Short-range repulsion in hypernuclei and calculation using K harmonics

S. P. Verma* and D. P. Sural
Department of Physics, Jadavpur University, Calcutta-700 032, India
(Received 5 November 1979)

The K-harmonics method is used to determine the Λ separation energy B_Λ for the hypertriton and the $\Lambda\Lambda$ separation energy $B_{\Lambda\Lambda}$ for ${}^{6}_{\Lambda\Lambda}$He. B_Λ is calculated by retaining twelve harmonics and using Λ-N and N-N local potentials of the Volkov type fitted to Nagel's meson theoretic (model B) values of the scattering lengths and effective ranges. It gets reduced from 0.33 to 0.13 MeV by the addition of suitable repulsive terms. Retaining only nine harmonics, a convergent result for $B_{\Lambda\Lambda}$ is obtained with Tang and Herndon's α-Λ potential and a two-term Λ-Λ potential of the Volkov type. The experimental $B_{\Lambda\Lambda}$ value of 10.8 MeV can be reproduced with a Λ-Λ potential that gives rise to the scattering length of -2.23 fm and an effective range of 3.31 fm.

[NUCLEAR STRUCTURE Short-range repulsion, K harmonics, ${}^{3}_{\Lambda}$H, Λ separation energy, ${}^{6}_{\Lambda\Lambda}$He, $\Lambda\Lambda$ separation energy, scattering length, effective range.]

I. INTRODUCTION

A few years ago Gibson and Lehman[1] calculated the binding energy of the hypertriton for one-term s-wave nonlocal separable (NLS) Λn and Λp potentials whose parameters were chosen to reproduce the scattering lengths and effective ranges derived earlier by Nagels *et al.*[2] The latter authors constructed two different types of meson theoretic hyperon-nucleon potentials by using two models for the interaction, which were called A and B, respectively. While the two pion exchange process was included in the model A, the one boson exchange mechanism was considered in the other model, B. (Both the models incorporated the exchange of a number of other mesons.) For the ${}^{3}_{\Lambda}$H system, Gibson and Lehman found considerable difference in the Λ separation energy (B_Λ) values obtained with the two sets of one-term NLS potentials used as approximation to the model A and model B potentials. Schick[3] suggested that the approximation to a meson theoretic ΛN potential should contain, besides the long-range attractive part, a short-range repulsion as well. He used two-term NLS potentials that yielded the singlet and triplet ΛN (average of ΛN and Λp) scattering lengths and effective ranges of the model A, and showed that the inclusion of a repulsive term led to a significant reduction in B_Λ for the hypertriton. However, using a repulsive term of a different shape, Gibson and Lehman[4] found a much smaller value for such reduction. They used low energy ΛN scattering parameters which were not exactly the same as those used by Schick, but this cannot wholly account for the difference between the two findings. To investigate this problem, Arnold *et al.*[5] have recently used a Fredholm determinant approach to the radial wave equation for a nonlocal potential. They have shown that, although the two-term nonlocal ΛN potentials used by Schick[3] and by Gibson and Lehman[4] are approximately on-shell T matrix equivalent, the former (specifically, the singlet potential) gives rise to an extra node in the wave function. This results in a difference between the B_Λ values obtained with the two potentials.

It may be noted that, in the original Gibson-Lehman calculation,[1] the one-term NLS potentials approximating the model B ΛN potential gave a better value for the Λ separation energy. However, no calculation within the framework of the model B was made to investigate the effect of a short-range ΛN repulsion on the binding energy of the hypertriton. Roy-Choudhury and Gautam[6] have, of course, studied such effects using two-term NLS potentials fitted to reproduce the low-energy scattering parameters of Herndon and Tang[7] and of Alexander *et al.*[8] The three-body calculations mentioned in this and the earlier paragraph are all based on the Faddeev formalism.

