Radiative muon capture

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It is shown by relating the transition amplitude of radiative muon capture to that of radiative pion capture, that the transition amplitude of radiative muon capture proposed recently by Hwang and Primakoff differs from the others mainly by Low's counter terms. Despite the fact that the "original" transition amplitude does not violate seriously the conservation of the hadronic electromagnetic current, Low's counter terms, as introduced via Low's prescription to secure the presence of small conservation-of-hadronic-electromagnetic-current-breaking terms, are confirmed to be of numerical importance. Further, it is found in the "elementary-particle" treatment of radiative muon capture that the uncertainty arising from the nuclear structure can be reduced to become negligible. Therefore, an exclusive radiative muon capture experiment can in principle differentiate the Hwang-Primakoff theory from the others and yet provide a comprehensive test of partial conservation of axial-vector current.

[RADIOACTIVITY Theories of radiative muon capture, linearity hypothesis versus Low's prescription; nuclear structure and PCAC.]

I. INTRODUCTION

Recently, the feasibility in performing radiative muon capture experiments has aroused much theoretical interest. In view of its sensitivity to the pseudoscalar form factor $F_p(q^2)$, exclusive radiative muon capture in light nuclei has often been considered as a serious candidate which can offer us a *definitive* test of the partial conservation of the hadronic charge weak axial-vector current (PCAC). However, the interpretation of the forthcoming high-precision experiment versus the validity of PCAC can suffer severely from pitfalls in common theoretical practices. Even in the simplest case of radiative muon capture by a proton, several forms of the transition amplitude have been proposed as the description of the same process, viz: One common form of this amplitude, which was adopted in the early literature[1-7] as well as by some recent articles,[8,9] is taken as the sum of possible Feynman diagrams, subject to the necessity of restoring the conservation of the hadronic electromagnetic current (CEC) by Low's prescription.[10] Since the conservation of the hadronic charge weak polar current (CVC) as well as PCAC is also at stake for the description of this process, Adler and Dothan[11] proposed a transition amplitude which is constructed in accord with CEC, CVC, and PCAC. The Adler-Dothan procedure,[11] which represents essentially an elaboration of Low's prescription,[10] was recently reiterated by Christillin and Servadio in a slightly different fashion.[12] Yet, Hwang and Primakoff[13,14] introduced a simplifying dynamical approximation, the so-called "linearity hypothesis" (LH), so that the constraint equations derived from CEC, CVC, and PCAC are used to determine the overall amplitude from the knowledge of a few input *radiative* form factors. Therefore, we need to learn how to choose the appropriate transition amplitude before the hope to test PCAC via a radiative muon capture experiment can be realized.

In the case of radiative muon capture by a nucleus, there are more uncertainties arising from the choice of the nuclear wave functions and, as in some practical calculations, the invocation of the impulse approximation. If the *nucleon* pseudoscalar form factor is the entity of our ultimate concern, the adoption of the "elementary-particle" treatment (EPT) in the study of radiative muon capture[8,13,14] becomes inadequate, since the connection between the *nuclear* and *nucleon* form factors must eventually be determined. We need to clarify, in quantitative terms, whether the validity of PCAC remains a legitimate question to address despite these uncertainties.

It is the main task of this paper to present some results from analyzing the various forms of the transition amplitude for radiative muon capture by a proton, i.e., $\mu^- p \rightarrow \nu_\mu n \gamma$. We begin with a detailed review of the consistency question in relating the amplitude of radiative muon capture to that of radiative pion capture (or pion photoproduction). To use the CEC, CVC, and PCAC constraints in a consistent manner, we need *either* to perform an unwilling generalization of the on-shell-pion photoproduction amplitude to the off-shell-pion photoproduction amplitude *or* to introduce some "seagull" terms. For the purpose of analyzing the Hwang-Primakoff amplitude, we make a näive choice of the relationship, Eq. (19), which complies with the soft-pion theorems. However, this point must be confronted further if the Adler-Dothan or Christillin-Servadio amplitude is at stake. We make no further attempt in analyzing the Adler-Dothan or Christillin-Servadio amplitude, since, as should become clear from the text, the primary reason for the differences among the various amplitudes is that the Hwang-Primakoff approach does not comply with Low's prescription.

It is understood that, once the overall amplitude contains a CEC-breaking term of the form

$$G(q^2)B\cdot k\left[\alpha_i\,\frac{(Q-q)\cdot\epsilon^*}{(Q-q)\cdot k}-\alpha_f\,\frac{(Q+q)\cdot\epsilon^*}{(Q+q)\cdot k}\right]. \quad (1)$$

Low's prescription[10] restores CEC by adding to the amplitude a term specified by

$$-G(q^2)B\cdot\epsilon^*(\alpha_i-\alpha_f)\,. \quad (2)$$

Here $q\equiv p^{(f)}-p^{(i)}$ and $Q\equiv p^{(f)}+p^{(i)}$, with $p^{(i)}$ and $p^{(f)}$ the momenta of the initial and final hadrons, k and ϵ the momentum and polarization of the photon, and $G(q^2)$, B_μ, α_i and α_f arbitrary parameters. Whereas Low's prescription applies to an arbitrary set of Feynman diagrams, the LH in the Hwang-Primakoff approach is a dynamical approximation characterizing how the sum of all possible Feynman diagrams saturates the CEC, CVC, and PCAC sum rules. It is somewhat *misleading* to contrast the conventional amplitudes[7-9] directly with the Hwang-Primakoff amplitude, since the conventional approach deals essentially with a specific set of Feynman diagrams. However, we shall make such a comparison since Low's "counter" terms, as referred to those terms which are introduced to secure the presence of CEC-breaking terms, explain the difference between the Hwang-Primakoff amplitude and the others.[7-9]

It is somewhat beyond intuition why one should invoke a quantity of order unity to secure the presence of a CEC-breaking term of order $|\vec{q}|/M$ [see Eqs. (1) and (2)]. This feature gives rise to our assertion that Low's counter terms arising from the $(q+k)^2$ dependence of the weak magnetism form factor F_M are not negligible. It is also not clear whether Low's counter terms to the contributions from the excited states and from the meson-exchange currents are indeed negligible.

On the contrary, the various CEC-breaking terms in the form of Eq. (1) are neglected consistently via the adoption of LH in the Hwang-Primakoff approach. LH can be at most approximate simply because the contributions from the excited states and from the meson-exchange currents are generally at variance with LH. However, gross violation of LH is not expected, both since these contributions are not important numerically, and since they modify the radiative form factors in a fairly symmetrical fashion. Should the Hwang-Primakoff amplitude be confirmed experimentally, it indicates simply that the effect due to violation of CEC by the "original" amplitude be negligible.

As regard the uncertainties in the study of radiative muon capture by a nucleus, EPT is invoked to relate radiative muon capture by a nucleus, $\mu^- N_i\to\nu_\mu N_f\gamma$, *directly* to its corresponding nonradiative processes, and the sensitivity of physical quantities to the nuclear wave functions is investigated in $\mu^-\,{}^{12}C\to\nu_\mu{}^{12}\mathrm{B}\gamma$. It is found that the Cohen-Kurath model and the single-particle shell model yield almost the same predictions in radiative muon capture (except the absolute overall normalization). The extraction of the *nucleon* pseudoscalar form factor out of the nuclear form factors is also considered. The difference between the Hwang-Primakoff and (approximate) conventional amplitudes is confirmed to be substantial. As our final judgement, we emphasize that the radiative muon capture experiment can differentiate easily between the two theories and yet provide a comprehensive test of PCAC.

In what follows, we shall always use the same notations as in the papers of Hwang and Primakoff.[13,14] For the purpose of our discussions, some results will be quoted without duplicating the corresponding definitions and derivations. Although this paper itself is intended to be self-contained, reference to these papers is urged if some confusion arises.

II. RADIATIVE MUON CAPTURE AND RADIATIVE PION CAPTURE

Making use of standard reduction formulas, we can write the transition amplitude $\mathcal{T}$ for the radiative muon capture

$$\mu^-(p^{(\mu)},s^{(\mu)})+N_i(p^{(i)},s^{(i)})\to\nu_\mu(p^{(\nu)},s^{(\nu)})+N_f(p^{(f)},s^{(f)})+\gamma(k,\epsilon) \quad (3)$$

as follows:

$$\mathcal{T}=\mathcal{T}^{(l)}+\mathcal{T}^{(h)}, \tag{4}$$

where

$$\mathcal{T}^{(l)}=-\frac{Ge}{\sqrt{2}}\langle N_f(p^{(f)},s^{(f)})|[V_\lambda(0)+A_\lambda(0)]|N_i(p^{(i)},s^{(i)})\rangle \times\frac{1}{(2k_0)^{1/2}}\bar{u}^{(\nu)}(p^{(\nu)},s^{(\nu)})\gamma_\lambda(1+\gamma_5)i\frac{m_\mu-i(\not p^{(\mu)}-\not k)}{m_\mu^2+(p^{(\mu)}+k)^2}\not\epsilon^* u^{(\mu)}(p^{(\mu)},s^{(\mu)}), \tag{5}$$

$$\mathcal{T}^{(h)}=-\frac{Ge}{\sqrt{2}}\bar{u}^{(\nu)}(p^{(\nu)},s^{(\nu)})\gamma_\lambda(1+\gamma_5)u^{(\mu)}(p^{(\mu)},s^{(\mu)})\frac{1}{m_p}\frac{\epsilon_\mu^*}{(2k_0)^{1/2}}[V_{\mu\lambda}(k,q,Q)+A_{\mu\lambda}(k,q,Q)], \tag{6}$$

$$V_{\mu\lambda}(k,q,Q)=-im_p\int d^4x\, e^{-k\cdot x}\langle N_f;\text{out}|T[J_\mu(x)V_\lambda(0)]|N_i;\text{in}\rangle, \tag{6a}$$

$$A_{\mu\lambda}(k,q,Q)=-im_p\int d^4x\, e^{-k\cdot x}\langle N_f;\text{out}|T[J_\mu(x)A_\lambda(0)]|N_i;\text{in}\rangle, \tag{6b}$$

where $J_\mu(x)$, $V_\lambda(x)$, and $A_\lambda(x)$ are, respectively, the hadronic electromagnetic, charge weak polar, and charge weak axial-vector currents. Hereafter, the momentum and spin variables will always be suppressed, i.e., $N_i\equiv N_i(p^{(i)},s^{(i)})$ and $N_f\equiv N_f(p^{(f)},s^{(f)})$, since the formalism applies also to the cases other than nuclear spin and isospin doublets. The constraints arising from CEC, CVC, and PCAC are given as follows[13]:

