbar\omega$ admixtures since the $\vec{\sigma}\tau_3$ contribution is dominant and the SU(4) character of the wave function remains almost unchanged. In actual fact, the $B(M1)$ changes from 0.63 to 0.71 W.u., but this change comes about because the balance of $[4^3 1]$ and $[4^3 32]$ symmetry, $S=\frac{1}{2}$ and $S=\frac{3}{2}$, etc., in the $0\hbar\omega$ wave functions is disturbed when only the restricted set of $2\hbar\omega$ configurations specified above is included.

B. Transitions involving the even-parity states

The bound $\frac{1}{2}^+$ and $\frac{5}{2}^+$ levels have long been described[33-38] by the weak coupling of an (sd)-shell nucleon to the 0_1^+ and 2_1^+ states of the ^{12}C core. The coupling must be weak to account[35] for the strong $E1$ transitions listed in Table III. Such weak coupling occurs in all light $A=4n+1$ systems with $T=\frac{1}{2}$ and is a natural consequence[33,39] of the strong space exchange component in the effective NN interaction. Shell-model calculations in a $1\hbar\omega$ space naturally produce[40] this weak coupling structure. The present calculation gives, in weak-coupling notation,

$$|\tfrac{1}{2}{}_1^+\rangle = 0.945\,|0^+\times\tfrac{1}{2}\rangle + 0.251\,|2^+\times\tfrac{5}{2}^+\rangle + \cdots + 2\%\,|p^8(T=1)(sd)\rangle + 1.5\%\,|s^3p^{10}\rangle$$
$$|\tfrac{5}{2}{}_1^+\rangle = 0.897\,|0^+\times\tfrac{5}{2}^+\rangle + 0.357\,|2^+\times\tfrac{5}{2}^+\rangle + \cdots + 2.5\%\,|p^8(T=1)(sd)\rangle\,. \quad (6)$$

The $0\hbar\omega\to 1\hbar\omega$ electromagnetic transition strengths listed in Table III result from the calculation described by Millener and Kurath.[41] These numbers differ slightly from those quoted by Teeters and Kurath[42] due to small modifications to the particle-hole interaction and the single particle energies. The calculations use Cohen and Kurath's[27] (8–16)2*BME* effective interaction for the s^4p^9 states. The correspondence between theory and experiment for transitions involving the positive parity levels is generally good. The predicted $B(E1)$ values are weaker than those deduced from experiment in common with the results of most other calculations. Kurath[43] has demonstrated the sensitivity of $B(E1;\frac{1}{2}^+\to\frac{1}{2}^-)$ to small admixtures into the dominant weak coupling component. He obtains

$$B(E1;\tfrac{1}{2}^+\to\tfrac{1}{2}^-) = \frac{3}{4\pi}[0.371\alpha_s - 0.778\alpha_d]^2 e^2\mathrm{fm}^2\,, \quad (7)$$

where α_s and α_d are the amplitudes of $|0^+\times\frac{1}{2}^+\rangle$ and $|2^+\times\frac{5}{2}^+\rangle$ in the $\frac{1}{2}^+$ wave function. It should be emphasized that α_s and α_d are not completely free parameters. For $\alpha_s=1$ the $\frac{1}{2}^+$ wave function [see Eq. (6)] contains 5.1% spurious center-of-mass components. Thus small amplitudes of other weak coupling wave functions are required to ensure that the $\frac{1}{2}^+$ wave function be free from spurious components. It is for this reason that the full $1\hbar\omega$ shell-model calculations are to be preferred over the limited basis weak-coupling-model calculations, especially when $B(E1)$ and $B(M2)$ values are to be calculated.

For the $\frac{5}{2}^+\to\frac{1}{2}^-$ transition the $M2$ component is adequately described by the theory but the predicted $E3$ component is weaker than observed. An $E2$ isoscalar effective charge of $1.0e$ has been used, this being a value typically required in light nuclei, e.g., the $3_1^-,0\to 0_1^+,0$ $E3$ in ^{12}C, calculated with the same Hamiltonian and effective charge, is in excellent agreement with experiment. Finally, we note that, as expected, the $B(E2)$ for the $\frac{5}{2}^+\to\frac{1}{2}^+$ transition would be almost zero were it not for the enhanced isoscalar effective charge of $1.0e$.

V. CONCLUSIONS

A. Precision energy measurements

The results of Table I establish the ^{13}C γ rays of 3684 and 3854 keV as useful calibration sources. Of course, it would be desirable to verify the present measurements as well as to increase their accuracy.