In the present work we have used the model B scattering lengths and effective ranges, and have constructed two-body local potentials of the Volkov[9] type that yield them. The two-body potentials, containing long-range attractive and short-range repulsive parts, are utilized to find B_Λ in ${}^{3}_{\Lambda}$H by employing the method of hyperspherical or K harmonics.[10] The procedure consists in expanding the three-body wave function in a complete set of orthonormal functions (K harmonics) of five angular variables and getting a set of coupled integral equations for functions of a single variable, the hyperradius, which is the length of a six-dimensional vector. The three-body binding energy is obtained by solving the set of equations after a suitable truncation. In a previous work[11] (to be henceforth referred to as paper I), we applied the method of K harmonics to study the Λ binding in ${}^3_\Lambda$H and ${}^9_\Lambda$Be (in α-α-Λ model). The method was found to be quite suitable for simple attractive potentials of square-well, Gaussian, exponential and Yukawa shape. For our present problem, with the short-range repulsion taken into account, we find that the K-harmonics method works well, but the number of harmonics to be retained in the expansion for the hypertriton wave function needs to be increased.

We have also employed the method of K harmonics to work out the $\Lambda\Lambda$ separation energy $B_{\Lambda\Lambda}$ in the double hypernucleus ${}_{\Lambda\Lambda}^{\;6}$He using the Λ-Λ-α model. ${}_{\Lambda\Lambda}^{\;6}$He is the lightest double hypernucleus detected experimentally,[12] and the Λ-Λ-α model has been found to be a good approximation for it. Conventional variational calculations for the Λ-Λ-α system were done by Tang and Herndon,[13] Ali and Bodmer,[14] and Bhamathi *et al.*[15] The Faddeev method was used for it by Roy-Choudhury *et al.*[16] We have, in this work, used the Tang-Herndon potential B^{13} for the $\alpha\Lambda$ interaction. The $\Lambda\Lambda$ interaction is represented by a two-term local Volkov-type potential. The parameters for the $\Lambda\Lambda$ potential that reproduce the experimental $B_{\Lambda\Lambda}$ value are determined and utilized to calculate the scattering length and effective range. In this connection we want to mention that, because of the difficulty of performing a $\Lambda\Lambda$ scattering experiment, $\Lambda\Lambda$ scattering parameters are usually obtained from investigations of the binding energies of observed double hypernuclei.

The theory of our method and the results obtained are discussed. respectively, in Secs. II and III of the paper.

II. THEORY

The method of K harmonics as applied to the ${}^3_\Lambda$H problem was discussed in detail in paper I. We shall follow the notations used in that paper and shall briefly describe the theory with the additions that are necessary for the present work. Introducing the Jacobi coordinates

$$\vec{\xi}=\frac{(6mm_\Lambda)^{1/2}}{2m+m_\Lambda}\left(\frac{\vec{r}_1+\vec{r}_2}{2}-\vec{r}_3\right), \tag{1a}$$

$$\vec{\eta}=\left[\frac{3m}{2(2m+m_\Lambda)}\right]^{1/2}(\vec{r}_1-\vec{r}_2), \tag{1b}$$

where m and m_Λ are respectively, the nucleon and Λ-particle mass, and $\vec{r}_1$, $\vec{r}_2$, and $\vec{r}_3$ are, respectively, the position vectors of the neutron, proton, and Λ-particle, the space part of the ground-state wave function (in the c.m. system) of the hypertriton, assuming $L=0$, satisfies the equation

$$\left\{-\left[\frac{3}{2(2m+m_\Lambda)}\right](\nabla_\xi^2+\nabla_\eta^2)+V_T^{(12)}(r_{12})+\sum_{i=1}^{2}\left[\tfrac{3}{4}V_S^{(i3)}(r_{i3})+\tfrac{1}{4}V_T^{(i3)}(r_{i3})\right]-E\right\}\psi(\vec{\xi},\vec{\eta})=0. \tag{2}$$