CEC,

$$\frac{k_\mu}{m_p}V_{\mu\lambda}(k,q,Q)=\langle N_f;\text{out}|V_\lambda(0)|N_i;\text{in}\rangle, \tag{7a}$$

$$\frac{k_\mu}{m_p}A_{\mu\lambda}(k,q,Q)=\langle N_f;\text{out}|A_\lambda(0)|N_i;\text{in}\rangle, \tag{7b}$$

CVC,

$$\frac{(k_\lambda+q_\lambda)}{m_p}V_{\mu\lambda}(k,q,Q)=\langle N_f;\text{out}|V_\mu(0)|N_i;\text{in}\rangle, \tag{8}$$

PCAC,

$$\frac{(k_\lambda+q_\lambda)}{m_p}A_{\mu\lambda}(k,q,Q)=\langle N_f;\text{out}|A_\mu(0)|N_i;\text{in}\rangle+D_\mu(k,q,Q), \tag{9}$$

with

$$D_\mu(k,q,Q)=\int d^4x\, e^{-ik\cdot x}\times\langle N_f;\text{out}|T[J_\mu(x)\partial_\lambda A_\lambda(0)]|N_i;\text{in}\rangle, \tag{9a}$$

and

$$k_\mu D_\mu(k,q,Q)=i\langle N_f;\text{out}|\partial_\lambda A_\lambda(0)|N_i;\text{in}\rangle. \tag{9b}$$

We note that Eq. (9b) is simply a consequence of Eqs. (7b) and (9).

Along the same line, we can write the transition amplitude $\mathcal{T}^{(\pi)}$ for the radiative pion capture

$$\pi^-(p^{(\pi)})+N_i(p^{(i)},s^{(i)})\to N_f(p^{(f)},s^{(f)})+\gamma(k,\epsilon) \tag{10}$$

as follows:

$$\mathcal{T}^{(\pi)}=\frac{1}{(2E_\pi)^{1/2}}\langle N_f,\gamma;\text{out}|\mathcal{J}^{(\pi)}(0)|N_i;\text{in}\rangle =\frac{1}{(2E_\pi)^{1/2}}\frac{ie}{(2k_0)^{1/2}}\epsilon_\mu^*(T_\mu+S_\mu), \tag{11}$$

with

$$T_\mu=\int d^4x\, e^{-ik\cdot x}\langle N_f;\text{out}|T[J_\mu(x)\mathcal{J}^{(\pi)}(0)]|N_i;\text{in}\rangle, \tag{11a}$$

$$S_\mu=e^{-1}\int d^4x\, e^{-ik\cdot x}\times\langle N_f;\text{out}|\delta(x_0)[ik_0B_\mu(x)-\partial_0B_\mu(x),\mathcal{J}^{(\pi)}(0)]|N_i;\text{in}\rangle. \tag{11b}$$

Here $p^{(\pi)}\equiv(\vec{p}_\pi,iE_\pi)$ is the pion four-momentum, $\mathcal{J}^{(\pi)}(0)$ is the pion-source current, and $B_\mu(x)$ is the interacting photon field such that $-\partial_\lambda\partial_\lambda B_\mu(x)=eJ_\mu(x)$. As a practical example, we apply the pseudoscalar theory to radiative pion capture in nuclear spin and isospin doublets and obtain

$$T_\mu+S_\mu=-i\bar{u}^{(f)}(p^{(f)},s^{(f)})\Bigg[(e_f\gamma_\mu+\frac{\mu_f}{2M_f}\sigma_{\mu\nu}k_\nu)i\frac{M_f-i(\not p^{(f)}+\not k)}{M_f^2+(p^{(f)}+k)^2}f_{\pi N_iN_f}\gamma_5 +f_{\pi N_iN_f}\gamma_5 i\frac{M_i-i(\not p^{(i)}-\not k)}{M_i^2+(p^{(i)}-k)^2}(e_i\gamma_\mu+\frac{\mu_i}{2M_i}\sigma_{\mu\nu}k_\nu) +f_{\pi N_iN_f}\gamma_5\frac{e_\pi(2q_\mu+k_\mu)}{m_\pi^2+q^2}\Bigg]u^{(i)}(p^{(i)},s^{(i)}), \tag{12}$$

where $f_{\pi N_i N_f}$ are the $\pi^- + N_i \to N_f$ vertex function or form factor; e_i, e_f, and e_π are charge form factors for the initial nucleus, the final nucleus, and the pion; and μ_i and μ_f are the anomalous magnetic moment form factors for the initial and final nuclei. It should be noted that none of these form factors is on shell, viz.: $f_{\pi N_i N_f}$ in the first term of Eq. (12) stands for

$$f_{\pi N_i N_f}[(q+k)^2, (p^{(i)})^2, (p^{(f)}+k)^2],$$

e_π stands for $e_\pi(k^2, p_\pi^2, q^2)$, etc. In order that the Born amplitude specified by Eq. (12) be a sensible approximation at low energy, the three $f_{\pi N_i N_f}$ are taken simply as

$$f_{\pi N_i N_f}[(q+k)^2] \equiv f_{\pi N_i N_f}[(q+k)^2, -M_i^2, -M_f^2]$$

and all the other off-shell effects are also neglected. In this way, we find

$$k_\mu(T_\mu + S_\mu) = 0, \tag{13}$$

which ensures gauge invariance, i.e., CEC, of the transition amplitude $\mathcal{T}^{(\pi)}$. We need to note also that the literature on radiative pion capture or pion photoproduction has been rather extensive.[15] The Chew-Goldberger-Low-Nambu (CGLN) form (for on-shell photons[16]) or the Fubini-Nambu-Wataghin (FNW) form (for on- or off-shell photons[17]) of the transition amplitude is in accord with Eq. (13).

Making another use of the standard reduction formula, we obtain

$$\langle N_f, \gamma; \text{out}|\partial_\lambda A_\lambda(0)|N_i; \text{in}\rangle = \frac{ie}{(2k_0)^{1/2}}\epsilon_\mu^*[D_\mu(k,q,Q) + E_\mu(k,q,Q)], \tag{14}$$

where $D_\mu(k,q,Q)$ is already specified by Eq. (9a) and $E_\mu(k,q,Q)$ is the seagull term, viz.:

$$E_\mu(k,q,Q) = e^{-1}\int d^4x\, e^{-ik\cdot x}\langle N_f; \text{out}|\delta(x_0)[ik_0 B_\mu(x) - \partial_0 B_\mu(x), \partial_\lambda A_\lambda(0)]|N_i; \text{in}\rangle. \tag{15}$$

Further, PCAC yields

$$\langle N_f, \gamma; \text{out}|\partial_\lambda A_\lambda(0)|N_i; \text{in}\rangle = \frac{a_\pi m_\pi^3}{m_\pi^2 + (q+k)^2}\langle N_f, \gamma; \text{out}|\mathcal{J}^{(\pi)}(0)|N_i; \text{in}\rangle, \tag{16}$$

with a_π the pion-decay constant. If the seagull term $E_\mu(k,q,Q)$ is zero, then we obtain from Eqs. (11), (14), and (16)

$$D_\mu(k,q,Q) = \frac{a_\pi m_\pi^3}{m_\pi^2 + (q+k)^2} \times (T_\mu + S_\mu + ak_\mu + bt_\mu), \tag{17}$$

where t_μ is a timelike unit vector with $t\cdot\epsilon^* = 0$ and $t\cdot k = 0$, and a and b are arbitrary functions. With $a = b = 0$ (as suggested by a manipulation directly over D_μ, T_μ, and S_μ), Eq. (17) is the well-known gauge condition which has been adopted in the derivation of Adler-Dothan or Christillin-Servadio amplitude.[11,12] We note that, if $T_\mu + S_\mu$ were chosen to be that of Eq. (12) or the CGLN form[16] or the FNW form,[17] we would obtain from Eqs. (13) and (17)

$$k_\mu D_\mu(k,q,Q) = 0, \tag{18}$$

which is inconsistent with Eq. (9b). Accordingly, $T_\mu + S_\mu$ in Eq. (17) is interpreted as the amplitude for off-shell pions and a difference between the amplitudes for off-shell pions and on-shell pions is introduced to restore the validity of Eq. (9b). As indicated by Eq. (9b), the off-shell contribution $\delta(T_\mu + S_\mu)$ contracted by k_μ is independent of k_μ as $k_\mu \to 0$. To find out an explicit solution to the off-shell term $\delta(T_\mu + S_\mu)$, we need to invoke a procedure which is essentially the *converse* of Low's prescription. However, it is not unreasonable to speculate that the seagull terms and Schwinger terms[18] do not cancel exactly in the derivation of Eqs. (7)-(9). Alternatively, it might also be assumed that $E_\mu(k,q,Q)$ differs from zero. Therefore, a procedure contradictory to the spirit of Low's prescription can be avoided. In any case, there is some uncertainty to rewrite Eq. (17) properly. For our purpose, we simply take

$$D_\mu(k,q,Q) \cong \frac{a_\pi m_\pi^3}{m_\pi^2 + (q+k)^2} T'_\mu \tag{19}$$

where T'_μ is obtained from Eq. (12) by deleting the pion-pole term, i.e., the term in e_π. Equation (19), just as Eq. (17) with $a = b = 0$, leads to the soft-pion theorems for photoproduction and electroproduction,[19] which are justified reasonably well by the data. We do not consider Eq. (19) as a final resolution to the above-mentioned consistency question. The temptation in our mind is to see whether the Hwang-Primakoff amplitude for radiative muon capture complies with the soft-pion theorems.[19] The uncertainty in extrapolating from the soft-pion limit to physical pions[20] is beyond the scope of the present work.

