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Collapse of the conventional shell-model ordering in the very-neutron-rich isotopes of Na and Mg

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Recent experimental data on the structure of the very-neutron-rich isotopes of Na and Mg show sharp departures at $N = 20$ from the predictions of conventional $d_{5/2}$-$s_{1/2}$-$d_{3/2}$ shell-model calculations.

[NUCLEAR STRUCTURE Isotopes of Na and Mg; calculated ground state energies, magnetic dipole moments, and excitations of 2^+ states; $0d_{5/2}$-$1s_{1/2}$-$0d_{3/2}$ shell model, Chung-Wildenthal Hamiltonians.]

It has often been speculated that study of far-from-stability nuclei would reveal exciting and instructive extensions to our knowledge of nuclear structure as it has been accumulated from the study of systems with stable or near-stable neutron/proton ratios. Results emerging from an ongoing research program on the neutron-rich Na isotopes and their decay products[1-3] seem to fulfill such expectations. The first data to emerge from this program were the masses of the Na isotopes up through ^{32}Na.[1] The values which were obtained for ^{30}Na and ^{31}Na indicated significantly more binding energy for these systems than was predicted by conventional shell-model calculations[4] based on the then most firmly based effective interaction.[5] Subsequent Hartree-Fock calculations[6] suggested that these mass anomalies, together with the large binding energy of ^{32}Na, are associated with a shape transition which results in the filling, at large prolate deformations, of the negative-parity $f_{7/2}$ orbit in preference to the $d_{3/2}$ orbit.

Additional experimental information about these systems—the ground-state spins and magnetic moments of the Na isotopes[2] and the energies of the first excited states of some Mg isotopes[3]—has recently become available. We examine here the amalgam of these data in comparison to newer, more extensive shell-model calculations.[7] Our aim is to delineate in detail the degree to which these experimental data on "exotic" nuclei contradict conventional nuclear-structure expectations.

We utilize two complementary sets of shell-model results. Each set systematically utilizes the complete $d_{5/3}$-$s_{1/2}$-$d_{3/2}$ basis space and is founded upon the assumption that $0p$ and $0f1p$ degrees of freedom can be neglected in the description of lowest-lying states in the region $8 \leq N$, $Z \leq 20$. One set of results is based on an effective interaction (the "particle" Hamiltonian) determined by a fit to 200 level energies taken predominantly from the $A = 18$–24 mass region. The other is based on an interaction (the "hole" Hamiltonian) analogously determined from a fit to level energies taken predominantly from the $A = 32$–38 region. The predictions of these calculations explain a wide variety of structural features in the $A = 18$–26 and $A = 30$–38 regions rather accurately and have been used in the $A = 28$ region with fair success. Neither calculation is tailored to deal with the exotic Na and Mg isotopes, but inspection of how the agreement between these calculations and experimental data extrapolates away from the regions in which the effective interactions were determined suggests that their predictions should explain the qualitative aspects of the observed structure as long as the dominant degrees of freedom are those of the *sd* shell.

We consider first the experimentally assigned ground-state spins of the Na isotopes. Since shell-model calculations yield a definite energy-ordered sequence of nuclear spins, it is implicit that the spin of the observed ground state should correspond to that of the calculated ground state. In the case of close-lying ground-state multiplets, however, it is not critical if the theoretical and experimental spin orderings are not identical so long as the appropriate identifications are made. Up through ^{30}Na, the calculated and measured[2,8,9] ground-state spins (^{20}Na-2^+, ^{21}Na-$\frac{3}{2}^+$, ^{22}Na-3^+, ^{23}Na-$\frac{3}{2}^+$, ^{24}Na-4^+, ^{25}Na-$\frac{5}{2}^+$, ^{26}Na-3^+, ^{27}Na-$\frac{5}{2}^+$, ^{28}Na-1^+, ^{29}Na-$\frac{3}{2}^+$, ^{30}Na-2^+) are in agreement, even though the energy differences between the $J^\pi = \frac{5}{2}^+$ and $\frac{3}{2}^+$ states in the odd isotopes are small and the odd-neutron, odd-proton even-mass isotopes have

the characteristically closely spaced ground-state multiplets. For ^{31}Na, however, the calculated ground state is $J^\pi = \frac{5}{2}^+$ and the calculated $J^\pi = \frac{3}{2}^+$, first-excited state is at 0.6 MeV. The experimentally assigned spin of the ^{31}Na ground state is $\frac{3}{2}$ and this constitutes a significant contradiction to the assumption that the structure of the $Z = 11$, $N = 20$ system is of the same type as those of the less-neutron-rich systems.