A reason for measuring the energy of the ^{13}C 3684-keV transition not discussed in the Introduction has to do with comparison of the wavelength and mass-based energy scales. The γ-ray energy standards of Helmer *et al.*[6,7] are based on the measurement by Kessler *et al.*[44] of the absolute wavelength of the 411-keV γ ray from the decay of ^{198}Au. A second mass-based energy scale is derivable from precisely measured doublet mass differences[45-47] and cascade sums in the (n,γ) reaction with slow neutrons. Perhaps the most promising comparison of these scales is via γ rays of ^{13}C.[30] The binding energy S_n of the neutron in ^{13}C derived from the mass doublet is 4946.329(24) keV.[48] The ^{12}C$(n,\gamma)^{13}$C reaction proceeds through cascade via the 3684-keV state. Thus, a comparison of the two scales entails, for instance, a comparison of the transition energy

$$1261.847(33) = 4946.329(24) - 3684.482(23)\ \mathrm{keV}\,,$$

(where the 3684-keV level energy is from Table I) with a value measured relative to wavelength-derived standards utilizing the ^{12}C$(n,\gamma)^{13}$C reaction. The two measurements reported to date, 1261.99(6) keV (Ref. 49) and 1261.76(7) (Ref. 50) are in poor agreement, and in any case lack the necessary accuracy.

B. Shell-model calculations

Shell-model calculations restricted to the full $0\hbar\omega$ and $1\hbar\omega$ spaces for the negative and positive parity states, respectively, with proper elimination of spurious center-of-mass states in the latter case, adequately describe the measured electromagnetic matrix elements connecting the bound states of ^{13}C. The necessity of the isoscalar effective charge enhancements used for the $E2$ and $E3$ transitions reflect the omission of $2\hbar\omega$ and $3\hbar\omega$ configurations (at least) from the negative and positive parity shell-model spaces. A preliminary calculation in a $(0+2)\hbar\omega$ model space reproduces the modest enhancement of the $\frac{3}{2}^- \rightarrow \frac{1}{2}^-$ $E2$ matrix element observed experimentally. Whether an extension of the shell-model space would lead to enhancement of the $E1$ transitions is an open (and difficult) question.

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[1]D. J. Millener (unpublished).

[2]F. Ajzenberg-Selove, Nucl. Phys. A248, 1 (1975); A268, 1 (1976); A281, 1 (1977).
[3]J. A. J. Hermans, G. A. P. Engelbertink, L. P. Ekström, H. H. Eggenhuisen, and M. A. Van Driel, Nucl. Phys. A284, 307 (1977).
[4]E. K. Warburton, J. W. Olness, and C. J. Lister, Phys. Rev. C 20, 619 (1979).

[5]For the lifetime of the ^{13}C 3854-keV level we adopt the weighted mean of the Doppler shift line shape result, 12.6 ps, of Hermans, Engelbertink, Ekström, Eggenhuisen, and Van Driel (Ref. 3) with the weighted average, 12.4±0.6 ps, quoted by these authors for the five previous recoil distance measurements. For the line shape value of Hermans *et al.* we adopt an uncertainty of ±0.7 ps rather than the ±0.3 ps quoted by them. The larger uncertainty is due to the inclusion of a 2% uncertainty in the nuclear stopping power and the designation of the 5% uncertainty in stopping power as a systematic error common to all stopping materials (see footnote 37 of Ref. 3). Our adopted value is 12.5 ±0.5 ps.

[6]R. G. Helmer, R. C. Greenwood, and R. J. Gehrke, Nucl. Instrum. and Methods 155, 189 (1978); 77, 141 (1970).
[7]R. C. Greenwood, R. G. Helmer, and R. J. Gehrke, Nucl. Instrum. Methods 159, 465 (1979).
[8]R. G. Helmer, Nucl. Instrum. Methods 164, 355 (1979).
[9]R. A. Meyer, LLL Report No. M-100, 1978 (unpublished). Additional references reported within.
[10]A. E. Litherland, T. K. Alexander, and A. T. Jeffs, Can. J. Phys. 47, 1061 (1969).
[11]S. Tryti, T. Holtebekk, and F. Ugletveit, Nucl. Phys. A251, 206 (1975).
[12]R. E. Pixley, J. V. Kane, and D. H. Wilkinson, Phys. Rev. 120, 943 (1960).
[13]J. V. Kane, R. E. Pixley, and D. H. Wilkinson, Philos. Mag. 5, 365 (1960).

[14]The partial widths at this resonance for n_0, p_0, p_1, p_2, p_3, and d, are ≤0.6, 0.18, 0.085, 0.44, 9.6, and 2.0 keV, respectively (see Ref. 2).