III. LINEARITY HYPOTHESIS AND LOW'S PRESCRIPTION

We proceed to compare in detail the implication of the LH with that of Low's prescription. For our purpose, the meaning of LH will be described, making use of radiative muon capture in nuclear spin and isospin doublets[8] as an illustrative example.

In Hwang-Primakoff approach, one constructs the most general Lorentz-covariant expressions for $V_{\mu\lambda}(k,q,Q)$, $A_{\mu\lambda}(k,q,Q)$, and $D_\mu(k,q,Q)$ from which approximate expressions for $\epsilon_\mu^* V_{\mu\lambda}(k,q,Q)$ and $\epsilon_\mu^* A_{\mu\lambda}(k,q,Q)$ will be obtained with the aid of LH and the CEC, CVC, and PCAC constraint equations. For instance, one writes[13]

$$\begin{aligned}
V_{\mu\lambda}(k,q,Q)
&=\bar u^{(f)}(p^{(f)},s^{(f)})\Big\{\sigma_{\mu\lambda}F_{00}^{(a)}+i\sigma_{\mu\lambda}\frac{\not k}{m_p}F_{00}^{(b)}+\frac{\gamma_\mu}{m_p}[k_\lambda F_{11}^{(a)}+Q_\lambda F_{12}^{(a)}+q_\lambda F_{13}^{(a)}]+\frac{\gamma_\lambda}{m_p}(k_\mu F_{11}^{(b)}+Q_\mu F_{12}^{(b)}+q_\mu F_{13}^{(b)})\\
&\quad+\frac{\sigma_{\mu\nu}k_\nu}{m_p^2}(k_\lambda F_{21}^{(a)}+Q_\lambda F_{22}^{(a)}+q_\lambda F_{23}^{(a)})+\frac{\sigma_{\lambda\nu}k_\nu}{m_p^2}(k_\mu F_{21}^{(b)}+Q_\mu F_{22}^{(b)}+q_\mu F_{23}^{(b)})\\
&\quad+\Big[\delta_{\mu\lambda}F_{00}^{(c)}+\frac{k_\mu}{m_p^2}(k_\lambda F_{11}^{(c)}+Q_\lambda F_{12}^{(c)}+q_\lambda F_{13}^{(c)})+\frac{Q_\mu}{m_p^2}(k_\lambda F_{21}^{(c)}+Q_\lambda F_{22}^{(c)}+q_\lambda F_{23}^{(c)})\\
&\qquad+\frac{q_\mu}{m_p^2}(k_\lambda F_{31}^{(c)}+Q_\lambda F_{32}^{(c)}+q_\lambda F_{33}^{(c)})\Big]\\
&\quad+i\frac{\not k}{m_p}\Big[\delta_{\mu\lambda}F_{00}^{(d)}+\frac{k_\mu}{m_p^2}(k_\lambda F_{11}^{(d)}+Q_\lambda F_{12}^{(d)}+q_\lambda F_{13}^{(d)})\\
&\qquad+\frac{Q_\mu}{m_p^2}(k_\lambda F_{21}^{(d)}+Q_\lambda F_{22}^{(d)}+q_\lambda F_{23}^{(d)})\\
&\qquad+\frac{q_\mu}{m_p^2}(k_\lambda F_{31}^{(d)}+Q_\lambda F_{32}^{(d)}+q_\lambda F_{33}^{(d)})\Big]\Big\}u^{(i)}(p^{(i)},s^{(i)})
\end{aligned}\tag{20a}$$

and

$$\begin{aligned}
D_\mu(k,q,Q)&=\bar u^{(f)}(p^{(f)},s^{(f)})\Big[f_A\gamma_\mu\gamma_5+f_P i\frac{2Mq_\mu\gamma_5}{m_\pi^2}-f_E\frac{\gamma_5\sigma_{\mu\nu}q_\nu}{2m_p}+\tilde f_P i\frac{2Mk_\mu\gamma_5}{m_\pi^2}-\tilde f_E\frac{\gamma_5\sigma_{\mu\nu}k_\nu}{2m_p}\\
&\quad+i\frac{\gamma_5\not k}{m_p^2}(k_\mu f_1+Q_\mu f_2+q_\mu f_3)\Big]u^{(i)}(p^{(i)},s^{(i)}).
\end{aligned}\tag{20b}$$

Here, each of the radiative form factors $R\equiv F_{ij}^{(a),(b),(c),(d)}, f_{A,P,E,}, \tilde f_{P,E}$, and f_i is, in general, a function of the three Lorentz invariants q^2, $Q\cdot k$, and $q\cdot k$. One then applies CEC, CVC, and PCAC, as described by Eqs. (7)–(9), to $V_{\mu\lambda}(k,q,Q)$, $A_{\mu\lambda}(k,q,Q)$, and $D_\mu(k,q,Q)$ as expressed in terms of radiative form factors. In this way, a number of constraint equations relating the various radiative form factors are obtained. For instance, one has

$$F_{22}^{(b)}Q\cdot k+F_{23}^{(b)}q\cdot k=m_p^2F_{00}^{(a)},\tag{21a}$$

$$F_{12}^{(b)}Q\cdot k+F_{13}^{(b)}q\cdot k=m_p^2\left[F_V(q^2)+\frac{M}{m_p}F_M(q^2)\right],\tag{21b}$$

$$f_2Q\cdot k+f_3q\cdot k=-im_p^2f_A,\tag{21c}$$

$$\frac{1}{2M}\frac{1}{2m_p}f_EQ\cdot k+\frac{1}{m_\pi^2}f_Pq\cdot k=F_A(q^2)+\frac{q^2}{m_\pi^2}F_P(q^2),\tag{21d}$$

where $M\equiv\frac{1}{2}(M_f+M_i)\cong M_f\cong M_i$ and $F_{V,M,A,P}(q^2)$ are respectively, the vector, weak magnetism, axial-vector, and pseudoscalar form factors characterizing

$$\langle N_f;\text{out}|V_\lambda(0)|N_i;\text{in}\rangle \text{ and } \langle N_f;\text{out}|A_\lambda(0)|N_i;\text{in}\rangle$$

(see Ref. 13).

The new step taken by Hwang and Primakoff is the introduction of the LH which states that the radiative form factors $R(q^2,Q\cdot k,q\cdot k)$ can always be approximated by

$$R(q^2,Q\cdot k,q\cdot k)=\frac{R^+(q^2)}{(Q+q)\cdot k}+\frac{R^-(q^2)}{(Q-q)\cdot k}+R^0(q^2),\tag{22}$$

where $R^+(q^2)$ is linear in $F_{V,M,A,P}(q^2)$ and in e_f, μ_f; $R^-(q^2)$ is linear in $F_{V,M,A,P}(q^2)$ and in e_i, μ_i; and $R^0(q^2)$ is linear in $F_{V,M,A,P}(q^2)$ and in e_i, e_f, μ_i, μ_f. (Here e_i, e_f, μ_i, and μ_f are considered to be on shell.)

It should be noted that, in a perturbation-theory calculation as based upon a particular set of Feynman diagrams, the form factors $F_{V,M,A,P}$, e_i, e_f, μ_i, and μ_f enter the amplitude in a more complicated manner, viz.: Whereas their dependence on q^2 is not so simple, these form factors are also slightly off shell. Furthermore, the choice of a specific set of Feynman diagrams appears to be somewhat arbitrary since the overall amplitude does not satisfy CEC automatically.

The meaning of LH versus the above mentioned complications has been explored to some extent in the work of Hwang and Primakoff.[13] In what follows, we try to contrast the implication of LH with that of Low's prescription.[10] In addition, we make use of Eq. (19) to relate the amplitude of radiative muon capture to that of radiative pion capture.