We consider the masses of the Mg and Na isotopes in terms of the nuclear part of the binding energy relative to ^{16}O. In the lower portion of Fig. 1 we plot the differences between the measured and calculated binding energies for the Ne, Na, and Mg isotopes. The predictions of the particle Hamiltonian are used up to $A = 28$ and the average of the particle and hole predictions used for $A \geq 28$. The agreement between experiment and calculation deteriorates as the masses increase away from the region for which the Hamiltonian was designed, but the only clear anomaly is for ^{31}Na ($N = 20$). The degree of this discrepancy, and the suddenness of its onset, again seem unexpected in the context of a continuous validity of the *sd*-shell degrees of freedom over this entire mass range.

In the upper portion of Fig. 1 are plotted the calculated excitation energies of the lowest $J^\pi = 2^+$ states of the Mg isotopes, $A = 20$–32, and the experimental values to which they are either assigned[8,9] or assumed[3] to correspond. (For ^{20}Mg we use the energy from the mirror ^{20}O spectrum.[8] The theoretical excitations are consistently larger than the experimental values, a result principally of the fact that the Hamiltonians we use appear systematically to generate too much binding energy for $J^\pi = 0^+$ ground states. Nonetheless, the trend of the observed $J^\pi = 2^+$ excitation energies is well reproduced by the calculations for $A = 20$ up through $A = 30$. However, the energy assumed[3] to represent the lowest 2^+ state in ^{32}Mg ($N = 20$) completely deviates from the pattern established for the $N < 20$ systems. The present calculations yield a quantitative foundation to the simple qualitative theoretical principle that the freezing out of a degree of freedom, in this case the loss of neu-

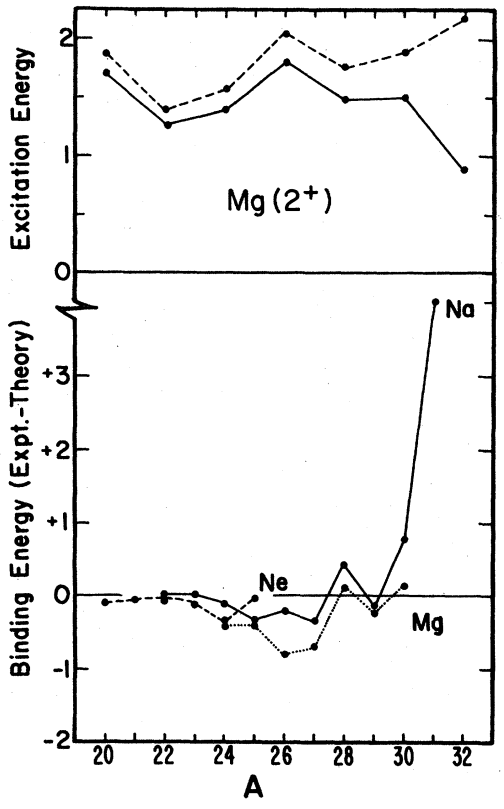

FIG. 1. Plots of the deviations between calculated and measured nuclear binding energies for the $N \geq Z$ isotopes of Ne, Na, and Mg (lower portion) and of the calculated and measured excitation energies of the lowest $J^\pi = 2^+$ states of the even Mg isotopes (upper portion). Units are MeV.