[15]R. G. Helmer, R. J. Gehrke, and R. C. Greenwood, Nucl. Instrum. Methods 123, 51 (1975).
[16]J. T. Routti and S. G. Prussin, Nucl. Instrum. Methods 72, 125 (1969).
[17]D. B. Fossan and E. K. Warburton, in *Nuclear Spectroscopy and Reactions*, edited by J. Cerny (Academic, New York, 1974), Part C, p. 307.

[18]See Ref. 11 for an exploitation of these line shapes for determining the angular distribution of the $^{12}C(d, p)^{13}C$ reaction.

[19]D. E. Alburger, E. K. Warburton, A. Gallmann, and D. H. Wilkinson, Phys. Rev. 185, 1242 (1969).
[20]*Table of Isotopes*, Seventh Ed., edited by C. M. Lederer and V. S. Shirley (Wiley, New York, 1978).
[21]A. R. Poletti, J. W. Olness, and E. K. Warburton, Phys. Rev. 151, 812 (1966).
[22]P. M. Endt, At. Data Nucl. Data Tables 23, 3 (1979).
[23]R. J. Mackin, W. R. Mills, and J. Thirion, Phys. Rev. 102, 802 (1956).
[24]S. J. Skorka, J. Hertel, and T. W. Retz-Schmidt, Nucl. Data A2, 347 (1966).
[25]G. Wittwer, H.-G. Clerc, and G. A. Beer, Z. Phys. 234, 120 (1970).
[26]H. J. Rose and D. M. Brink, Rev. Mod. Phys. 39, 306 (1967).
[27]S. Cohen and D. Kurath, Nucl. Phys. 73, 1 (1965).
[28]P. S. Hauge and S. Maripuu, Phys. Rev. C 8, 1609 (1973).
[29]A. R. Poletti, E. K. Warburton, and D. Kurath, Phys. Rev. 155, 1096 (1967).

[30]There are many difficulties with such calculations. In particular, the mixing between the $0\hbar\omega$ and 1p-1h $2\hbar\omega$ configurations involves $\sum_{i<j} v_{ij} - \sum_i u_i$ where u_i is, in our case, the single particle oscillator potential. Thus many of the matrix elements are obtained as the difference between two large numbers. Also it is well known, e.g., Refs. 31 and 32, that in extended shell-model spaces convergence as a function of N, where $N\hbar\omega$ is the maximum unperturbed energy of the configurations included, is slow.

[31]F. Arickx, J. Broeckhove, and E. Deumens, Nucl. Phys. A318, 269 (1979).
[32]T. Tomoda and A. Arima, Nucl. Phys. A303, 217 (1978).
[33]A. M. Lane, Proc. Phys. Soc. London A68, 197 (1955).
[34]A. M. Lane, Rev. Mod. Phys. 32, 519 (1960).
[35]D. Kurath and R. D. Lawson, Nucl. Phys. 23, 5 (1961).
[36]F. C. Barker, Nucl. Phys. 28, 96 (1961).
[37]T. Sebe, Prog. Theor. Phys. 30, 290 (1963).
[38]M. R. Meder and J. E. Purcell, Phys. Rev. C 12, 2056 (1975).
[39]V. G. Neudatchin and Yu. F. Smirnov, At. Energy Rev. 3, 157 (1965).

[40]H. V. Jäger, H. R. Kissener, and R. A. Eramzhian, Nucl. Phys. A171, 16 (1971).
[41]D. J. Millener and D. Kurath, Nucl. Phys. A255, 315 (1975).
[42]W. D. Teeters and D. Kurath, Nucl. Phys. A275, 61 (1977).
[43]D. Kurath, Phys. Rev. Lett. 35, 1546 (1975).
[44]E. G. Kessler, R. D. Deslattes, A. Henins, and W. C. Sauder, Phys. Rev. Lett. 40, 171 (1978).
[45]L. G. Smith, Phys. Rev. C 4, 22 (1971).
[46]L. G. Smith and A. H. Wapstra, Phys. Rev. C 11, 1392 (1975).
[47]A. H. Wapstra and K. Bos, At. Data Nucl. Data Tables 19, 177 (1977); 20, 1 (1977).
[48]R. C. Greenwood, and R. E. Chrien, in *Proceedings of Third International Symposium on Neutron Capture Gamma-ray Spectroscopy and Related Topics*, edited by R. E. Chrien and W. R. Kane (Plenum, New York, 1979), p. 618; and private communication.
[49]W. V. Prestwich, R. E. Cote, and G. E. Thomas, Phys. Rev. 161, 1080 (1967).
[50]P. Spilling, H. Gruppelaar, H. F. DeVries, and A. M. J. Spits, Nucl. Phys. A113, 395 (1968).