LH is restrictive. If LH is valid, $F_{12}^{(b)}$, $F_{13}^{(b)}$, f_E, and f_P are readily determined from Eqs. (21b) and (21d). This yields

$$F_{12}^{(b)} = m_p^2\left[F_V(q^2) + \frac{M}{m_p} F_M(q^2)\right] \times\left[\frac{e_i}{(Q-q)\cdot k} - \frac{e_f}{(Q+q)\cdot k}\right], \tag{23a}$$

$$F_{13}^{(b)} = -m_p^2\left[F_V(q^2) + \frac{M}{m_p} F_M(q^2)\right] \times\left[\frac{e_i}{(Q-q)\cdot k} + \frac{e_f}{(Q+q)\cdot k}\right], \tag{23b}$$

$$f_E = 4Mm_p\left[F_A(q^2) + \frac{q^2}{m_\pi^2} F_P(q^2)\right] \times\left[\frac{e_i}{(Q-q)\cdot k} - \frac{e_f}{(Q+q)\cdot k}\right], \tag{23c}$$

$$f_P = -m_\pi^2\left[F_A(q^2) + \frac{q^2}{m_\pi^2} F_P(q^2)\right] \times\left[\frac{e_i}{(Q-q)\cdot k} + \frac{e_f}{(Q+q)\cdot k}\right]. \tag{23d}$$

Similarly, the validity of LH together with Eqs. (21a) and (21c) require that $F_{00}^{(a)}$ and f_A depend only on q^2. Once $F_{00}^{(a)}$ and f_A are given in accord with LH, the validity of LH together with Eqs. (21a) and (21c) determines $F_{22}^{(b)}$, $F_{23}^{(b)}$, f_2, and f_3 completely. Accordingly, despite a large number of radiative form factors introduced in the definition of $V_{\mu\lambda}(k,q,Q)$ and $A_{\mu\lambda}(k,q,Q)$, $\epsilon_\mu^* V_{\mu\lambda}(k,q,Q)$ and $\epsilon_\mu^* A_{\mu\lambda}(k,q,Q)$, which enter linearly into $\mathcal{T}^{(h)}$, are determined from a small number of *input* form factors. As a remarkable example, we note that $\epsilon_\mu^* A_{\mu\lambda}(k,q,Q)$ in the case of $\mu^- {}^{12}\text{C} \to \nu_\mu {}^{12}\text{B}\gamma$ is determined completely from only one input form factor, i.e., $G_{01}^{(a)}=0$ (see Appendix A of Ref. 14).

Can LH be viewed as a reasonable approximation? Let us consider the evaluation of the various radiative form factors from Feynman diagrams corresponding to $\mu^- N_i \to \mu^- X\gamma \to \nu_\mu N_f\gamma$ and $\mu^- N_i \to \nu_\mu Y \to \nu_\mu N_f \gamma$ (where the photon is emitted "before" and "after" the neutrino; they will be denoted simply by X and Y) as well as from the "box" diagrams (BD) (where the photon and neutrino are emitted "simultaneously"). For our purpose, we define

$$L_\mu \equiv \frac{1}{m_p}\,(Q_\mu F_{12}^{(b)} + q_\mu F_{13}^{(b)}), \tag{24a}$$

$$M_{\mu\lambda} \equiv \sigma_{\mu\lambda} F_{00}^{(a)} + \frac{\sigma_{\lambda\nu}k_\nu}{m_p^2}\,(Q_\mu F_{22}^{(b)} + q_\mu F_{23}^{(b)}), \tag{24b}$$

$$N_\mu^{(1)} \equiv \frac{2M}{m_\pi^2} q_\mu f_P + \frac{Q_\mu}{2m_p} f_E, \tag{24c}$$

$$N_\mu^{(2)} \equiv f_A\gamma_\mu - \frac{i\not{k}}{m_p^2}\,(Q_\mu f_2 + q_\mu f_3). \tag{24d}$$

The following sum rules are *exact*:

$$k_\mu\left(\sum_X \{L_\mu\}_X + \sum_Y \{L_\mu\}_Y + \{L_\mu\}_{\text{BD}}\right) = m_p\left[F_V(q^2) + \frac{M}{m_p} F_M(q^2)\right], \tag{25a}$$

$$k_\mu\left(\sum_X \{M_{\mu\lambda}\}_X + \sum_Y \{M_{\mu\lambda}\}_Y + \{M_{\mu\lambda}\}_{\text{BD}}\right) = 0, \tag{25b}$$

$$k_\mu\left(\sum_X \{N_\mu^{(1)}\}_X + \sum_Y \{N_\mu^{(1)}\}_Y + \{N_\mu^{(1)}\}_{\text{BD}}\right) = 2M\left[F_A(q^2) + \frac{q^2}{m_\pi^2} F_P(q^2)\right], \tag{25c}$$

$$k_\mu\left(\sum_X \{N_\mu^{(2)}\}_X + \sum_Y \{N_\mu^{(2)}\}_Y + \{N_\mu^{(2)}\}_{\text{BD}}\right) = 0. \tag{25d}$$

Additional sum rules can be inferred from the work of Hwang and Primakoff.[13,14] In the case of radiative muon capture by a complex nucleus, contributions from the excited states, i.e., $X \neq N_i$ or $Y \neq N_f$, are no longer negligible, so that the sum rules, as determined from CEC, CVC, and PCAC, become important guidelines. In essence, LH provides the most näive realization of these sum rules. Nonetheless, the CEC sum rules are ensured by Low's prescription if all possible Feynman diagrams are taken into account. Likewise, the additional CVC and PCAC sum rules are evident by applying the Adler-Dothan or Christillin-Servadio procedure to the sum of all possible Feynman diagrams. In a complete (nonlocal) theory, the sum of all possible Feynman diagrams is already in accord with CEC, CVC, and PCAC so that the additional terms introduced by Low's prescription cancel among themselves. It remains to be seen if the approximate validity of LH, as inferred from the näive perturbation-theory calculations, can be violated drastically by such a theory.

To pursue the problem further, we consider only the two diagrams with $X=N_i$ and $Y=N_f$ and so unjustifiably neglect the contributions from the excited states as well as from the "calculable" box diagrams.[21] In this case, we find[13]

$$\begin{aligned} F_{12}^{(b),0} = m_p^2 & \left\{ F_V[(q+k)^2, (p^{(i)}-k)^2, -M_f^2] + \frac{M}{m_p} F_M[(q+k)^2, (p^{(i)}-k)^2, -M_f^2] \right\} \\ & \times e_i[0, -M_i^2, (p^{(i)}-k)^2]/(Q-q)\cdot k \\ & - m_p^2 \left\{ F_V[(q+k)^2, -M_i^2, (p^{(f)}+k)^2] + \frac{M}{m_p} F_M[(q+k)^2, -M_i^2, (p^{(f)}+k)^2] \right\} \\ & \times e_f[0, (p^{(f)}+k)^2, -M_f^2]/(Q+q)\cdot k \,, \end{aligned} \tag{26a}$$

$$\begin{aligned} F_{13}^{(b),0} = -m_p^2 & \left\{ F_V[(q+k)^2, (p^{(i)}-k)^2, -M_f^2] + \frac{M}{m_p} F_M[(q+k)^2, (p^{(i)}-k)^2, -M_f^2] \right\} \\ & \times e_i[0, -M_i^2, (p^{(i)}-k)^2]/(Q-q)\cdot k \\ & - m_p^2 \left\{ F_V[(q+k)^2, -M_i^2, (p^{(f)}+k)^2] + \frac{M}{m_p} F_M[(q+k)^2, -M_i^2, (p^{(f)}+k)^2] \right\} \\ & \times e_f[0, (p^{(f)}+k)^2, -M_f^2]/(Q+q)\cdot k \,, \end{aligned} \tag{26b}$$

where the superscripts "0" denote the contributions from the above mentioned two diagrams. We expand the above $F_{V,M}$, e_i, and e_f around the points specified by $F_{V,M}(q^2) \equiv F_{V,M}(q^2, -M_i^2, -M_f^2)$, $e_i \equiv e_i(0, -M_i^2, -M_i^2)$, and $e_f \equiv e_f(0, -M_f^2, -M_f^2)$, and then substitute the resultant $F_{12}^{(b),0}$ and $F_{13}^{(b),0}$ into the L_μ of Eq. (24a). The resultant L_μ^0 is at variance with CEC [Eq. (21b) or (25a)]. The CEC-breaking term induced by the off-shell effects is independent of k. The remaining CEC-breaking term arising from the expansion of $F_{V,M}[(q+k)^2, -M_i^2, -M_f^2]$ around $F_{V,M}(q^2)$ is in the form of Eq. (1). We arbitrarily throw away the first CEC-breaking term and add a term in the form of Eq. (2) to secure the presence of the second CEC-breaking term. The resultant L_μ is what one expects from the application of Low's prescription.[10] Since this L_μ saturates the sum rule of Eq. (25a), we expect that δL_μ^0 as introduced by Low's prescription comes from all the other diagrams which we have neglected.

More explicitly, we find

$$\delta L_\mu^0 \equiv L_{\mu,\mathrm{Low}}^0 - L_{\mu,\mathrm{LH}} = 2q\cdot k m_p \left[F_V'(q^2) + \frac{M}{m_p} F_M'(q^2)\right]\left[\frac{e_i(Q-q)_\mu}{(Q-q)\cdot k} - \frac{e_f(Q+q)_\mu}{(Q+q)\cdot k}\right] - 2q_\mu m_p\left[F_V'(q^2) + \frac{M}{m_p} F_M'(q^2)\right], \tag{27}$$

where the last term is required by Low's prescription. We note that a term of order $m_\mu/m_p F_M(q^2)$ has been invoked by Low's prescription to secure the presence of a CEC-breaking term of order $(m_\mu/m_p)^2 F_M(q^2)$. In fact, the overall contribution to the transition amplitude [Eqs. (4)–(6)] from the extra terms required by Low's prescription is found to be of order $m_\mu/m_p F_M(q^2)$ and so is not negligible.[13] In the conventional approach,[1-9] the q^2 dependence of $F_{V,M,A}(q^2)$ was thought to be irrelevant, so that Low's "counter" terms are neglected. Here we find, on the contrary, that the smallness of the CEC-breaking terms does not justify the neglect of Low's counter terms.