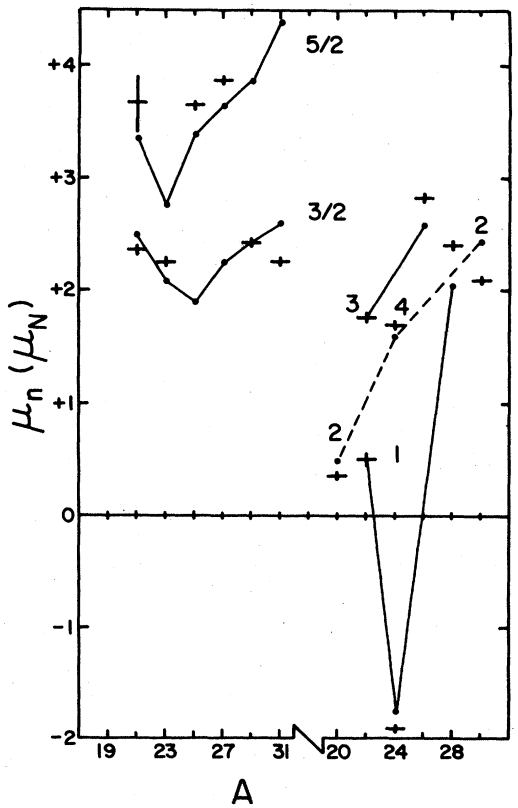

FIG. 2. Plots of the calculated and measured magnetic dipole moments of ground and excited states of various Na isotopes. The lines connect the theoretical values. The spins of the various states are indicated by the notation $\frac{3}{2}$, $\frac{5}{2}$, 1, etc.

tron excitations due to the $d_{3/2}$, $N=20$ shell closure, should raise the energy of excited states. The fact that the actual $N=20$ system shows a dramatically lowered energy for the first excited state is very suggestive of the dominance in the lowest energy configuration of ^{32}Mg of a mode of excitation different from that available in the lighter isotopes.

We lastly consider what information can be extracted from a comparison of the magnetic moments measured for various states of the Na isotopes with the predictions from our shell-model wave functions. The theoretical values are obtained from the free-nucleon moments of the neutron and proton as described in Ref. 10. Comparison between the observed and calculated results is made in Fig. 2. We note that the moments of the $J^\pi = \frac{3}{2}^+$ states are completely uncharacteristic of $d_{3/2}$ structure, which would yield a value of 0.1 μ_N. The moments of the odd-proton, odd-neutron even-mass Na isotopes show strong variations with A, the $J^\pi = 2^+$ states of ^{20}Na and ^{30}Na differing by a factor of 8, the $J^\pi = 1^+$ states of ^{22}Na, ^{24}Na, and ^{28}Na changing from +0.5 to −1.75 to +2.0 μ_N, and the $J^\pi = 3^+$ states of ^{22}Na and ^{26}Na differing by 50%. The level of experimental-theoretical agreement for these data from ^{20}Na through ^{31}Na is essentially independent of A, and is consistent, although a little worse overall, with the agreement between extensions of these calculations and the complete range of magnetic moment data in the *sd* shell.[10]

We conclude from this inspection of the data available on the very-neutron-rich isotopes in the *sd* shell the following:

(1) The features observed for ^{31}Na and ^{32}Mg, the $N=20$ isotones, are not to be explained within the *sd*-shell configuration space.

(2) The fewer data available on the $N=19$ isotones are not obviously incompatible with an *sd*-shell description.

(3) All the features of the $N=18$ and lighter exotic systems seem fully consistent with the structure of ordinary nuclear states in the $A=18-24$ and $A=32-28$ regions as this structure is parametrized and extrapolated via a model confined to *sd*-shell configurations.

Many aspects of this subject remain to be elucidated. Experimental determinations of the masses of the heavier exotic Ne, Mg, and Al isotopes, in particular ^{31}Mg and ^{32}Mg, are necessary. Comparable in importance to the ground-state masses are spectra of excited states, which greatly aid in discriminating between possible theoretical formulations. Further spectroscopic information such as has been provided by the Na magnetic moments will also be higher valuable in understanding what is happening in this new realm. The currently available isotope shifts and beta decay matrix elements for the Na isotopes should, when theoretically analyzed, provide immediate further insights. The new challenge for shell-model theory is to determine an effective Hamiltonian which, while operative in a space expanded beyond the confines of the *sd* shell, maintains *sd*-shell structure intact for the appropriate N, Z systems and then induces the radical changes currently observed at $Z=11-12$, $N=20$.

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