We use units such that $\hbar=1$. In Eq. (2), the triplet np potential $V_T^{(12)}$, and the singlet and triplet $N\Lambda$ potentials $V_S^{(i3)}$ and $V_T^{(i3)}$ have all been taken to be of the form

$$-V_A\exp\left(-\frac{r^2}{\beta_A^2}\right)+V_R\exp\left(-\frac{r^2}{\beta_R^2}\right), \tag{3}$$

consisting of an attractive and a repulsive part. $\psi(\vec{\xi},\vec{\eta})$ is then expanded in terms of a complete set of orthonormal functions $U_K^\nu(\hat{\rho})$ of the direction of the six-dimensional vector $\vec{\rho}(\vec{\xi},\vec{\eta})$. In our problem, ψ should be symmetric with respect to change in sign of $\vec{\eta}$, and the K harmonics of the necessary symmetry are given by[10]

$$U_K^\nu(\hat{\rho})=\left[1-\delta_{\nu 0}\left(1-\frac{1}{\sqrt{2}}\right)\right]\left(\frac{K+2}{\pi^3}\right)^{1/2}\times\cos\left[\nu\arctan\left(\frac{2\vec{\xi}\cdot\vec{\eta}}{\eta^2-\xi^2}\right)\right]\times A^\nu P_{K/4-\nu/2}^{(\nu,0)}(1-2A^2), \tag{4}$$

where

$$A=[(\eta^2-\xi^2)^2+4(\vec{\xi}\cdot\vec{\eta})^2]^{1/2}/\rho^2,$$

K is an even positive integer, and ν takes all positive values starting from $\frac{1}{2}K$ and decreasing in steps of two. In Eq. (4), $P_n^{(\alpha,\beta)}(x)$ represents a Jacobi polynomial. The ground-state energy E of ${}^3_\Lambda$H is obtained from the set of integral equations

$$\sum_{K',\nu'}\int_0^\infty I_{K+2}^{(x_<)}K_{K+2}^{(x_>)}M_{KK'}^{\nu\nu'}\left(\frac{x'}{\kappa}\right)x'R_{K'}^{\nu'}\left(\frac{x'}{\kappa}\right)dx'=ER_K^\nu\left(\frac{x}{\kappa}\right), \tag{5}$$

where $\kappa=[-\frac{2}{3}(2m+m_\Lambda)E]^{1/2}$ and $M_{KK'}^{\nu\nu'}(\rho)$ are matrix elements of the potential occurring in Eq. (2) between two K-harmonics of the type given in Eq. (4). The procedure for determining E from Eq. (5) was described in paper I and will not be repeated. We only want to mention that convergent result was obtained in our previous work on retaining nine

K harmonics in the expansion for the three-body wave function. In the present work, with short-range repulsion included in the interaction, the basis set has to be enlarged in the case of the Λnp system. The final result for the hypertriton is obtained with twelve harmonics retained. The three additional harmonics included are U_{10}^{1}, U_{10}^{3}, and U_{10}^{5}.

For the ${}_{\Lambda\Lambda}^{6}$He problem, we note that the ground state is a state with $J=0$. Assuming the Λ-Λ-α model and zero total orbital angular momentum for the ground state, the two Λ hyperons must together be in singlet spin state. The space part of the total wave function should therefore be symmetric with respect to the exchange of the Λ particles. If we denote the position vectors of the two Λ particles by $\vec{r}_1$ and $\vec{r}_2$ and that of the α particle by $\vec{r}_3$, the formalism developed for the hypertriton can be used with m and m_Λ being, respectively, replaced by m_Λ and m_α (α-particle mass). In this case also, we need consider only those wave functions which remain unchanged when $\vec{\eta}$ is changed to $-\vec{\eta}$. The potentials occurring in Eq. (2), and hence in the matrix elements $M_{KK'}^{\nu\nu'}$, will, of course, have to be changed to

$$V_T^{(12)}(r_{12}) \rightarrow V_S^{\Lambda\Lambda}(r_{12}) \tag{6a}$$

and

$$\tfrac{3}{4}V_S^{(i3)}(r_{i3}) + \tfrac{1}{4}V_T^{(i3)}(r_{i3}) \rightarrow V^{\alpha\Lambda}(r_{i3}), \quad i=1,2\,. \tag{6b}$$

As stated earlier, the $\alpha\Lambda$ potential has been taken from Tang and Herndon's work.[13] The form of the $\Lambda\Lambda$ potential used by us is again given by Eq. (3). Convergent values for the ground-state energy of the Λ-Λ-α system can be obtained by retaining nine harmonics.