The situation is almost identical in the evaluation of $\epsilon_\mu^* A_{\mu\lambda}(k,q,Q)$. The CEC and PCAC sum rules are important if contributions from the excited states as well as from the box diagrams should be taken into account. LH is again a näive way to realize these sum rules. Neither LH nor Low's prescription can be justified definitely in the absence of a fundamental theory.

The relationship between the amplitude of radiative muon capture and that of radiative pion capture deserves some attention. Before Eq. (19) is applied to this problem, it is important to note that both $D_\mu(k,q,Q)$ and T_μ receive contributions from the excited states and from the box diagrams. In what follows, we evaluate $D_\mu(k,q,Q)$ *only* from T_μ^0, which is taken as the sum of the first two terms of Eq. (12). The off-shell effects are neglected. In the case of nuclear spin and isospin doublets, we find

$$f_A^0=\left\{F_A[(q+k)^2]+\frac{(q+k)^2}{m_\pi^2}F_P[(q+k)^2]\right\}(\mu_i-\mu_f), \tag{28a}$$

$$f_P^0=-m_\pi^2\left\{F_A[(q+k)^2]+\frac{(q+k)^2}{m_\pi^2}F_P[(q+k)^2]\right\}\times\left[\frac{e_i}{(Q-q)\cdot k}+\frac{e_f}{(Q+q)\cdot k}\right], \tag{28b}$$

$$f_E^0=4Mm_p\left\{F_A[(q+k)^2]+\frac{(q+k)^2}{m_\pi^2}F_P[(q+k)^2]\right\}\times\left[\frac{e_i}{(Q-q)\cdot k}-\frac{e_f}{(Q+q)\cdot k}\right], \tag{28c}$$

$$f_2^0=-im_p^2\left\{F_A[(q+k)^2]+\frac{(q+k)^2}{m_\pi^2}F_P[(q+k)^2]\right\}\times\left[\frac{\mu_i}{(Q-q)\cdot k}-\frac{\mu_f}{(Q+q)\cdot k}\right], \tag{28d}$$

$$f_3^0=im_p^2\left\{F_A[(q+k)^2]+\frac{(q+k)^2}{m_\pi^2}F_P[(q+k)^2]\right\}\times\left[\frac{\mu_i}{(Q-q)\cdot k}+\frac{\mu_f}{(Q+q)\cdot k}\right], \tag{28e}$$

$$\tilde f_E^0=4Mm_p\left\{F_A[(q+k)^2]+\frac{(q+k)^2}{m_\pi^2}F_P[(q+k)^2]\right\}\times\left[\frac{e_i+\mu_i}{(Q-q)\cdot k}-\frac{e_f+\mu_f}{(Q+q)\cdot k}\right], \tag{28f}$$

where we have used the PCAC relationship,[13]

$$F_A(q^2)+\frac{q^2}{m_\pi^2}F_P(q^2)=\frac{a_\pi m_\pi^2 f_{\pi N_i N_f}(q^2)}{m_\pi^2+q^2}. \tag{29}$$

The agreement between Eqs. (28a)–(28f) and the Hwang-Primakoff result [Eqs. (32a′)–(32f′) of Ref. 13] is remarkable. If the variable $(q+k)^2$ is shifted to q^2 in Eqs. (28a)–(28f), the Hwang-Primakoff expressions for $f_{A,P,E}$, f_2, and f_3 are reproduced, while the resultant $\tilde f_E$ differs from theirs only by a small term,

$$2m_pF_A(q^2)\left(\frac{\mu_i}{2M_i}+\frac{\mu_f}{2M_f}\right).$$

The shift in the q^2 variable is needed since Eqs. (28b) and (28c) are not in accord with the CEC constraint equation, Eq. (21d). Alternatively, Low's prescription can be invoked to remove such inconsistency. This yields

$$N_{\mu,\mathrm{Low}}^{(1),0}-N_{\mu,\mathrm{LH}}^{(1)}=4Mq\cdot k\left[F_A(q^2)+\frac{q^2}{m_\pi^2}F_P(q^2)\right]'\left[\frac{e_i(Q-q)_\mu}{(Q-q)\cdot k}-\frac{e_f(Q+q)_\mu}{(Q+q)\cdot k}\right]-4Mq_\mu\left[F_A(q^2)+\frac{q^2}{m_\pi^2}F_P(q^2)\right]', \tag{30}$$

where the last term is induced again by Low's prescription. Low's counter term in Eq. (30) is by no means small since

$$\begin{aligned}\left[F_A(q^2)+\frac{q^2}{m_\pi^2}F_P(q^2)\right]'&=\frac{d}{dq^2}\left[\frac{a_\pi m_\pi^2 f_{\pi N_i N_f}(q^2)}{m_\pi^2+q^2}\right]\\&\cong-\frac{a_\pi m_\pi^2 f_{\pi N_i N_f}(q^2)}{(m_\pi^2+q^2)^2}\\&\cong\frac{F_P(q^2)}{m_\pi^2+q^2}.\end{aligned} \tag{31}$$

This is probably the reason why the Hwang-Primakoff predictions are somewhat lower than those of Beder.[22] In passing from Beder's expression for $1/m_p\, \epsilon_\mu^* A_{\mu\lambda}(k,q,Q)$ to that of Hwang and Primakoff, a major difference seems to be a flip in the overall sign of the expression specified by

$$i\frac{2M}{m_\pi^2}F_P(q^2)\bar u^{(f)}(p^{(f)},s^{(f)})\gamma_5\left[k_\lambda\left(\frac{e_ip^{(i)}\cdot\epsilon^*}{p^{(i)}\cdot k}-\frac{e_fp^{(f)}\cdot\epsilon^*}{p^{(f)}\cdot k}\right)-\epsilon_\lambda^*\right]u^{(i)}(p^{(i)},s^{(i)}).$$

The same sign flip can also be observed in $1/m_p \times\epsilon_\mu^* A_{\mu\lambda}(k,q,Q)$ in the case of $\mu^{-12}\mathrm{C}\to\nu_\mu{}^{12}\mathrm{B}\gamma$ where $G_{01}^{(a)}=0$ is the only input to generate an entirely nontrivial result.[14] The sign in the Hwang-Primakoff amplitudes can be understood intuitively from the PCAC constraint equation (9), by noting that, unless Eq. (19) does not yield an approximate expression for $D_\mu(k,q,Q)$, the $F_P(q^2)$ term in $\langle N_f;\,\mathrm{out}|A_\mu(0)|N_i;\mathrm{in}\rangle$ must be offset by a term in $A_{\mu\lambda}(k,q,Q)$. If Low's counter term specified by Eqs. (30) and (31) were added to the Hwang-Primakoff amplitude, we would recover the sign in the

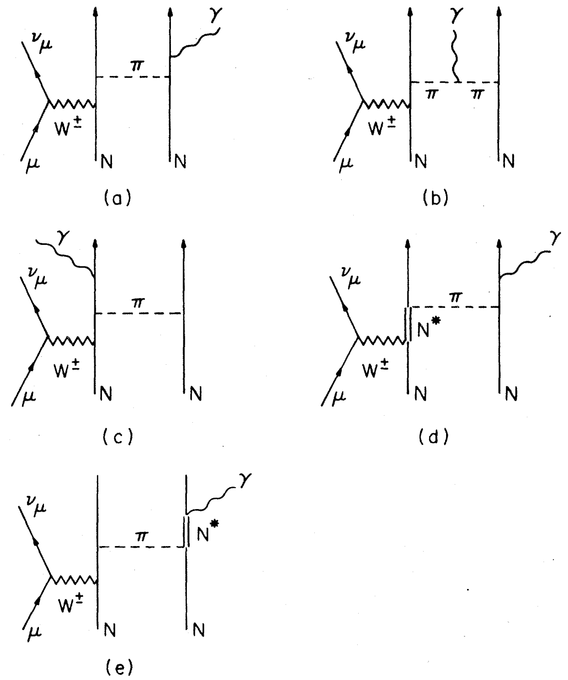

FIG. 1. Typical examples of the meson-exchange diagrams in radiative muon capture by a complex nucleus. These contributions can be extremely important if the impulse approximation starting with a transition amplitude for $\mu^- p \to \nu_\mu n\gamma$ is adopted.

conventional amplitude. In the next section, we shall confirm the numerical significance of such sign difference.

IV. PCAC AND NUCLEAR STRUCTURE

In view of an apparent A dependence of the ordinary and radiative muon capture rates, a possible test of PCAC has often been considered in an experiment with a complex nucleus.[23] It is a common practice to assume[24,9] that the sensitivity to the nuclear wave functions is not so important for an observable which is, by definition, to be some ratio. Examples of such an observable are the average polarization P_{av} in ordinary muon capture, the branching ratio R of radiative muon capture with respect to ordinary muon capture, etc. As should become clear from what follows, such an assumption is not justified in general if the question at stake is the validity of PCAC.

The reason is not unexpected. An assumed 100% violation of PCAC causes only a change of order (10–30)% on numerical predictions of many quantities in this category, including the above-mentioned average polarization P_{av} and branching ratio R. Unfortunately, there have been many effects which are of the same order of magnitude, viz.: The invocation of the impulse approximation (IA) has its own theoretical uncertainty, the nuclear wave functions are not so well known, the effects due to *nuclear* excited states and meson-exchange contributions can hardly be assessed, and so on. We can avoid some of these uncertainties by resorting to the so-called elementary-particle treatment.[25,8,13,14] However, if the *nucleon* pseudoscalar form factor is the entity of our ultimate concern, the connection between the *nuclear* and *nucleon* form factors must be determined via the invocation of a specific dynamical model such as the impulse approximation with/without meson-exchange corrections.

A direct IA calculation of radiative muon capture by a nucleus, as based upon a given amplitude for $\mu^- p \to \nu_\mu n\gamma$, is not very satisfactory since it is a formidable task to calculate the meson-exchange diagrams. Some examples of the meson-exchange diagrams are illustrated by Fig. 1. It is forseeable that many of these meson-exchange contributions can be enhanced considerably (in comparison with the meson-exchange contribution to a nonradiative process). Clearly, it is desirable that EPT is invoked to relate radiative muon capture by a nucleus, $\mu^- N_i \to \nu_\mu N_f \gamma$, *directly* to its corresponding nonradiative processes.[8,13,14] In this way, the only major uncertainty has to do with the connection between the nuclear and nucleon form factors. In general, the *nuclear* form factors can be expressed in terms of the *nucleon* form factors if IA with/without meson-exchange corrections is assumed.[26] Here it is important to distinguish the invocation of IA in this context from the above mentioned IA calculation of $\mu^- N_i \to \nu_\mu N_f \gamma$ from a given amplitude for $\mu^- p \to \nu_\mu n\gamma$. The uncertainty in question is that the nuclear wave functions (nuclear structure) must enter in the determination of the nuclear form factors in terms of the nucleon form factors. In what follows, we investigate quantitatively how a test of PCAC is subject to such uncertainty. Apart from the average polarization P_{av} in ordinary muon capture by ^{12}C, we analyze the impact of nuclear structure on radiative muon capture, $\mu^- {}^{12}C \to \nu_\mu {}^{12}B\gamma$.

To describe the $^{12}C \leftrightarrow {}^{12}B$ transitions, we introduce the nuclear weak magnetism (M), axial-vector (A), pseudoscalar (P), and weak electricity (E) form factors $F_{M,A,P,E}(q^2)$ as in Ref. 14 or 26. These form factors can be represented as follows[27,26]:

$$\sqrt{2}\,F_M(q^2) = [f_V(q^2) + f_M(q^2)]\,\frac{1}{\sqrt{3}}\left([\sigma]^{01} - \frac{1}{\sqrt{2}}[\sigma]^{21}\right) - f_V(q^2)\,\xi\,\frac{\sqrt{2}\,m_p}{|\vec{q}\,|}[\nabla/m_p]^{11} + [\sqrt{2}\,F_M(q^2)]_{MEC} + [\sqrt{2}\,F_M(q^2)]_{EM}\,, \tag{32a}$$

$$\sqrt{2}\,F_A(q^2)=f_A(q^2)\frac{1}{\sqrt{3}}\left([\sigma]^{01}-\frac{1}{\sqrt{2}}[\sigma]^{21}\right)+[\sqrt{2}\,F_A(q^2)]_{MEC}+[\sqrt{2}\,F_A(q^2)]_{EM}\,, \tag{32b}$$

$$\sqrt{2}\,F_P(q^2)=f_P(q^2)\frac{1}{\sqrt{3}}([\sigma]^{01}+\sqrt{2}\,[\sigma]^{21})+f_A(q^2)\sqrt{\tfrac{3}{2}}\,\frac{m_\pi^2}{|\vec{q}\,|^2}[\sigma]^{21}+[\sqrt{2}\,F_P(q^2)]_{MEC}+[\sqrt{2}\,F_P(q^2)]_{EM}\,, \tag{32c}$$

$$\sqrt{2}\,F_E(q^2)=\left[f_A(q^2)+\frac{2m_p\Delta}{m_\pi^2}(1-\zeta)f_P(q^2)\right]\frac{1}{\sqrt{3}}([\sigma]^{01}+\sqrt{2}\,[\sigma]^{21})+f_A(q^2)\zeta\frac{2m_p}{|\vec{q}\,|}[\vec{\sigma}\cdot\vec{\nabla}/m_p]^{11}$$
$$+f_A(q^2)\sqrt{\tfrac{3}{2}}\,\frac{2m_p\Delta}{|\vec{q}\,|^2}[\sigma]^{21}+[\sqrt{2}\,F_E(q^2)]_{MEC}+[\sqrt{2}\,F_E(q^2)]_{EM}\,, \tag{32d}$$

where $\Delta\equiv M(^{12}\mathrm{B})-M(^{12}\mathrm{C})=13.881$ MeV, and $f_{V,M,A,P}(q^2)$ are, respectively, the *nucleon* vector (V), weak magnetism (M), axial vector (A), and pseudoscalar (P) form factors, the various reduced matrix elements $[\sigma]^{01}$, $[\sigma]^{21}$, $[\nabla/m_p]^{11}$, and $[\vec{\sigma}\cdot\vec{\nabla}/m_p]^{11}$ are defined as in the paper of Delorme[26] or of Hwang and Henley,[27] and $\zeta\equiv 1-1/A=\frac{11}{12}$ is a factor arising from the extraction of the c.m. coordinates in the definition of the impulse approximation.[27] Further, $\{F_{M,A,P,E}\}_{MEC}$ and $\{F_{M,A,P,E}\}_{EM}$ are the meson-exchange corrections and residual electromagnetic corrections, respectively.[27]

It has been demonstrated explicitly that the residual electromagnetic corrections $\{F_{M,A,P,E}\}_{EM}$ be negligibly small.[28] On the other hand, the meson-exchange correction to the weak electricity form factor $F_E(q^2)$ can be as large as 30% of its impulse approximation value[29] and the meson-exchange correction to the ratio $\xi(q^2)\equiv F_P(q^2)/F_A(q^2)$ is expected to be about -17%.[30] In the low energy regime of our interest, i.e., $0\lesssim q^2\lesssim 0.74m_\mu^2$, we obtain from Eqs. (32a)–(32d)

$$\lambda(q^2)\equiv\frac{F_M(q^2)}{F_A(q^2)}\cong 3.76\,[1+\tfrac{11}{12}(x-1)]+[\lambda(q^2)]_{MEC}, \tag{33a}$$

$$\eta(q^2)\equiv\frac{F_E(q^2)}{F_A(q^2)}\cong 0.886\left(1+\frac{q^2\delta}{m_\pi^2}\right)+\tfrac{11}{12}(y-1)\left(1+\frac{1}{3}\frac{q^2\delta}{m_\pi^2}+\frac{q^2\tilde{\delta}}{m_\pi^2}\right)+\frac{2m_p\Delta}{m_\pi^2}\delta+[\eta(q^2)]_{MEC}, \tag{33b}$$

$$\xi(q^2)\equiv\frac{F_P(q^2)}{F_A(q^2)}\cong-\frac{1.02}{1+q^2/m_\pi^2}\left[1+\frac{q^2\delta}{m_\pi^2}+\epsilon(q^2)\right]+\delta\,, \tag{33c}$$

with

$$x-1\equiv-\lim_{|\vec{q}|\to 0}\frac{\sqrt{6}}{4.71}\frac{m_p}{|\vec{q}\,|}\frac{[\nabla/m_p]^{11}}{[\sigma]^{01}}\,, \tag{33d}$$

$$y-1\equiv\lim_{|\vec{q}|\to 0}\frac{2\sqrt{3}\,m_p}{|\vec{q}\,|}\frac{[\vec{\sigma}\cdot\vec{\nabla}/m_p]^{11}}{[\sigma]^{01}}\,, \tag{33e}$$

$$\delta\equiv\lim_{|\vec{q}|\to 0}\frac{3}{\sqrt{2}}\frac{m_\pi^2}{|\vec{q}\,|^2}\frac{[\sigma]^{21}}{[\sigma]^{01}}. \tag{33f}$$

Here $\{\lambda(q^2)\}_{MEC}$, $\{\eta(q^2)\}_{MEC}$, and $\epsilon(q^2)$ are the meson-exchange corrections, and the factor $q^2\tilde{\delta}/m_\pi^2$ in Eq. (33b) signifies the difference between the q^2 dependence of $(m_p/|\vec{q}|)[\vec{\sigma}\cdot\vec{\nabla}/m_p]^{11}$ and that of $[\sigma]^{01}$. Numerically, we find

$$+0.05\lesssim\frac{q^2\tilde{\delta}}{m_\pi^2}\lesssim+0.10 \text{ for } q^2=0.74m_\mu^2\,, \tag{34}$$

depending upon the radial dependence of the p-shell single-particle wave function.