III. RESULTS AND DISCUSSION

A. The hypertriton

We have first solved numerically the two-body Schrödinger equations with local potentials of the Volkov type [Eq. (3)]. The potential parameters have been determined so as to reproduce the NN and ΛN scattering lengths and effective ranges of Nagel's model B potentials. In order to reduce the magnitude of the numerical work, we have followed Schick[3] (who, however, considered the model A potentials) and have taken the average of the Λp and Λn scattering parameters to represent the effective ΛN parameters. The singlet and triplet ΛN scattering parameters used in the present work are

$$\begin{aligned} a_s &= -2.29 \text{ fm}, \quad r_s = 3.14 \text{ fm}, \\ a_t &= -1.77 \text{ fm}, \quad r_t = 3.25 \text{ fm}\,. \end{aligned} \tag{7}$$

TABLE I. ΛN singlet and triplet potential parameters [see Eq. (3) of text] and the corresponding Λ separation energy (B_Λ) for the hypertriton.

		V_A (MeV)	β_A (fm)	V_R (MeV)	β_R (fm)	B_Λ (MeV)
Set 1	singlet	92.94	1.20	132.10	0.82	
	triplet	73.66	1.20	92.07	0.82	0.24
Set 2	singlet	72.03	1.25	95.55	0.82	
	triplet	57.03	1.25	63.23	0.82	0.28
Set 3	singlet	167.34	1.10	246.80	0.82	
	triplet	132.41	1.10	181.68	0.82	0.13
Set 4	singlet	26.17	1.50			
	triplet	25.65	1.44			0.33

Table I shows the strengths and the ranges of four different sets of ΛN potentials that yield the above parameters. In the first three sets, the range for the repulsive part has been kept at the same value of 0.82 fm, but different values for the attractive range have been used. The last set does not contain any repulsive core. The ranges for the repulsive and attractive parts of the np interaction are taken to be 0.82 and 1.34 fm, respectively. In the calculation with set 4 ΛN potential, we have used an np interaction without core.

We now apply the method of K harmonics to find out the B_Λ values for the hypertriton using the two-body potentials described above. These B_Λ values are given in the last column of Table I. It may be pointed out that the Λ separation energy of 0.33 MeV which we have obtained for the set 4 potentials without repulsion is close to the value of 0.37 MeV found by Gibson and Lehman[1] with model B parameters. As explained in the Introduction, Gibson and Lehman employed the Faddeev method with one-term attractive Λn and Λp potentials. The inclusion of the short-range repulsion in our work reduces the B_Λ values. Table I illustrates the nature of the variation in the magnitude of this reduction when the range of the ΛN attraction is changed. The set 3 potentials, in particular, reproduce the experimental value[17] for Λ binding in the hypertriton.

B. $\Lambda\Lambda$ scattering parameters

In dealing with the double Λ hypernucleus ${}_{\Lambda\Lambda}^{6}$He in the Λ-Λ-α model, we have used the $\alpha\Lambda$ potential B of Tang and Herndon[13] which can account for an α-Λ bound state at -3.04 MeV. Two-term $\Lambda\Lambda$ singlet potentials of the Volkov type with $\beta_R = 0.82$ fm and $\beta_A = 1.20$ fm but varying strength parameters are considered. We find that the K-

harmonics method yields the observed[12] $\Lambda\Lambda$ separation energy of -10.8 MeV when the attractive and repulsive strengths are 93.75 and 148.00 MeV, respectively. The $\Lambda\Lambda$ singlet potential found in this way is now utilized to calculate the scattering length and effective range. The values found are -2.23 and 3.31 fm, respectively. The test of the reliability of these parameters must await further development of the theory and experiment on $\Lambda\Lambda$ interaction.

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*On leave from Patna University.

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