The parameters x and y were introduced by Morita *et al.*[31] and the importance of the parameter δ was stressed by Hwang.[27] Averaging the results obtained by Morita *et al.*[31] for the three configurations of the Cohen-Kurath model,[32] we have

$$x=0.975,\quad y=3.61,\quad \delta=-0.282\,. \tag{35a}$$

In the single-particle shell model, we find[31]

$$x=0.894,\quad y=1.50,\quad \delta=-0.164\,. \tag{35b}$$

As already mentioned above, the meson-exchange correction to $\eta(q^2)$, i.e., to the time component of the weak axial current, is not negligible. For our purpose, we take[29,27]

$$[\eta(q^2)]_{MEC}\cong[\eta(0)]_{MEC}\cong 1.0\pm 0.2\,, \tag{36}$$

where the quoted uncertainty may even be larger. The meson-exchange corrections to the spatial components of the weak polar and axial current are expected to be of less importance. Since a previous estimate $\epsilon(q^2)=-0.17$ was obtained via the quenching argument,[30] and since the quenching of the nuclear axial form factor $F_A(0)$ has been used as an input in the Cohen-Kurath model,[32] it is possible that the estimate $\epsilon(q^2)=-0.17$ has been absorbed, partly or almost completely, by the specific wave functions. Thus, we simply neglect both $\{\lambda(q^2)\}_{MEC}$ and $\epsilon(q^2)$, but admit the importance of carrying out a consistent, though extremely complicated, calculation of all the meson-exchange corrections. That is, we take

$$[\lambda(q^2)]_{MEC}\cong 0,\quad \epsilon(q^2)\cong 0\,. \tag{37}$$

Equations (33a)–(33c) together with Eqs. (34)–(37) allow us to evaluate $\lambda(q^2)$, $\eta(q^2)$, and $\xi(q^2)$, with an uncertainty of order 1%, for $0 \lesssim q^2 \lesssim 0.74 m_\mu^2$. We note that the Kim-Primakoff relation as specified by[26]

$$\frac{\mu(q^2)}{\mu(0)} \simeq \frac{F_M(q^2)}{F_M(0)} \simeq \frac{F_A(q^2)}{F_A(0)} \tag{38}$$

has been justified by these considerations. It remains to note that, if the nucleon pseudoscalar form factor $f_P(q^2)$ were doubled, then the first term in the $\xi(q^2)$ of Eq. (33c) would be doubled while the coefficient 0.886 in Eq. (33b) would be replaced by $1-2\times(1-0.886)=0.772$. This allows us to investigate the sensitivity of a given physical quantity to the *nucleon* pseudoscalar form factor $f_P(q^2)$, although such investigation suffers slightly from the unknown $\epsilon(q^2)$.

Although the predicted value of $F_A(0)$ in the single-particle model is larger than its realistic value (as obtained from the beta decay rate) by about 60%, it is obvious that such an intolerable discrepancy cannot show up in the prediction of an observable which is, by definition, to be some ratio. This led some of us to conjecture that the nuclear structure would not be so important in the study of such an observable. The recent disputes over the existence of second-class axial currents already revealed the danger of adopting such a conjecture. Using Eqs. (33)–(38) and the formulas of the asymmetry coefficients $\alpha_\mp$ in the beta decays of ^{12}B and ^{12}N (see Refs. 26, 27, and 31), we find for the Cohen-Kurath model

$$\alpha_- = -0.08/\text{GeV}, \quad \alpha_+ = -2.69/\text{GeV}, \tag{39a}$$

and for the single-particle shell model

$$\alpha_- = 0.45/\text{GeV}, \quad \alpha_+ = 1.96/\text{GeV}. \tag{39b}$$

The predicted value of $\alpha_\mp$ in the Cohen-Kurath model agrees well with the data,[33] but the previous large discrepancy between the predictions of the single-particle shell model and the data is also removed to some extent by the inclusion of $\{\eta(0)\}_{MEC}$. For our purpose, we note that the 30% difference between the two α_+ of Eqs. (39a) and (39b) is merely a manifestation of the nuclear structure.

Similarly, we use Eqs. (33)–(38) to calculate the average polarization P_{av} in ordinary muon capture by ^{12}C and obtain

$$\begin{aligned} P_{av} &= 0.51, \quad \text{Cohen-Kurath} \\ &= 0.56, \quad \text{single particle}. \end{aligned} \tag{40a}$$

A variation in the nuclear structure can induce a 10% change in the predicted value of P_{av}. If the *nucleon* pseudoscalar form factor $f_P(q^2)$ were doubled, we would find, instead of Eq. (40a),

$$\begin{aligned} P_{av} &= 0.36, \quad \text{Cohen-Kurath} \\ &= 0.42, \quad \text{single particle}. \end{aligned} \tag{40b}$$

A 100% violation of PCAC induces a 30% change in the predicted value of P_{av}. It is evident that, if the validity of PCAC is at stake, the uncertainty arising from the nuclear structure should be taken into account.[34]

In the case of radiative muon capture by ^{12}C, we use Eqs. (33)–(38) to investigate the impact of the nuclear structure on a possible test of PCAC. Since it remains to be an experimental question to verify whether the Hwang-Primakoff amplitude is a realistic choice, it is useful to pin down the size of the effect due to the sign flip mentioned at the end of the previous section [see Eq. (31) *et seq*.]. Nevertheless, we use the Hwang-Primakoff amplitude in the analysis of the nuclear structure problem.

The quantities of prime experimental interest are the photon spectrum

$$d\Gamma(\mu^{-12}\text{C} \to \nu_\mu\, {}^{12}\text{B}\gamma)/dk_0,$$

the branching ratio

$$R \equiv \Gamma(\mu^-\, {}^{12}\text{C} \to \nu_\mu\, {}^{12}\text{B}\gamma)/\Gamma(\mu^-\, {}^{12}\text{C} \to \nu_\mu\, {}^{12}\text{B}),$$

and the photon asymmetry $\mathfrak{A}_{\hat{k},\hat{z}}$ (with respect to the direction of the residual muon polarization $P_\mu \hat{z}$). The explicit formulas for these quantities can be found in the paper of Hwang-Primakoff.

Using Eqs. (33)–(38), we obtain for the Cohen-Kurath model

$$\begin{aligned} R &= \frac{\alpha}{12\pi}\left(\frac{k_m}{m_\mu}\right)^2 \frac{0.559}{0.425} = 1.91\times 10^{-4}, \\ \mathfrak{A}_{\hat{k},\hat{z}}(x=\tfrac{2}{3}) &= 0.79 P_\mu, \end{aligned} \tag{41a}$$

and for the single-particle shell model

$$\begin{aligned} R &= \frac{\alpha}{12\pi}\left(\frac{k_m}{m_\mu}\right)^2 \frac{0.556}{0.431} = 1.87\times 10^{-4}, \\ \mathfrak{A}_{\hat{k},\hat{z}}(x=\tfrac{2}{3}) &= 0.82 P_\mu \end{aligned} \tag{41b}$$

where k_m is the maximum photon energy and $x \equiv k_0/k_m$. The photon spectrum is not considered in Eqs. (41a) and (41b) since the single-particle shell model fails to predict the absolute overall normalization of the photon spectrum. On the other hand, if the *nucleon* pseudoscalar form factor were doubled, we would obtain for the Cohen-Kurath model

$$\begin{aligned} R &= \frac{\alpha}{12\pi}\left(\frac{k_m}{m_\mu}\right)^2 \frac{0.603}{0.407} = 2.15\times 10^{-4}, \\ \mathfrak{A}_{\hat{k},\hat{z}}(x=\tfrac{2}{3}) &= 0.58 P_\mu, \end{aligned} \tag{42a}$$

and for the single-particle shell model

$$R=\frac{\alpha}{12\pi}\left(\frac{k_m}{m_\mu}\right)^2\frac{0.569}{0.401}=2.06\times10^{-4}\,,$$
$$\mathcal{A}_{\hat{k},\hat{z}}(x=\tfrac{2}{3})=0.60P_\mu\,. \qquad (42b)$$

A variation in the nuclear structure appears to have little impact on the numerical predictions of the branching ratio R and the photon asymmetry $\mathcal{A}_{\hat{k},\hat{z}}$. Unfortunately, such a feature might happen to occur only in the case of $\mu^- {}^{12}C\to\nu_\mu {}^{12}B\gamma$ and an arbitrary generalization to the other cases could be dangerous. Nevertheless, it is sensible to note that, since the photon asymmetry (not the branching ratio) is very sensitive to the *nucleon* pseudoscalar form factor $f_P(q^2)$, a test of PCAC via a measurement of the photon asymmetry suffers little from the uncertainty due to the nuclear structure.

We conclude our investigations by considering the effect due to the sign flip mentioned at the end of the previous section [see Eq. (31) *et seq.*]. Since the validity of PCAC is the main motive in the study of radiative muon capture, we present the photon spectrum, the radiative capture rate, and the photon asymmetry in the following form[14]:

$$\frac{d\Gamma(\mu^- {}^{12}C\to\nu_\mu {}^{12}B\gamma)}{dx}=\frac{\alpha}{12\pi}\Gamma_0\left(\frac{k_m}{m_\mu}\right)^2[12x(1-x)^2]D(\xi,x)\,, \qquad (43a)$$

$$\Gamma(\mu^- {}^{12}C\to\nu_\mu {}^{12}B\gamma)=\frac{\alpha}{12\pi}\Gamma_0\left(\frac{k_m}{m_\mu}\right)^2K(\bar{\xi})\,, \qquad (43b)$$

$$\mathcal{A}_{\hat{k},\hat{z}}=\left[1-\frac{N(\xi,x)}{D(\xi,x)}\right]P_\mu\,, \qquad (43c)$$

where $\xi\equiv\xi(q^2)$ with $q^2\cong m_\mu{}^2(1-2x+2x^2)$, $\bar{\xi}\equiv\xi(0.45\,m_\mu{}^2)$, and[35]

$$D(\xi,x)=W_1^{(0)}(x)+\tfrac{1}{3}W_1^{(2)}(x)\,,$$
$$N(\xi,x)=W_1^{(0)}(x)-W_2^{(0)}(x)+\tfrac{1}{3}\times[W_1^{(2)}(x)-W_2^{(2)}(x)-W_3^{(1)}(x)]\,,$$
$$K(\bar{\xi})=\int_0^1 12x(1-x)^2D(\bar{\xi},x)\,dx\,, \qquad (43d)$$
$$\Gamma_0=\frac{G^2m_\mu{}^5}{2\pi}\left(\frac{k_m}{m_\mu}\right)^2\left(1-\frac{m_\mu}{m_\mu+M_f}\right)\times C_i\left(e_i\alpha\frac{M_i}{m_\mu+M_i}\right)^3,\quad C_i=0.841\,.$$

Here $W_i^{(n)}(x)$ are the structure functions introduced by Hwang and Primakoff.[14] It is a sufficient approximation to write, from Eqs. (33)–(38),

$$\lambda(q^2)\cong\lambda(0.45\,m_\mu{}^2)=3.67\,,$$
$$\eta(q^2)\cong\eta(0.45\,m_\mu{}^2)=3.87\,, \qquad (44)$$
$$\xi(q^2)\cong0.93[f_P(q^2)/f_A(q^2)][1+\epsilon(q^2)]-0.28\,.$$

The overall normalization is determined by the choice of $F_A(q^2)=F_A(0.45\,m_\mu{}^2)=0.432$. Hereafter, we consider only the Cohen-Kurath model since this model is in good agreement with the data in the beta decays of ^{12}B and ^{12}C and in ordinary muon capture. At this connection, it is important to note that it is the combination $[f_P(q^2)/f_A(q^2)][1+\epsilon(q^2)]$ which can be extracted from the experimental data.

The results for the Hwang-Primakoff amplitude can be recorded as follows:

$$D(\xi,x)=(0.933-0.357x+0.036x^2)+\xi(0.398+0.055x-0.106x^2)+\xi^2(0.240-0.384x+0.384x^2)\,, \qquad (45a)$$
$$N(\xi,x)=0.016+\xi(0.026-0.104x)+\xi^2(0.096-0.256x+0.256x^2)\,, \qquad (45b)$$
$$K(\bar{\xi})=0.797+0.399\bar{\xi}+0.163\bar{\xi}^2\,. \qquad (45c)$$

For the amplitude obtained from the Hwang-Primakoff amplitude by the designated sign flip, we find

$$D(\xi,x)=(0.933-0.357x+0.036x^2)+\xi(-0.013+0.064x-0.012x^2)+\xi^2(0.048-0.128x+0.128x^2)\,, \qquad (46a)$$
$$N(\xi,x)=0.016-\xi(0.026-0.104x)+\xi^2(0.096-0.256x+0.256x^2)\,, \qquad (46b)$$
$$K(\bar{\xi})=0.797+0.010\bar{\xi}+0.022\bar{\xi}^2\,. \qquad (46c)$$

This amplitude is referred as "the approximate conventional amplitude". As compared to the conventional amplitude,[4-9] it does not include the pion-exchange diagram and, yet, it does not have the $(q+k)^2$ dependence in the nonradiative weak form factors. Nonetheless, its numerical predictions are not expected to differ drastically from the conventional amplitude (see Ref. 22).

The difference between the Hwang-Primakoff and approximate conventional amplitudes is substantial. If the validity of PCAC is assumed for the nucleon form factors [Eq. (33c)], the radiative capture rate in the approximate conventional amplitude is larger than that of Hwang and Primakoff by 45%. In Figs. 2 and 3, the photon spectrum in units of $\alpha/12\pi\;\Gamma_0(k_m/m_\mu)^2$ and the photon asymmetry are plotted for these two amplitudes with the validity of PCAC assumed and $\epsilon(q^2)\cong0$. The differences are again substantial. At $x=\frac{2}{3}$, the spectrum in the approximate conventional amplitude is larger than that of Hwang and Primakoff by 49% and the photon asymmetry is $0.98P_\mu$ for the approximate conventional amplitude and $0.79P_\mu$ for the Hwang-Primakoff amplitude. It

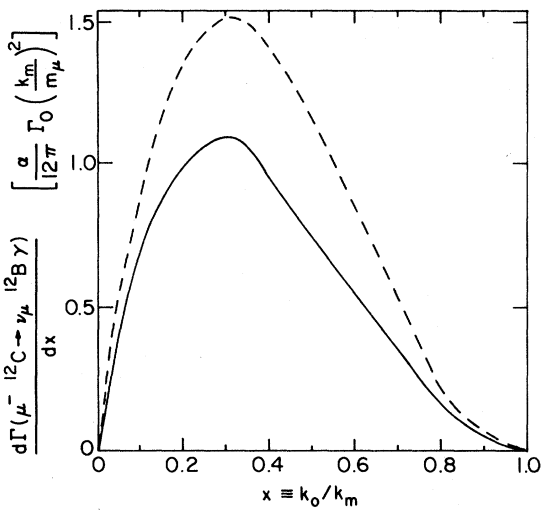

FIG. 2. The photon spectra predicted by the Hwang-Primakoff amplitude (solid curve) and by the approximate conventional amplitude (dash curve). PCAC and $\varepsilon(q^2)\cong 0$ are used. The curves are normalized to $\alpha/12\pi\ \Gamma_0(k_m/m_\mu)^2$.

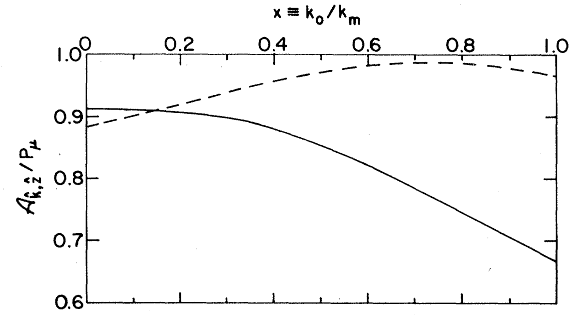

FIG. 3. The photon asymmetry predicted by the Hwang-Primakoff amplitude (solid curve) and by the approximate conventional amplitude (dash curve). PCAC and $\varepsilon(q^2)\cong 0$ are used.

is evident from Eqs. (45) and (46) that the predictions in the Hwang-Primakoff theory are much more sensitive to the pseudoscalar form factor. These numerical results are quite similar to those Fearing[22] obtained for $\mu^- p\to\nu_\mu n\gamma$ and $\mu^-\,{}^3\mathrm{He}\to\nu_\mu\,{}^3\mathrm{H}\gamma$, except that the difference between the two amplitudes in our case measures more precisely the effect due to the sign flip of our interest. In any event, the radiative muon capture experiment can differentiate easily between the two amplitudes and, yet, provide a comprehensive test of PCAC.

V. CONCLUSIONS

In conclusion, the main results of this paper are recorded, viz.:

(i) The relationship between the transition amplitude of radiative muon capture and that of radiative pion capture should be derived consistently such that Eq. (9b) is not violated. A näive choice of such relationship is given approximately by Eq. (19), which leads to the well-known soft-pion theorems.

(ii) In the Hwang-Primakoff approach,[13,14] a simplifying dynamical approximation, the so-called "linearity hypothesis" (LH), is adopted to realize the various CEC, CVC, and PCAC sum rules. The näive relationship, Eq. (19), between radiative muon capture and radiative pion capture is satisfied approximately by the Hwang-Primakoff amplitudes.

(iii) The difference between the Hwang-Primakoff and conventional amplitudes[7-9] arises from the fact that some Low's counter terms, as introduced to secure the presence of small CEC-breaking terms in the amplitude, are not negligible.[36] It is also found that, if Low's prescription is used consistently, some Low's counter terms with numerical significance should be added to the conventional amplitude which is currently adopted[8,9,22] [see Eq. (27) *et seq*.].

(iv) Despite the importance of the nuclear structure in the beta decays of ^{12}B(g.s.) and ^{12}N(g.s.) and in ordinary muon capture by ^{12}C(g.s), it is found in the elementary particle treatment[14] that, apart from the absolute overall normalization, a variation from the Cohen-Kurath model to the single-particle shell model *does not* induce any significant change in the numerical predictions on radiative muon capture by ^{12}C. Since the difference between the predictions of the Hwang-Primakoff amplitude and of the ("approximate") conventional amplitude is confirmed to be very substantial,[22] a radiative muon capture experiment can *easily* pin down the correct theory, and yet provide a comprehensive test of PCAC.

To date, the observed photon spectrum in inclusive radiative muon capture by ^{40}Ca appears to be relatively low, such that the sign of a particularly important $F_P(q^2)$ term in the transition amplitude is consistent with the one suggested by the Hwang-Primakoff approach.[37] However, this last result is at most suggestive since there involve more theoretical uncertainties in inclusive radiative muon capture.[37] In view of its grave importance in the description of any process in which both CEC and PCAC are at stake, such sign ambiguity must be settled as soon as possible by an exclusive radiative muon capture experiment.

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[35]In Ref. 14, the factor k_m^2/m_μ^2 is missing from the definition of Γ_0 and $\frac{1}{3}W_1^{(2)}(x)$ in the definition of the photon spectrum is misprinted as $\frac{1}{2}W_1^{(2)}(x)$ [see Eq. (45)].

[36]It is evident that the major difference between the Adler-Dothan (Ref. 11) or Christillin-Servadio (Ref. 12) amplitude and the Hwang-Primakoff amplitude is again the presence of Low's counter terms.

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